INSTRUCTOR'S SOLUTIONS MANUAL

FOR

SERWAY AND JEWETT'S

PHYSICS

FOR SCIENTISTS AND ENGINEERS

SIXTH EDITION, VOLUME ONE

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Physics and Measurement

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q1.1 Atomic clocks are based on electromagnetic waves which atoms emit. Also, pulsars are highly regular astronomical clocks.

Q1.2 Density varies with temperature and pressure. It would be necessary to measure both mass and volume very accurately in order to use the density of water as a standard.

Q1.3 People have different size hands. Defining the unit precisely would be cumbersome.

Q1.4 (a) 0.3 millimeters (b) 50 microseconds (c) 7.2 kilograms

Q1.5 (b) and (d). You cannot add or subtract quantities of different dimension.

Q1.6 A dimensionally correct equation need not be true. Example: 1 chimpanzee = 2 chimpanzee is dimensionally correct. If an equation is not dimensionally correct, it cannot be correct.

Q1.7 If I were a runner, I might walk or run 10^1 miles per day. Since I am a college professor, I walk about 10^0 miles per day. I drive about 40 miles per day on workdays and up to 200 miles per day on vacation.

Q1.8 On February 7, 2001, I am 55 years and 39 days old.

$$55\text{ yr}\left(\frac{365.25\text{ d}}{1\text{ yr}}\right)+39\text{ d}=20\,128\text{ d}\left(\frac{86\,400\text{ s}}{1\text{ d}}\right)=1.74\times10^{9}\text{ s}\sim10^{9}\text{ s}.$$

Many college students are just approaching 1 Gs.

Q1.9 Zero digits. An order-of-magnitude calculation is accurate only within a factor of 10.

Q1.10 The mass of the forty-six chapter textbook is on the order of 10^0 kg.

Q1.11 With one datum known to one significant digit, we have 80 million yr + 24 yr = 80 million yr.

Section 1.1 **Standards of Length, Mass, and Time**

No problems in this section

Section 1.2 **Matter and Model-Building**

P1.1 From the figure, we may see that the spacing between diagonal planes is half the distance between diagonally adjacent atoms on a flat plane. This diagonal distance may be obtained from the Pythagorean theorem, $L_{\text{diag}} = \sqrt{L^2 + L^2}$. Thus, since the atoms are separated by a distance $L = 0.200 \text{ nm}$, the diagonal planes are separated by $\frac{1}{2}\sqrt{L^2 + L^2} = \boxed{0.141 \text{ nm}}$.

Section 1.3 **Density and Atomic Mass**

***P1.2** Modeling the Earth as a sphere, we find its volume as $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(6.37 \times 10^6 \text{ m}\right)^3 = 1.08 \times 10^{21} \text{ m}^3$. Its density is then $\rho = \frac{m}{V} = \frac{5.98 \times 10^{24} \text{ kg}}{1.08 \times 10^{21} \text{ m}^3} = \boxed{5.52 \times 10^3 \text{ kg/m}^3}$. This value is intermediate between the tabulated densities of aluminum and iron. Typical rocks have densities around 2 000 to 3 000 kg/m^3. The average density of the Earth is significantly higher, so higher-density material must be down below the surface.

P1.3 With $V = (\text{base area})(\text{height})$ $V = \left(\pi r^2\right)h$ and $\rho = \frac{m}{V}$, we have

$$\rho = \frac{m}{\pi r^2 h} = \frac{1 \text{ kg}}{\pi (19.5 \text{ mm})^2 (39.0 \text{ mm})}\left(\frac{10^9 \text{ mm}^3}{1 \text{ m}^3}\right)$$

$$\rho = \boxed{2.15 \times 10^4 \text{ kg/m}^3}.$$

***P1.4** Let V represent the volume of the model, the same in $\rho = \frac{m}{V}$ for both. Then $\rho_{\text{iron}} = 9.35 \text{ kg}/V$ and $\rho_{\text{gold}} = \frac{m_{\text{gold}}}{V}$. Next, $\frac{\rho_{\text{gold}}}{\rho_{\text{iron}}} = \frac{m_{\text{gold}}}{9.35 \text{ kg}}$ and $m_{\text{gold}} = 9.35 \text{ kg}\left(\frac{19.3 \times 10^3 \text{ kg/m}^3}{7.86 \times 10^3 \text{ kg/m}^3}\right) = \boxed{23.0 \text{ kg}}$.

P1.5
$$V = V_o - V_i = \frac{4}{3}\pi\left(r_2^3 - r_1^3\right)$$

$$\rho = \frac{m}{V}, \text{ so } m = \rho V = \rho\left(\frac{4}{3}\pi\right)\left(r_2^3 - r_1^3\right) = \boxed{\frac{4\pi\rho\left(r_2^3 - r_1^3\right)}{3}}.$$

P1.6 For either sphere the volume is $V=\frac{4}{3}\pi r^3$ and the mass is $m=\rho V=\rho\frac{4}{3}\pi r^3$. We divide this equation for the larger sphere by the same equation for the smaller:

$$\frac{m_\ell}{m_s}=\frac{\rho 4\pi r_\ell^3 3}{\rho 4\pi r_s^3 3}=\frac{r_\ell^3}{r_s^3}=5.$$

Then $r_\ell = r_s\sqrt[3]{5}=4.50\ \text{cm}(1.71)=\boxed{7.69\ \text{cm}}$.

P1.7 Use $1\ \text{u}=1.66\times10^{-24}\ \text{g}$.

(a) For He, $m_0=4.00\ \text{u}\left(\frac{1.66\times10^{-24}\ \text{g}}{1\ \text{u}}\right)=\boxed{6.64\times10^{-24}\ \text{g}}$.

(b) For Fe, $m_0=55.9\ \text{u}\left(\frac{1.66\times10^{-24}\ \text{g}}{1\ \text{u}}\right)=\boxed{9.29\times10^{-23}\ \text{g}}$.

(c) For Pb, $m_0=207\ \text{u}\left(\frac{1.66\times10^{-24}\ \text{g}}{1\ \text{u}}\right)=\boxed{3.44\times10^{-22}\ \text{g}}$.

***P1.8** (a) The mass of any sample is the number of atoms in the sample times the mass m_0 of one atom: $m=Nm_0$. The first assertion is that the mass of one aluminum atom is

$$m_0=27.0\ \text{u}=27.0\ \text{u}\times1.66\times10^{-27}\ \text{kg/1 u}=4.48\times10^{-26}\ \text{kg}.$$

Then the mass of 6.02×10^{23} atoms is

$$m=Nm_0=6.02\times10^{23}\times4.48\times10^{-26}\ \text{kg}=0.027\,0\ \text{kg}=27.0\ \text{g}.$$

Thus the first assertion implies the second. Reasoning in reverse, the second assertion can be written $m=Nm_0$.

$$0.027\,0\ \text{kg}=6.02\times10^{23}m_0,\ \text{so}\ m_0=\frac{0.027\ \text{kg}}{6.02\times10^{23}}=4.48\times10^{-26}\ \text{kg},$$

in agreement with the first assertion.

(b) The general equation $m=Nm_0$ applied to one mole of any substance gives $M\ \text{g}=NM\ \text{u}$, where M is the numerical value of the atomic mass. It divides out exactly for all substances, giving $1.000\,000\,0\times10^{-3}\ \text{kg}=N1.660\,540\,2\times10^{-27}\ \text{kg}$. With eight-digit data, we can be quite sure of the result to seven digits. For one mole the number of atoms is

$$N=\left(\frac{1}{1.660\,540\,2}\right)10^{-3+27}=\boxed{6.022\,137\times10^{23}}.$$

(c) The atomic mass of hydrogen is 1.008 0 u and that of oxygen is 15.999 u. The mass of one molecule of H_2O is $2(1.008\,0)+15.999\ \text{u}=18.0\ \text{u}$. Then the molar mass is $\boxed{18.0\ \text{g}}$.

(d) For CO_2 we have $12.011\ \text{g}+2(15.999\ \text{g})=\boxed{44.0\ \text{g}}$ as the mass of one mole.

P1.9 Mass of gold abraded: $|\Delta m| = 3.80 \text{ g} - 3.35 \text{ g} = 0.45 \text{ g} = (0.45 \text{ g})\left(\frac{1 \text{ kg}}{10^3 \text{ g}}\right) = 4.5 \times 10^{-4} \text{ kg}$.

Each atom has mass $m_0 = 197 \text{ u} = 197 \text{ u}\left(\frac{1.66 \times 10^{-27} \text{ kg}}{1 \text{ u}}\right) = 3.27 \times 10^{-25} \text{ kg}$.

Now, $|\Delta m| = |\Delta N| m_0$, and the number of atoms missing is

$$|\Delta N| = \frac{|\Delta m|}{m_0} = \frac{4.5 \times 10^{-4} \text{ kg}}{3.27 \times 10^{-25} \text{ kg}} = 1.38 \times 10^{21} \text{ atoms}.$$

The rate of loss is

$$\frac{|\Delta N|}{\Delta t} = \frac{1.38 \times 10^{21} \text{ atoms}}{50 \text{ yr}}\left(\frac{1 \text{ yr}}{365.25 \text{ d}}\right)\left(\frac{1 \text{ d}}{24 \text{ h}}\right)\left(\frac{1 \text{ h}}{60 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$$

$$\frac{|\Delta N|}{\Delta t} = \boxed{8.72 \times 10^{11} \text{ atoms/s}}.$$

P1.10 (a) $m = \rho L^3 = (7.86 \text{ g/cm}^3)(5.00 \times 10^{-6} \text{ cm})^3 = \boxed{9.83 \times 10^{-16} \text{ g}} = 9.83 \times 10^{-19} \text{ kg}$

(b) $N = \frac{m}{m_0} = \frac{9.83 \times 10^{-19} \text{ kg}}{55.9 \text{ u}(1.66 \times 10^{-27} \text{ kg/1 u})} = \boxed{1.06 \times 10^7 \text{ atoms}}$

P1.11 (a) The cross-sectional area is

$$A = 2(0.150 \text{ m})(0.010 \text{ m}) + (0.340 \text{ m})(0.010 \text{ m})$$
$$= 6.40 \times 10^{-3} \text{ m}^2.$$

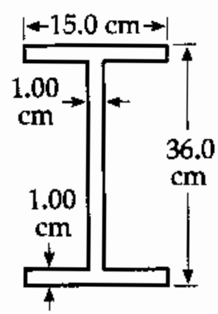


FIG. P1.11

The volume of the beam is

$$V = AL = (6.40 \times 10^{-3} \text{ m}^2)(1.50 \text{ m}) = 9.60 \times 10^{-3} \text{ m}^3.$$

Thus, its mass is

$$m = \rho V = (7.56 \times 10^3 \text{ kg/m}^3)(9.60 \times 10^{-3} \text{ m}^3) = \boxed{72.6 \text{ kg}}.$$

(b) The mass of one typical atom is $m_0 = (55.9 \text{ u})\left(\frac{1.66 \times 10^{-27} \text{ kg}}{1 \text{ u}}\right) = 9.28 \times 10^{-26} \text{ kg}$. Now $m = Nm_0$ and the number of atoms is $N = \frac{m}{m_0} = \frac{72.6 \text{ kg}}{9.28 \times 10^{-26} \text{ kg}} = \boxed{7.82 \times 10^{26} \text{ atoms}}$.

P1.12 (a) The mass of one molecule is $m_0 = 18.0\ \text{u}\left(\frac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}}\right) = 2.99\times10^{-26}\ \text{kg}$. The number of molecules in the pail is

$$N_{\text{pail}} = \frac{m}{m_0} = \frac{1.20\ \text{kg}}{2.99\times10^{-26}\ \text{kg}} = \boxed{4.02\times10^{25}\ \text{molecules}}.$$

(b) Suppose that enough time has elapsed for thorough mixing of the hydrosphere.

$$N_{\text{both}} = N_{\text{pail}}\left(\frac{m_{\text{pail}}}{M_{\text{total}}}\right) = (4.02\times10^{25}\ \text{molecules})\left(\frac{1.20\ \text{kg}}{1.32\times10^{21}\ \text{kg}}\right),$$

or

$$N_{\text{both}} = \boxed{3.65\times10^{4}\ \text{molecules}}.$$

Section 1.4 **Dimensional Analysis**

P1.13 The term x has dimensions of L, a has dimensions of LT^{-2}, and t has dimensions of T. Therefore, the equation $x = ka^m t^n$ has dimensions of

$$\text{L} = \left(\text{LT}^{-2}\right)^m (\text{T})^n \text{ or } \text{L}^1\text{T}^0 = \text{L}^m\text{T}^{n-2m}.$$

The powers of L and T must be the same on each side of the equation. Therefore,

$$\text{L}^1 = \text{L}^m \text{ and } \boxed{m = 1}.$$

Likewise, equating terms in T, we see that $n - 2m$ must equal 0. Thus, $\boxed{n = 2}$. The value of k, a dimensionless constant, $\boxed{\text{cannot be obtained by dimensional analysis}}$.

***P1.14** (a) Circumference has dimensions of L.

(b) Volume has dimensions of L^3.

(c) Area has dimensions of L^2.

Expression (i) has dimension $\text{L}\left(\text{L}^2\right)^{1/2} = \text{L}^2$, so this must be area (c).
Expression (ii) has dimension L, so it is (a).
Expression (iii) has dimension $\text{L}\left(\text{L}^2\right) = \text{L}^3$, so it is (b). Thus, $\boxed{\text{(a)} = \text{ii}; \quad \text{(b)} = \text{iii}, \ \text{(c)} = \text{i}}$.

P1.15 (a) $\boxed{\text{This is incorrect}}$ since the units of $[ax]$ are m^2/s^2, while the units of $[v]$ are m/s.

(b) $\boxed{\text{This is correct}}$ since the units of $[y]$ are m, and $\cos(kx)$ is dimensionless if $[k]$ is in m^{-1}.

***P1.16** (a) $a \propto \frac{\sum F}{m}$ or $a = k\frac{\sum F}{m}$ represents the proportionality of acceleration to resultant force and the inverse proportionality of acceleration to mass. If k has no dimensions, we have

$$[a] = [k]\frac{[F]}{[m]},\ \frac{\text{L}}{\text{T}^2} = 1\frac{[\text{F}]}{\text{M}},\ [\text{F}] = \boxed{\frac{\text{M}\cdot\text{L}}{\text{T}^2}}.$$

(b) In units, $\frac{\text{M}\cdot\text{L}}{\text{T}^2} = \frac{\text{kg}\cdot\text{m}}{\text{s}^2}$, so $\boxed{1\text{ newton} = 1\text{ kg}\cdot\text{m/s}^2}$.

P1.17 Inserting the proper units for everything except G,

$$\left[\frac{\text{kg m}}{\text{s}^2}\right] = \frac{G[\text{kg}]^2}{[\text{m}]^2}.$$

Multiply both sides by $[\text{m}]^2$ and divide by $[\text{kg}]^2$; the units of G are $\boxed{\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}}$.

Section 1.5 Conversion of Units

***P1.18** Each of the four walls has area $(8.00\text{ ft})(12.0\text{ ft}) = 96.0\text{ ft}^2$. Together, they have area

$$4\left(96.0\text{ ft}^2\right)\left(\frac{1\text{ m}}{3.28\text{ ft}}\right)^2 = \boxed{35.7\text{ m}^2}.$$

P1.19 Apply the following conversion factors:

$$1\text{ in} = 2.54\text{ cm},\ 1\text{ d} = 86\,400\text{ s},\ 100\text{ cm} = 1\text{ m},\text{ and }10^9\text{ nm} = 1\text{ m}$$

$$\left(\frac{1}{32}\text{ in/day}\right)\frac{(2.54\text{ cm/in})\left(10^{-2}\text{ m/cm}\right)\left(10^9\text{ nm/m}\right)}{86\,400\text{ s/day}} = \boxed{9.19\text{ nm/s}}.$$

This means the proteins are assembled at a rate of many layers of atoms each second!

***P1.20** $8.50\text{ in}^3 = 8.50\text{ in}^3\left(\frac{0.025\,4\text{ m}}{1\text{ in}}\right)^3 = \boxed{1.39\times10^{-4}\text{ m}^3}$

P1.21 *Conceptualize*: We must calculate the area and convert units. Since a meter is about 3 feet, we should expect the area to be about $A \approx (30 \text{ m})(50 \text{ m}) = 1\,500 \text{ m}^2$.

Categorize: We model the lot as a perfect rectangle to use Area = Length × Width. Use the conversion: $1 \text{ m} = 3.281 \text{ ft}$.

Analyze: $A = LW = (100 \text{ ft})\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right)(150 \text{ ft})\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 1\,390 \text{ m}^2 = \boxed{1.39 \times 10^3 \text{ m}^2}$.

Finalize: Our calculated result agrees reasonably well with our initial estimate and has the proper units of m^2. Unit conversion is a common technique that is applied to many problems.

P1.22 (a) $V = (40.0 \text{ m})(20.0 \text{ m})(12.0 \text{ m}) = 9.60 \times 10^3 \text{ m}^3$

$V = 9.60 \times 10^3 \text{ m}^3 (3.28 \text{ ft/1 m})^3 = \boxed{3.39 \times 10^5 \text{ ft}^3}$

(b) The mass of the air is

$$m = \rho_{\text{air}} V = (1.20 \text{ kg/m}^3)(9.60 \times 10^3 \text{ m}^3) = 1.15 \times 10^4 \text{ kg}.$$

The student must look up weight in the index to find

$$F_g = mg = (1.15 \times 10^4 \text{ kg})(9.80 \text{ m/s}^2) = 1.13 \times 10^5 \text{ N}.$$

Converting to pounds,

$$F_g = (1.13 \times 10^5 \text{ N})(1 \text{ lb}/4.45 \text{ N}) = \boxed{2.54 \times 10^4 \text{ lb}}.$$

P1.23 (a) Seven minutes is 420 seconds, so the rate is

$$r = \frac{30.0 \text{ gal}}{420 \text{ s}} = \boxed{7.14 \times 10^{-2} \text{ gal/s}}.$$

(b) Converting gallons first to liters, then to m^3,

$$r = (7.14 \times 10^{-2} \text{ gal/s})\left(\frac{3.786 \text{ L}}{1 \text{ gal}}\right)\left(\frac{10^{-3} \text{ m}^3}{1 \text{ L}}\right)$$

$$r = \boxed{2.70 \times 10^{-4} \text{ m}^3/\text{s}}.$$

(c) At that rate, to fill a 1-m^3 tank would take

$$t = \left(\frac{1 \text{ m}^3}{2.70 \times 10^{-4} \text{ m}^3/\text{s}}\right)\left(\frac{1 \text{ h}}{3\,600}\right) = \boxed{1.03 \text{ h}}.$$

***P1.24** (a) Length of Mammoth Cave $= 348 \text{ mi}\left(\frac{1.609 \text{ km}}{1 \text{ mi}}\right) = \boxed{560 \text{ km} = 5.60\times10^5 \text{ m} = 5.60\times10^7 \text{ cm}}$.

(b) Height of Ribbon Falls $= 1\,612 \text{ ft}\left(\frac{0.304\,8 \text{ m}}{1 \text{ ft}}\right) = \boxed{491 \text{ m} = 0.491 \text{ km} = 4.91\times10^4 \text{ cm}}$.

(c) Height of Denali $= 20\,320 \text{ ft}\left(\frac{0.304\,8 \text{ m}}{1 \text{ ft}}\right) = \boxed{6.19 \text{ km} = 6.19\times10^3 \text{ m} = 6.19\times10^5 \text{ cm}}$.

(d) Depth of King's Canyon $= 8\,200 \text{ ft}\left(\frac{0.304\,8 \text{ m}}{1 \text{ ft}}\right) = \boxed{2.50 \text{ km} = 2.50\times10^3 \text{ m} = 2.50\times10^5 \text{ cm}}$.

P1.25 From Table 1.5, the density of lead is $1.13\times10^4 \text{ kg/m}^3$, so we should expect our calculated value to be close to this number. This density value tells us that lead is about 11 times denser than water, which agrees with our experience that lead sinks.

Density is defined as mass per volume, in $\rho = \frac{m}{V}$. We must convert to SI units in the calculation.

$$\rho = \frac{23.94 \text{ g}}{2.10 \text{ cm}^3}\left(\frac{1 \text{ kg}}{1\,000 \text{ g}}\right)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = \boxed{1.14\times10^4 \text{ kg/m}^3}$$

At one step in the calculation, we note that *one million* cubic centimeters make one cubic meter. Our result is indeed close to the expected value. Since the last reported significant digit is not certain, the difference in the two values is probably due to measurement uncertainty and should not be a concern. One important common-sense check on density values is that objects which sink in water must have a density greater than 1 g/cm^3, and objects that float must be less dense than water.

P1.26 It is often useful to remember that the 1 600-m race at track and field events is approximately 1 mile in length. To be precise, there are 1 609 meters in a mile. Thus, 1 acre is equal in area to

$$(1 \text{ acre})\left(\frac{1 \text{ mi}^2}{640 \text{ acres}}\right)\left(\frac{1\,609 \text{ m}}{\text{mi}}\right)^2 = \boxed{4.05\times10^3 \text{ m}^2}.$$

***P1.27** The weight flow rate is $1\,200\ \frac{\text{ton}}{\text{h}}\left(\frac{2\,000 \text{ lb}}{\text{ton}}\right)\left(\frac{1 \text{ h}}{60 \text{ min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{667 \text{ lb/s}}$.

P1.28 $1 \text{ mi} = 1\,609 \text{ m} = 1.609 \text{ km}$; thus, to go from mph to km/h, multiply by 1.609.

(a) $1 \text{ mi/h} = \boxed{1.609 \text{ km/h}}$

(b) $55 \text{ mi/h} = \boxed{88.5 \text{ km/h}}$

(c) $65 \text{ mi/h} = 104.6 \text{ km/h}$. Thus, $\Delta v = \boxed{16.1 \text{ km/h}}$.

P1.29 (a) $\left(\dfrac{6\times 10^{12}\ \$}{1\,000\ \$/\text{s}}\right)\left(\dfrac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\dfrac{1\ \text{day}}{24\ \text{h}}\right)\left(\dfrac{1\ \text{yr}}{365\ \text{days}}\right)=\boxed{190\ \text{years}}$

(b) The circumference of the Earth at the equator is $2\pi\left(6.378\times 10^{3}\ \text{m}\right)=4.01\times 10^{7}\ \text{m}$. The length of one dollar bill is 0.155 m so that the length of 6 trillion bills is $9.30\times 10^{11}\ \text{m}$. Thus, the 6 trillion dollars would encircle the Earth

$$\frac{9.30\times 10^{11}\ \text{m}}{4.01\times 0^{7}\ \text{m}}=\boxed{2.32\times 10^{4}\ \text{times}}.$$

P1.30 $N_{\text{atoms}}=\dfrac{m_{\text{Sun}}}{m_{\text{atom}}}=\dfrac{1.99\times 10^{30}\ \text{kg}}{1.67\times 10^{-27}\ \text{kg}}=\boxed{1.19\times 10^{57}\ \text{atoms}}$

P1.31 $V=At$ so $t=\dfrac{V}{A}=\dfrac{3.78\times 10^{-3}\ \text{m}^{3}}{25.0\ \text{m}^{2}}=\boxed{1.51\times 10^{-4}\ \text{m (or 151 }\mu\text{m)}}$

P1.32
$$V=\frac{1}{3}Bh=\frac{\left[(13.0\ \text{acres})\left(43\,560\ \text{ft}^{2}/\text{acre}\right)\right]}{3}(481\ \text{ft})$$
$$=9.08\times 10^{7}\ \text{ft}^{3},$$

or

$$V=\left(9.08\times 10^{7}\ \text{ft}^{3}\right)\left(\frac{2.83\times 10^{-2}\ \text{m}^{3}}{1\ \text{ft}^{3}}\right)$$
$$=\boxed{2.57\times 10^{6}\ \text{m}^{3}}$$

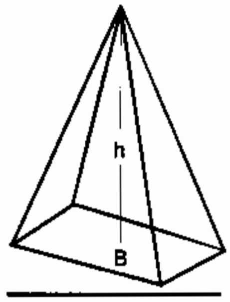


FIG. P1.32

P1.33 $F_{g}=(2.50\ \text{tons/block})\left(2.00\times 10^{6}\ \text{blocks}\right)(2\,000\ \text{lb/ton})=\boxed{1.00\times 10^{10}\ \text{lbs}}$

***P1.34** The area covered by water is

$$A_{w}=0.70A_{\text{Earth}}=(0.70)\left(4\pi R_{\text{Earth}}^{2}\right)=(0.70)(4\pi)\left(6.37\times 10^{6}\ \text{m}\right)^{2}=3.6\times 10^{14}\ \text{m}^{2}.$$

The average depth of the water is

$$d=(2.3\ \text{miles})(1\,609\ \text{m/1 mile})=3.7\times 10^{3}\ \text{m}.$$

The volume of the water is

$$V=A_{w}d=\left(3.6\times 10^{14}\ \text{m}^{2}\right)\left(3.7\times 10^{3}\ \text{m}\right)=1.3\times 10^{18}\ \text{m}^{3}$$

and the mass is

$$m=\rho V=\left(1\,000\ \text{kg/m}^{3}\right)\left(1.3\times 10^{18}\ \text{m}^{3}\right)=\boxed{1.3\times 10^{21}\ \text{kg}}.$$

P1.35 (a) $d_{\text{nucleus, scale}} = d_{\text{nucleus, real}}\left(\dfrac{d_{\text{atom, scale}}}{d_{\text{atom, real}}}\right) = \left(2.40\times10^{-15}\ \text{m}\right)\left(\dfrac{300\ \text{ft}}{1.06\times10^{-10}\ \text{m}}\right) = 6.79\times10^{-3}\ \text{ft}$, or

$$d_{\text{nucleus, scale}} = \left(6.79\times10^{-3}\ \text{ft}\right)\left(304.8\ \text{mm/1 ft}\right) = \boxed{2.07\ \text{mm}}$$

(b) $$\frac{V_{\text{atom}}}{V_{\text{nucleus}}} = \frac{\frac{4\pi r_{\text{atom}}^3}{3}}{\frac{4\pi r_{\text{nucleus}}^3}{3}} = \left(\frac{r_{\text{atom}}}{r_{\text{nucleus}}}\right)^3 = \left(\frac{d_{\text{atom}}}{d_{\text{nucleus}}}\right)^3 = \left(\frac{1.06\times10^{-10}\ \text{m}}{2.40\times10^{-15}\ \text{m}}\right)^3$$

$$= \boxed{8.62\times10^{13}\ \text{times as large}}$$

***P1.36** $$\begin{matrix}\text{scale distance}\\ \text{between}\end{matrix} = \begin{pmatrix}\text{real}\\ \text{distance}\end{pmatrix}\begin{pmatrix}\text{scale}\\ \text{factor}\end{pmatrix} = \left(4.0\times10^{13}\ \text{km}\right)\left(\frac{7.0\times10^{-3}\ \text{m}}{1.4\times10^{9}\ \text{m}}\right) = \boxed{200\ \text{km}}$$

P1.37 The scale factor used in the "dinner plate" model is

$$S = \frac{0.25\ \text{m}}{1.0\times10^{5}\ \text{lightyears}} = 2.5\times10^{-6}\ \text{m/lightyears}.$$

The distance to Andromeda in the scale model will be

$$D_{\text{scale}} = D_{\text{actual}}S = \left(2.0\times10^{6}\ \text{lightyears}\right)\left(2.5\times10^{-6}\ \text{m/lightyears}\right) = \boxed{5.0\ \text{m}}.$$

P1.38 (a) $$\frac{A_{\text{Earth}}}{A_{\text{Moon}}} = \frac{4\pi r_{\text{Earth}}^2}{4\pi r_{\text{Moon}}^2} = \left(\frac{r_{\text{Earth}}}{r_{\text{Moon}}}\right)^2 = \left(\frac{\left(6.37\times10^{6}\ \text{m}\right)\left(100\ \text{cm/m}\right)}{1.74\times10^{8}\ \text{cm}}\right)^2 = \boxed{13.4}$$

(b) $$\frac{V_{\text{Earth}}}{V_{\text{Moon}}} = \frac{\frac{4\pi r_{\text{Earth}}^3}{3}}{\frac{4\pi r_{\text{Moon}}^3}{3}} = \left(\frac{r_{\text{Earth}}}{r_{\text{Moon}}}\right)^3 = \left(\frac{\left(6.37\times10^{6}\ \text{m}\right)\left(100\ \text{cm/m}\right)}{1.74\times10^{8}\ \text{cm}}\right)^3 = \boxed{49.1}$$

P1.39 To balance, $m_{\text{Fe}} = m_{\text{Al}}$ or $\rho_{\text{Fe}}V_{\text{Fe}} = \rho_{\text{Al}}V_{\text{Al}}$

$$\rho_{\text{Fe}}\left(\frac{4}{3}\right)\pi r_{\text{Fe}}^{\ 3} = \rho_{\text{Al}}\left(\frac{4}{3}\right)\pi r_{\text{Al}}^{\ 3}$$

$$r_{\text{Al}} = r_{\text{Fe}}\left(\frac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}\right)^{1/3} = (2.00\ \text{cm})\left(\frac{7.86}{2.70}\right)^{1/3} = \boxed{2.86\ \text{cm}}.$$

P1.40 The mass of each sphere is

$$m_{\text{Al}} = \rho_{\text{Al}} V_{\text{Al}} = \frac{4\pi \rho_{\text{Al}} r_{\text{Al}}^3}{3}$$

and

$$m_{\text{Fe}} = \rho_{\text{Fe}} V_{\text{Fe}} = \frac{4\pi \rho_{\text{Fe}} r_{\text{Fe}}^3}{3}.$$

Setting these masses equal,

$$\frac{4\pi \rho_{\text{Al}} r_{\text{Al}}^3}{3} = \frac{4\pi \rho_{\text{Fe}} r_{\text{Fe}}^3}{3} \text{ and } \boxed{r_{\text{Al}} = r_{\text{Fe}} \sqrt[3]{\frac{\rho_{\text{Fe}}}{\rho_{\text{Al}}}}}.$$

Section 1.6 Estimates and Order-of-Magnitude Calculations

P1.41 Model the room as a rectangular solid with dimensions 4 m by 4 m by 3 m, and each ping-pong ball as a sphere of diameter 0.038 m. The volume of the room is $4 \times 4 \times 3 = 48 \text{ m}^3$, while the volume of one ball is

$$\frac{4\pi}{3}\left(\frac{0.038 \text{ m}}{2}\right)^3 = 2.87 \times 10^{-5} \text{ m}^3.$$

Therefore, one can fit about $\frac{48}{2.87 \times 10^{-5}} \sim \boxed{10^6}$ ping-pong balls in the room.

As an aside, the actual number is smaller than this because there will be a lot of space in the room that cannot be covered by balls. In fact, even in the best arrangement, the so-called "best packing fraction" is $\frac{1}{6}\pi\sqrt{2} = 0.74$ so that at least 26% of the space will be empty. Therefore, the above estimate reduces to $1.67 \times 10^6 \times 0.740 \sim 10^6$.

P1.42 A reasonable guess for the diameter of a tire might be 2.5 ft, with a circumference of about 8 ft. Thus, the tire would make $(50\,000 \text{ mi})(5\,280 \text{ ft/mi})(1 \text{ rev}/8 \text{ ft}) = 3 \times 10^7 \text{ rev} \sim \boxed{10^7 \text{ rev}}$.

P1.43 In order to reasonably carry on photosynthesis, we might expect a blade of grass to require at least $\frac{1}{16} \text{ in}^2 = 43 \times 10^{-5} \text{ ft}^2$. Since 1 acre $= 43\,560 \text{ ft}^2$, the number of blades of grass to be expected on a quarter-acre plot of land is about

$$n = \frac{\text{total area}}{\text{area per blade}} = \frac{(0.25 \text{ acre})\left(43\,560 \text{ ft}^2/\text{acre}\right)}{43 \times 10^{-5} \text{ ft}^2/\text{blade}} = 2.5 \times 10^7 \text{ blades} \sim \boxed{10^7 \text{ blades}}.$$

P1.44 A typical raindrop is spherical and might have a radius of about 0.1 inch. Its volume is then approximately $4\times10^{-3}\ \text{in}^3$. Since 1 acre $= 43\,560\ \text{ft}^2$, the volume of water required to cover it to a depth of 1 inch is

$$(1\ \text{acre})(1\ \text{inch}) = (1\ \text{acre}\cdot\text{in})\left(\frac{43\,560\ \text{ft}^2}{1\ \text{acre}}\right)\left(\frac{144\ \text{in}^2}{1\ \text{ft}^2}\right) \approx 6.3\times10^6\ \text{in}^3.$$

The number of raindrops required is

$$n = \frac{\text{volume of water required}}{\text{volume of a single drop}} = \frac{6.3\times10^6\ \text{in}^3}{4\times10^{-3}\ \text{in}^3} = 1.6\times10^9 \sim \boxed{10^9}.$$

***P1.45** Assume the tub measures 1.3 m by 0.5 m by 0.3 m. One-half of its volume is then

$$V = (0.5)(1.3\ \text{m})(0.5\ \text{m})(0.3\ \text{m}) = 0.10\ \text{m}^3.$$

The mass of this volume of water is

$$m_{\text{water}} = \rho_{\text{water}}V = \left(1\,000\ \text{kg/m}^3\right)\left(0.10\ \text{m}^3\right) = 100\ \text{kg}\ \boxed{\sim 10^2\ \text{kg}}.$$

Pennies are now mostly zinc, but consider copper pennies filling 50% of the volume of the tub. The mass of copper required is

$$m_{\text{copper}} = \rho_{\text{copper}}V = \left(8\,920\ \text{kg/m}^3\right)\left(0.10\ \text{m}^3\right) = 892\ \text{kg}\ \boxed{\sim 10^3\ \text{kg}}.$$

P1.46 The typical person probably drinks 2 to 3 soft drinks daily. Perhaps half of these were in aluminum cans. Thus, we will estimate 1 aluminum can disposal per person per day. In the U.S. there are ~250 million people, and 365 days in a year, so

$$\left(250\times10^6\ \text{cans/day}\right)(365\ \text{days/year}) \cong \boxed{10^{11}\ \text{cans}}$$

are thrown away or recycled each year. Guessing that each can weighs around $1/10$ of an ounce, we estimate this represents

$$\left(10^{11}\ \text{cans}\right)(0.1\ \text{oz/can})(1\ \text{lb/16 oz})(1\ \text{ton/2\,000 lb}) \approx 3.1\times10^5\ \text{tons/year}.\ \boxed{\sim 10^5\ \text{tons}}$$

P1.47 Assume: Total population $= 10^7$; one out of every 100 people has a piano; one tuner can serve about 1 000 pianos (about 4 per day for 250 weekdays, assuming each piano is tuned once per year). Therefore,

$$\#\ \text{tuners} \sim \left(\frac{1\ \text{tuner}}{1\,000\ \text{pianos}}\right)\left(\frac{1\ \text{piano}}{100\ \text{people}}\right)\left(10^7\ \text{people}\right) = \boxed{100}.$$

Section 1.7 **Significant Figures**

***P1.48** METHOD ONE

We treat the best value with its uncertainty as a binomial (21.3 ± 0.2) cm (9.8 ± 0.1) cm,

$$A = \left[21.3(9.8) \pm 21.3(0.1) \pm 0.2(9.8) \pm (0.2)(0.1)\right] \text{cm}^2 .$$

The first term gives the best value of the area. The cross terms add together to give the uncertainty and the fourth term is negligible.

$$A = \boxed{209 \text{ cm}^2 \pm 4 \text{ cm}^2} .$$

METHOD TWO

We add the fractional uncertainties in the data.

$$A = (21.3 \text{ cm})(9.8 \text{ cm}) \pm \left(\frac{0.2}{21.3} + \frac{0.1}{9.8}\right) = 209 \text{ cm}^2 \pm 2\% = 209 \text{ cm}^2 \pm 4 \text{ cm}^2$$

P1.49 (a)

$$\pi r^2 = \pi(10.5 \text{ m} \pm 0.2 \text{ m})^2$$
$$= \pi\left[(10.5 \text{ m})^2 \pm 2(10.5 \text{ m})(0.2 \text{ m}) + (0.2 \text{ m})^2\right]$$
$$= \boxed{346 \text{ m}^2 \pm 13 \text{ m}^2}$$

(b) $2\pi r = 2\pi(10.5 \text{ m} \pm 0.2 \text{ m}) = \boxed{66.0 \text{ m} \pm 1.3 \text{ m}}$

P1.50 (a) $\boxed{3}$ (b) $\boxed{4}$ (c) $\boxed{3}$ (d) $\boxed{2}$

P1.51 $r = (6.50 \pm 0.20) \text{ cm} = (6.50 \pm 0.20) \times 10^{-2} \text{ m}$

$m = (1.85 \pm 0.02) \text{ kg}$

$$\rho = \frac{m}{\left(\frac{4}{3}\right)\pi r^3}$$

also,

$$\frac{\delta\rho}{\rho} = \frac{\delta m}{m} + \frac{3\delta r}{r} .$$

In other words, the percentages of uncertainty are cumulative. Therefore,

$$\frac{\delta\rho}{\rho} = \frac{0.02}{1.85} + \frac{3(0.20)}{6.50} = 0.103 ,$$

$$\rho = \frac{1.85}{\left(\frac{4}{3}\right)\pi\left(6.5 \times 10^{-2} \text{ m}\right)^3} = \boxed{1.61 \times 10^3 \text{ kg/m}^3}$$

and

$$\rho \pm \delta\rho = \boxed{(1.61 \pm 0.17) \times 10^3 \text{ kg/m}^3} = (1.6 \pm 0.2) \times 10^3 \text{ kg/m}^3 .$$

P1.52 (a)

$$\begin{array}{r} 756.?? \\ 37.2? \\ 0.83 \\ \underline{+\ 2.5?} \\ 796.53 \end{array} = \boxed{797}$$

(b) $0.003\,2(2\text{ s.f.}) \times 356.3(4\text{ s.f.}) = 1.140\,16 = (2\text{ s.f.})\ \boxed{1.1}$

(c) $5.620(4\text{ s.f.}) \times \pi(>4\text{ s.f.}) = 17.656 = (4\text{ s.f.})\ \boxed{17.66}$

***P1.53** We work to nine significant digits:

$$1\text{ yr} = 1\text{ yr}\left(\frac{365.242\,199\text{ d}}{1\text{ yr}}\right)\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{31\,556\,926.0\text{ s}}.$$

P1.54 The distance around is $38.44\text{ m} + 19.5\text{ m} + 38.44\text{ m} + 19.5\text{ m} = 115.88\text{ m}$, but this answer must be rounded to 115.9 m because the distance 19.5 m carries information to only one place past the decimal. $\boxed{115.9\text{ m}}$

P1.55

$$V = 2V_1 + 2V_2 = 2(V_1 + V_2)$$

$$V_1 = (17.0\text{ m} + 1.0\text{ m} + 1.0\text{ m})(1.0\text{ m})(0.09\text{ m}) = 1.70\text{ m}^3$$

$$V_2 = (10.0\text{ m})(1.0\text{ m})(0.090\text{ m}) = 0.900\text{ m}^3$$

$$V = 2\left(1.70\text{ m}^3 + 0.900\text{ m}^3\right) = \boxed{5.2\text{ m}^3}$$

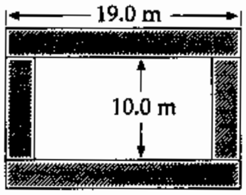


FIG. P1.55

$$\left.\begin{aligned} \frac{\delta \ell_1}{\ell_1} &= \frac{0.12\text{ m}}{19.0\text{ m}} = 0.0063 \\ \frac{\delta w_1}{w_1} &= \frac{0.01\text{ m}}{1.0\text{ m}} = 0.010 \\ \frac{\delta t_1}{t_1} &= \frac{0.1\text{ cm}}{9.0\text{ cm}} = 0.011 \end{aligned}\right\} \frac{\delta V}{V} = 0.006 + 0.010 + 0.011 = 0.027 = \boxed{3\%}$$

Additional Problems

P1.56 It is desired to find the distance x such that

$$\frac{x}{100\text{ m}} = \frac{1\,000\text{ m}}{x}$$

(i.e., such that x is the same multiple of 100 m as the multiple that 1 000 m is of x). Thus, it is seen that

$$x^2 = (100\text{ m})(1\,000\text{ m}) = 1.00 \times 10^5\text{ m}^2$$

and therefore

$$x = \sqrt{1.00 \times 10^5\text{ m}^2} = \boxed{316\text{ m}}.$$

***P1.57** Consider one cubic meter of gold. Its mass from Table 1.5 is 19 300 kg. One atom of gold has mass

$$m_0 = (197\ \text{u})\left(\frac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}}\right) = 3.27\times10^{-25}\ \text{kg}.$$

So, the number of atoms in the cube is

$$N = \frac{19\,300\ \text{kg}}{3.27\times10^{-25}\ \text{kg}} = 5.90\times10^{28}.$$

The imagined cubical volume of each atom is

$$d^3 = \frac{1\ \text{m}^3}{5.90\times10^{28}} = 1.69\times10^{-29}\ \text{m}^3.$$

So

$$d = \boxed{2.57\times10^{-10}\ \text{m}}.$$

P1.58

$$A_{\text{total}} = (N)(A_{\text{drop}}) = \left(\frac{V_{\text{total}}}{V_{\text{drop}}}\right)(A_{\text{drop}}) = \left(\frac{V_{\text{total}}}{\frac{4\pi r^3}{3}}\right)(4\pi r^2)$$

$$A_{\text{total}} = \left(\frac{3V_{\text{total}}}{r}\right) = 3\left(\frac{30.0\times10^{-6}\ \text{m}^3}{2.00\times10^{-5}\ \text{m}}\right) = \boxed{4.50\ \text{m}^2}$$

P1.59 One month is

$$1\ \text{mo} = (30\ \text{day})(24\ \text{h/day})(3\,600\ \text{s/h}) = 2.592\times10^{6}\ \text{s}.$$

Applying units to the equation,

$$V = \left(1.50\ \text{Mft}^3/\text{mo}\right)t + \left(0.008\,00\ \text{Mft}^3/\text{mo}^2\right)t^2.$$

Since $1\ \text{Mft}^3 = 10^6\ \text{ft}^3$,

$$V = \left(1.50\times10^{6}\ \text{ft}^3/\text{mo}\right)t + \left(0.008\,00\times10^{6}\ \text{ft}^3/\text{mo}^2\right)t^2.$$

Converting months to seconds,

$$V = \frac{1.50\times10^{6}\ \text{ft}^3/\text{mo}}{2.592\times10^{6}\ \text{s/mo}}t + \frac{0.008\,00\times10^{6}\ \text{ft}^3/\text{mo}^2}{\left(2.592\times10^{6}\ \text{s/mo}\right)^2}t^2.$$

Thus, $\boxed{V\,[\text{ft}^3] = \left(0.579\ \text{ft}^3/\text{s}\right)t + \left(1.19\times10^{-9}\ \text{ft}^3/\text{s}^2\right)t^2}$.

P1.60

α'(deg)	α(rad)	$\tan(\alpha)$	$\sin(\alpha)$	difference
15.0	0.262	0.268	0.259	3.47%
20.0	0.349	0.364	0.342	6.43%
25.0	0.436	0.466	0.423	10.2%
24.0	0.419	0.445	0.407	9.34%
24.4	0.426	0.454	0.413	9.81%
24.5	0.428	0.456	0.415	9.87%
24.6	0.429	0.458	0.416	9.98%
24.7	0.431	0.460	0.418	10.1%

$\boxed{24.6°}$

P1.61 $2\pi r = 15.0\text{ m}$

$r = 2.39\text{ m}$

$\frac{h}{r} = \tan 55.0°$

$h = (2.39\text{ m})\tan(55.0°) = \boxed{3.41\text{ m}}$

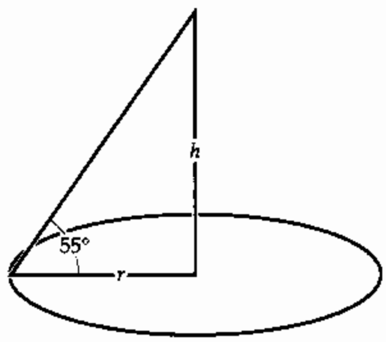


FIG. P1.61

***P1.62** Let d represent the diameter of the coin and h its thickness. The mass of the gold is

$$m = \rho V = \rho A t = \rho\left(\frac{2\pi d^2}{4} + \pi d h\right)t$$

where t is the thickness of the plating.

$$m = 19.3\left[2\pi\frac{(2.41)^2}{4} + \pi(2.41)(0.178)\right](0.18\times 10^{-4})$$

$$= 0.003\,64\text{ grams}$$

$$\text{cost} = 0.003\,64\text{ grams} \times \$10/\text{gram} = \$0.036\,4 = \boxed{3.64\text{ cents}}$$

This is negligible compared to \$4.98.

P1.63 The actual number of seconds in a year is

$$(86\,400\text{ s/day})(365.25\text{ day/yr}) = 31\,557\,600\text{ s/yr}.$$

The percent error in the approximation is

$$\frac{\left|(\pi\times 10^7\text{ s/yr}) - (31\,557\,600\text{ s/yr})\right|}{31\,557\,600\text{ s/yr}} \times 100\% = \boxed{0.449\%}.$$

P1.64 (a) $[V] = L^3,\ [A] = L^2,\ [h] = L$

$$[V] = [A][h]$$

$L^3 = L^2L = L^3$. Thus, the equation is dimensionally correct.

(b) $V_{\text{cylinder}} = \pi R^2 h = \left(\pi R^2\right)h = Ah$, where $\boxed{A = \pi R^2}$

$V_{\text{rectangular object}} = \ell wh = (\ell w)h = Ah$, where $\boxed{A = \ell w}$

P1.65 (a) The speed of rise may be found from

$$v = \frac{\text{(Vol rate of flow)}}{\left(\text{Area: } \frac{\pi D^2}{4}\right)} = \frac{16.5\ \text{cm}^3/\text{s}}{\frac{\pi(6.30\ \text{cm})^2}{4}} = \boxed{0.529\ \text{cm/s}}.$$

(b) Likewise, at a 1.35 cm diameter,

$$v = \frac{16.5\ \text{cm}^3/\text{s}}{\frac{\pi(1.35\ \text{cm})^2}{4}} = \boxed{11.5\ \text{cm/s}}.$$

P1.66 (a) 1 cubic meter of water has a mass

$$m = \rho V = \left(1.00\times10^{-3}\ \text{kg/cm}^3\right)\left(1.00\ \text{m}^3\right)\left(10^2\ \text{cm/m}\right)^3 = \boxed{1\,000\ \text{kg}}$$

(b) As a rough calculation, we treat each item as if it were 100% water.

cell: $m = \rho V = \rho\left(\frac{4}{3}\pi R^3\right) = \rho\left(\frac{1}{6}\pi D^3\right) = \left(1\,000\ \text{kg/m}^3\right)\left(\frac{1}{6}\pi\right)\left(1.0\times10^{-6}\ \text{m}\right)^3$

$= \boxed{5.2\times10^{-16}\ \text{kg}}$

kidney: $m = \rho V = \rho\left(\frac{4}{3}\pi R^3\right) = \left(1.00\times10^{-3}\ \text{kg/cm}^3\right)\left(\frac{4}{3}\pi\right)(4.0\ \text{cm})^3$

$= \boxed{0.27\ \text{kg}}$

fly: $m = \rho\left(\frac{\pi}{4}D^2h\right) = \left(1\times10^{-3}\ \text{kg/cm}^3\right)\left(\frac{\pi}{4}\right)(2.0\ \text{mm})^2(4.0\ \text{mm})\left(10^{-1}\ \text{cm/mm}\right)^3$

$= \boxed{1.3\times10^{-5}\ \text{kg}}$

P1.67 $V_{20\text{ mpg}} = \dfrac{(10^8\ \text{cars})(10^4\ \text{mi/yr})}{20\ \text{mi/gal}} = 5.0\times10^{10}\ \text{gal/yr}$

$V_{25\text{ mpg}} = \dfrac{(10^8\ \text{cars})(10^4\ \text{mi/yr})}{25\ \text{mi/gal}} = 4.0\times10^{10}\ \text{gal/yr}$

Fuel saved $= V_{25\text{ mpg}} - V_{20\text{ mpg}} = \boxed{1.0\times10^{10}\ \text{gal/yr}}$

P1.68 $$v=\left(5.00\frac{\text{furlongs}}{\text{fortnight}}\right)\left(\frac{220\text{ yd}}{1\text{ furlong}}\right)\left(\frac{0.9144\text{ m}}{1\text{ yd}}\right)\left(\frac{1\text{ fortnight}}{14\text{ days}}\right)\left(\frac{1\text{ day}}{24\text{ hrs}}\right)\left(\frac{1\text{ hr}}{3\,600\text{ s}}\right)=\boxed{8.32\times10^{-4}\text{ m/s}}$$

This speed is almost 1 mm/s; so we might guess the creature was a snail, or perhaps a sloth.

P1.69 The volume of the galaxy is

$$\pi r^2 t=\pi\left(10^{21}\text{ m}\right)^2\left(10^{19}\text{ m}\right)\sim10^{61}\text{ m}^3.$$

If the distance between stars is 4×10^{16} m, then there is one star in a volume on the order of

$$\left(4\times10^{16}\text{ m}\right)^3\sim10^{50}\text{ m}^3.$$

The number of stars is about $\dfrac{10^{61}\text{ m}^3}{10^{50}\text{ m}^3/\text{star}}\sim\boxed{10^{11}\text{ stars}}$.

P1.70 The density of each material is $\rho=\dfrac{m}{V}=\dfrac{m}{\pi r^2 h}=\dfrac{4m}{\pi D^2 h}$.

Al: $\rho=\dfrac{4(51.5\text{ g})}{\pi(2.52\text{ cm})^2(3.75\text{ cm})}=\boxed{2.75\dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(2.70\dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{2\%}$ smaller.

Cu: $\rho=\dfrac{4(56.3\text{ g})}{\pi(1.23\text{ cm})^2(5.06\text{ cm})}=\boxed{9.36\dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(8.92\dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{5\%}$ smaller.

Brass: $\rho=\dfrac{4(94.4\text{ g})}{\pi(1.54\text{ cm})^2(5.69\text{ cm})}=\boxed{8.91\dfrac{\text{g}}{\text{cm}^3}}$

Sn: $\rho=\dfrac{4(69.1\text{ g})}{\pi(1.75\text{ cm})^2(3.74\text{ cm})}=\boxed{7.68\dfrac{\text{g}}{\text{cm}^3}}$

Fe: $\rho=\dfrac{4(216.1\text{ g})}{\pi(1.89\text{ cm})^2(9.77\text{ cm})}=\boxed{7.88\dfrac{\text{g}}{\text{cm}^3}}$ The tabulated value $\left(7.86\dfrac{\text{g}}{\text{cm}^3}\right)$ is $\boxed{0.3\%}$ smaller.

P1.71 (a) $(3\,600\text{ s/hr})(24\text{ hr/day})(365.25\text{ days/yr})=\boxed{3.16\times10^7\text{ s/yr}}$

(b) $V_{mm}=\dfrac{4}{3}\pi r^3=\dfrac{4}{3}\pi\left(5.00\times10^{-7}\text{ m}\right)^3=5.24\times10^{-19}\text{ m}^3$

$$\frac{V_{cube}}{V_{mm}}=\frac{1\text{ m}^3}{5.24\times10^{-19}\text{ m}^3}=1.91\times10^{18}\text{ micrometeorites}$$

This would take $\dfrac{1.91\times10^{18}\text{ micrometeorites}}{3.16\times10^7\text{ micrometeorites/yr}}=\boxed{6.05\times10^{10}\text{ yr}}$.

P1.2 5.52×10^3 kg/m^3, between the densities of aluminum and iron, and greater than the densities of surface rocks.

P1.4 23.0 kg

P1.6 7.69 cm

P1.8 (a) and (b) see the solution, $N_A = 6.022\,137 \times 10^{23}$; (c) 18.0 g; (d) 44.0 g

P1.10 (a) 9.83×10^{-16} g; (b) 1.06×10^7 atoms

P1.12 (a) 4.02×10^{25} molecules; (b) 3.65×10^4 molecules

P1.14 (a) ii; (b) iii; (c) i

P1.16 (a) $\frac{M \cdot L}{T^2}$; (b) 1 newton = 1 kg·m/s^2

P1.18 35.7 m^2

P1.20 1.39×10^{-4} m^3

P1.22 (a) 3.39×10^5 ft^3; (b) 2.54×10^4 lb

P1.24 (a) 560 km = 5.60×10^5 m = 5.60×10^7 cm; (b) 491 m = 0.491 km = 4.91×10^4 cm; (c) 6.19 km = 6.19×10^3 m = 6.19×10^5 cm; (d) 2.50 km = 2.50×10^3 m = 2.50×10^5 cm

P1.26 4.05×10^3 m^2

P1.28 (a) 1 mi/h = 1.609 km/h; (b) 88.5 km/h; (c) 16.1 km/h

P1.30 1.19×10^{57} atoms

P1.32 2.57×10^6 m^3

P1.34 1.3×10^{21} kg

P1.36 200 km

P1.38 (a) 13.4; (b) 49.1

P1.40 $r_{Al} = r_{Fe}\left(\frac{\rho_{Fe}}{\rho_{Al}}\right)^{1/3}$

P1.42 $\sim 10^7$ rev

P1.44 $\sim 10^9$ raindrops

P1.46 $\sim 10^{11}$ cans; $\sim 10^5$ tons

P1.48 (209 ± 4) cm^2

P1.50 (a) 3; (b) 4; (c) 3; (d) 2

P1.52 (a) 797; (b) 1.1; (c) 17.66

P1.54 115.9 m

P1.56 316 m

P1.58 4.50 m^2

P1.60 see the solution; 24.6°

P1.62 3.64 cents; no

P1.64 see the solution

P1.66 (a) 1 000 kg; (b) 5.2×10^{-16} kg; 0.27 kg; 1.3×10^{-5} kg

P1.68 8.32×10^{-4} m/s; a snail

P1.70 see the solution

2

Motion in One Dimension

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q2.1 If I count 5.0 s between lightning and thunder, the sound has traveled $(331\ \text{m/s})(5.0\ \text{s}) = 1.7\ \text{km}$. The transit time for the light is smaller by

$$\frac{3.00\times 10^8\ \text{m/s}}{331\ \text{m/s}} = 9.06\times 10^5 \text{ times,}$$

so it is negligible in comparison.

Q2.2 Yes. Yes, if the particle winds up in the $+x$ region at the end.

Q2.3 Zero.

Q2.4 Yes. Yes.

Q2.5 No. Consider a sprinter running a straight-line race. His average velocity would simply be the length of the race divided by the time it took for him to complete the race. If he stops along the way to tie his shoe, then his instantaneous velocity at that point would be zero.

Q2.6 We assume the object moves along a straight line. If its average velocity is zero, then the displacement must be zero over the time interval, according to Equation 2.2. The object might be stationary throughout the interval. If it is moving to the right at first, it must later move to the left to return to its starting point. Its velocity must be zero as it turns around. The graph of the motion shown to the right represents such motion, as the initial and final positions are the same. In an x vs. t graph, the instantaneous velocity at any time t is the slope of the curve at that point. At t_0 in the graph, the slope of the curve is zero, and thus the instantaneous velocity at that time is also zero.

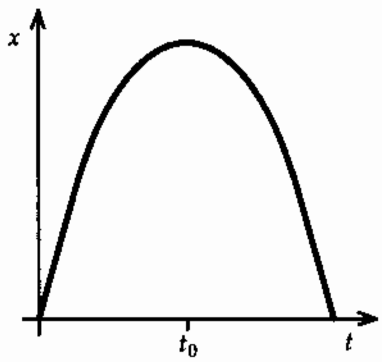


FIG. Q2.6

Q2.7 Yes. If the velocity of the particle is nonzero, the particle is in motion. If the acceleration is zero, the velocity of the particle is unchanging, or is a constant.

Q2.8 Yes. If you drop a doughnut from rest $(v=0)$, then its acceleration is not zero. A common misconception is that immediately after the doughnut is released, both the velocity and acceleration are zero. If the acceleration were zero, then the velocity would not change, leaving the doughnut floating at rest in mid-air.

Q2.9 No: Car A might have greater acceleration than B, but they might both have zero acceleration, or otherwise equal accelerations; or the driver of B might have tramped hard on the gas pedal in the recent past.

Q2.10 Yes. Consider throwing a ball straight up. As the ball goes up, its velocity is upward $(v>0)$, and its acceleration is directed down $(a<0)$. A graph of v vs. t for this situation would look like the figure to the right. The acceleration is the slope of a v vs. t graph, and is always negative in this case, even when the velocity is positive.

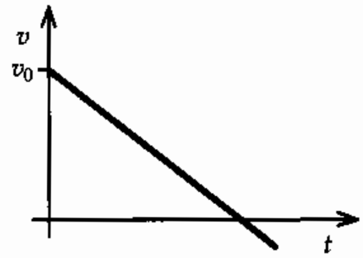


FIG. Q2.10

Q2.11

(a)	Accelerating East	(b)	Braking East	(c)	Cruising East
(d)	Braking West	(e)	Accelerating West	(f)	Cruising West

(g) Stopped but starting to move East

(h) Stopped but starting to move West

Q2.12 No. Constant acceleration only. Yes. Zero is a constant.

Q2.13 The position does depend on the origin of the coordinate system. Assume that the cliff is 20 m tall, and that the stone reaches a maximum height of 10 m above the top of the cliff. If the origin is taken as the top of the cliff, then the maximum height reached by the stone would be 10 m. If the origin is taken as the bottom of the cliff, then the maximum height would be 30 m.

The velocity is independent of the origin. Since the *change* in position is used to calculate the instantaneous velocity in Equation 2.5, the choice of origin is arbitrary.

Q2.14 Once the objects leave the hand, both are in free fall, and both experience the same downward acceleration equal to the free-fall acceleration, $-g$.

Q2.15 They are the same. After the first ball reaches its apex and falls back downward past the student, it will have a downward velocity equal to v_i. This velocity is the same as the velocity of the second ball, so after they fall through equal heights their impact speeds will also be the same.

Q2.16 With $h=\frac{1}{2}gt^2$,

(a) $0.5h=\frac{1}{2}g(0.707t)^2$. The time is later than $0.5t$.

(b) The distance fallen is $0.25h=\frac{1}{2}g(0.5t)^2$. The elevation is $0.75h$, greater than $0.5h$.

Q2.17 Above. Your ball has zero initial speed and smaller average speed during the time of flight to the passing point.

Solutions to Problems

Section 2.1 Position, Velocity, and Speed

P2.1 (a) $\bar{v} = \boxed{2.30 \text{ m/s}}$

(b) $v = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5 \text{ m} - 9.20 \text{ m}}{3.00 \text{ s}} = \boxed{16.1 \text{ m/s}}$

(c) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{57.5 \text{ m} - 0 \text{ m}}{5.00 \text{ s}} = \boxed{11.5 \text{ m/s}}$

***P2.2** (a) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{20 \text{ ft}}{1 \text{ yr}}\left(\dfrac{1 \text{ m}}{3.281 \text{ ft}}\right)\left(\dfrac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}}\right) = \boxed{2 \times 10^{-7} \text{ m/s}}$ or in particularly windy times

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{100 \text{ ft}}{1 \text{ yr}}\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right)\left(\frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}}\right) = \boxed{1 \times 10^{-6} \text{ m/s}}.$$

(b) The time required must have been

$$\Delta t = \frac{\Delta x}{\bar{v}} = \frac{3\,000 \text{ mi}}{10 \text{ mm/yr}}\left(\frac{1\,609 \text{ m}}{1 \text{ mi}}\right)\left(\frac{10^3 \text{ mm}}{1 \text{ m}}\right) = \boxed{5 \times 10^8 \text{ yr}}.$$

P2.3 (a) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{10 \text{ m}}{2 \text{ s}} = \boxed{5 \text{ m/s}}$

(b) $\bar{v} = \dfrac{5 \text{ m}}{4 \text{ s}} = \boxed{1.2 \text{ m/s}}$

(c) $\bar{v} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{5 \text{ m} - 10 \text{ m}}{4 \text{ s} - 2 \text{ s}} = \boxed{-2.5 \text{ m/s}}$

(d) $\bar{v} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{-5 \text{ m} - 5 \text{ m}}{7 \text{ s} - 4 \text{ s}} = \boxed{-3.3 \text{ m/s}}$

(e) $\bar{v} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{0 - 0}{8 - 0} = \boxed{0 \text{ m/s}}$

P2.4 $x = 10t^2$: For

t(s)	=	2.0	2.1	3.0
x(m)	=	40	44.1	90

(a) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{50 \text{ m}}{1.0 \text{ s}} = \boxed{50.0 \text{ m/s}}$

(b) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{4.1 \text{ m}}{0.1 \text{ s}} = \boxed{41.0 \text{ m/s}}$

P2.5 (a) Let d represent the distance between A and B. Let t_1 be the time for which the walker has the higher speed in $5.00\ \text{m/s} = \frac{d}{t_1}$. Let t_2 represent the longer time for the return trip in $-3.00\ \text{m/s} = -\frac{d}{t_2}$. Then the times are $t_1 = \frac{d}{(5.00\ \text{m/s})}$ and $t_2 = \frac{d}{(3.00\ \text{m/s})}$. The average speed is:

$$\bar{v} = \frac{\text{Total distance}}{\text{Total time}} = \frac{d+d}{\frac{d}{(5.00\ \text{m/s})} + \frac{d}{(3.00\ \text{m/s})}} = \frac{2d}{\frac{(8.00\ \text{m/s})d}{(15.0\ \text{m}^2/\text{s}^2)}}$$

$$\bar{v} = \frac{2(15.0\ \text{m}^2/\text{s}^2)}{8.00\ \text{m/s}} = \boxed{3.75\ \text{m/s}}$$

(b) She starts and finishes at the same point A. With total displacement = 0, average velocity $= \boxed{0}$.

Section 2.2 **Instantaneous Velocity and Speed**

P2.6 (a) At any time, t, the position is given by $x = (3.00\ \text{m/s}^2)t^2$.
Thus, at $t_i = 3.00\ \text{s}$: $x_i = (3.00\ \text{m/s}^2)(3.00\ \text{s})^2 = \boxed{27.0\ \text{m}}$.

(b) At $t_f = 3.00\ \text{s} + \Delta t$: $x_f = (3.00\ \text{m/s}^2)(3.00\ \text{s} + \Delta t)^2$, or

$$x_f = \boxed{27.0\ \text{m} + (18.0\ \text{m/s})\Delta t + (3.00\ \text{m/s}^2)(\Delta t)^2}.$$

(c) The instantaneous velocity at $t = 3.00\ \text{s}$ is:

$$v = \lim_{\Delta t \to 0}\left(\frac{x_f - x_i}{\Delta t}\right) = \lim_{\Delta t \to 0}\left(18.0\ \text{m/s} + (3.00\ \text{m/s}^2)\Delta t\right) = \boxed{18.0\ \text{m/s}}.$$

P2.7 (a) at $t_i = 1.5\ \text{s}$, $x_i = 8.0\ \text{m}$ (Point A)
at $t_f = 4.0\ \text{s}$, $x_f = 2.0\ \text{m}$ (Point B)

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0)\ \text{m}}{(4 - 1.5)\ \text{s}} = -\frac{6.0\ \text{m}}{2.5\ \text{s}} = \boxed{-2.4\ \text{m/s}}$$

(b) The slope of the tangent line is found from points C and D. ($t_C = 1.0\ \text{s}$, $x_C = 9.5\ \text{m}$) and ($t_D = 3.5\ \text{s}$, $x_D = 0$),

$$v \cong \boxed{-3.8\ \text{m/s}}.$$

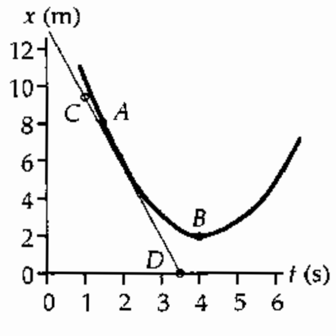
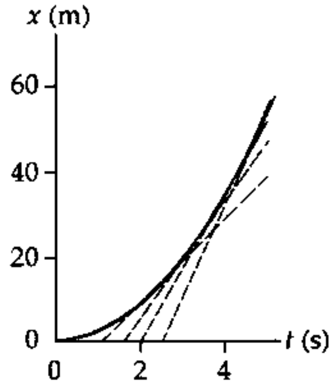
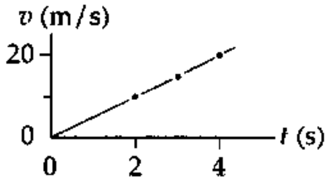


FIG. P2.7

(c) The velocity is zero when x is a minimum. This is at $t \cong \boxed{4\ \text{s}}$.

P2.8 (a)





(b) At $t = 5.0$ s, the slope is $v \cong \dfrac{58 \text{ m}}{2.5 \text{ s}} \cong \boxed{23 \text{ m/s}}$.

At $t = 4.0$ s, the slope is $v \cong \dfrac{54 \text{ m}}{3 \text{ s}} \cong \boxed{18 \text{ m/s}}$.

At $t = 3.0$ s, the slope is $v \cong \dfrac{49 \text{ m}}{3.4 \text{ s}} \cong \boxed{14 \text{ m/s}}$.

At $t = 2.0$ s, the slope is $v \cong \dfrac{36 \text{ m}}{4.0 \text{ s}} \cong \boxed{9.0 \text{ m/s}}$.

(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} \cong \dfrac{23 \text{ m/s}}{5.0 \text{ s}} \cong \boxed{4.6 \text{ m/s}^2}$

(d) Initial velocity of the car was $\boxed{\text{zero}}$.

P2.9 (a) $v = \dfrac{(5-0) \text{ m}}{(1-0) \text{ s}} = \boxed{5 \text{ m/s}}$

(b) $v = \dfrac{(5-10) \text{ m}}{(4-2) \text{ s}} = \boxed{-2.5 \text{ m/s}}$

(c) $v = \dfrac{(5 \text{ m} - 5 \text{ m})}{(5 \text{ s} - 4 \text{ s})} = \boxed{0}$

(d) $v = \dfrac{0-(-5 \text{ m})}{(8 \text{ s} - 7 \text{ s})} = \boxed{+5 \text{ m/s}}$

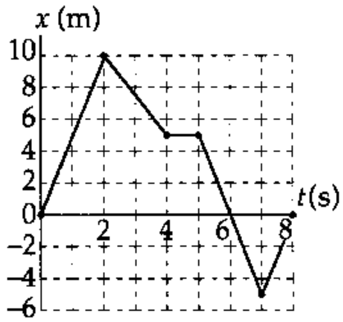


FIG. P2.9

***P2.10** Once it resumes the race, the hare will run for a time of

$$t = \frac{x_f - x_i}{v_x} = \frac{1\,000 \text{ m} - 800 \text{ m}}{8 \text{ m/s}} = 25 \text{ s}.$$

In this time, the tortoise can crawl a distance

$$x_f - x_i = (0.2 \text{ m/s})(25 \text{ s}) = \boxed{5.00 \text{ m}}.$$

Section 2.3 Acceleration

P2.11 Choose the positive direction to be the outward direction, perpendicular to the wall.

$$v_f = v_i + at:\ a = \frac{\Delta v}{\Delta t} = \frac{22.0\ \text{m/s} - (-25.0\ \text{m/s})}{3.50 \times 10^{-3}\ \text{s}} = \boxed{1.34 \times 10^4\ \text{m/s}^2}.$$

P2.12 (a) Acceleration is constant over the first ten seconds, so at the end,

$$v_f = v_i + at = 0 + (2.00\ \text{m/s}^2)(10.0\ \text{s}) = \boxed{20.0\ \text{m/s}}.$$

Then $a = 0$ so v is constant from $t = 10.0$ s to $t = 15.0$ s. And over the last five seconds the velocity changes to

$$v_f = v_i + at = 20.0\ \text{m/s} + (3.00\ \text{m/s}^2)(5.00\ \text{s}) = \boxed{5.00\ \text{m/s}}.$$

(b) In the first ten seconds,

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}(2.00\ \text{m/s}^2)(10.0\ \text{s})^2 = 100\ \text{m}.$$

Over the next five seconds the position changes to

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 100\ \text{m} + (20.0\ \text{m/s})(5.00\ \text{s}) + 0 = 200\ \text{m}.$$

And at $t = 20.0$ s,

$$x_f = x_i + v_i t + \frac{1}{2}at^2 = 200\ \text{m} + (20.0\ \text{m/s})(5.00\ \text{s}) + \frac{1}{2}(-3.00\ \text{m/s}^2)(5.00\ \text{s})^2 = \boxed{262\ \text{m}}.$$

***P2.13** (a) The average speed during a time interval Δt is $\bar{v} = \frac{\text{distance traveled}}{\Delta t}$. During the first quarter mile segment, Secretariat's average speed was

$$\bar{v}_1 = \frac{0.250\ \text{mi}}{25.2\ \text{s}} = \frac{1\,320\ \text{ft}}{25.2\ \text{s}} = \boxed{52.4\ \text{ft/s}}\quad (35.6\ \text{mi/h}).$$

During the second quarter mile segment,

$$\bar{v}_2 = \frac{1\,320\ \text{ft}}{24.0\ \text{s}} = \boxed{55.0\ \text{ft/s}}\quad (37.4\ \text{mi/h}).$$

For the third quarter mile of the race,

$$\bar{v}_3 = \frac{1\,320\ \text{ft}}{23.8\ \text{s}} = \boxed{55.5\ \text{ft/s}}\quad (37.7\ \text{mi/h}),$$

and during the final quarter mile,

$$\bar{v}_4 = \frac{1\,320\ \text{ft}}{23.0\ \text{s}} = \boxed{57.4\ \text{ft/s}}\quad (39.0\ \text{mi/h}).$$

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(b) Assuming that $v_f = \bar{v}_4$ and recognizing that $v_i = 0$, the average acceleration during the race was

$$\bar{a} = \frac{v_f - v_i}{\text{total elapsed time}} = \frac{57.4 \text{ ft/s} - 0}{(25.2 + 24.0 + 23.8 + 23.0) \text{ s}} = \boxed{0.598 \text{ ft/s}^2}.$$

P2.14 (a) Acceleration is the slope of the graph of v vs t.

For $0 < t < 5.00$ s, $a = 0$.

For $15.0 \text{ s} < t < 20.0$ s, $a = 0$.

For $5.0 \text{ s} < t < 15.0$ s, $a = \frac{v_f - v_i}{t_f - t_i}$.

$$a = \frac{8.00 - (-8.00)}{15.0 - 5.00} = 1.60 \text{ m/s}^2$$

We can plot $a(t)$ as shown.

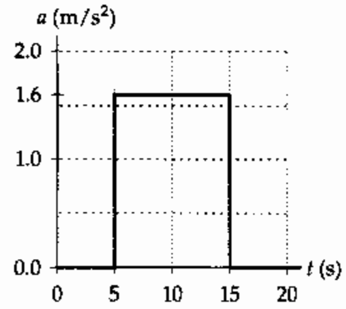


FIG. P2.14

(b) $a = \frac{v_f - v_i}{t_f - t_i}$

(i) For $5.00 \text{ s} < t < 15.0$ s, $t_i = 5.00$ s, $v_i = -8.00$ m/s,

$$t_f = 15.0 \text{ s}$$

$$v_f = 8.00 \text{ m/s}$$

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{15.0 - 5.00} = \boxed{1.60 \text{ m/s}^2}.$$

(ii) $t_i = 0$, $v_i = -8.00$ m/s, $t_f = 20.0$ s, $v_f = 8.00$ m/s

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 - (-8.00)}{20.0 - 0} = \boxed{0.800 \text{ m/s}^2}$$

P2.15 $x = 2.00 + 3.00t - t^2$, $v = \frac{dx}{dt} = 3.00 - 2.00t$, $a = \frac{dv}{dt} = -2.00$

At $t = 3.00$ s:

(a) $x = (2.00 + 9.00 - 9.00) \text{ m} = \boxed{2.00 \text{ m}}$

(b) $v = (3.00 - 6.00) \text{ m/s} = \boxed{-3.00 \text{ m/s}}$

(c) $a = \boxed{-2.00 \text{ m/s}^2}$

P2.16 (a) At $t = 2.00$ s, $x = \left[3.00(2.00)^2 - 2.00(2.00) + 3.00\right]$ m $= 11.0$ m.

At $t = 3.00$ s, $x = \left[3.00(9.00)^2 - 2.00(3.00) + 3.00\right]$ m $= 24.0$ m

so

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{24.0 \text{ m} - 11.0 \text{ m}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{13.0 \text{ m/s}}.$$

(b) At all times the instantaneous velocity is

$$v = \frac{d}{dt}\left(3.00t^2 - 2.00t + 3.00\right) = (6.00t - 2.00) \text{ m/s}$$

At $t = 2.00$ s, $v = [6.00(2.00) - 2.00]$ m/s $= \boxed{10.0 \text{ m/s}}$.
At $t = 3.00$ s, $v = [6.00(3.00) - 2.00]$ m/s $= \boxed{16.0 \text{ m/s}}$.

(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{16.0 \text{ m/s} - 10.0 \text{ m/s}}{3.00 \text{ s} - 2.00 \text{ s}} = \boxed{6.00 \text{ m/s}^2}$

(d) At all times $a = \dfrac{d}{dt}(6.00 - 2.00) = \boxed{6.00 \text{ m/s}^2}$. (This includes both $t = 2.00$ s and $t = 3.00$ s).

P2.17 (a) $a = \dfrac{\Delta v}{\Delta t} = \dfrac{8.00 \text{ m/s}}{6.00 \text{ s}} = \boxed{1.3 \text{ m/s}^2}$

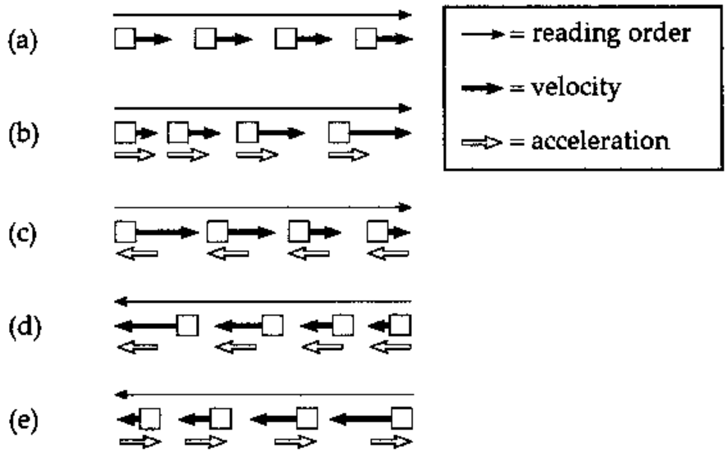
(b) Maximum positive acceleration is at $t = 3$ s, and is approximately $\boxed{2 \text{ m/s}^2}$.

(c) $a = 0$, at $\boxed{t = 6 \text{ s}}$, and also for $\boxed{t > 10 \text{ s}}$.

(d) Maximum negative acceleration is at $t = 8$ s, and is approximately $\boxed{-1.5 \text{ m/s}^2}$.

Section 2.4 Motion Diagrams

P2.18



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(f) One way of phrasing the answer: The spacing of the successive positions would change with less regularity.
Another way: The object would move with some combination of the kinds of motion shown in (a) through (e). Within one drawing, the accelerations vectors would vary in magnitude and direction.

Section 2.5 One-Dimensional Motion with Constant Acceleration

P2.19 From $v_f^2 = v_i^2 + 2ax$, we have $(10.97\times10^3\ \text{m/s})^2 = 0 + 2a(220\ \text{m})$, so that $\boxed{a = 2.74\times10^5\ \text{m/s}^2}$ which is $\boxed{a = 2.79\times10^4 \text{ times } g}$.

P2.20 (a) $x_f - x_i = \frac{1}{2}(v_i + v_f)t$ becomes $40\ \text{m} = \frac{1}{2}(v_i + 2.80\ \text{m/s})(8.50\ \text{s})$ which yields $v_i = \boxed{6.61\ \text{m/s}}$.

(b) $a = \dfrac{v_f - v_i}{t} = \dfrac{2.80\ \text{m/s} - 6.61\ \text{m/s}}{8.50\ \text{s}} = \boxed{-0.448\ \text{m/s}^2}$

P2.21 Given $v_i = 12.0\ \text{cm/s}$ when $x_i = 3.00\ \text{cm}(t = 0)$, and at $t = 2.00\ \text{s}$, $x_f = -5.00\ \text{cm}$,

$$x_f - x_i = v_i t + \frac{1}{2}at^2:\ -5.00 - 3.00 = 12.0(2.00) + \frac{1}{2}a(2.00)^2$$
$$-8.00 = 24.0 + 2a \qquad a = -\frac{32.0}{2} = \boxed{-16.0\ \text{cm/s}^2}.$$

***P2.22** (a) Let *i* be the state of moving at 60 mi/h and *f* be at rest

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$
$$0 = (60\ \text{mi/h})^2 + 2a_x(121\ \text{ft} - 0)\left(\frac{1\ \text{mi}}{5\,280\ \text{ft}}\right)$$
$$a_x = \frac{-3\,600\ \text{mi}}{242\ \text{h}^2}\left(\frac{5\,280\ \text{ft}}{1\ \text{mi}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{-21.8\ \text{mi/h}\cdot\text{s}}$$
$$= -21.8\ \text{mi/h}\cdot\text{s}\left(\frac{1\,609\ \text{m}}{1\ \text{mi}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{-9.75\ \text{m/s}^2}.$$

(b) Similarly,

$$0 = (80\ \text{mi/h})^2 + 2a_x(211\ \text{ft} - 0)$$
$$a_x = -\frac{6\,400(5\,280)}{422(3\,600)}\ \text{mi/h}\cdot\text{s} = \boxed{-22.2\ \text{mi/h}\cdot\text{s} = -9.94\ \text{m/s}^2}.$$

(c) Let *i* be moving at 80 mi/h and *f* be moving at 60 mi/h.

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$
$$(60\ \text{mi/h})^2 = (80\ \text{mi/h})^2 + 2a_x(211\ \text{ft} - 121\ \text{ft})$$
$$a_x = -\frac{2\,800(5\,280)}{2(90)(3\,600)}\ \text{mi/h}\cdot\text{s} = \boxed{-22.8\ \text{mi/h}\cdot\text{s} = -10.2\ \text{m/s}^2}.$$

***P2.23** (a) Choose the initial point where the pilot reduces the throttle and the final point where the boat passes the buoy:

$$x_i = 0,\ x_f = 100\ \text{m},\ v_{xi} = 30\ \text{m/s},\ v_{xf} = ?,\ a_x = -3.5\ \text{m/s}^2,\ t = ?$$

$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$:

$$100\ \text{m} = 0 + (30\ \text{m/s})t + \frac{1}{2}(-3.5\ \text{m/s}^2)t^2$$

$$(1.75\ \text{m/s}^2)t^2 - (30\ \text{m/s})t + 100\ \text{m} = 0.$$

We use the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{30\ \text{m/s} \pm \sqrt{900\ \text{m}^2/\text{s}^2 - 4(1.75\ \text{m/s}^2)(100\ \text{m})}}{2(1.75\ \text{m/s}^2)} = \frac{30\ \text{m/s} \pm 14.1\ \text{m/s}}{3.5\ \text{m/s}^2} = 12.6\ \text{s or } \boxed{4.53\ \text{s}}.$$

The smaller value is the physical answer. If the boat kept moving with the same acceleration, it would stop and move backward, then gain speed, and pass the buoy again at 12.6 s.

(b) $v_{xf} = v_{xi} + a_x t = 30\ \text{m/s} - (3.5\ \text{m/s}^2)4.53\ \text{s} = \boxed{14.1\ \text{m/s}}$

P2.24 (a) Total displacement = area under the $(v,\ t)$ curve from $t = 0$ to 50 s.

$$\Delta x = \frac{1}{2}(50\ \text{m/s})(15\ \text{s}) + (50\ \text{m/s})(40 - 15)\ \text{s} + \frac{1}{2}(50\ \text{m/s})(10\ \text{s})$$

$$\Delta x = \boxed{1\,875\ \text{m}}$$

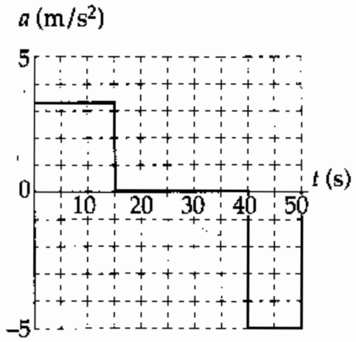


FIG. P2.24

(b) From $t = 10$ s to $t = 40$ s, displacement is

$$\Delta x = \frac{1}{2}(50\ \text{m/s} + 33\ \text{m/s})(5\ \text{s}) + (50\ \text{m/s})(25\ \text{s}) = \boxed{1\,457\ \text{m}}.$$

(c) $0 \le t \le 15\ \text{s}$: $a_1 = \frac{\Delta v}{\Delta t} = \frac{(50 - 0)\ \text{m/s}}{15\ \text{s} - 0} = \boxed{3.3\ \text{m/s}^2}$

$15\ \text{s} < t < 40\ \text{s}$: $\boxed{a_2 = 0}$

$40\ \text{s} \le t \le 50\ \text{s}$: $a_3 = \frac{\Delta v}{\Delta t} = \frac{(0 - 50)\ \text{m/s}}{50\ \text{s} - 40\ \text{s}} = \boxed{-5.0\ \text{m/s}^2}$

continued on next page

(d) (i) $x_1 = 0 + \frac{1}{2}a_1t^2 = \frac{1}{2}(3.3\ \text{m/s}^2)t^2$ or $\boxed{x_1 = (1.67\ \text{m/s}^2)t^2}$

(ii) $x_2 = \frac{1}{2}(15\ \text{s})[50\ \text{m/s} - 0] + (50\ \text{m/s})(t - 15\ \text{s})$ or $\boxed{x_2 = (50\ \text{m/s})t - 375\ \text{m}}$

(iii) For $40\ \text{s} \le t \le 50\ \text{s}$,

$$x_3 = \begin{pmatrix}\text{area under } v \text{ vs } t \\ \text{from } t = 0 \text{ to } 40\ \text{s}\end{pmatrix} + \frac{1}{2}a_3(t - 40\ \text{s})^2 + (50\ \text{m/s})(t - 40\ \text{s})$$

or

$$x_3 = 375\ \text{m} + 1\,250\ \text{m} + \frac{1}{2}(-5.0\ \text{m/s}^2)(t - 40\ \text{s})^2 + (50\ \text{m/s})(t - 40\ \text{s})$$

which reduces to

$$\boxed{x_3 = (250\ \text{m/s})t - (2.5\ \text{m/s}^2)t^2 - 4\,375\ \text{m}}.$$

(e) $\bar{v} = \dfrac{\text{total displacement}}{\text{total elapsed time}} = \dfrac{1\,875\ \text{m}}{50\ \text{s}} = \boxed{37.5\ \text{m/s}}$

P2.25 (a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form

$$x_f = x_i + v_it + \frac{1}{2}at^2$$

to recognize that $x_i = 2.00$ m, $v_i = 3.00$ m/s, and $a = -8.00\ \text{m/s}^2$. The velocity equation, $v_f = v_i + at$, is then

$$v_f = 3.00\ \text{m/s} - (8.00\ \text{m/s}^2)t.$$

The particle changes direction when $v_f = 0$, which occurs at $t = \frac{3}{8}$ s. The position at this time is:

$$x = 2.00\ \text{m} + (3.00\ \text{m/s})\left(\frac{3}{8}\ \text{s}\right) - (4.00\ \text{m/s}^2)\left(\frac{3}{8}\ \text{s}\right)^2 = \boxed{2.56\ \text{m}}.$$

(b) From $x_f = x_i + v_it + \frac{1}{2}at^2$, observe that when $x_f = x_i$, the time is given by $t = -\frac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00\ \text{m/s})}{-8.00\ \text{m/s}^2} = \frac{3}{4}\ \text{s}$$

and the velocity is $v_f = 3.00\ \text{m/s} - (8.00\ \text{m/s}^2)\left(\frac{3}{4}\ \text{s}\right) = \boxed{-3.00\ \text{m/s}}$.

***P2.26** The time for the Ford to slow down we find from

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$$
$$t = \frac{2\Delta x}{v_{xi} + v_{xf}} = \frac{2(250\text{ m})}{71.5\text{ m/s} + 0} = 6.99\text{ s}.$$

Its time to speed up is similarly

$$t = \frac{2(350\text{ m})}{0 + 71.5\text{ m/s}} = 9.79\text{ s}.$$

The whole time it is moving at less than maximum speed is $6.99\text{ s} + 5.00\text{ s} + 9.79\text{ s} = 21.8\text{ s}$. The Mercedes travels

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t = 0 + \frac{1}{2}(71.5 + 71.5)(\text{m/s})(21.8\text{ s})$$
$$= 1\,558\text{ m}$$

while the Ford travels $250 + 350\text{ m} = 600\text{ m}$, to fall behind by $1\,558\text{ m} - 600\text{ m} = \boxed{958\text{ m}}$.

P2.27 (a) $v_i = 100\text{ m/s}$, $a = -5.00\text{ m/s}^2$, $v_f = v_i + at$ so $0 = 100 - 5t$, $v_f^2 = v_i^2 + 2a(x_f - x_i)$ so $0 = (100)^2 - 2(5.00)(x_f - 0)$. Thus $x_f = 1\,000\text{ m}$ and $t = \boxed{20.0\text{ s}}$.

(b) At this acceleration the plane would overshoot the runway: $\boxed{\text{No}}$.

P2.28 (a) Take $t_i = 0$ at the bottom of the hill where $x_i = 0$, $v_i = 30.0\text{ m/s}$, $a = -2.00\text{ m/s}^2$. Use these values in the general equation

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to find

$$x_f = 0 + (30.0t\text{ m/s}) + \frac{1}{2}(-2.00\text{ m/s}^2)t^2$$

when t is in seconds

$$\boxed{x_f = (30.0t - t^2)\text{ m}}.$$

To find an equation for the velocity, use $v_f = v_i + at = 30.0\text{ m/s} + (-2.00\text{ m/s}^2)t$,

$$\boxed{v_f = (30.0 - 2.00t)\text{ m/s}}.$$

(b) The distance of travel x_f becomes a maximum, x_{max}, when $v_f = 0$ (turning point in the motion). Use the expressions found in part (a) for v_f to find the value of t when x_f has its maximum value:

From $v_f = (3.00 - 2.00t)\text{ m/s}$, $v_f = 0$ when $t = 15.0\text{ s}$. Then

$$x_{max} = (30.0t - t^2)\text{ m} = (30.0)(15.0) - (15.0)^2 = \boxed{225\text{ m}}.$$

P2.29 In the simultaneous equations:

$$\left\{\begin{array}{c} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t \end{array}\right\} \text{ we have } \left\{\begin{array}{c} v_{xf} = v_{xi} - (5.60 \text{ m/s}^2)(4.20 \text{ s}) \\ 62.4 \text{ m} = \frac{1}{2}(v_{xi} + v_{xf})(4.20 \text{ s}) \end{array}\right\}.$$

So substituting for v_{xi} gives $62.4 \text{ m} = \frac{1}{2}\left[v_{xf} + (56.0 \text{ m/s}^2)(4.20 \text{ s}) + v_{xf}\right](4.20 \text{ s})$

$$14.9 \text{ m/s} = v_{xf} + \frac{1}{2}(5.60 \text{ m/s}^2)(4.20 \text{ s}).$$

Thus

$$v_{xf} = \boxed{3.10 \text{ m/s}}.$$

P2.30 Take any two of the standard four equations, such as $\left\{\begin{array}{c} v_{xf} = v_{xi} + a_x t \\ x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t \end{array}\right\}$. Solve one for v_{xi}, and substitute into the other: $v_{xi} = v_{xf} - a_x t$

$$x_f - x_i = \frac{1}{2}(v_{xf} - a_x t + v_{xf})t.$$

Thus

$$\boxed{x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2}.$$

Back in problem 29, $62.4 \text{ m} = v_{xf}(4.20 \text{ s}) - \frac{1}{2}(-5.60 \text{ m/s}^2)(4.20 \text{ s})^2$

$$v_{xf} = \frac{62.4 \text{ m} - 49.4 \text{ m}}{4.20 \text{ s}} = \boxed{3.10 \text{ m/s}}.$$

P2.31 (a) $a = \frac{v_f - v_i}{t} = \frac{632\left(\frac{5\,280}{3\,600}\right)}{1.40} = \boxed{-662 \text{ ft/s}^2} = -202 \text{ m/s}^2$

(b) $x_f = v_i t + \frac{1}{2}at^2 = (632)\left(\frac{5\,280}{3\,600}\right)(1.40) - \frac{1}{2}(662)(1.40)^2 = \boxed{649 \text{ ft}} = \boxed{198 \text{ m}}$

P2.32 (a) The time it takes the truck to reach 20.0 m/s is found from $v_f = v_i + at$. Solving for t yields

$$t = \frac{v_f - v_i}{a} = \frac{20.0 \text{ m/s} - 0 \text{ m/s}}{2.00 \text{ m/s}^2} = 10.0 \text{ s}.$$

The total time is thus

$$10.0 \text{ s} + 20.0 \text{ s} + 5.00 \text{ s} = \boxed{35.0 \text{ s}}.$$

(b) The average velocity is the total distance traveled divided by the total time taken. The distance traveled during the first 10.0 s is

$$x_1 = \bar{v}t = \left(\frac{0 + 20.0}{2}\right)(10.0) = 100 \text{ m}.$$

With a being 0 for this interval, the distance traveled during the next 20.0 s is

$$x_2 = v_i t + \frac{1}{2}at^2 = (20.0)(20.0) + 0 = 400 \text{ m}.$$

The distance traveled in the last 5.00 s is

$$x_3 = \bar{v}t = \left(\frac{20.0 + 0}{2}\right)(5.00) = 50.0 \text{ m}.$$

The total distance $x = x_1 + x_2 + x_3 = 100 + 400 + 50 = 550 \text{ m}$, and the average velocity is given by $\bar{v} = \frac{x}{t} = \frac{550}{35.0} = \boxed{15.7 \text{ m/s}}$.

P2.33 We have $v_i = 2.00 \times 10^4 \text{ m/s}$, $v_f = 6.00 \times 10^6 \text{ m/s}$, $x_f - x_i = 1.50 \times 10^{-2} \text{ m}$.

(a) $$x_f - x_i = \frac{1}{2}(v_i + v_f)t: \; t = \frac{2(x_f - x_i)}{v_i + v_f} = \frac{2(1.50 \times 10^{-2} \text{ m})}{2.00 \times 10^4 \text{ m/s} + 6.00 \times 10^6 \text{ m/s}} = \boxed{4.98 \times 10^{-9} \text{ s}}$$

(b) $v_f^2 = v_i^2 + 2a_x(x_f - x_i)$:

$$a_x = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(6.00 \times 10^6 \text{ m/s})^2 - (2.00 \times 10^4 \text{ m/s})^2}{2(1.50 \times 10^{-2} \text{ m})} = \boxed{1.20 \times 10^{15} \text{ m/s}^2}$$

***P2.34** (a) $v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$: $\left[0.01(3\times10^8 \text{ m/s})\right]^2 = 0 + 2a_x(40 \text{ m})$

$$a_x = \frac{(3\times10^6 \text{ m/s})^2}{80 \text{ m}} = \boxed{1.12\times10^{11} \text{ m/s}^2}$$

(b) We must find separately the time t_1 for speeding up and the time t_2 for coasting:

$$x_f - x_i = \frac{1}{2}(v_{xf} + v_{xi})t_1: \quad 40 \text{ m} = \frac{1}{2}(3\times10^6 \text{ m/s} + 0)t_1$$
$$t_1 = 2.67\times10^{-5} \text{ s}$$

$$x_f - x_i = \frac{1}{2}(v_{xf} + v_{xi})t_2: \quad 60 \text{ m} = \frac{1}{2}(3\times10^6 \text{ m/s} + 3\times10^6 \text{ m/s})t_2$$
$$t_2 = 2.00\times10^{-5} \text{ s}$$

total time $= \boxed{4.67\times10^{-5} \text{ s}}$.

***P2.35** (a) Along the time axis of the graph shown, let $i = 0$ and $f = t_m$. Then $v_{xf} = v_{xi} + a_x t$ gives $v_c = 0 + a_m t_m$

$$\boxed{a_m = \frac{v_c}{t_m}}.$$

(b) The displacement between 0 and t_m is

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2 = 0 + \frac{1}{2}\frac{v_c}{t_m}t_m^2 = \frac{1}{2}v_c t_m.$$

The displacement between t_m and t_0 is

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2 = v_c(t_0 - t_m) + 0.$$

The total displacement is

$$\Delta x = \frac{1}{2}v_c t_m + v_c t_0 - v_c t_m = \boxed{v_c\left(t_0 - \frac{1}{2}t_m\right)}.$$

(c) For constant v_c and t_0, Δx is minimized by maximizing t_m to $t_m = t_0$. Then $\Delta x_{\min} = v_c\left(t_0 - \frac{1}{2}t_0\right) = \boxed{\frac{v_c t_0}{2}}$.

(e) This is realized by having the servo motor on all the time.

(d) We maximize Δx by letting t_m approach zero. In the limit $\Delta x = v_c(t_0 - 0) = \boxed{v_c t_0}$.

(e) This cannot be attained because the acceleration must be finite.

***P2.36** Let the glider enter the photogate with velocity v_i and move with constant acceleration a. For its motion from entry to exit,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$
$$\ell = 0 + v_i\Delta t_d + \frac{1}{2}a\Delta t_d^2 = v_d\Delta t_d$$
$$v_d = v_i + \frac{1}{2}a\Delta t_d$$

(a) The speed halfway through the photogate in space is given by

$$v_{hs}^2 = v_i^2 + 2a\left(\frac{\ell}{2}\right) = v_i^2 + av_d\Delta t_d.$$

$v_{hs} = \sqrt{v_i^2 + av_d\Delta t_d}$ and this is not equal to v_d unless $a = 0$.

(b) The speed halfway through the photogate in time is given by $v_{ht} = v_i + a\left(\frac{\Delta t_d}{2}\right)$ and this is equal to v_d as determined above.

P2.37 (a) Take initial and final points at top and bottom of the incline. If the ball starts from rest,

$$v_i = 0,\ a = 0.500\ \text{m/s}^2,\ x_f - x_i = 9.00\ \text{m}.$$

Then

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = 0^2 + 2(0.500\ \text{m/s}^2)(9.00\ \text{m})$$
$$v_f = \boxed{3.00\ \text{m/s}}.$$

(b) $x_f - x_i = v_i t + \frac{1}{2}at^2$

$$9.00 = 0 + \frac{1}{2}(0.500\ \text{m/s}^2)t^2$$
$$t = \boxed{6.00\ \text{s}}$$

(c) Take initial and final points at the bottom of the planes and the top of the second plane, respectively:

$$v_i = 3.00\ \text{m/s},\ v_f = 0,\ x_f - x_i = 15.00\ \text{m}.$$

$v_f^2 = v_i^2 + 2a(x_f - x_i)$ gives

$$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{\left[0 - (3.00\ \text{m/s})^2\right]}{2(15.0\ \text{m})} = \boxed{-0.300\ \text{m/s}^2}.$$

(d) Take the initial point at the bottom of the planes and the final point 8.00 m along the second: $v_i = 3.00\ \text{m/s},\ x_f - x_i = 8.00\ \text{m},\ a = -0.300\ \text{m/s}^2$

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = (3.00\ \text{m/s})^2 + 2(-0.300\ \text{m/s}^2)(8.00\ \text{m}) = 4.20\ \text{m}^2/\text{s}^2$$
$$v_f = \boxed{2.05\ \text{m/s}}.$$

P2.38 Take the original point to be when Sue notices the van. Choose the origin of the x-axis at Sue's car. For her we have $x_{is} = 0$, $v_{is} = 30.0$ m/s, $a_s = -2.00$ m/s^2 so her position is given by

$$x_s(t) = x_{is} + v_{is}t + \frac{1}{2}a_s t^2 = (30.0 \text{ m/s})t + \frac{1}{2}(-2.00 \text{ m/s}^2)t^2.$$

For the van, $x_{iv} = 155$ m, $v_{iv} = 5.00$ m/s, $a_v = 0$ and

$$x_v(t) = x_{iv} + v_{iv}t + \frac{1}{2}a_v t^2 = 155 + (5.00 \text{ m/s})t + 0.$$

To test for a collision, we look for an instant t_c when both are at the same place:

$$30.0t_c - t_c^2 = 155 + 5.00t_c$$
$$0 = t_c^2 - 25.0t_c + 155.$$

From the quadratic formula

$$t_c = \frac{25.0 \pm \sqrt{(25.0)^2 - 4(155)}}{2} = 13.6 \text{ s or } \boxed{11.4 \text{ s}}.$$

The smaller value is the collision time. (The larger value tells when the van would pull ahead again if the vehicles could move through each other). The wreck happens at position

$$155 \text{ m} + (5.00 \text{ m/s})(11.4 \text{ s}) = \boxed{212 \text{ m}}.$$

***P2.39** As in the algebraic solution to Example 2.8, we let t represent the time the trooper has been moving. We graph

$$x_{\text{car}} = 45 + 45t$$

and

$$x_{\text{trooper}} = 1.5t^2.$$

They intersect at

$$t = \boxed{31 \text{ s}}.$$

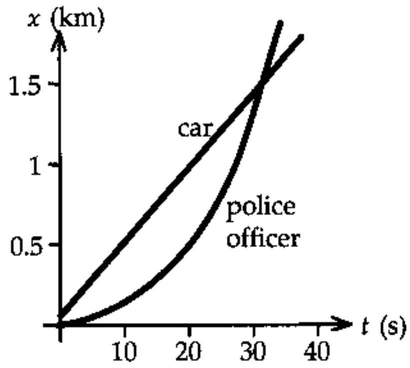


FIG. P2.39

Section 2.6 Freely Falling Objects

P2.40 Choose the origin ($y=0$, $t=0$) at the starting point of the ball and take upward as positive. Then $y_i=0$, $v_i=0$, and $a=-g=-9.80\ \text{m/s}^2$. The position and the velocity at time t become:

$$y_f-y_i=v_it+\frac{1}{2}at^2:\ y_f=-\frac{1}{2}gt^2=-\frac{1}{2}\left(9.80\ \text{m/s}^2\right)t^2$$

and

$$v_f=v_i+at:\ v_f=-gt=-\left(9.80\ \text{m/s}^2\right)t.$$

(a) at $t=1.00\ \text{s}$: $y_f=-\frac{1}{2}\left(9.80\ \text{m/s}^2\right)(1.00\ \text{s})^2=\boxed{-4.90\ \text{m}}$

at $t=2.00\ \text{s}$: $y_f=-\frac{1}{2}\left(9.80\ \text{m/s}^2\right)(2.00\ \text{s})^2=\boxed{-19.6\ \text{m}}$

at $t=3.00\ \text{s}$: $y_f=-\frac{1}{2}\left(9.80\ \text{m/s}^2\right)(3.00\ \text{s})^2=\boxed{-44.1\ \text{m}}$

(b) at $t=1.00\ \text{s}$: $v_f=-\left(9.80\ \text{m/s}^2\right)(1.00\ \text{s})=\boxed{-9.80\ \text{m/s}}$

at $t=2.00\ \text{s}$: $v_f=-\left(9.80\ \text{m/s}^2\right)(2.00\ \text{s})=\boxed{-19.6\ \text{m/s}}$

at $t=3.00\ \text{s}$: $v_f=-\left(9.80\ \text{m/s}^2\right)(3.00\ \text{s})=\boxed{-29.4\ \text{m/s}}$

P2.41 Assume that air resistance may be neglected. Then, the acceleration at all times during the flight is that due to gravity, $a=-g=-9.80\ \text{m/s}^2$. During the flight, Goff went 1 mile (1 609 m) up and then 1 mile back down. Determine his speed just after launch by considering his upward flight:

$$v_f^2=v_i^2+2a\left(y_f-y_i\right):\quad 0=v_i^2-2\left(9.80\ \text{m/s}^2\right)(1\,609\ \text{m})$$
$$v_i=178\ \text{m/s}.$$

His time in the air may be found by considering his motion from just after launch to just before impact:

$$y_f-y_i=v_it+\frac{1}{2}at^2:\ 0=(178\ \text{m/s})t-\frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t^2.$$

The root $t=0$ describes launch; the other root, $t=36.2\ \text{s}$, describes his flight time. His rate of pay may then be found from

$$\text{pay rate}=\frac{\$1.00}{36.2\ \text{s}}=(0.027\,6\ \$/\text{s})(3\,600\ \text{s/h})=\boxed{\$99.3/\text{h}}.$$

We have assumed that the workman's flight time, "a mile", and "a dollar", were measured to three-digit precision. We have interpreted "up in the sky" as referring to the free fall time, not to the launch and landing times. Both the takeoff and landing times must be several seconds away from the job, in order for Goff to survive to resume work.

P2.42 We have $y_f = -\frac{1}{2}gt^2 + v_i t + y_i$

$$0 = -\left(4.90 \text{ m/s}^2\right)t^2 - (8.00 \text{ m/s})t + 30.0 \text{ m}.$$

Solving for t,

$$t = \frac{8.00 \pm \sqrt{64.0 + 588}}{-9.80}.$$

Using only the positive value for t, we find that $t = \boxed{1.79 \text{ s}}$.

P2.43 (a) $y_f - y_i = v_i t + \frac{1}{2}at^2$: $4.00 = (1.50)v_i - (4.90)(1.50)^2$ and $v_i = \boxed{10.0 \text{ m/s upward}}$.

(b) $v_f = v_i + at = 10.0 - (9.80)(1.50) = -4.68 \text{ m/s}$

$$v_f = \boxed{4.68 \text{ m/s downward}}$$

P2.44 The bill starts from rest $v_i = 0$ and falls with a downward acceleration of 9.80 m/s^2 (due to gravity). Thus, in 0.20 s it will fall a distance of

$$\Delta y = v_i t - \frac{1}{2}gt^2 = 0 - \left(4.90 \text{ m/s}^2\right)(0.20 \text{ s})^2 = -0.20 \text{ m}.$$

This distance is about twice the distance between the center of the bill and its top edge ($\cong 8$ cm). $\boxed{\text{Thus, David will be unsuccessful}}$.

***P2.45** (a) From $\Delta y = v_i t + \frac{1}{2}at^2$ with $v_i = 0$, we have

$$t = \sqrt{\frac{2(\Delta y)}{a}} = \sqrt{\frac{2(-23 \text{ m})}{-9.80 \text{ m/s}^2}} = \boxed{2.17 \text{ s}}.$$

(b) The final velocity is $v_f = 0 + \left(-9.80 \text{ m/s}^2\right)(2.17 \text{ s}) = \boxed{-21.2 \text{ m/s}}$.

(c) The time take for the sound of the impact to reach the spectator is

$$t_{\text{sound}} = \frac{\Delta y}{v_{\text{sound}}} = \frac{23 \text{ m}}{340 \text{ m/s}} = 6.76 \times 10^{-2} \text{ s},$$

so the total elapsed time is $t_{\text{total}} = 2.17 \text{ s} + 6.76 \times 10^{-2} \text{ s} \approx \boxed{2.23 \text{ s}}$.

P2.46 At any time t, the position of the ball released from rest is given by $y_1 = h - \frac{1}{2}gt^2$. At time t, the position of the ball thrown vertically upward is described by $y_2 = v_i t - \frac{1}{2}gt^2$. The time at which the first ball has a position of $y_1 = \frac{h}{2}$ is found from the first equation as $\frac{h}{2} = h - \frac{1}{2}gt^2$, which yields $t = \sqrt{\frac{h}{g}}$. To require that the second ball have a position of $y_2 = \frac{h}{2}$ at this time, use the second equation to obtain $\frac{h}{2} = v_i\sqrt{\frac{h}{g}} - \frac{1}{2}g\left(\frac{h}{g}\right)$. This gives the required initial upward velocity of the second ball as $\boxed{v_i = \sqrt{gh}}$.

P2.47 (a) $v_f = v_i - gt$: $v_f = 0$ when $t = 3.00\text{ s}$, $g = 9.80\text{ m/s}^2$. Therefore,

$$v_i = gt = (9.80\text{ m/s}^2)(3.00\text{ s}) = \boxed{29.4\text{ m/s}}.$$

(b) $y_f - y_i = \frac{1}{2}(v_f + v_i)t$

$$y_f - y_i = \frac{1}{2}(29.4\text{ m/s})(3.00\text{ s}) = \boxed{44.1\text{ m}}$$

***P2.48** (a) Consider the upward flight of the arrow.

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$
$$0 = (100\text{ m/s})^2 + 2(-9.8\text{ m/s}^2)\Delta y$$
$$\Delta y = \frac{10\,000\text{ m}^2/\text{s}^2}{19.6\text{ m/s}^2} = \boxed{510\text{ m}}$$

(b) Consider the whole flight of the arrow.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$0 = 0 + (100\text{ m/s})t + \frac{1}{2}(-9.8\text{ m/s}^2)t^2$$

The root $t = 0$ refers to the starting point. The time of flight is given by

$$t = \frac{100\text{ m/s}}{4.9\text{ m/s}^2} = \boxed{20.4\text{ s}}.$$

P2.49 Time to fall 3.00 m is found from Eq. 2.12 with $v_i = 0$, $3.00\text{ m} = \frac{1}{2}(9.80\text{ m/s}^2)t^2$, $t = 0.782\text{ s}$.

(a) With the horse galloping at 10.0 m/s, the horizontal distance is $vt = \boxed{7.82\text{ m}}$.

(b) $t = \boxed{0.782\text{ s}}$

P2.50 Take downward as the positive y direction.

(a) While the woman was in free fall,

$$\Delta y = 144 \text{ ft}, \; v_i = 0, \text{ and } a = g = 32.0 \text{ ft/s}^2.$$

Thus, $\Delta y = v_i t + \frac{1}{2}at^2 \to 144 \text{ ft} = 0 + (16.0 \text{ ft/s}^2)t^2$ giving $t_{fall} = 3.00$ s. Her velocity just before impact is:

$$v_f = v_i + gt = 0 + (32.0 \text{ ft/s}^2)(3.00 \text{ s}) = \boxed{96.0 \text{ ft/s}}.$$

(b) While crushing the box, $v_i = 96.0$ ft/s, $v_f = 0$, and $\Delta y = 18.0 \text{ in.} = 1.50$ ft. Therefore,

$$a = \frac{v_f^2 - v_i^2}{2(\Delta y)} = \frac{0 - (96.0 \text{ ft/s})^2}{2(1.50 \text{ ft})} = -3.07 \times 10^3 \text{ ft/s}^2, \text{ or } \boxed{a = 3.07 \times 10^3 \text{ ft/s}^2 \text{ upward}}.$$

(c) Time to crush box: $\Delta t = \frac{\Delta y}{\bar{v}} = \frac{\Delta y}{\frac{v_f + v_i}{2}} = \frac{2(1.50 \text{ ft})}{0 + 96.0 \text{ ft/s}}$ or $\boxed{\Delta t = 3.13 \times 10^{-2} \text{ s}}$.

P2.51 $y = 3.00t^3$: At $t = 2.00$ s, $y = 3.00(2.00)^3 = 24.0$ m and

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0 \text{ m/s} \uparrow.$$

If the helicopter releases a small mailbag at this time, the equation of motion of the mailbag is

$$y_b = y_{bi} + v_i t - \frac{1}{2}gt^2 = 24.0 + 36.0t - \frac{1}{2}(9.80)t^2.$$

Setting $y_b = 0$,

$$0 = 24.0 + 36.0t - 4.90t^2.$$

Solving for t, (only positive values of t count), $\boxed{t = 7.96 \text{ s}}$.

***P2.52** Consider the last 30 m of fall. We find its speed 30 m above the ground:

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$0 = 30 \text{ m} + v_{yi}(1.5 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(1.5 \text{ s})^2$$

$$v_{yi} = \frac{-30 \text{ m} + 11.0 \text{ m}}{1.5 \text{ s}} = -12.6 \text{ m/s}.$$

Now consider the portion of its fall above the 30 m point. We assume it starts from rest

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

$$(-12.6 \text{ m/s})^2 = 0 + 2(-9.8 \text{ m/s}^2)\Delta y$$

$$\Delta y = \frac{160 \text{ m}^2/\text{s}^2}{-19.6 \text{ m/s}^2} = -8.16 \text{ m}.$$

Its original height was then $30 \text{ m} + |-8.16 \text{ m}| = \boxed{38.2 \text{ m}}$.

Section 2.7 **Kinematic Equations Derived from Calculus**

P2.53 (a) $J = \frac{da}{dt} = \text{constant}$

$$da = Jdt$$

$$a = J\int dt = Jt + c_1$$

but $a = a_i$ when $t = 0$ so $c_1 = a_i$. Therefore, $\boxed{a = Jt + a_i}$

$$a = \frac{dv}{dt}$$
$$dv = adt$$
$$v = \int adt = \int (Jt + a_i)dt = \frac{1}{2}Jt^2 + a_i t + c_2$$

but $v = v_i$ when $t = 0$, so $c_2 = v_i$ and $\boxed{v = \frac{1}{2}Jt^2 + a_i t + v_i}$

$$v = \frac{dx}{dt}$$
$$dx = vdt$$
$$x = \int vdt = \int \left(\frac{1}{2}Jt^2 + a_i t + v_i\right)dt$$
$$x = \frac{1}{6}Jt^3 + \frac{1}{2}a_i t^2 + v_i t + c_3$$
$$x = x_i$$

when $t = 0$, so $c_3 = x_i$. Therefore, $\boxed{x = \frac{1}{6}Jt^3 + \frac{1}{2}a_i t^2 + v_i t + x_i}$.

(b) $a^2 = (Jt + a_i)^2 = J^2t^2 + a_i^2 + 2Ja_i t$

$a^2 = a_i^2 + \left(J^2t^2 + 2Ja_i t\right)$

$a^2 = a_i^2 + 2J\left(\frac{1}{2}Jt^2 + a_i t\right)$

Recall the expression for v: $v = \frac{1}{2}Jt^2 + a_i t + v_i$. So $(v - v_i) = \frac{1}{2}Jt^2 + a_i t$. Therefore,

$$\boxed{a^2 = a_i^2 + 2J(v - v_i)}.$$

P2.54 (a) See the graphs at the right.

Choose $x = 0$ at $t = 0$.

At $t = 3$ s, $x = \frac{1}{2}(8\ \text{m/s})(3\ \text{s}) = 12$ m.

At $t = 5$ s, $x = 12\ \text{m} + (8\ \text{m/s})(2\ \text{s}) = 28$ m.

At $t = 7$ s, $x = 28\ \text{m} + \frac{1}{2}(8\ \text{m/s})(2\ \text{s}) = 36$ m.

(b) For $0 < t < 3$ s, $a = \frac{8\ \text{m/s}}{3\ \text{s}} = 2.67\ \text{m/s}^2$.

For $3 < t < 5$ s, $a = 0$.

(c) For $5\ \text{s} < t < 9$ s, $a = -\frac{16\ \text{m/s}}{4\ \text{s}} = \boxed{-4\ \text{m/s}^2}$.

(d) At $t = 6$ s, $x = 28\ \text{m} + (6\ \text{m/s})(1\ \text{s}) = \boxed{34\ \text{m}}$.

(e) At $t = 9$ s, $x = 36\ \text{m} + \frac{1}{2}(-8\ \text{m/s})(2\ \text{s}) = \boxed{28\ \text{m}}$.

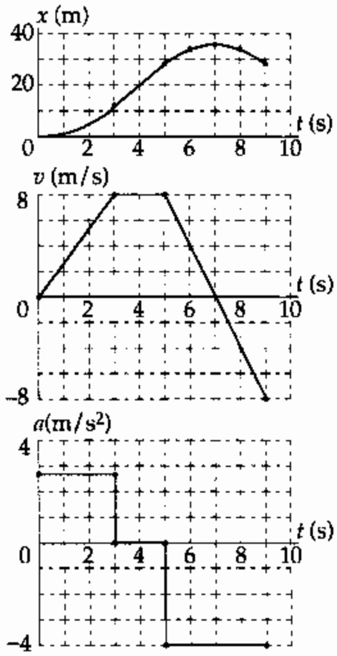


FIG. P2.54

P2.55 (a) $a = \frac{dv}{dt} = \frac{d}{dt}\left[-5.00\times 10^7 t^2 + 3.00\times 10^5 t\right]$

$$\boxed{a = -\left(10.0\times 10^7\ \text{m/s}^3\right)t + 3.00\times 10^5\ \text{m/s}^2}$$

Take $x_i = 0$ at $t = 0$. Then $v = \frac{dx}{dt}$

$$x - 0 = \int_0^t v\,dt = \int_0^t \left(-5.00\times 10^7 t^2 + 3.00\times 10^5 t\right)dt$$

$$x = -5.00\times 10^7 \frac{t^3}{3} + 3.00\times 10^5 \frac{t^2}{2}$$

$$\boxed{x = -\left(1.67\times 10^7\ \text{m/s}^3\right)t^3 + \left(1.50\times 10^5\ \text{m/s}^2\right)t^2}.$$

(b) The bullet escapes when $a = 0$, at $-\left(10.0\times 10^7\ \text{m/s}^3\right)t + 3.00\times 10^5\ \text{m/s}^2 = 0$

$$t = \frac{3.00\times 10^5\ \text{s}}{10.0\times 10^7} = \boxed{3.00\times 10^{-3}\ \text{s}}.$$

(c) New $v = \left(-5.00\times 10^7\right)\left(3.00\times 10^{-3}\right)^2 + \left(3.00\times 10^5\right)\left(3.00\times 10^{-3}\right)$

$$v = -450\ \text{m/s} + 900\ \text{m/s} = \boxed{450\ \text{m/s}}.$$

(d) $x = -\left(1.67\times 10^7\right)\left(3.00\times 10^{-3}\right)^3 + \left(1.50\times 10^5\right)\left(3.00\times 10^{-3}\right)^2$

$x = -0.450\ \text{m} + 1.35\ \text{m} = \boxed{0.900\ \text{m}}$

P2.56 $a = \frac{dv}{dt} = -3.00v^2$, $v_i = 1.50$ m/s

Solving for v, $\frac{dv}{dt} = -3.00v^2$

$$\int_{v=v_i}^{v} v^{-2}dv = -3.00 \int_{t=0}^{t} dt$$

$$-\frac{1}{v} + \frac{1}{v_i} = -3.00t \text{ or } 3.00t = \frac{1}{v} - \frac{1}{v_i}.$$

When $v = \frac{v_i}{2}$, $t = \frac{1}{3.00v_i} = \boxed{0.222 \text{ s}}$.

Additional Problems

***P2.57** The distance the car travels at constant velocity, v_0, during the reaction time is $(\Delta x)_1 = v_0 \Delta t_r$. The time for the car to come to rest, from initial velocity v_0, after the brakes are applied is

$$t_2 = \frac{v_f - v_i}{a} = \frac{0 - v_0}{a} = -\frac{v_0}{a}$$

and the distance traveled during this braking period is

$$(\Delta x)_2 = \bar{v}t_2 = \left(\frac{v_f + v_i}{2}\right)t_2 = \left(\frac{0 + v_0}{2}\right)\left(-\frac{v_0}{a}\right) = -\frac{v_0^2}{2a}.$$

Thus, the total distance traveled before coming to a stop is

$$s_{\text{stop}} = (\Delta x)_1 + (\Delta x)_2 = \boxed{v_0 \Delta t_r - \frac{v_0^2}{2a}}.$$

***P2.58** (a) If a car is a distance $s_{\text{stop}} = v_0 \Delta t_r - \frac{v_0^2}{2a}$ (See the solution to Problem 2.57) from the intersection of length s_i when the light turns yellow, the distance the car must travel before the light turns red is

$$\Delta x = s_{\text{stop}} + s_i = v_0 \Delta t_r - \frac{v_0^2}{2a} + s_i.$$

Assume the driver does not accelerate in an attempt to "beat the light" (an extremely dangerous practice!). The time the light should remain yellow is then the time required for the car to travel distance Δx at constant velocity v_0. This is

$$\Delta t_{\text{light}} = \frac{\Delta x}{v_0} = \frac{v_0 \Delta t_r - \frac{v_0^2}{2a} + s_i}{v_0} = \boxed{\Delta t_r - \frac{v_0}{2a} + \frac{s_i}{v_0}}.$$

(b) With $s_i = 16$ m, $v = 60$ km/h, $a = -2.0$ m/s^2, and $\Delta t_r = 1.1$ s,

$$\Delta t_{\text{light}} = 1.1 \text{ s} - \frac{60 \text{ km/h}}{2(-2.0 \text{ m/s}^2)}\left(\frac{0.278 \text{ m/s}}{1 \text{ km/h}}\right) + \frac{16 \text{ m}}{60 \text{ km/h}}\left(\frac{1 \text{ km/h}}{0.278 \text{ m/s}}\right) = \boxed{6.23 \text{ s}}.$$

***P2.59** (a) As we see from the graph, from about –50 s to 50 s Acela is cruising at a constant positive velocity in the $+x$ direction. From 50 s to 200 s, Acela accelerates in the $+x$ direction reaching a top speed of about 170 mi/h. Around 200 s, the engineer applies the brakes, and the train, still traveling in the $+x$ direction, slows down and then stops at 350 s. Just after 350 s, Acela reverses direction (v becomes negative) and steadily gains speed in the $-x$ direction.

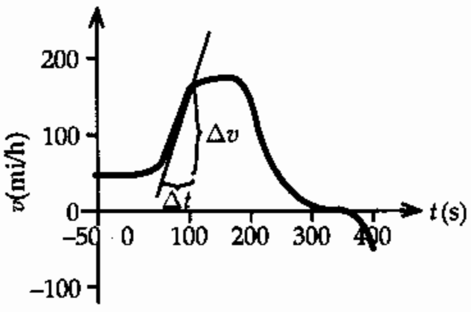


FIG. P2.59(a)

(b) The peak acceleration between 45 and 170 mi/h is given by the slope of the steepest tangent to the v versus t curve in this interval. From the tangent line shown, we find

$$a = \text{slope} = \frac{\Delta v}{\Delta t} = \frac{(155-45)\ \text{mi/h}}{(100-50)\ \text{s}} = \boxed{2.2\ (\text{mi/h})/\text{s}} = 0.98\ \text{m/s}^2.$$

(c) Let us use the fact that the area under the v versus t curve equals the displacement. The train's displacement between 0 and 200 s is equal to the area of the gray shaded region, which we have approximated with a series of triangles and rectangles.

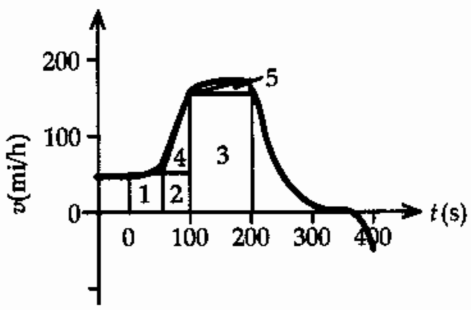


FIG. P2.59(c)

$$\begin{aligned}\Delta x_{0\to 200\ \text{s}} &= \text{area}_1 + \text{area}_2 + \text{area}_3 + \text{area}_4 + \text{area}_5 \\ &\approx (50\ \text{mi/h})(50\ \text{s}) + (50\ \text{mi/h})(50\ \text{s}) \\ &+ (160\ \text{mi/h})(100\ \text{s}) \\ &+ \frac{1}{2}(50\ \text{s})(100\ \text{mi/h}) \\ &+ \frac{1}{2}(100\ \text{s})(170\ \text{mi/h} - 160\ \text{mi/h}) \\ &= 24\,000(\text{mi/h})(\text{s})\end{aligned}$$

Now, at the end of our calculation, we can find the displacement in miles by converting hours to seconds. As 1 h = 3 600 s,

$$\Delta x_{0\to 200\ \text{s}} \approx \left(\frac{24\,000\ \text{mi}}{3\,600\ \text{s}}\right)(\text{s}) = \boxed{6.7\ \text{mi}}.$$

***P2.60** Average speed of every point on the train as the first car passes Liz:

$$\frac{\Delta x}{\Delta t} = \frac{8.60 \text{ m}}{1.50 \text{ s}} = 5.73 \text{ m/s}.$$

The train has this as its instantaneous speed halfway through the 1.50 s time. Similarly, halfway through the next 1.10 s, the speed of the train is $\frac{8.60 \text{ m}}{1.10 \text{ s}} = 7.82 \text{ m/s}$. The time required for the speed to change from 5.73 m/s to 7.82 m/s is

$$\frac{1}{2}(1.50 \text{ s}) + \frac{1}{2}(1.10 \text{ s}) = 1.30 \text{ s}$$

so the acceleration is: $a_x = \frac{\Delta v_x}{\Delta t} = \frac{7.82 \text{ m/s} - 5.73 \text{ m/s}}{1.30 \text{ s}} = \boxed{1.60 \text{ m/s}^2}$.

P2.61 The rate of hair growth is a velocity and the rate of its increase is an acceleration. Then $v_{xi} = 1.04 \text{ mm/d}$ and $a_x = 0.132\left(\frac{\text{mm/d}}{\text{w}}\right)$. The increase in the length of the hair (i.e., displacement) during a time of $t = 5.00 \text{ w} = 35.0 \text{ d}$ is

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$

$$\Delta x = (1.04 \text{ mm/d})(35.0 \text{ d}) + \frac{1}{2}(0.132 \text{ mm/d}\cdot\text{w})(35.0 \text{ d})(5.00 \text{ w})$$

or $\boxed{\Delta x = 48.0 \text{ mm}}$.

P2.62 Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact. The data in the table are found for each phase of the rocket's motion.

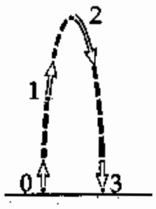


FIG. P2.62

(0 to 1) $v_f^2 - (80.0)^2 = 2(4.00)(1\,000)$ so $v_f = 120 \text{ m/s}$
$120 = 80.0 + (4.00)t$ giving $t = 10.0 \text{ s}$

(1 to 2) $0 - (120)^2 = 2(-9.80)(x_f - x_i)$ giving $x_f - x_i = 735 \text{ m}$
$0 - 120 = -9.80t$ giving $t = 12.2 \text{ s}$
This is the time of maximum height of the rocket.

(2 to 3) $v_f^2 - 0 = 2(-9.80)(-1\,735)$
$v_f = -184 = (-9.80)t$ giving $t = 18.8 \text{ s}$

(a) $t_{\text{total}} = 10 + 12.2 + 18.8 = \boxed{41.0 \text{ s}}$

(b) $(x_f - x_i)_{\text{total}} = \boxed{1.73 \text{ km}}$

continued on next page

(c) $v_{\text{final}} = \boxed{-184 \text{ m/s}}$

		t	x	v	a
0	Launch	0.0	0	80	+4.00
#1	End Thrust	10.0	1 000	120	+4.00
#2	Rise Upwards	22.2	1 735	0	–9.80
#3	Fall to Earth	41.0	0	–184	–9.80

P2.63 Distance traveled by motorist $= (15.0 \text{ m/s})t$

Distance traveled by policeman $= \frac{1}{2}(2.00 \text{ m/s}^2)t^2$

(a) intercept occurs when $15.0t = t^2$, or $t = \boxed{15.0 \text{ s}}$

(b) $v(\text{officer}) = (2.00 \text{ m/s}^2)t = \boxed{30.0 \text{ m/s}}$

(c) $x(\text{officer}) = \frac{1}{2}(2.00 \text{ m/s}^2)t^2 = \boxed{225 \text{ m}}$

P2.64 Area A_1 is a rectangle. Thus, $A_1 = hw = v_{xi}t$.

Area A_2 is triangular. Therefore $A_2 = \frac{1}{2}bh = \frac{1}{2}t(v_x - v_{xi})$.

The total area under the curve is

$$A = A_1 + A_2 = v_{xi}t + \frac{(v_x - v_{xi})t}{2}$$

and since $v_x - v_{xi} = a_x t$

$$\boxed{A = v_{xi}t + \frac{1}{2}a_x t^2}.$$

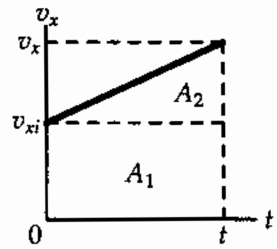


FIG. P2.64

The displacement given by the equation is: $x = v_{xi}t + \frac{1}{2}a_x t^2$, the same result as above for the total area.

P2.65 (a) Let x be the distance traveled at acceleration a until maximum speed v is reached. If this is achieved in time t_1 we can use the following three equations:

$$x=\frac{1}{2}(v+v_i)t_1,\ 100-x=v(10.2-t_1)\text{ and }v=v_i+at_1.$$

The first two give

$$100=\left(10.2-\frac{1}{2}t_1\right)v=\left(10.2-\frac{1}{2}t_1\right)at_1$$

$$a=\frac{200}{(20.4-t_1)t_1}.$$

$$\text{For Maggie: } a=\frac{200}{(18.4)(2.00)}=\boxed{5.43\ \text{m/s}^2}$$

$$\text{For Judy: } a=\frac{200}{(17.4)(3.00)}=\boxed{3.83\ \text{m/s}^2}$$

(b) $v=a_1t$

$$\text{Maggie: } v=(5.43)(2.00)=\boxed{10.9\ \text{m/s}}$$

$$\text{Judy: } v=(3.83)(3.00)=\boxed{11.5\ \text{m/s}}$$

(c) At the six-second mark

$$x=\frac{1}{2}at_1^2+v(6.00-t_1)$$

$$\text{Maggie: } x=\frac{1}{2}(5.43)(2.00)^2+(10.9)(4.00)=54.3\text{ m}$$

$$\text{Judy: } x=\frac{1}{2}(3.83)(3.00)^2+(11.5)(3.00)=51.7\text{ m}$$

Maggie is ahead by $\boxed{2.62\text{ m}}$.

P2.66 $a_1=0.100\ \text{m/s}^2$

$a_2=-0.500\ \text{m/s}^2$

$$x=1\,000\text{ m}=\frac{1}{2}a_1t_1^2+v_1t_2+\frac{1}{2}a_2t_2^2$$

$t=t_1+t_2$ and $v_1=a_1t_1=-a_2t_2$

$$1\,000=\frac{1}{2}a_1t_1^2+a_1t_1\left(-\frac{a_1t_1}{a_2}\right)+\frac{1}{2}a_2\left(\frac{a_1t_1}{a_2}\right)^2$$

$$1\,000=\frac{1}{2}a_1\left(1-\frac{a_1}{a_2}\right)t_1^2$$

$$t_1=\sqrt{\frac{20\,000}{1.20}}=\boxed{129\text{ s}}$$

$$t_2=\frac{a_1t_1}{-a_2}=\frac{12.9}{0.500}\approx 26\text{ s}$$

Total time $=t=\boxed{155\text{ s}}$

P2.67 Let the ball fall 1.50 m. It strikes at speed given by

$$v_{xf}^2 = v_{xi}^2 + 2a(x_f - x_i):$$

$$v_{xf}^2 = 0 + 2(-9.80\ \text{m/s}^2)(-1.50\ \text{m})$$

$$v_{xf} = -5.42\ \text{m/s}$$

and its stopping is described by

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$0 = (-5.42\ \text{m/s})^2 + 2a_x(-10^{-2}\ \text{m})$$

$$a_x = \frac{-29.4\ \text{m}^2/\text{s}^2}{-2.00 \times 10^{-2}\ \text{m}} = +1.47 \times 10^3\ \text{m/s}^2.$$

Its maximum acceleration will be larger than the average acceleration we estimate by imagining constant acceleration, but will still be of order of magnitude $\boxed{\sim 10^3\ \text{m/s}^2}$.

***P2.68** (a) $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$. We assume the package starts from rest.

$$-145\ \text{m} = 0 + 0 + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2$$

$$t = \sqrt{\frac{2(-145\ \text{m})}{-9.80\ \text{m/s}^2}} = \boxed{5.44\ \text{s}}$$

(b) $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 0 + 0 + \frac{1}{2}(-9.80\ \text{m/s}^2)(5.18\ \text{s})^2 = -131\ \text{m}$

$$\text{distance fallen} = |x_f| = \boxed{131\ \text{m}}$$

(c) speed $= |v_{xf}| = |v_{xi} + a_x t| = |0 + (-9.8\ \text{m/s}^2)5.18\ \text{s}| = \boxed{50.8\ \text{m/s}}$

(d) The remaining distance is

$$145\ \text{m} - 131.5\ \text{m} = 13.5\ \text{m}.$$

During deceleration,

$$v_{xi} = -50.8\ \text{m/s},\ v_{xf} = 0,\ x_f - x_i = -13.5\ \text{m}$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i):$$

$$0 = (-50.8\ \text{m/s})^2 + 2a_x(-13.5\ \text{m})$$

$$a_x = \frac{-2\,580\ \text{m}^2/\text{s}^2}{2(-13.5\ \text{m})} = +95.3\ \text{m/s}^2 = \boxed{95.3\ \text{m/s}^2\ \text{upward}}.$$

P2.69 (a) $y_f = v_{i1}t + \frac{1}{2}at^2 = 50.0 = 2.00t + \frac{1}{2}(9.80)t^2,$

$4.90t^2 + 2.00t - 50.0 = 0$

$$t = \frac{-2.00 + \sqrt{2.00^2 - 4(4.90)(-50.0)}}{2(4.90)}$$

Only the positive root is physically meaningful:

$$t = \boxed{3.00\text{ s}} \text{ after the first stone is thrown.}$$

(b) $y_f = v_{i2}t + \frac{1}{2}at^2$ and $t = 3.00 - 1.00 = 2.00$ s

substitute $50.0 = v_{i2}(2.00) + \frac{1}{2}(9.80)(2.00)^2$:

$$v_{i2} = \boxed{15.3\text{ m/s}} \text{ downward}$$

(c) $v_{1f} = v_{i1} + at = 2.00 + (9.80)(3.00) = \boxed{31.4\text{ m/s}}$ downward

$v_{2f} = v_{i2} + at = 15.3 + (9.80)(2.00) = \boxed{34.8\text{ m/s}}$ downward

P2.70 (a) $d = \frac{1}{2}(9.80)t_1^2$

$d = 336t_2$

$t_1 + t_2 = 2.40$

$336t_2 = 4.90(2.40 - t_2)^2$

$4.90t_2^2 - 359.5t_2 + 28.22 = 0$

$$t_2 = \frac{359.5 \pm \sqrt{359.5^2 - 4(4.90)(28.22)}}{9.80}$$

$t_2 = \frac{359.5 \pm 358.75}{9.80} = 0.0765\text{ s}$ so $d = 336t_2 = \boxed{26.4\text{ m}}$

(b) Ignoring the sound travel time, $d = \frac{1}{2}(9.80)(2.40)^2 = 28.2$ m, an error of $\boxed{6.82\%}$.

P2.71 (a) In walking a distance Δx, in a time Δt, the length of rope ℓ is only increased by $\Delta x \sin\theta$.

$\therefore$ The pack lifts at a rate $\frac{\Delta x}{\Delta t}\sin\theta$.

$$v = \frac{\Delta x}{\Delta t}\sin\theta = v_{\text{boy}}\frac{x}{\ell} = \boxed{v_{\text{boy}}\frac{x}{\sqrt{x^2 + h^2}}}$$

(b) $a = \frac{dv}{dt} = \frac{v_{\text{boy}}}{\ell}\frac{dx}{dt} + v_{\text{boy}}x\frac{d}{dt}\left(\frac{1}{\ell}\right)$

$a = v_{\text{boy}}\frac{v_{\text{boy}}}{\ell} - \frac{v_{\text{boy}}x}{\ell^2}\frac{d\ell}{dt}$, but $\frac{d\ell}{dt} = v = v_{\text{boy}}\frac{x}{\ell}$

$$\therefore a = \frac{v_{\text{boy}}^2}{\ell}\left(1 - \frac{x^2}{\ell^2}\right) = \frac{v_{\text{boy}}^2}{\ell}\frac{h^2}{\ell^2} = \boxed{\frac{h^2 v_{\text{boy}}^2}{\left(x^2 + h^2\right)^{3/2}}}$$

(c) $\frac{v_{\text{boy}}^2}{h}$, 0

(d) v_{boy}, 0

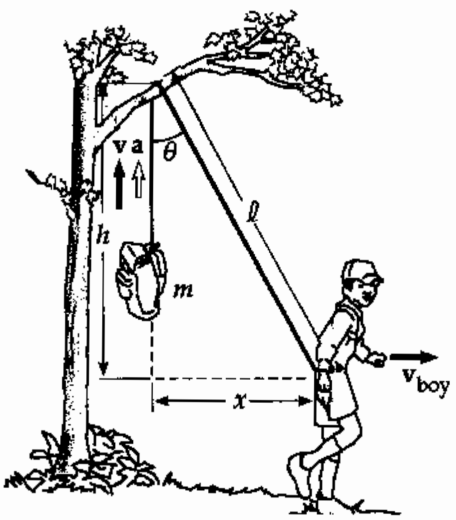


FIG. P2.71

P2.72 $h = 6.00$ m, $v_{boy} = 2.00$ m/s $v = \frac{\Delta x}{\Delta t}\sin\theta = v_{boy}\frac{x}{\ell} = \frac{v_{boy}x}{\left(x^2+h^2\right)^{1/2}}$.

However, $x = v_{boy}t$: $\therefore v = \frac{v_{boy}^2 t}{\left(v_{boy}^2 t^2 + h^2\right)^{1/2}} = \frac{4t}{\left(4t^2+36\right)^{1/2}}$.

(a)

t(s)	v(m/s)
0	0
0.5	0.32
1	0.63
1.5	0.89
2	1.11
2.5	1.28
3	1.41
3.5	1.52
4	1.60
4.5	1.66
5	1.71

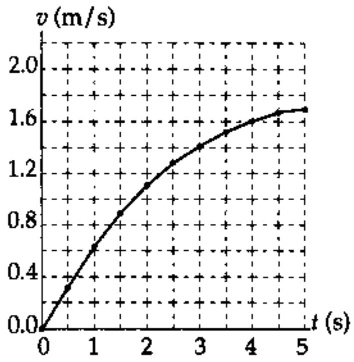


FIG. P2.72(a)

(b) From problem 2.71 above, $a = \frac{h^2 v_{boy}^2}{\left(x^2+h^2\right)^{3/2}} = \frac{h^2 v_{boy}^2}{\left(v_{boy}^2 t^2 + h^2\right)^{3/2}} = \frac{144}{\left(4t^2+36\right)^{3/2}}$.

t(s)	$a\left(\text{m/s}^2\right)$
0	0.67
0.5	0.64
1	0.57
1.5	0.48
2	0.38
2.5	0.30
3.	0.24
3.5	0.18
4.	0.14
4.5	0.11
5	0.09

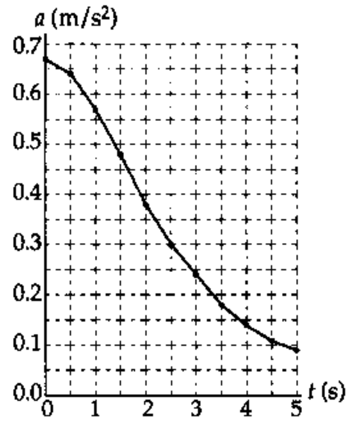


FIG. P2.72(b)

P2.73 (a) We require $x_s = x_k$ when $t_s = t_k + 1.00$

$$x_s = \frac{1}{2}\left(3.50\text{ m/s}^2\right)\left(t_k + 1.00\right)^2 = \frac{1}{2}\left(4.90\text{ m/s}^2\right)\left(t_k\right)^2 = x_k$$

$$t_k + 1.00 = 1.183 t_k$$

$$t_k = \boxed{5.46\text{ s}}.$$

(b) $x_k = \frac{1}{2}\left(4.90\text{ m/s}^2\right)(5.46\text{ s})^2 = \boxed{73.0\text{ m}}$

(c) $v_k = \left(4.90\text{ m/s}^2\right)(5.46\text{ s}) = \boxed{26.7\text{ m/s}}$

$v_s = \left(3.50\text{ m/s}^2\right)(6.46\text{ s}) = \boxed{22.6\text{ m/s}}$

P2.74

Time t (s)	Height h (m)	Δh (m)	Δt (s)	$\bar{v}$ (m/s)	midpt time t (s)
0.00	5.00				
		0.75	0.25	3.00	0.13
0.25	5.75				
		0.65	0.25	2.60	0.38
0.50	6.40				
		0.54	0.25	2.16	0.63
0.75	6.94				
		0.44	0.25	1.76	0.88
1.00	7.38				
		0.34	0.25	1.36	1.13
1.25	7.72				
		0.24	0.25	0.96	1.38
1.50	7.96				
		0.14	0.25	0.56	1.63
1.75	8.10				
		0.03	0.25	0.12	1.88
2.00	8.13				
		–0.06	0.25	–0.24	2.13
2.25	8.07				
		–0.17	0.25	–0.68	2.38
2.50	7.90				
		–0.28	0.25	–1.12	2.63
2.75	7.62				
		–0.37	0.25	–1.48	2.88
3.00	7.25				
		–0.48	0.25	–1.92	3.13
3.25	6.77				
		–0.57	0.25	–2.28	3.38
3.50	6.20				
		–0.68	0.25	–2.72	3.63
3.75	5.52				
		–0.79	0.25	–3.16	3.88
4.00	4.73				
		–0.88	0.25	–3.52	4.13
4.25	3.85				
		–0.99	0.25	–3.96	4.38
4.50	2.86				
		–1.09	0.25	–4.36	4.63
4.75	1.77				
		–1.19	0.25	–4.76	4.88
5.00	0.58				

TABLE P2.74

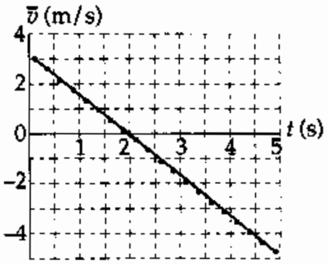


FIG. P2.74

acceleration = slope of line is constant.

$$\bar{a} = -1.63 \text{ m/s}^2 = \boxed{1.63 \text{ m/s}^2 \text{ downward}}$$

P2.75 The distance x and y are always related by $x^2+y^2=L^2$. Differentiating this equation with respect to time, we have

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

Now $\frac{dy}{dt}$ is v_B, the unknown velocity of B; and $\frac{dx}{dt}=-v$.

From the equation resulting from differentiation, we have

$$\frac{dy}{dt}=-\frac{x}{y}\left(\frac{dx}{dt}\right)=-\frac{x}{y}(-v).$$

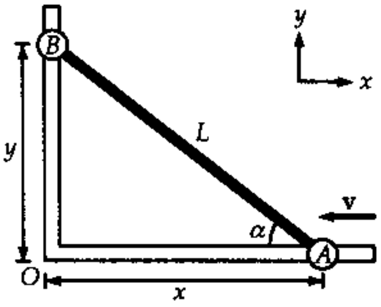


FIG. P2.75

But $\frac{y}{x}=\tan\alpha$ so $v_B=\left(\frac{1}{\tan\alpha}\right)v$. When $\alpha=60.0°$, $v_B=\frac{v}{\tan 60.0°}=\frac{v\sqrt{3}}{3}=\boxed{0.577v}$.

ANSWERS TO EVEN PROBLEMS

P2.2 (a) 2×10^{-7} m/s; 1×10^{-6} m/s; (b) 5×10^{8} yr

P2.4 (a) 50.0 m/s; (b) 41.0 m/s

P2.6 (a) 27.0 m; (b) $27.0\text{ m}+(18.0\text{ m/s})\Delta t+(3.00\text{ m/s}^2)(\Delta t)^2$; (c) 18.0 m/s

P2.8 (a), (b), (c) see the solution; 4.6 m/s^2; (d) 0

P2.10 5.00 m

P2.12 (a) 20.0 m/s; 5.00 m/s; (b) 262 m

P2.14 (a) see the solution; (b) 1.60 m/s^2; 0.800 m/s^2

P2.16 (a) 13.0 m/s; (b) 10.0 m/s; 16.0 m/s; (c) 6.00 m/s^2; (d) 6.00 m/s^2

P2.18 see the solution

P2.20 (a) 6.61 m/s; (b) −0.448 m/s^2

P2.22 (a) $-21.8\text{ mi/h}\cdot\text{s}=-9.75\text{ m/s}^2$; (b) $-22.2\text{ mi/h}\cdot\text{s}=-9.94\text{ m/s}^2$; (c) $-22.8\text{ mi/h}\cdot\text{s}=-10.2\text{ m/s}^2$

P2.24 (a) 1.88 km; (b) 1.46 km; (c) see the solution; (d) (i) $x_1=(1.67\text{ m/s}^2)t^2$; (ii) $x_2=(50\text{ m/s})t-375\text{ m}$; (iii) $x_3=(250\text{ m/s})t-(2.5\text{ m/s}^2)t^2-4\,375\text{ m}$; (e) 37.5 m/s

P2.26 958 m

P2.28 (a) $x_f=(30.0t-t^2)$ m; $v_f=(30.0-2t)$ m/s; (b) 225 m

P2.30 $x_f-x_i=v_{xf}t-\frac{1}{2}a_xt^2$; 3.10 m/s

P2.32 (a) 35.0 s; (b) 15.7 m/s

P2.34 (a) $1.12\times10^{11}\text{ m/s}^2$; (b) 4.67×10^{-5} s

P2.36 (a) False unless the acceleration is zero; see the solution; (b) True

P2.38 Yes; 212 m; 11.4 s

P2.40 (a) −4.90 m; −19.6 m; −44.1 m; (b) −9.80 m/s; −19.6 m/s; −29.4 m/s

P2.42 1.79 s

P2.44 No; see the solution

P2.46 The second ball is thrown at speed $v_i = \sqrt{gh}$

P2.48 (a) 510 m; (b) 20.4 s

P2.50 (a) $96.0\ \text{ft/s}$;
(b) $a = 3.07 \times 10^3\ \text{ft/s}^2$ upward;
(c) $\Delta t = 3.13 \times 10^{-2}\ \text{s}$

P2.52 38.2 m

P2.54 (a) and (b) see the solution; (c) $-4\ \text{m/s}^2$;
(d) 34 m; (e) 28 m

P2.56 0.222 s

P2.58 (a) see the solution; (b) 6.23 s

P2.60 $1.60\ \text{m/s}^2$

P2.62 (a) 41.0 s; (b) 1.73 km; (c) $-184\ \text{m/s}$

P2.64 $v_{xi}t + \frac{1}{2}a_x t^2$; displacements agree

P2.66 155 s; 129 s

P2.68 (a) 5.44 s; (b) 131 m; (c) $50.8\ \text{m/s}$;
(d) $95.3\ \text{m/s}^2$ upward

P2.70 (a) 26.4 m; (b) 6.82%

P2.72 see the solution

P2.74 see the solution; $a_x = -1.63\ \text{m/s}^2$

3

Vectors

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q3.1 No. The sum of two vectors can only be zero if they are in opposite directions and have the same magnitude. If you walk 10 meters north and then 6 meters south, you won't end up where you started.

Q3.2 No, the magnitude of the displacement is always less than or equal to the distance traveled. If two displacements in the same direction are added, then the magnitude of their sum will be equal to the distance traveled. Two vectors in any other orientation will give a displacement less than the distance traveled. If you first walk 3 meters east, and then 4 meters south, you will have walked a total distance of 7 meters, but you will only be 5 meters from your starting point.

Q3.3 The largest possible magnitude of $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is 7 units, found when **A** and **B** point in the same direction. The smallest magnitude of $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is 3 units, found when **A** and **B** have opposite directions.

Q3.4 Only force and velocity are vectors. None of the other quantities requires a direction to be described.

Q3.5 If the direction-angle of **A** is between 180 degrees and 270 degrees, its components are both negative. If a vector is in the second quadrant or the fourth quadrant, its components have opposite signs.

Q3.6 The book's displacement is zero, as it ends up at the point from which it started. The distance traveled is 6.0 meters.

Q3.7 85 miles. The magnitude of the displacement is the distance from the starting point, the 260-mile mark, to the ending point, the 175-mile mark.

Q3.8 Vectors **A** and **B** are perpendicular to each other.

Q3.9 No, the magnitude of a vector is always positive. A minus sign in a vector only indicates direction, not magnitude.

Q3.10 Any vector that points along a line at 45° to the x and y axes has components equal in magnitude.

Q3.11 $A_x = B_x$ and $A_y = B_y$.

Q3.12 Addition of a vector to a scalar is not defined. Think of apples and oranges.

Q3.13 One difficulty arises in determining the individual components. The relationships between a vector and its components such as $A_x = A\cos\theta$, are based on right-triangle trigonometry. Another problem would be in determining the magnitude or the direction of a vector from its components. Again, $A = \sqrt{A_x^2 + A_y^2}$ only holds true if the two component vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, are perpendicular.

Q3.14 If the direction of a vector is specified by giving the angle of the vector measured clockwise from the positive y-axis, then the x-component of the vector is equal to the sine of the angle multiplied by the magnitude of the vector.

SOLUTIONS TO PROBLEMS

Section 3.1 Coordinate Systems

P3.1 $x = r\cos\theta = (5.50\text{ m})\cos 240° = (5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}}$

$y = r\sin\theta = (5.50\text{ m})\sin 240° = (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}}$

P3.2 (a) $x = r\cos\theta$ and $y = r\sin\theta$, therefore

$x_1 = (2.50\text{ m})\cos 30.0°$, $y_1 = (2.50\text{ m})\sin 30.0°$, and

$(x_1, y_1) = \boxed{(2.17,\ 1.25)\text{ m}}$

$x_2 = (3.80\text{ m})\cos 120°$, $y_2 = (3.80\text{ m})\sin 120°$, and

$(x_2, y_2) = \boxed{(-1.90,\ 3.29)\text{ m}}$.

(b) $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{16.6 + 4.16} = \boxed{4.55\text{ m}}$

P3.3 The x distance out to the fly is 2.00 m and the y distance up to the fly is 1.00 m.

(a) We can use the Pythagorean theorem to find the distance from the origin to the fly.

$$\text{distance} = \sqrt{x^2 + y^2} = \sqrt{(2.00\text{ m})^2 + (1.00\text{ m})^2} = \sqrt{5.00\text{ m}^2} = \boxed{2.24\text{ m}}$$

(b) $\theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6°$; $\mathbf{r} = \boxed{2.24\text{ m},\ 26.6°}$

P3.4 (a) $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2.00 - [-3.00])^2 + (-4.00 - 3.00)^2}$

$d = \sqrt{25.0 + 49.0} = \boxed{8.60 \text{ m}}$

(b) $r_1 = \sqrt{(2.00)^2 + (-4.00)^2} = \sqrt{20.0} = \boxed{4.47 \text{ m}}$

$\theta_1 = \tan^{-1}\left(-\frac{4.00}{2.00}\right) = \boxed{-63.4°}$

$r_2 = \sqrt{(-3.00)^2 + (3.00)^2} = \sqrt{18.0} = \boxed{4.24 \text{ m}}$

$\theta_2 = \boxed{135°}$ measured from the $+x$ axis.

P3.5 We have $2.00 = r\cos 30.0°$

$$r = \frac{2.00}{\cos 30.0°} = \boxed{2.31}$$

and $y = r\sin 30.0° = 2.31\sin 30.0° = \boxed{1.15}$.

P3.6 We have $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

(a) The radius for this new point is

$$\sqrt{(-x)^2 + y^2} = \sqrt{x^2 + y^2} = \boxed{r}$$

and its angle is

$$\tan^{-1}\left(\frac{y}{-x}\right) = \boxed{180° - \theta}.$$

(b) $\sqrt{(-2x)^2 + (-2y)^2} = \boxed{2r}$. This point is in the third quadrant if (x, y) is in the first quadrant or in the fourth quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{180° + \theta}$.

(c) $\sqrt{(3x)^2 + (-3y)^2} = \boxed{3r}$. This point is in the fourth quadrant if (x, y) is in the first quadrant or in the third quadrant if (x, y) is in the second quadrant. It is at an angle of $\boxed{-\theta}$.

Section 3.2 Vector and Scalar Quantities

Section 3.3 Some Properties of Vectors

P3.7 $\tan 35.0° = \dfrac{x}{100 \text{ m}}$

$$x = (100 \text{ m})\tan 35.0° = \boxed{70.0 \text{ m}}$$

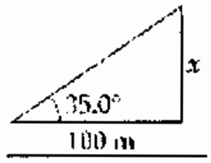


FIG. P3.7

P3.8 $R = \boxed{14 \text{ km}}$

$\theta = \boxed{65° \text{ N of E}}$

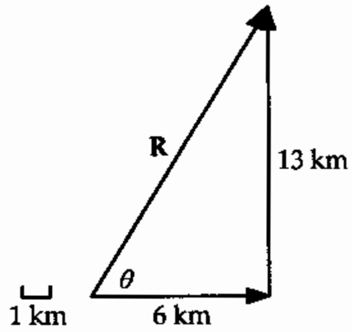


FIG. P3.8

P3.9 $-\mathbf{R} = \boxed{310 \text{ km at } 57° \text{ S of W}}$

(Scale: 1 unit = 20 km)

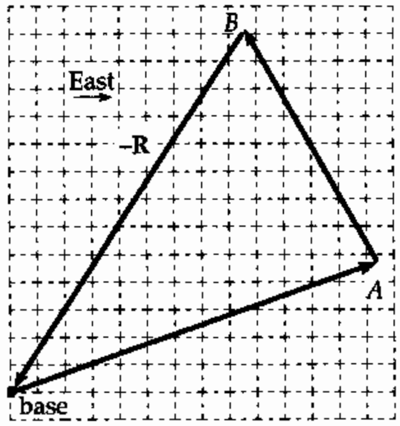


FIG. P3.9

P3.10 (a) Using graphical methods, place the tail of vector **B** at the head of vector **A**. The new vector **A** + **B** has a magnitude of $\boxed{6.1 \text{ at } 112°}$ from the x-axis.

(b) The vector difference **A** – **B** is found by placing the negative of vector **B** at the head of vector **A**. The resultant vector **A** – **B** has magnitude $\boxed{14.8}$ units at an $\boxed{\text{angle of } 22°}$ from the + x-axis.

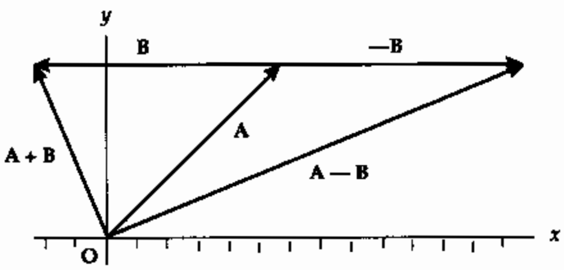


FIG. P3.10

P3.11 (a) $|\mathbf{d}| = |-10.0\hat{\mathbf{i}}| = \boxed{10.0 \text{ m}}$ since the displacement is in a straight line from point A to point B.

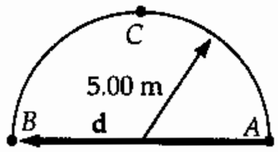


FIG. P3.11

(b) The actual distance skated is not equal to the straight-line displacement. The distance follows the curved path of the semi-circle (*ACB*).

$$s = \frac{1}{2}(2\pi r) = 5\pi = \boxed{15.7 \text{ m}}$$

(c) If the circle is complete, **d** begins and ends at point A. Hence, $|\mathbf{d}| = \boxed{0}$.

P3.12 Find the resultant $\mathbf{F}_1 + \mathbf{F}_2$ graphically by placing the tail of $\mathbf{F}_2$ at the head of $\mathbf{F}_1$. The resultant force vector $\mathbf{F}_1 + \mathbf{F}_2$ is of magnitude $\boxed{9.5 \text{ N}}$ and at an angle of $\boxed{57° \text{ above the } x\text{-axis}}$.

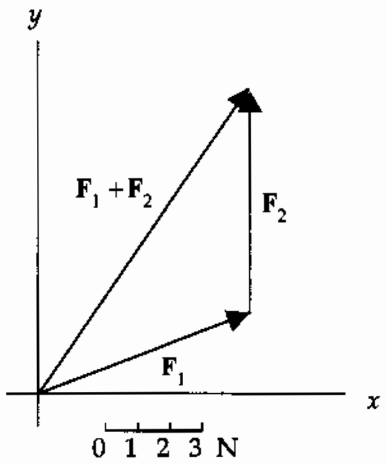


FIG. P3.12

P3.13 (a) The large majority of people are standing or sitting at this hour. Their instantaneous foot-to-head vectors have upward vertical components on the order of 1 m and randomly oriented horizontal components. The citywide sum will be $\boxed{\sim 10^5 \text{ m upward}}$.

(b) Most people are lying in bed early Saturday morning. We suppose their beds are oriented north, south, east, west quite at random. Then the horizontal component of their total vector height is very nearly zero. If their compressed pillows give their height vectors vertical components averaging 3 cm, and if one-tenth of one percent of the population are on-duty nurses or police officers, we estimate the total vector height as $\sim 10^5(0.03 \text{ m}) + 10^2(1 \text{ m})$

$\boxed{\sim 10^3 \text{ m upward}}$.

P3.14 Your sketch should be drawn to scale, and should look somewhat like that pictured to the right. The angle from the westward direction, θ, can be measured to be $\boxed{4° \text{ N of W}}$, and the distance R from the sketch can be converted according to the scale to be $\boxed{7.9 \text{ m}}$.

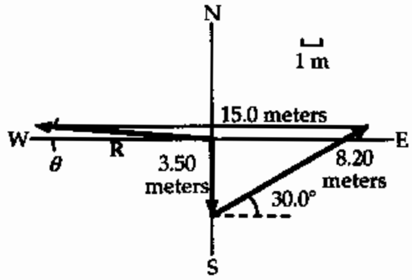


FIG. P3.14

P3.15 To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor. (Scale: 1 unit = 0.5 m)

(a) **A** + **B** = 5.2 m at 60°

(b) **A** − **B** = 3.0 m at 330°

(c) **B** − **A** = 3.0 m at 150°

(d) **A** − 2**B** = 5.2 m at 300°.

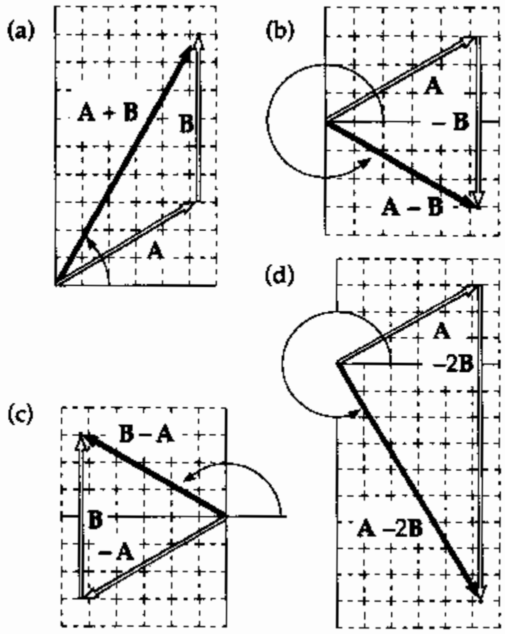


FIG. P3.15

***P3.16** The three diagrams shown below represent the graphical solutions for the three vector sums: $\mathbf{R}_1 = \mathbf{A}+\mathbf{B}+\mathbf{C}$, $\mathbf{R}_2 = \mathbf{B}+\mathbf{C}+\mathbf{A}$, and $\mathbf{R}_3 = \mathbf{C}+\mathbf{B}+\mathbf{A}$. You should observe that $\mathbf{R}_1 = \mathbf{R}_2 = \mathbf{R}_3$, illustrating that the sum of a set of vectors is not affected by the order in which the vectors are added.

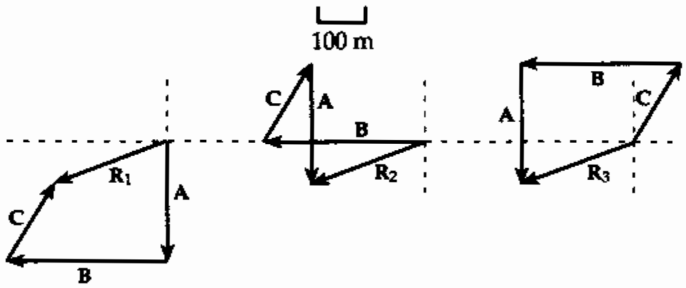
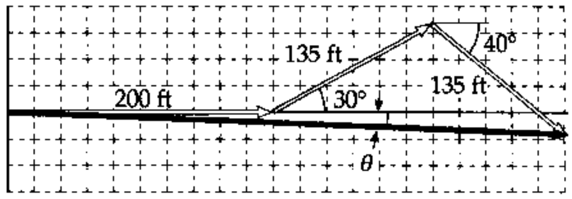


FIG. P3.16

P3.17 The scale drawing for the graphical solution should be similar to the figure to the right. The magnitude and direction of the final displacement from the starting point are obtained by measuring d and θ on the drawing and applying the scale factor used in making the drawing. The results should be

$$\boxed{d = 420 \text{ ft and } \theta = -3°}$$



(Scale: 1 unit = 20 ft)

FIG. P3.17

Section 3.4 Components of a Vector and Unit Vectors

P3.18 Coordinates of the super-hero are:

$$x = (100 \text{ m})\cos(-30.0°) = \boxed{86.6 \text{ m}}$$
$$y = (100 \text{ m})\sin(-30.0°) = \boxed{-50.0 \text{ m}}$$

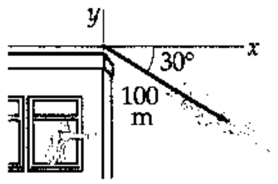


FIG. P3.18

P3.19 $A_x = -25.0$

$A_y = 40.0$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2} = \boxed{47.2 \text{ units}}$$

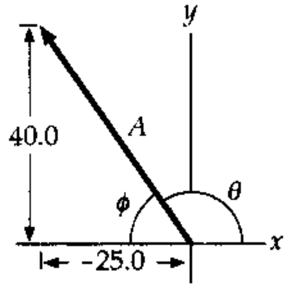


FIG. P3.19

We observe that

$$\tan\phi = \frac{|A_y|}{|A_x|}.$$

So

$$\phi = \tan^{-1}\left(\frac{A_y}{|A_x|}\right) = \tan\frac{40.0}{25.0} = \tan^{-1}(1.60) = 58.0°.$$

The diagram shows that the angle from the $+x$ axis can be found by subtracting from 180°:

$$\theta = 180° - 58° = \boxed{122°}.$$

P3.20 The person would have to walk $3.10\sin(25.0°) = \boxed{1.31 \text{ km north}}$, and

$$3.10\cos(25.0°) = \boxed{2.81 \text{ km east}}.$$

P3.21 $x = r\cos\theta$ and $y = r\sin\theta$, therefore:

(a) $x = 12.8\cos 150°$, $y = 12.8\sin 150°$, and $(x,\ y) = \left(-11.1\hat{\mathbf{i}} + 6.40\hat{\mathbf{j}}\right)$ m

(b) $x = 3.30\cos 60.0°$, $y = 3.30\sin 60.0°$, and $(x,\ y) = \left(1.65\hat{\mathbf{i}} + 2.86\hat{\mathbf{j}}\right)$ cm

(c) $x = 22.0\cos 215°$, $y = 22.0\sin 215°$, and $(x,\ y) = \left(-18.0\hat{\mathbf{i}} - 12.6\hat{\mathbf{j}}\right)$ in

P3.22
$$x = d\cos\theta = (50.0\text{ m})\cos(120) = -25.0\text{ m}$$
$$y = d\sin\theta = (50.0\text{ m})\sin(120) = 43.3\text{ m}$$
$$\mathbf{d} = \boxed{(-25.0\text{ m})\hat{\mathbf{i}} + (43.3\text{ m})\hat{\mathbf{j}}}$$

***P3.23** (a) Her net x (east-west) displacement is $-3.00 + 0 + 6.00 = +3.00$ blocks, while her net y (north-south) displacement is $0 + 4.00 + 0 = +4.00$ blocks. The magnitude of the resultant displacement is

$$R = \sqrt{(x_{\text{net}})^2 + (y_{\text{net}})^2} = \sqrt{(3.00)^2 + (4.00)^2} = 5.00\text{ blocks}$$

and the angle the resultant makes with the x-axis (eastward direction) is

$$\theta = \tan^{-1}\left(\frac{4.00}{3.00}\right) = \tan^{-1}(1.33) = 53.1°.$$

The resultant displacement is then $\boxed{5.00\text{ blocks at }53.1°\text{ N of E}}$.

(b) The total distance traveled is $3.00 + 4.00 + 6.00 = \boxed{13.0\text{ blocks}}$.

***P3.24** Let $\hat{\mathbf{i}} = \text{east}$ and $\hat{\mathbf{j}} = \text{north}$. The unicyclist's displacement is, in meters

$$280\hat{\mathbf{j}} + 220\hat{\mathbf{i}} + 360\hat{\mathbf{j}} - 300\hat{\mathbf{i}} - 120\hat{\mathbf{j}} + 60\hat{\mathbf{i}} - 40\hat{\mathbf{j}} - 90\hat{\mathbf{i}} + 70\hat{\mathbf{j}}.$$

$$\mathbf{R} = -110\hat{\mathbf{i}} + 550\hat{\mathbf{j}}$$
$$= \sqrt{(110\text{ m})^2 + (550\text{ m})^2}\ \text{at}\ \tan^{-1}\frac{110\text{ m}}{550\text{ m}}\ \text{west of north}$$
$$= 561\text{ m at }11.3°\text{ west of north}.$$

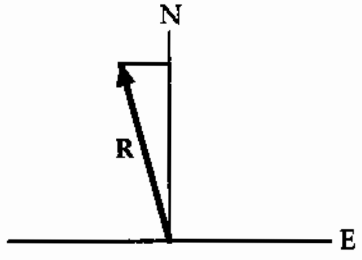


FIG. P3.24

The crow's velocity is

$$\mathbf{v} = \frac{\Delta\mathbf{x}}{\Delta t} = \frac{561\text{ m at }11.3°\text{ W of N}}{40\text{ s}}$$
$$= \boxed{14.0\text{ m/s at }11.3°\text{ west of north}}.$$

P3.25 $+x$ East, $+y$ North

$$\sum x = 250 + 125\cos 30° = 358 \text{ m}$$
$$\sum y = 75 + 125\sin 30° - 150 = -12.5 \text{ m}$$
$$d = \sqrt{\left(\sum x\right)^2 + \left(\sum y\right)^2} = \sqrt{(358)^2 + (-12.5)^2} = 358 \text{ m}$$
$$\tan\theta = \frac{\left(\sum y\right)}{\left(\sum x\right)} = -\frac{12.5}{358} = -0.0349$$
$$\theta = -2.00°$$
$$\boxed{\mathbf{d} = 358 \text{ m at } 2.00° \text{ S of E}}$$

P3.26 The east and north components of the displacement from Dallas (D) to Chicago (C) are the sums of the east and north components of the displacements from Dallas to Atlanta (A) and from Atlanta to Chicago. In equation form:

$$d_{\text{DC east}} = d_{\text{DA east}} + d_{\text{AC east}} = 730\cos 5.00° - 560\sin 21.0° = 527 \text{ miles}.$$
$$d_{\text{DC north}} = d_{\text{DA north}} + d_{\text{AC north}} = 730\sin 5.00° + 560\cos 21.0° = 586 \text{ miles}.$$

By the Pythagorean theorem, $d = \sqrt{(d_{\text{DC east}})^2 + (d_{\text{DC north}})^2} = 788 \text{ mi}$.

Then $\tan\theta = \frac{d_{\text{DC north}}}{d_{\text{DC east}}} = 1.11$ and $\theta = 48.0°$.

Thus, Chicago is $\boxed{\text{788 miles at 48.0° northeast of Dallas}}$.

P3.27 (a) See figure to the right.

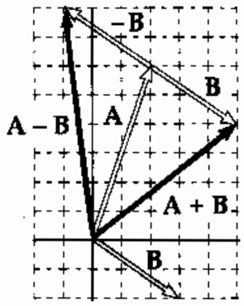


FIG. P3.27

(b) $\mathbf{C} = \mathbf{A} + \mathbf{B} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} = \boxed{5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}}$

$\mathbf{C} = \sqrt{25.0 + 16.0}$ at $\tan^{-1}\left(\frac{4}{5}\right) = \boxed{6.40 \text{ at } 38.7°}$

$\mathbf{D} = \mathbf{A} - \mathbf{B} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} = \boxed{-1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}}$

$\mathbf{D} = \sqrt{(-1.00)^2 + (8.00)^2}$ at $\tan^{-1}\left(\frac{8.00}{-1.00}\right)$

$\mathbf{D} = 8.06$ at $(180° - 82.9°) = \boxed{8.06 \text{ at } 97.2°}$

P3.28 $d = \sqrt{(x_1 + x_2 + x_3)^2 + (y_1 + y_2 + y_3)^2}$

$= \sqrt{(3.00 - 5.00 + 6.00)^2 + (2.00 + 3.00 + 1.00)^2} = \sqrt{52.0} = \boxed{7.21 \text{ m}}$

$\theta = \tan^{-1}\left(\frac{6.00}{4.00}\right) = \boxed{56.3°}$

P3.29 We have $\mathbf{B} = \mathbf{R} - \mathbf{A}$:

$$A_x = 150\cos 120° = -75.0 \text{ cm}$$
$$A_y = 150\sin 120° = 130 \text{ cm}$$
$$R_x = 140\cos 35.0° = 115 \text{ cm}$$
$$R_y = 140\sin 35.0° = 80.3 \text{ cm}$$

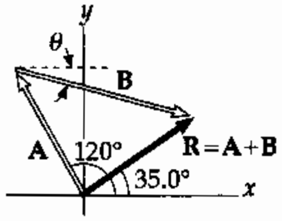


FIG. P3.29

Therefore,

$$\mathbf{B} = [115 - (-75)]\hat{\mathbf{i}} + [80.3 - 130]\hat{\mathbf{j}} = (190\hat{\mathbf{i}} - 49.7\hat{\mathbf{j}}) \text{ cm}$$
$$|\mathbf{B}| = \sqrt{190^2 + 49.7^2} = \boxed{196 \text{ cm}}$$
$$\theta = \tan^{-1}\left(-\frac{49.7}{190}\right) = \boxed{-14.7°}.$$

P3.30 $\mathbf{A} = -8.70\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}$ and $\mathbf{B} = 13.2\hat{\mathbf{i}} - 6.60\hat{\mathbf{j}}$

$\mathbf{A} - \mathbf{B} + 3\mathbf{C} = 0$:

$$3\mathbf{C} = \mathbf{B} - \mathbf{A} = 21.9\hat{\mathbf{i}} - 21.6\hat{\mathbf{j}}$$
$$\mathbf{C} = 7.30\hat{\mathbf{i}} - 7.20\hat{\mathbf{j}}$$

or

$$C_x = \boxed{7.30 \text{ cm}}; \; C_y = \boxed{-7.20 \text{ cm}}$$

P3.31 (a) $(\mathbf{A} + \mathbf{B}) = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + (-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = \boxed{2\hat{\mathbf{i}} - 6\hat{\mathbf{j}}}$

(b) $(\mathbf{A} - \mathbf{B}) = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) - (-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}) = \boxed{4\hat{\mathbf{i}} + 2\hat{\mathbf{j}}}$

(c) $|\mathbf{A} + \mathbf{B}| = \sqrt{2^2 + 6^2} = \boxed{6.32}$

(d) $|\mathbf{A} - \mathbf{B}| = \sqrt{4^2 + 2^2} = \boxed{4.47}$

(e) $\theta_{|\mathbf{A}+\mathbf{B}|} = \tan^{-1}\left(-\frac{6}{2}\right) = -71.6° = \boxed{288°}$

$\theta_{|\mathbf{A}-\mathbf{B}|} = \tan^{-1}\left(\frac{2}{4}\right) = \boxed{26.6°}$

P3.32 (a) $\mathbf{D} = \mathbf{A} + \mathbf{B} + \mathbf{C} = 2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$

$|\mathbf{D}| = \sqrt{2^2 + 4^2} = \boxed{4.47 \text{ m at } \theta = 63.4°}$

(b) $\mathbf{E} = -\mathbf{A} - \mathbf{B} + \mathbf{C} = -6\hat{\mathbf{i}} + 6\hat{\mathbf{j}}$

$|\mathbf{E}| = \sqrt{6^2 + 6^2} = \boxed{8.49 \text{ m at } \theta = 135°}$

P3.33 $d_1 = \left(-3.50\hat{\mathbf{j}}\right)$ m

$d_2 = 8.20\cos 45.0°\,\hat{\mathbf{i}} + 8.20\sin 45.0°\,\hat{\mathbf{j}} = \left(5.80\hat{\mathbf{i}} + 5.80\hat{\mathbf{j}}\right)$ m

$d_3 = \left(-15.0\hat{\mathbf{i}}\right)$ m

$\mathbf{R} = d_1 + d_2 + d_3 = (-15.0 + 5.80)\hat{\mathbf{i}} + (5.80 - 3.50)\hat{\mathbf{j}} = \boxed{\left(-9.20\hat{\mathbf{i}} + 2.30\hat{\mathbf{j}}\right)\text{ m}}$

(or 9.20 m west and 2.30 m north)

The magnitude of the resultant displacement is

$$|\mathbf{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(-9.20)^2 + (2.30)^2} = \boxed{9.48\text{ m}}.$$

The direction is $\theta = \arctan\left(\dfrac{2.30}{-9.20}\right) = \boxed{166°}$.

P3.34 Refer to the sketch

$$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} = -10.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}} + 50.0\hat{\mathbf{i}}$$
$$= 40.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}$$
$$|\mathbf{R}| = \left[(40.0)^2 + (-15.0)^2\right]^{1/2} = \boxed{42.7\text{ yards}}$$

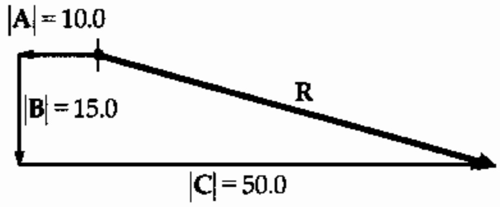


FIG. P3.34

P3.35 (a) $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$

$\mathbf{F} = 120\cos(60.0°)\hat{\mathbf{i}} + 120\sin(60.0°)\hat{\mathbf{j}} - 80.0\cos(75.0°)\hat{\mathbf{i}} + 80.0\sin(75.0°)\hat{\mathbf{j}}$

$\mathbf{F} = 60.0\hat{\mathbf{i}} + 104\hat{\mathbf{j}} - 20.7\hat{\mathbf{i}} + 77.3\hat{\mathbf{j}} = \left(39.3\hat{\mathbf{i}} + 181\hat{\mathbf{j}}\right)$ N

$|\mathbf{F}| = \sqrt{39.3^2 + 181^2} = \boxed{185\text{ N}}$

$\theta = \tan^{-1}\left(\dfrac{181}{39.3}\right) = \boxed{77.8°}$

(b) $\mathbf{F}_3 = -\mathbf{F} = \boxed{\left(-39.3\hat{\mathbf{i}} - 181\hat{\mathbf{j}}\right)\text{ N}}$

P3.36

East x	West y
0 m	4.00 m
1.41	1.41
−0.500	−0.866
+0.914	4.55

$|\mathbf{R}| = \sqrt{|x|^2 + |y|^2} = \boxed{4.64\text{ m at }78.6°\text{ N of E}}$

P3.37 $\mathbf{A} = 3.00 \text{ m}, \theta_A = 30.0°$ $\mathbf{B} = 3.00 \text{ m}, \theta_B = 90.0°$

$A_x = A\cos\theta_A = 3.00\cos 30.0° = 2.60 \text{ m}$ $A_y = A\sin\theta_A = 3.00\sin 30.0° = 1.50 \text{ m}$

$\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = (2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}) \text{ m}$

$B_x = 0, B_y = 3.00 \text{ m}$ so $\mathbf{B} = 3.00\hat{\mathbf{j}} \text{ m}$

$\mathbf{A} + \mathbf{B} = (2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}) + 3.00\hat{\mathbf{j}} = \boxed{(2.60\hat{\mathbf{i}} + 4.50\hat{\mathbf{j}}) \text{ m}}$

P3.38 Let the positive x-direction be eastward, the positive y-direction be vertically upward, and the positive z-direction be southward. The total displacement is then

$$\mathbf{d} = (4.80\hat{\mathbf{i}} + 4.80\hat{\mathbf{j}}) \text{ cm} + (3.70\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}) \text{ cm} = (4.80\hat{\mathbf{i}} + 8.50\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}) \text{ cm}.$$

(a) The magnitude is $d = \sqrt{(4.80)^2 + (8.50)^2 + (-3.70)^2} \text{ cm} = \boxed{10.4 \text{ cm}}$.

(b) Its angle with the y-axis follows from $\cos\theta = \frac{8.50}{10.4}$, giving $\boxed{\theta = 35.5°}$.

P3.39 $\mathbf{B} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} = 4.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{k}}$

$|\mathbf{B}| = \sqrt{4.00^2 + 6.00^2 + 3.00^2} = \boxed{7.81}$

$\alpha = \cos^{-1}\left(\frac{4.00}{7.81}\right) = \boxed{59.2°}$

$\beta = \cos^{-1}\left(\frac{6.00}{7.81}\right) = \boxed{39.8°}$

$\gamma = \cos^{-1}\left(\frac{3.00}{7.81}\right) = \boxed{67.4°}$

P3.40 The y coordinate of the airplane is constant and equal to 7.60×10^3 m whereas the x coordinate is given by $x = v_i t$ where v_i is the constant speed in the horizontal direction.

At $t = 30.0$ s we have $x = 8.04 \times 10^3$, so $v_i = 268$ m/s. The position vector as a function of time is

$$\mathbf{P} = (268 \text{ m/s})t\hat{\mathbf{i}} + (7.60 \times 10^3 \text{ m})\hat{\mathbf{j}}.$$

At $t = 45.0$ s, $\mathbf{P} = \left[1.21 \times 10^4\hat{\mathbf{i}} + 7.60 \times 10^3\hat{\mathbf{j}}\right]$ m. The magnitude is

$$P = \sqrt{(1.21 \times 10^4)^2 + (7.60 \times 10^3)^2} \text{ m} = \boxed{1.43 \times 10^4 \text{ m}}$$

and the direction is

$$\theta = \arctan\left(\frac{7.60 \times 10^3}{1.21 \times 10^4}\right) = \boxed{32.2° \text{ above the horizontal}}.$$

P3.41 (a) $\mathbf{A} = \boxed{8.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}}$

(b) $\mathbf{B} = \dfrac{\mathbf{A}}{4} = \boxed{2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{k}}}$

(c) $\mathbf{C} = -3\mathbf{A} = \boxed{-24.0\hat{\mathbf{i}} - 36.0\hat{\mathbf{j}} + 12.0\hat{\mathbf{k}}}$

P3.42 $\mathbf{R} = 75.0\cos 240°\,\hat{\mathbf{i}} + 75.0\sin 240°\,\hat{\mathbf{j}} + 125\cos 135°\,\hat{\mathbf{i}} + 125\sin 135°\,\hat{\mathbf{j}} + 100\cos 160°\,\hat{\mathbf{i}} + 100\sin 160°\,\hat{\mathbf{j}}$

$\mathbf{R} = -37.5\hat{\mathbf{i}} - 65.0\hat{\mathbf{j}} - 88.4\hat{\mathbf{i}} + 88.4\hat{\mathbf{j}} - 94.0\hat{\mathbf{i}} + 34.2\hat{\mathbf{j}}$

$\mathbf{R} = \boxed{-220\hat{\mathbf{i}} + 57.6\hat{\mathbf{j}}}$

$R = \sqrt{(-220)^2 + 57.6^2}$ at $\arctan\left(\dfrac{57.6}{220}\right)$ above the $-x$-axis

$\mathbf{R} = \boxed{227 \text{ paces at } 165°}$

P3.43 (a) $\mathbf{C} = \mathbf{A} + \mathbf{B} = \boxed{\left(5.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right) \text{m}}$

$|\mathbf{C}| = \sqrt{(5.00)^2 + (1.00)^2 + (3.00)^2} \text{ m} = \boxed{5.92 \text{ m}}$

(b) $\mathbf{D} = 2\mathbf{A} - \mathbf{B} = \boxed{\left(4.00\hat{\mathbf{i}} - 11.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{k}}\right) \text{m}}$

$|\mathbf{D}| = \sqrt{(4.00)^2 + (11.0)^2 + (15.0)^2} \text{ m} = \boxed{19.0 \text{ m}}$

P3.44 The position vector from radar station to ship is

$$\mathbf{S} = \left(17.3\sin 136°\,\hat{\mathbf{i}} + 17.3\cos 136°\,\hat{\mathbf{j}}\right) \text{km} = \left(12.0\hat{\mathbf{i}} - 12.4\hat{\mathbf{j}}\right) \text{km}.$$

From station to plane, the position vector is

$$\mathbf{P} = \left(19.6\sin 153°\,\hat{\mathbf{i}} + 19.6\cos 153°\,\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right) \text{km},$$

or

$$\mathbf{P} = \left(8.90\hat{\mathbf{i}} - 17.5\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right) \text{km}.$$

(a) To fly to the ship, the plane must undergo displacement

$$\mathbf{D} = \mathbf{S} - \mathbf{P} = \boxed{\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right) \text{km}}.$$

(b) The distance the plane must travel is

$$D = |\mathbf{D}| = \sqrt{(3.12)^2 + (5.02)^2 + (2.20)^2} \text{ km} = \boxed{6.31 \text{ km}}.$$

P3.45 The hurricane's first displacement is $\left(\frac{41.0 \text{ km}}{\text{h}}\right)(3.00 \text{ h})$ at 60.0° N of W, and its second displacement is $\left(\frac{25.0 \text{ km}}{\text{h}}\right)(1.50 \text{ h})$ due North. With $\hat{\mathbf{i}}$ representing east and $\hat{\mathbf{j}}$ representing north, its total displacement is:

$$\left(41.0\frac{\text{km}}{\text{h}}\cos 60.0°\right)(3.00 \text{ h})\left(-\hat{\mathbf{i}}\right)+\left(41.0\frac{\text{km}}{\text{h}}\sin 60.0°\right)(3.00 \text{ h})\hat{\mathbf{j}}+\left(25.0\frac{\text{km}}{\text{h}}\right)(1.50 \text{ h})\hat{\mathbf{j}} = 61.5 \text{ km}\left(-\hat{\mathbf{i}}\right) + 144 \text{ km}\,\hat{\mathbf{j}}$$

with magnitude $\sqrt{(61.5 \text{ km})^2 + (144 \text{ km})^2} = \boxed{157 \text{ km}}$.

P3.46 (a) $\mathbf{E} = (17.0 \text{ cm})\cos 27.0°\,\hat{\mathbf{i}} + (17.0 \text{ cm})\sin 27.0°\,\hat{\mathbf{j}}$

$\mathbf{E} = \boxed{\left(15.1\hat{\mathbf{i}} + 7.72\hat{\mathbf{j}}\right) \text{cm}}$

(b) $\mathbf{F} = -(17.0 \text{ cm})\sin 27.0°\,\hat{\mathbf{i}} + (17.0 \text{ cm})\cos 27.0°\,\hat{\mathbf{j}}$

$\mathbf{F} = \boxed{\left(-7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right) \text{cm}}$

(c) $\mathbf{G} = +(17.0 \text{ cm})\sin 27.0°\,\hat{\mathbf{i}} + (17.0 \text{ cm})\cos 27.0°\,\hat{\mathbf{j}}$

$\mathbf{G} = \boxed{\left(+7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right) \text{cm}}$

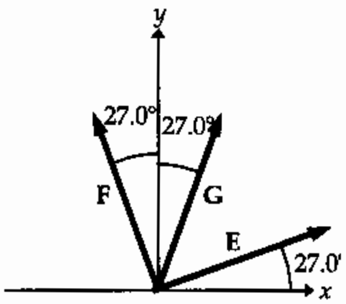


FIG. P3.46

P3.47 $A_x = -3.00$, $A_y = 2.00$

(a) $\mathbf{A} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = \boxed{-3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}}}$

(b) $|\mathbf{A}| = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3.00)^2 + (2.00)^2} = \boxed{3.61}$

$$\tan\theta = \frac{A_y}{A_x} = \frac{2.00}{(-3.00)} = -0.667,\ \tan^{-1}(-0.667) = -33.7°$$

θ is in the 2nd quadrant, so $\theta = 180° + (-33.7°) = \boxed{146°}$.

(c) $\mathbf{R}_x = 0$, $\mathbf{R}_y = -4.00$, $\mathbf{R} = \mathbf{A} + \mathbf{B}$ thus $\mathbf{B} = \mathbf{R} - \mathbf{A}$ and

$B_x = R_x - A_x = 0 - (-3.00) = 3.00$, $B_y = R_y - A_y = -4.00 - 2.00 = -6.00$.

Therefore, $\mathbf{B} = \boxed{3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}}$.

P3.48 Let $+x=$ East, $+y=$ North,

x	y
300	0
–175	303
0	150
125	453

(a) $\theta=\tan^{-1}\frac{y}{x}=\boxed{74.6^\circ \text{ N of E}}$

(b) $|\mathbf{R}|=\sqrt{x^2+y^2}=\boxed{470 \text{ km}}$

P3.49 (a) $R_x=40.0\cos 45.0^\circ+30.0\cos 45.0^\circ=49.5$

$R_y=40.0\sin 45.0^\circ-30.0\sin 45.0^\circ+20.0=27.1$

$\mathbf{R}=\boxed{49.5\hat{\mathbf{i}}+27.1\hat{\mathbf{j}}}$

(b) $|\mathbf{R}|=\sqrt{(49.5)^2+(27.1)^2}=\boxed{56.4}$

$\theta=\tan^{-1}\left(\frac{27.1}{49.5}\right)=\boxed{28.7^\circ}$

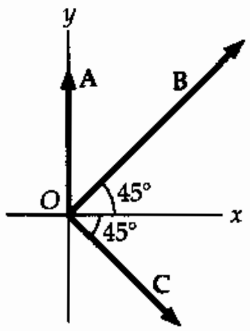


FIG. P3.49

P3.50 Taking components along $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get two equations:

$$6.00a-8.00b+26.0=0$$

and

$$-8.00a+3.00b+19.0=0.$$

Solving simultaneously,

$$\boxed{a=5.00,\ b=7.00}.$$

Therefore,

$$5.00\mathbf{A}+7.00\mathbf{B}+\mathbf{C}=0.$$

Additional Problems

P3.51 Let θ represent the angle between the directions of **A** and **B**. Since **A** and **B** have the same magnitudes, **A**, **B**, and $\mathbf{R} = \mathbf{A} + \mathbf{B}$ form an isosceles triangle in which the angles are $180° - \theta$, $\frac{\theta}{2}$, and $\frac{\theta}{2}$. The magnitude of **R** is then $R = 2A\cos\left(\frac{\theta}{2}\right)$. [*Hint:* apply the law of cosines to the isosceles triangle and use the fact that $B = A$.]

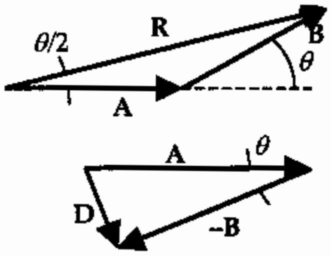


FIG. P3.51

Again, **A**, **–B**, and $\mathbf{D} = \mathbf{A} - \mathbf{B}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$(1 - \cos\theta) = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of **D** as $D = 2A\sin\left(\frac{\theta}{2}\right)$.

The problem requires that $R = 100D$.

Thus, $2A\cos\left(\frac{\theta}{2}\right) = 200A\sin\left(\frac{\theta}{2}\right)$. This gives $\tan\left(\frac{\theta}{2}\right) = 0.010$ and $\boxed{\theta = 1.15°}$.

P3.52 Let θ represent the angle between the directions of **A** and **B**. Since **A** and **B** have the same magnitudes, **A**, **B**, and $\mathbf{R} = \mathbf{A} + \mathbf{B}$ form an isosceles triangle in which the angles are $180° - \theta$, $\frac{\theta}{2}$, and $\frac{\theta}{2}$. The magnitude of **R** is then $R = 2A\cos\left(\frac{\theta}{2}\right)$. [*Hint:* apply the law of cosines to the isosceles triangle and use the fact that $B = A$.]

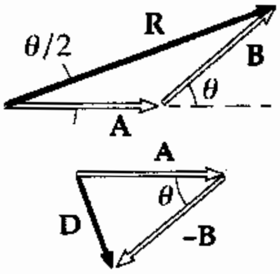


FIG. P3.52

Again, **A**, **–B**, and $\mathbf{D} = \mathbf{A} - \mathbf{B}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$(1 - \cos\theta) = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of **D** as $D = 2A\sin\left(\frac{\theta}{2}\right)$.

The problem requires that $R = nD$ or $\cos\left(\frac{\theta}{2}\right) = n\sin\left(\frac{\theta}{2}\right)$ giving

$\boxed{\theta = 2\tan^{-1}\left(\frac{1}{n}\right)}$.

P3.53 (a) $R_x = \boxed{2.00}$, $R_y = \boxed{1.00}$, $R_z = \boxed{3.00}$

(b) $|\mathbf{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{4.00 + 1.00 + 9.00} = \sqrt{14.0} = \boxed{3.74}$

(c)
$$\cos\theta_x = \frac{R_x}{|\mathbf{R}|} \Rightarrow \theta_x = \cos^{-1}\left(\frac{R_x}{|\mathbf{R}|}\right) = \boxed{57.7^\circ \text{ from } +x}$$
$$\cos\theta_y = \frac{R_y}{|\mathbf{R}|} \Rightarrow \theta_y = \cos^{-1}\left(\frac{R_y}{|\mathbf{R}|}\right) = \boxed{74.5^\circ \text{ from } +y}$$
$$\cos\theta_z = \frac{R_z}{|\mathbf{R}|} \Rightarrow \theta_z = \cos^{-1}\left(\frac{R_z}{|\mathbf{R}|}\right) = \boxed{36.7^\circ \text{ from } +z}$$

***P3.54** Take the x-axis along the tail section of the snake. The displacement from tail to head is

$$240 \text{ m}\hat{\mathbf{i}} + (420 - 240) \text{ m}\cos(180° - 105°)\hat{\mathbf{i}} - 180 \text{ m}\sin 75°\,\hat{\mathbf{j}} = 287 \text{ m}\hat{\mathbf{i}} - 174 \text{ m}\hat{\mathbf{j}}.$$

Its magnitude is $\sqrt{(287)^2 + (174)^2}$ m $= 335$ m. From $v = \frac{\text{distance}}{\Delta t}$, the time for each child's run is

$$\text{Inge: } \Delta t = \frac{\text{distance}}{v} = \frac{335 \text{ m(h)(1 km)(3 600 s)}}{(12 \text{ km)(1 000 m)(1 h)}} = 101 \text{ s}$$
$$\text{Olaf: } \Delta t = \frac{420 \text{ m}\cdot\text{s}}{3.33 \text{ m}} = 126 \text{ s}.$$

Inge wins by $126 - 101 = \boxed{25.4 \text{ s}}$.

***P3.55** The position vector from the ground under the controller of the first airplane is

$$\mathbf{r}_1 = (19.2 \text{ km})(\cos 25°)\hat{\mathbf{i}} + (19.2 \text{ km})(\sin 25°)\hat{\mathbf{j}} + (0.8 \text{ km})\hat{\mathbf{k}}$$
$$= \left(17.4\hat{\mathbf{i}} + 8.11\hat{\mathbf{j}} + 0.8\hat{\mathbf{k}}\right) \text{ km}.$$

The second is at

$$\mathbf{r}_2 = (17.6 \text{ km})(\cos 20°)\hat{\mathbf{i}} + (17.6 \text{ km})(\sin 20°)\hat{\mathbf{j}} + (1.1 \text{ km})\hat{\mathbf{k}}$$
$$= \left(16.5\hat{\mathbf{i}} + 6.02\hat{\mathbf{j}} + 1.1\hat{\mathbf{k}}\right) \text{ km}.$$

Now the displacement from the first plane to the second is

$$\mathbf{r}_2 - \mathbf{r}_1 = \left(-0.863\hat{\mathbf{i}} - 2.09\hat{\mathbf{j}} + 0.3\hat{\mathbf{k}}\right) \text{ km}$$

with magnitude

$$\sqrt{(0.863)^2 + (2.09)^2 + (0.3)^2} = \boxed{2.29 \text{ km}}.$$

***P3.56** Let A represent the distance from island 2 to island 3. The displacement is $\mathbf{A} = A$ at $159°$. Represent the displacement from 3 to 1 as $\mathbf{B} = B$ at $298°$. We have 4.76 km at $37° \; +\mathbf{A} + \mathbf{B} = 0$.

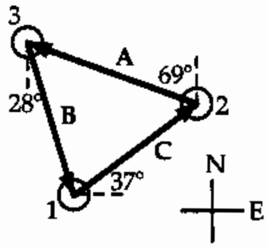


FIG. P3.56

For x-components

$$(4.76 \text{ km})\cos 37° + A\cos 159° + B\cos 298° = 0$$
$$3.80 \text{ km} - 0.934A + 0.469B = 0$$
$$B = -8.10 \text{ km} + 1.99A$$

For y-components,

$$(4.76 \text{ km})\sin 37° + A\sin 159° + B\sin 298° = 0$$
$$2.86 \text{ km} + 0.358A - 0.883B = 0$$

(a) We solve by eliminating B by substitution:

$$2.86 \text{ km} + 0.358A - 0.883(-8.10 \text{ km} + 1.99A) = 0$$
$$2.86 \text{ km} + 0.358A + 7.15 \text{ km} - 1.76A = 0$$
$$10.0 \text{ km} = 1.40A$$
$$A = \boxed{7.17 \text{ km}}$$

(b) $B = -8.10 \text{ km} + 1.99(7.17 \text{ km}) = \boxed{6.15 \text{ km}}$

***P3.57** (a) We first express the corner's position vectors as sets of components

$$\mathbf{A} = (10 \text{ m})\cos 50° \,\hat{\mathbf{i}} + (10 \text{ m})\sin 50° \,\hat{\mathbf{j}} = 6.43 \text{ m}\hat{\mathbf{i}} + 7.66 \text{ m}\hat{\mathbf{j}}$$
$$\mathbf{B} = (12 \text{ m})\cos 30° \,\hat{\mathbf{i}} + (12 \text{ m})\sin 30° \,\hat{\mathbf{j}} = 10.4 \text{ m}\hat{\mathbf{i}} + 6.00 \text{ m}\hat{\mathbf{j}}.$$

The horizontal width of the rectangle is

$$10.4 \text{ m} - 6.43 \text{ m} = 3.96 \text{ m}.$$

Its vertical height is

$$7.66 \text{ m} - 6.00 \text{ m} = 1.66 \text{ m}.$$

Its perimeter is

$$2(3.96 + 1.66) \text{ m} = \boxed{11.2 \text{ m}}.$$

(b) The position vector of the distant corner is $B_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = 10.4 \text{ m}\hat{\mathbf{i}} + 7.66 \text{ m}\hat{\mathbf{j}} = \sqrt{10.4^2 + 7.66^2}$ m at $\tan^{-1}\dfrac{7.66 \text{ m}}{10.4 \text{ m}} = \boxed{12.9 \text{ m at } 36.4°}$.

P3.58 Choose the $+x$-axis in the direction of the first force. The total force, in newtons, is then

$$12.0\hat{\mathbf{i}}+31.0\hat{\mathbf{j}}-8.40\hat{\mathbf{i}}-24.0\hat{\mathbf{j}}=\boxed{\left(3.60\hat{\mathbf{i}}\right)+\left(7.00\hat{\mathbf{j}}\right)\text{ N}}.$$

The magnitude of the total force is

$$\sqrt{(3.60)^2+(7.00)^2}\text{ N}=\boxed{7.87\text{ N}}$$

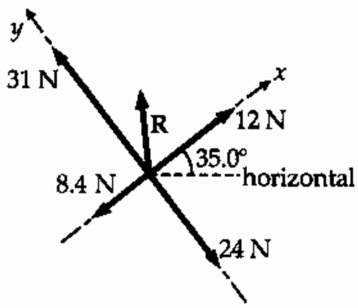


FIG. P3.58

and the angle it makes with our $+x$-axis is given by $\tan\theta=\frac{(7.00)}{(3.60)}$, $\theta=62.8°$. Thus, its angle counterclockwise from the horizontal is $35.0°+62.8°=\boxed{97.8°}$.

P3.59

$\mathbf{d}_1=100\hat{\mathbf{i}}$

$\mathbf{d}_2=-300\hat{\mathbf{j}}$

$\mathbf{d}_3=-150\cos(30.0°)\hat{\mathbf{i}}-150\sin(30.0°)\hat{\mathbf{j}}=-130\hat{\mathbf{i}}-75.0\hat{\mathbf{j}}$

$\mathbf{d}_4=-200\cos(60.0°)\hat{\mathbf{i}}+200\sin(60.0°)\hat{\mathbf{j}}=-100\hat{\mathbf{i}}+173\hat{\mathbf{j}}$

$\mathbf{R}=\mathbf{d}_1+\mathbf{d}_2+\mathbf{d}_3+\mathbf{d}_4=\boxed{\left(-130\hat{\mathbf{i}}-202\hat{\mathbf{j}}\right)\text{ m}}$

$|\mathbf{R}|=\sqrt{(-130)^2+(-202)^2}=\boxed{240\text{ m}}$

$\phi=\tan^{-1}\left(\frac{202}{130}\right)=57.2°$

$\theta=180+\phi=\boxed{237°}$

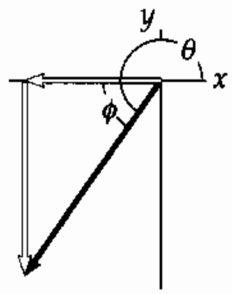


FIG. P3.59

P3.60 $\frac{d\mathbf{r}}{dt}=\frac{d\left(4\hat{\mathbf{i}}+3\hat{\mathbf{j}}-2t\hat{\mathbf{j}}\right)}{dt}=0+0-2\hat{\mathbf{j}}=\boxed{-(2.00\text{ m/s})\hat{\mathbf{j}}}$

The position vector at $t=0$ is $4\hat{\mathbf{i}}+3\hat{\mathbf{j}}$. At $t=1$ s, the position is $4\hat{\mathbf{i}}+1\hat{\mathbf{j}}$, and so on. The object is moving straight downward at 2 m/s, so

$$\frac{d\mathbf{r}}{dt}\text{ represents }\boxed{\text{its velocity vector}}.$$

P3.61

$\mathbf{v}=v_x\hat{\mathbf{i}}+v_y\hat{\mathbf{j}}=(300+100\cos 30.0°)\hat{\mathbf{i}}+(100\sin 30.0°)\hat{\mathbf{j}}$

$\mathbf{v}=\left(387\hat{\mathbf{i}}+50.0\hat{\mathbf{j}}\right)\text{ mi/h}$

$|\mathbf{v}|=\boxed{390\text{ mi/h at }7.37°\text{ N of E}}$

P3.62 (a) You start at point A: $\mathbf{r}_1 = \mathbf{r}_A = (30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}})$ m.

The displacement to B is

$$\mathbf{r}_B - \mathbf{r}_A = 60.0\hat{\mathbf{i}} + 80.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} + 20.0\hat{\mathbf{j}} = 30.0\hat{\mathbf{i}} + 100\hat{\mathbf{j}}.$$

You cover half of this, $(15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}})$ to move to $\mathbf{r}_2 = 30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}}$.

Now the displacement from your current position to C is

$$\mathbf{r}_C - \mathbf{r}_2 = -10.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} = -55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}.$$

You cover one-third, moving to

$$\mathbf{r}_3 = \mathbf{r}_2 + \Delta\mathbf{r}_{23} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}} + \frac{1}{3}(-55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}}.$$

The displacement from where you are to D is

$$\mathbf{r}_D - \mathbf{r}_3 = 40.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} - 26.7\hat{\mathbf{i}} - 16.7\hat{\mathbf{j}} = 13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}.$$

You traverse one-quarter of it, moving to

$$\mathbf{r}_4 = \mathbf{r}_3 + \frac{1}{4}(\mathbf{r}_D - \mathbf{r}_3) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}} + \frac{1}{4}(13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}) = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}.$$

The displacement from your new location to E is

$$\mathbf{r}_E - \mathbf{r}_4 = -70.0\hat{\mathbf{i}} + 60.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 55.0\hat{\mathbf{j}}$$

of which you cover one-fifth the distance, $-20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}}$, moving to

$$\mathbf{r}_4 + \Delta\mathbf{r}_{45} = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}} = 10.0\hat{\mathbf{i}} + 16.0\hat{\mathbf{j}}.$$

The treasure is at $\boxed{(10.0\text{ m}, 16.0\text{ m})}$.

(b) Following the directions brings you to the average position of the trees. The steps we took numerically in part (a) bring you to

$$\mathbf{r}_A + \frac{1}{2}(\mathbf{r}_B - \mathbf{r}_A) = \left(\frac{\mathbf{r}_A + \mathbf{r}_B}{2}\right)$$

then to $\dfrac{(\mathbf{r}_A + \mathbf{r}_B)}{2} + \dfrac{\mathbf{r}_C - \frac{(\mathbf{r}_A + \mathbf{r}_B)}{2}}{3} = \dfrac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C}{3}$

then to $\dfrac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)}{3} + \dfrac{\mathbf{r}_D - \frac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C)}{3}}{4} = \dfrac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D}{4}$

and last to $\dfrac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D)}{4} + \dfrac{\mathbf{r}_E - \frac{(\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D)}{4}}{5} = \dfrac{\mathbf{r}_A + \mathbf{r}_B + \mathbf{r}_C + \mathbf{r}_D + \mathbf{r}_E}{5}$.

This center of mass of the tree distribution is the same location whatever order we take the trees in.

***P3.63** (a) Let T represent the force exerted by each child. The x-component of the resultant force is

$$T\cos 0 + T\cos 120° + T\cos 240° = T(1) + T(-0.5) + T(-0.5) = 0.$$

The y-component is

$$T\sin 0 + T\sin 120 + T\sin 240 = 0 + 0.866T - 0.866T = 0.$$

Thus,

$$\sum \mathbf{F} = 0.$$

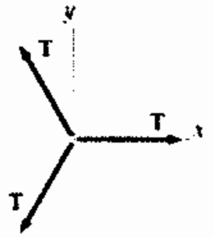


FIG. P3.63

(b) If the total force is not zero, it must point in some direction. When each child moves one space clockwise, the total must turn clockwise by that angle, $\frac{360°}{N}$. Since each child exerts the same force, the new situation is identical to the old and the net force on the tire must still point in the original direction. The contradiction indicates that we were wrong in supposing that the total force is not zero. The total force *must* be zero.

P3.64 (a) From the picture, $\mathbf{R}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$ and $|\mathbf{R}_1| = \sqrt{a^2 + b^2}$.

(b) $\mathbf{R}_2 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$; its magnitude is

$$\sqrt{|\mathbf{R}_1|^2 + c^2} = \sqrt{a^2 + b^2 + c^2}.$$

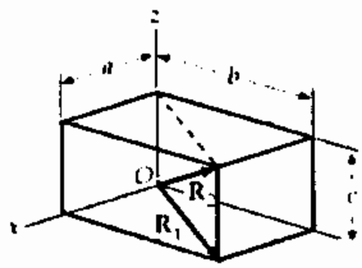


FIG. P3.64

P3.65 Since

$$\mathbf{A}+\mathbf{B}=6.00\hat{\mathbf{j}},$$

we have

$$(A_x+B_x)\hat{\mathbf{i}}+(A_y+B_y)\hat{\mathbf{j}}=0\hat{\mathbf{i}}+6.00\hat{\mathbf{j}}$$

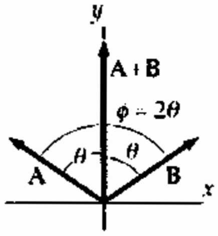


FIG. P3.65

giving

$$A_x+B_x=0 \text{ or } A_x=-B_x \qquad [1]$$

and

$$A_y+B_y=6.00. \qquad [2]$$

Since both vectors have a magnitude of 5.00, we also have

$$A_x^2+A_y^2=B_x^2+B_y^2=5.00^2.$$

From $A_x=-B_x$, it is seen that

$$A_x^2=B_x^2.$$

Therefore, $A_x^2+A_y^2=B_x^2+B_y^2$ gives

$$A_y^2=B_y^2.$$

Then, $A_y=B_y$ and Eq. [2] gives

$$A_y=B_y=3.00.$$

Defining θ as the angle between either **A** or **B** and the y axis, it is seen that

$$\cos\theta=\frac{A_y}{A}=\frac{B_y}{B}=\frac{3.00}{5.00}=0.600 \text{ and } \theta=53.1°.$$

The angle between **A** and **B** is then $\boxed{\phi=2\theta=106°}$.

***P3.66** Let θ represent the angle the x-axis makes with the horizontal. Since angles are equal if their sides are perpendicular right side to right side and left side to left side, θ is also the angle between the weight and our y axis. The x-components of the forces must add to zero:

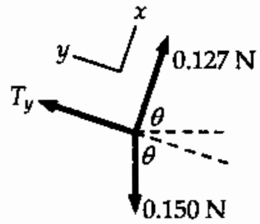


FIG. P3.66

$$-0.150\text{ N}\sin\theta + 0.127\text{ N} = 0\,.$$

(b) $\theta = \boxed{57.9°}$

(a) The y-components for the forces must add to zero:

$$+T_y - (0.150\text{ N})\cos 57.9° = 0,\; T_y = \boxed{0.079\,8\text{ N}}\,.$$

(c) The angle between the y axis and the horizontal is $90.0° - 57.9° = \boxed{32.1°}$.

P3.67 The displacement of point P is invariant under rotation of the coordinates. Therefore, $r = r'$ and $r^2 = (r')^2$ or, $x^2 + y^2 = (x')^2 + (y')^2$. Also, from the figure, $\beta = \theta - \alpha$

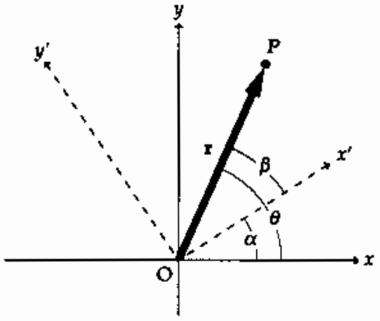


FIG. P3.67

$$\therefore \tan^{-1}\left(\frac{y'}{x'}\right) = \tan^{-1}\left(\frac{y}{x}\right) - \alpha$$

$$\frac{y'}{x'} = \frac{\left(\frac{y}{x}\right) - \tan\alpha}{1 + \left(\frac{y}{x}\right)\tan\alpha}$$

Which we simplify by multiplying top and bottom by $x\cos\alpha$. Then,

$$x' = x\cos\alpha + y\sin\alpha,\; y' = -x\sin\alpha + y\cos\alpha.$$

ANSWERS TO EVEN PROBLEMS

P3.2 (a) (2.17 m, 1.25 m); (−1.90 m, 3.29 m); (b) 4.55 m

P3.4 (a) 8.60 m; (b) 4.47 m at −63.4°; 4.24 m at 135°

P3.6 (a) r at $180° - \theta$; (b) $2r$ at $180° + \theta$; (c) $3r$ at $-\theta$

P3.8 14 km at 65° north of east

P3.10 (a) 6.1 at 112°; (b) 14.8 at 22°

P3.12 9.5 N at 57°

P3.14 7.9 m at 4° north of west

P3.16 see the solution

P3.18 86.6 m and −50.0 m

P3.20 1.31 km north; 2.81 km east

P3.22 $-25.0\text{ m}\,\hat{\mathbf{i}} + 43.3\text{ m}\,\hat{\mathbf{j}}$

P3.24 14.0 m/s at 11.3° west of north

P3.26 788 mi at 48.0° north of east

P3.28 7.21 m at 56.3°

P3.30 $\mathbf{C} = 7.30\text{ cm}\,\hat{\mathbf{i}} - 7.20\text{ cm}\,\hat{\mathbf{j}}$

P3.32 (a) 4.47 m at 63.4°; (b) 8.49 m at 135°

P3.34 42.7 yards

P3.36 4.64 m at 78.6°

P3.38 (a) 10.4 cm; (b) 35.5°

P3.40 1.43×10^4 m at 32.2° above the horizontal

P3.42 $-220\hat{\mathbf{i}} + 57.6\hat{\mathbf{j}} = 227$ paces at 165°

P3.44 (a) $\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right)$ km; (b) 6.31 km

P3.46 (a) $\left(15.1\hat{\mathbf{i}} + 7.72\hat{\mathbf{j}}\right)$ cm;
(b) $\left(-7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right)$ cm;
(c) $\left(+7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right)$ cm

P3.48 (a) 74.6° north of east; (b) 470 km

P3.50 $a = 5.00$, $b = 7.00$

P3.52 $2\tan^{-1}\left(\frac{1}{n}\right)$

P3.54 25.4 s

P3.56 (a) 7.17 km; (b) 6.15 km

P3.58 7.87 N at 97.8° counterclockwise from a horizontal line to the right

P3.60 $(-2.00 \text{ m/s})\hat{\mathbf{j}}$; its velocity vector

P3.62 (a) (10.0 m, 16.0 m); (b) see the solution

P3.64 (a) $\mathbf{R}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}$; $|\mathbf{R}_1| = \sqrt{a^2 + b^2}$;
(b) $\mathbf{R}_2 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}$; $|\mathbf{R}_2| = \sqrt{a^2 + b^2 + c^2}$

P3.66 (a) 0.079 8N; (b) 57.9°; (c) 32.1°

4

Motion in Two Dimensions

CHAPTER OUTLINE

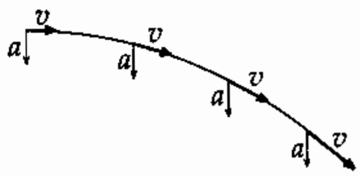
ANSWERS TO QUESTIONS

Q4.1 Yes. An object moving in uniform circular motion moves at a constant speed, but changes its direction of motion. An object cannot accelerate if its velocity is constant.

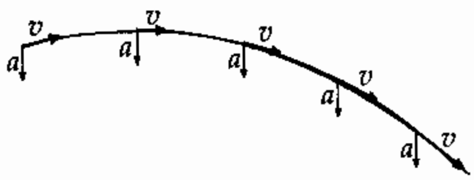
Q4.2 No, you cannot determine the instantaneous velocity. Yes, you can determine the average velocity. The points could be widely separated. In this case, you can only determine the average velocity, which is

$$\overline{\mathbf{v}} = \frac{\Delta \mathbf{x}}{\Delta t}.$$

Q4.3 (a)



(b)



Q4.4 (a) $10\hat{\mathbf{i}}$ m/s (b) $-9.80\hat{\mathbf{j}}$ m/s^2

Q4.5 The easiest way to approach this problem is to determine acceleration first, velocity second and finally position.

Vertical: In free flight, $a_y = -g$. At the top of a projectile's trajectory, $v_y = 0$. Using this, the maximum height can be found using $v_{fy}^2 = v_{iy}^2 + 2a_y(y_f - y_i)$.

Horizontal: $a_x = 0$, so v_x is always the same. To find the horizontal position at maximum height, one needs the flight time, t. Using the vertical information found previously, the flight time can be found using $v_{fy} = v_{iy} + a_y t$. The horizontal position is $x_f = v_{ix} t$.

If air resistance is taken into account, then the acceleration in both the x and y-directions would have an additional term due to the drag.

Q4.6 A parabola.

Q4.7 The balls will be closest together as the second ball is thrown. Yes, the first ball will always be moving faster, since its flight time is larger, and thus the vertical component of the velocity is larger. The time interval will be one second. No, since the vertical component of the motion determines the flight time.

Q4.8 The ball will have the greater speed. Both the rock and the ball will have the same vertical component of the velocity, but the ball will have the additional horizontal component.

Q4.9 (a) yes (b) no (c) no (d) yes (e) no

Q4.10 Straight up. Throwing the ball any other direction than straight up will give a nonzero speed at the top of the trajectory.

Q4.11 No. The projectile with the larger vertical component of the initial velocity will be in the air longer.

Q4.12 The projectile is in free fall. Its vertical component of acceleration is the downward acceleration of gravity. Its horizontal component of acceleration is zero.

Q4.13 (a) no (b) yes (c) yes (d) no

Q4.14 60°. The projection angle appears in the expression for horizontal range in the function $\sin 2\theta$. This function is the same for 30° and 60°.

Q4.15 The optimal angle would be less than 45°. The longer the projectile is in the air, the more that air resistance will change the components of the velocity. Since the vertical component of the motion determines the flight time, an angle less than 45° would increase range.

Q4.16 The projectile on the moon would have both the larger range and the greater altitude. *Apollo* astronauts performed the experiment with golf balls.

Q4.17 Gravity only changes the vertical component of motion. Since both the coin and the ball are falling from the same height with the same vertical component of the initial velocity, they must hit the floor at the same time.

Q4.18 (a) no (b) yes

In the second case, the particle is continuously changing the direction of its velocity vector.

Q4.19 The racing car rounds the turn at a constant *speed* of 90 miles per hour.

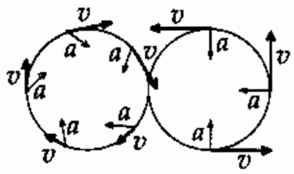
Q4.20 The acceleration cannot be zero because the pendulum does not remain at rest at the end of the arc.

Q4.21 (a) The velocity is not constant because the object is constantly changing the direction of its motion.

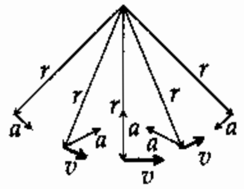
(b) The acceleration is not constant because the acceleration always points towards the center of the circle. The magnitude of the acceleration is constant, but not the direction.

Q4.22 (a) straight ahead (b) in a circle or straight ahead

Q4.23



Q4.24



Q4.25 The unit vectors $\hat{\mathbf{r}}$ and $\hat{\boldsymbol{\theta}}$ are in different directions at different points in the xy plane. At a location along the x-axis, for example, $\hat{\mathbf{r}} = \hat{\mathbf{i}}$ and $\hat{\boldsymbol{\theta}} = \hat{\mathbf{j}}$, but at a point on the y-axis, $\hat{\mathbf{r}} = \hat{\mathbf{j}}$ and $\hat{\boldsymbol{\theta}} = -\hat{\mathbf{i}}$. The unit vector $\hat{\mathbf{i}}$ is equal everywhere, and $\hat{\mathbf{j}}$ is also uniform.

Q4.26 The wrench will hit at the base of the mast. If air resistance is a factor, it will hit slightly leeward of the base of the mast, displaced in the direction in which air is moving relative to the deck. If the boat is scudding before the wind, for example, the wrench's impact point can be in front of the mast.

Q4.27 (a) The ball would move straight up and down as observed by the passenger. The ball would move in a parabolic trajectory as seen by the ground observer.

(b) Both the passenger and the ground observer would see the ball move in a parabolic trajectory, although the two observed paths would not be the same.

Q4.28 (a) g downward (b) g downward

The horizontal component of the motion does not affect the vertical acceleration.

SOLUTIONS TO PROBLEMS

Section 4.1 The Position, Velocity, and Acceleration Vectors

P4.1

$x(m)$	$y(m)$
0	−3 600
−3 000	0
−1 270	1 270
−4 270 m	−2 330 m

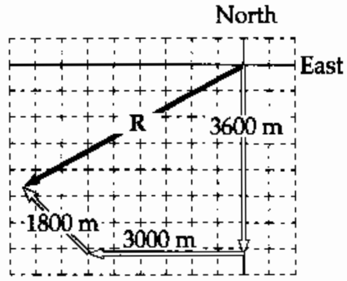


FIG. P4.1

(a) Net displacement $= \sqrt{x^2 + y^2}$

$$= \boxed{4.87 \text{ km at } 28.6° \text{ S of W}}$$

(b) $$\text{Average speed} = \frac{(20.0 \text{ m/s})(180 \text{ s}) + (25.0 \text{ m/s})(120 \text{ s}) + (30.0 \text{ m/s})(60.0 \text{ s})}{180 \text{ s} + 120 \text{ s} + 60.0 \text{ s}} = \boxed{23.3 \text{ m/s}}$$

(c) $$\text{Average velocity} = \frac{4.87 \times 10^3 \text{ m}}{360 \text{ s}} = \boxed{13.5 \text{ m/s along } \mathbf{R}}$$

P4.2 (a) $\mathbf{r} = \boxed{18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}}$

(b) $\mathbf{v} = \boxed{(18.0\ \text{m/s})\hat{\mathbf{i}} + \left[4.00\ \text{m/s} - \left(9.80\ \text{m/s}^2\right)t\right]\hat{\mathbf{j}}}$

(c) $\mathbf{a} = \boxed{\left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}}$

(d) $\mathbf{r}(3.00\ \text{s}) = \boxed{(54.0\ \text{m})\hat{\mathbf{i}} - (32.1\ \text{m})\hat{\mathbf{j}}}$

(e) $\mathbf{v}(3.00\ \text{s}) = \boxed{(18.0\ \text{m/s})\hat{\mathbf{i}} - (25.4\ \text{m/s})\hat{\mathbf{j}}}$

(f) $\mathbf{a}(3.00\ \text{s}) = \boxed{\left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}}$

***P4.3** The sun projects onto the ground the x-component of her velocity:

$$5.00\ \text{m/s}\ \cos(-60.0°) = \boxed{2.50\ \text{m/s}}.$$

P4.4 (a) From $x = -5.00 \sin \omega t$, the x-component of velocity is

$$v_x = \frac{dx}{dt} = \left(\frac{d}{dt}\right)(-5.00 \sin \omega t) = -5.00\omega \cos \omega t$$

and $a_x = \dfrac{dv_x}{dt} = +5.00\omega^2 \sin \omega t$

similarly, $v_y = \left(\dfrac{d}{dt}\right)(4.00 - 5.00 \cos \omega t) = 0 + 5.00\omega \sin \omega t$

and $a_y = \left(\dfrac{d}{dt}\right)(5.00\omega \sin \omega t) = 5.00\omega^2 \cos \omega t$.

At $t = 0$, $\mathbf{v} = -5.00\omega \cos 0\hat{\mathbf{i}} + 5.00\omega \sin 0\hat{\mathbf{j}} = \boxed{\left(5.00\omega\,\hat{\mathbf{i}} + 0\hat{\mathbf{j}}\right)\ \text{m/s}}$

and $\mathbf{a} = 5.00\omega^2 \sin 0\hat{\mathbf{i}} + 5.00\omega^2 \cos 0\hat{\mathbf{j}} = \boxed{\left(0\hat{\mathbf{i}} + 5.00\omega^2\hat{\mathbf{j}}\right)\ \text{m/s}^2}$.

(b) $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = \boxed{(4.00\ \text{m})\hat{\mathbf{j}} + (5.00\ \text{m})\left(-\sin \omega t\,\hat{\mathbf{i}} - \cos \omega t\,\hat{\mathbf{j}}\right)}$

$\mathbf{v} = \boxed{(5.00\ \text{m})\omega\left[-\cos \omega t\,\hat{\mathbf{i}} + \sin \omega t\,\hat{\mathbf{j}}\right]}$

$\mathbf{a} = \boxed{(5.00\ \text{m})\omega^2\left[\sin \omega t\,\hat{\mathbf{i}} + \cos \omega t\,\hat{\mathbf{j}}\right]}$

(c) The object moves in $\boxed{\text{a circle of radius 5.00 m centered at (0, 4.00 m)}}$.

Section 4.2 Two-Dimensional Motion with Constant Acceleration

P4.5 (a) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}\,t$

$$\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t} = \frac{\left(9.00\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right) - \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right)}{3.00} = \boxed{\left(2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\ \text{m/s}^2}$$

(b) $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}\,t^2 = \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right)t + \frac{1}{2}\left(2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)t^2$

$\boxed{x = \left(3.00t + t^2\right)\ \text{m}}$ and $\boxed{y = \left(1.50t^2 - 2.00t\right)\ \text{m}}$

P4.6 (a) $\mathbf{v} = \frac{d\mathbf{r}}{dt} = \left(\frac{d}{dt}\right)\left(3.00\hat{\mathbf{i}} - 6.00t^2\hat{\mathbf{j}}\right) = \boxed{-12.0t\,\hat{\mathbf{j}}\ \text{m/s}}$

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \left(\frac{d}{dt}\right)\left(-12.0t\,\hat{\mathbf{j}}\right) = \boxed{-12.0\hat{\mathbf{j}}\ \text{m/s}^2}$$

(b) $\boxed{\mathbf{r} = \left(3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}\right)\ \text{m};\ \mathbf{v} = -12.0\hat{\mathbf{j}}\ \text{m/s}}$

P4.7 $\mathbf{v}_i = \left(4.00\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}}\right)$ m/s and $\mathbf{v}(20.0) = \left(20.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)$ m/s

(a)
$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{20.0 - 4.00}{20.0}\ \text{m/s}^2 = \boxed{0.800\ \text{m/s}^2}$$
$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{-5.00 - 1.00}{20.0}\ \text{m/s}^2 = \boxed{-0.300\ \text{m/s}^2}$$

(b) $\theta = \tan^{-1}\left(\frac{-0.300}{0.800}\right) = -20.6° = \boxed{339°\ \text{from}\ +x\ \text{axis}}$

(c) At $t = 25.0$ s

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 10.0 + 4.00(25.0) + \frac{1}{2}(0.800)(25.0)^2 = \boxed{360\ \text{m}}$$
$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = -4.00 + 1.00(25.0) + \frac{1}{2}(-0.300)(25.0)^2 = \boxed{-72.7\ \text{m}}$$
$$v_{xf} = v_{xi} + a_x t = 4 + 0.8(25) = 24\ \text{m/s}$$
$$v_{yf} = v_{yi} + a_y t = 1 - 0.3(25) = -6.5\ \text{m/s}$$
$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-6.50}{24.0}\right) = \boxed{-15.2°}$$

P4.8 $\mathbf{a} = 3.00\hat{\mathbf{j}}\ \text{m/s}^2$; $\mathbf{v}_i = 5.00\hat{\mathbf{i}}\ \text{m/s}$; $\mathbf{r}_i = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$

(a) $$\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2 = \boxed{\left[5.00t\hat{\mathbf{i}} + \frac{1}{2}3.00t^2\hat{\mathbf{j}}\right]\ \text{m}}$$

$$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = \boxed{\left(5.00\hat{\mathbf{i}} + 3.00t\hat{\mathbf{j}}\right)\ \text{m/s}}$$

(b) $$t = 2.00\ \text{s},\ \mathbf{r}_f = 5.00(2.00)\hat{\mathbf{i}} + \frac{1}{2}(3.00)(2.00)^2\hat{\mathbf{j}} = \left(10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)\ \text{m}$$

so $x_f = \boxed{10.0\ \text{m}}$, $y_f = \boxed{6.00\ \text{m}}$

$$\mathbf{v}_f = 5.00\hat{\mathbf{i}} + 3.00(2.00)\hat{\mathbf{j}} = \left(5.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$v_f = \left|\mathbf{v}_f\right| = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00)^2 + (6.00)^2} = \boxed{7.81\ \text{m/s}}$$

***P4.9** (a) For the x-component of the motion we have $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$.

$$0.01\ \text{m} = 0 + \left(1.80\times10^7\ \text{m/s}\right)t + \frac{1}{2}\left(8\times10^{14}\ \text{m/s}^2\right)t^2$$

$$\left(4\times10^{14}\ \text{m/s}^2\right)t^2 + \left(1.80\times10^7\ \text{m/s}\right)t - 10^{-2}\ \text{m} = 0$$

$$t = \frac{-1.80\times10^7\ \text{m/s} \pm \sqrt{\left(1.8\times10^7\ \text{m/s}\right)^2 - 4\left(4\times10^{14}\ \text{m/s}^2\right)\left(-10^{-2}\ \text{m}\right)}}{2\left(4\times10^{14}\ \text{m/s}^2\right)}$$

$$= \frac{-1.8\times10^7 \pm 1.84\times10^7\ \text{m/s}}{8\times10^{14}\ \text{m/s}^2}$$

We choose the + sign to represent the physical situation

$$t = \frac{4.39\times10^5\ \text{m/s}}{8\times10^{14}\ \text{m/s}^2} = 5.49\times10^{-10}\ \text{s}.$$

Here

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = 0 + 0 + \frac{1}{2}\left(1.6\times10^{15}\ \text{m/s}^2\right)\left(5.49\times10^{-10}\ \text{s}\right)^2 = 2.41\times10^{-4}\ \text{m}.$$

So, $\boxed{\mathbf{r}_f = \left(10.0\ \hat{\mathbf{i}} + 0.241\ \hat{\mathbf{j}}\right)\ \text{mm}}$.

(b) $$\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = 1.80\times10^7\ \text{m/s}\,\hat{\mathbf{i}} + \left(8\times10^{14}\ \text{m/s}^2\ \hat{\mathbf{i}} + 1.6\times10^{15}\ \text{m/s}^2\ \hat{\mathbf{j}}\right)\left(5.49\times10^{-10}\ \text{s}\right)$$

$$= \left(1.80\times10^7\ \text{m/s}\right)\hat{\mathbf{i}} + \left(4.39\times10^5\ \text{m/s}\right)\hat{\mathbf{i}} + \left(8.78\times10^5\ \text{m/s}\right)\hat{\mathbf{j}}$$

$$= \boxed{\left(1.84\times10^7\ \text{m/s}\right)\hat{\mathbf{i}} + \left(8.78\times10^5\ \text{m/s}\right)\hat{\mathbf{j}}}$$

(c) $$\left|\mathbf{v}_f\right| = \sqrt{\left(1.84\times10^7\ \text{m/s}\right)^2 + \left(8.78\times10^5\ \text{m/s}\right)^2} = \boxed{1.85\times10^7\ \text{m/s}}$$

(d) $$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{8.78\times10^5}{1.84\times10^7}\right) = \boxed{2.73°}$$

Section 4.3 Projectile Motion

P4.10 $x = v_{xi}t = v_i \cos\theta_i t$

$$x = (300\ \text{m/s})(\cos 55.0°)(42.0\ \text{s})$$

$$x = \boxed{7.23 \times 10^3\ \text{m}}$$

$$y = v_{yi}t - \frac{1}{2}gt^2 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

$$y = (300\ \text{m/s})(\sin 55.0°)(42.0\ \text{s}) - \frac{1}{2}(9.80\ \text{m/s}^2)(42.0\ \text{s})^2 = \boxed{1.68 \times 10^3\ \text{m}}$$

P4.11 (a) The mug leaves the counter horizontally with a velocity v_{xi} (say). If time t elapses before it hits the ground, then since there is no horizontal acceleration, $x_f = v_{xi}t$, i.e.,

$$t = \frac{x_f}{v_{xi}} = \frac{(1.40\ \text{m})}{v_{xi}}$$

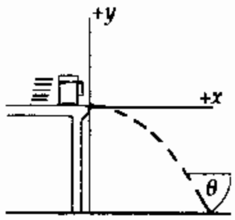


FIG. P4.11

In the same time it falls a distance of 0.860 m with acceleration downward of 9.80 m/s^2. Then

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2:\ 0 = 0.860\ \text{m} + \frac{1}{2}(-9.80\ \text{m/s}^2)\left(\frac{1.40\ \text{m}}{v_{xi}}\right)^2.$$

Thus,

$$v_{xi} = \sqrt{\frac{(4.90\ \text{m/s}^2)(1.96\ \text{m}^2)}{0.860\ \text{m}}} = \boxed{3.34\ \text{m/s}}.$$

(b) The vertical velocity component with which it hits the floor is

$$v_{yf} = v_{yi} + a_y t = 0 + (-9.80\ \text{m/s}^2)\left(\frac{1.40\ \text{m}}{3.34\ \text{m/s}}\right) = -4.11\ \text{m/s}.$$

Hence, the angle θ at which the mug strikes the floor is given by

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-4.11}{3.34}\right) = \boxed{-50.9°}.$$

P4.12 The mug is a projectile from just after leaving the counter until just before it reaches the floor. Taking the origin at the point where the mug leaves the bar, the coordinates of the mug at any time are

$$x_f = v_{xi}t + \frac{1}{2}a_x t^2 = v_{xi}t + 0 \text{ and } y_f = v_{yi}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}g t^2.$$

When the mug reaches the floor, $y_f = -h$ so

$$-h = -\frac{1}{2}g t^2$$

which gives the time of impact as

$$t = \sqrt{\frac{2h}{g}}.$$

(a) Since $x_f = d$ when the mug reaches the floor, $x_f = v_{xi}t$ becomes $d = v_{xi}\sqrt{\frac{2h}{g}}$ giving the initial velocity as

$$\boxed{v_{xi} = d\sqrt{\frac{g}{2h}}}.$$

(b) Just before impact, the x-component of velocity is still

$$v_{xf} = v_{xi}$$

while the y-component is

$$v_{yf} = v_{yi} + a_y t = 0 - g\sqrt{\frac{2h}{g}}.$$

Then the direction of motion just before impact is below the horizontal at an angle of

$$\theta = \tan^{-1}\left(\frac{|v_{yf}|}{v_{xf}}\right) = \tan^{-1}\left(\frac{g\sqrt{\frac{2h}{g}}}{d\sqrt{\frac{g}{2h}}}\right) = \boxed{\tan^{-1}\left(\frac{2h}{d}\right)}.$$

P4.13 (a) The time of flight of the first snowball is the nonzero root of $y_f = y_i + v_{yi}t_1 + \frac{1}{2}a_y t_1^2$

$$0 = 0 + (25.0\text{ m/s})(\sin 70.0°)t_1 - \frac{1}{2}(9.80\text{ m/s}^2)t_1^2$$

$$t_1 = \frac{2(25.0\text{ m/s})\sin 70.0°}{9.80\text{ m/s}^2} = 4.79\text{ s}.$$

The distance to your target is

$$x_f - x_i = v_{xi}t_1 = (25.0\text{ m/s})\cos 70.0°(4.79\text{ s}) = 41.0\text{ m}.$$

Now the second snowball we describe by

$$y_f = y_i + v_{yi}t_2 + \frac{1}{2}a_y t_2{}^2$$

$$0 = (25.0\text{ m/s})\sin\theta_2 t_2 - (4.90\text{ m/s}^2)t_2^2$$

$$t_2 = (5.10\text{ s})\sin\theta_2$$
$$x_f - x_i = v_{xi}\,t_2$$

$$41.0\text{ m} = (25.0\text{ m/s})\cos\theta_2(5.10\text{ s})\sin\theta_2 = (128\text{ m})\sin\theta_2\cos\theta_2$$

$$0.321 = \sin\theta_2\cos\theta_2$$

Using $\sin 2\theta = 2\sin\theta\cos\theta$ we can solve $0.321 = \frac{1}{2}\sin 2\theta_2$

$$2\theta_2 = \sin^{-1}0.643 \text{ and } \theta_2 = \boxed{20.0°}.$$

(b) The second snowball is in the air for time $t_2 = (5.10\text{ s})\sin\theta_2 = (5.10\text{ s})\sin 20° = 1.75\text{ s}$, so you throw it after the first by

$$t_1 - t_2 = 4.79\text{ s} - 1.75\text{ s} = \boxed{3.05\text{ s}}.$$

P4.14 From Equation 4.14 with $R = 15.0\text{ m}$, $v_i = 3.00\text{ m/s}$, $\theta_{max} = 45.0°$

$$\therefore g = \frac{v_i^2}{R} = \frac{9.00}{15.0} = \boxed{0.600\text{ m/s}^2}$$

P4.15 $h=\dfrac{v_i^2\sin^2\theta_i}{2g}$; $R=\dfrac{v_i^2(\sin 2\theta_i)}{g}$; $3h=R$,

so $\dfrac{3v_i^2\sin^2\theta_i}{2g}=\dfrac{v_i^2(\sin 2\theta_i)}{g}$

or $\dfrac{2}{3}=\dfrac{\sin^2\theta_i}{\sin 2\theta_i}=\dfrac{\tan\theta_i}{2}$

thus $\theta_i=\tan^{-1}\left(\dfrac{4}{3}\right)=\boxed{53.1°}$.

***P4.16** (a) To identify the maximum height we let *i* be the launch point and *f* be the highest point:

$$v_{yf}^2=v_{yi}^2+2a_y(y_f-y_i)$$
$$0=v_i^2\sin^2\theta_i+2(-g)(y_{max}-0)$$
$$y_{max}=\frac{v_i^2\sin^2\theta_i}{2g}.$$

To identify the range we let *i* be the launch and *f* be the impact point; where *t* is not zero:

$$y_f=y_i+v_{yi}t+\frac{1}{2}a_yt^2$$
$$0=0+v_i\sin\theta_i t+\frac{1}{2}(-g)t^2$$
$$t=\frac{2v_i\sin\theta_i}{g}$$
$$x_f=x_i+v_{xi}t+\frac{1}{2}a_xt^2$$
$$d=0+v_i\cos\theta_i\frac{2v_i\sin\theta_i}{g}+0.$$

For this rock, $d=y_{max}$

$$\frac{v_i^2\sin^2\theta_i}{2g}=\frac{2v_i^2\sin\theta_i\cos\theta_i}{g}$$
$$\frac{\sin\theta_i}{\cos\theta_i}=\tan\theta_i=4$$
$$\theta_i=\boxed{76.0°}$$

(b) Since g divides out, the answer is $\boxed{\text{the same}}$ on every planet.

(c) The maximum range is attained for $\theta_i=45°$:

$$\frac{d_{max}}{d}=\frac{v_i\cos 45°2v_i\sin 45°g}{gv_i\cos 76°2v_i\sin 76°}=2.125.$$

So $d_{max}=\boxed{\dfrac{17d}{8}}$.

P4.17 (a) $x_f = v_{xi}t = 8.00\cos 20.0°(3.00) = \boxed{22.6\text{ m}}$

(b) Taking y positive downwards,

$$y_f = v_{yi}t + \frac{1}{2}gt^2$$

$$y_f = 8.00\sin 20.0°(3.00) + \frac{1}{2}(9.80)(3.00)^2 = \boxed{52.3\text{ m}}.$$

(c) $10.0 = 8.00(\sin 20.0°)t + \frac{1}{2}(9.80)t^2$

$$4.90t^2 + 2.74t - 10.0 = 0$$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18\text{ s}}$$

***P4.18** We interpret the problem to mean that the displacement from fish to bug is

$$2.00\text{ m at }30° = (2.00\text{ m})\cos 30°\,\hat{\mathbf{i}} + (2.00\text{ m})\sin 30°\,\hat{\mathbf{j}} = (1.73\text{ m})\hat{\mathbf{i}} + (1.00\text{ m})\hat{\mathbf{j}}.$$

If the water should drop 0.03 m during its flight, then the fish must aim at a point 0.03 m above the bug. The initial velocity of the water then is directed through the point with displacement

$$(1.73\text{ m})\hat{\mathbf{i}} + (1.03\text{ m})\hat{\mathbf{j}} = 2.015\text{ m at }30.7°.$$

For the time of flight of a water drop we have

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$1.73\text{ m} = 0 + (v_i\cos 30.7°)t + 0$ so

$$t = \frac{1.73\text{ m}}{v_i\cos 30.7°}.$$

The vertical motion is described by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2.$$

The "drop on its path" is

$$-3.00\text{ cm} = \frac{1}{2}\left(-9.80\text{ m/s}^2\right)\left(\frac{1.73\text{ m}}{v_i\cos 30.7°}\right)^2.$$

Thus,

$$v_i = \frac{1.73\text{ m}}{\cos 30.7°}\sqrt{\frac{9.80\text{ m/s}^2}{2\times 0.03\text{ m}}} = 2.015\text{ m}\left(12.8\text{ s}^{-1}\right) = \boxed{25.8\text{ m/s}}.$$

P4.19 (a) We use the trajectory equation:

$$y_f = x_f \tan\theta_i - \frac{g x_f^2}{2v_i^2 \cos^2\theta_i}.$$

With

$$x_f = 36.0\text{ m},\ v_i = 20.0\text{ m/s, and } \theta = 53.0°$$

we find

$$y_f = (36.0\text{ m})\tan 53.0° - \frac{(9.80\text{ m/s}^2)(36.0\text{ m})^2}{2(20.0\text{ m/s})^2\cos^2(53.0°)} = 3.94\text{ m}.$$

The ball clears the bar by

$$(3.94 - 3.05)\text{ m} = \boxed{0.889\text{ m}}.$$

(b) The time the ball takes to reach the maximum height is

$$t_1 = \frac{v_i \sin\theta_i}{g} = \frac{(20.0\text{ m/s})(\sin 53.0°)}{9.80\text{ m/s}^2} = 1.63\text{ s}.$$

The time to travel 36.0 m horizontally is $t_2 = \frac{x_f}{v_{ix}}$

$$t_2 = \frac{36.0\text{ m}}{(20.0\text{ m/s})(\cos 53.0°)} = 2.99\text{ s}.$$

Since $t_2 > t_1$ $\boxed{\text{the ball clears the goal on its way down}}$.

P4.20 The horizontal component of displacement is $x_f = v_{xi}t = (v_i \cos\theta_i)t$. Therefore, the time required to reach the building a distance d away is $t = \frac{d}{v_i \cos\theta_i}$. At this time, the altitude of the water is

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2 = v_i \sin\theta_i\left(\frac{d}{v_i\cos\theta_i}\right) - \frac{g}{2}\left(\frac{d}{v_i\cos\theta_i}\right)^2.$$

Therefore the water strikes the building at a height h above ground level of

$$h = y_f = \boxed{d\tan\theta_i - \frac{gd^2}{2v_i^2\cos^2\theta_i}}.$$

***P4.21** (a) For the horizontal motion, we have

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$24\text{ m} = 0 + v_i(\cos 53°)(2.2\text{ s}) + 0$$

$$v_i = \boxed{18.1\text{ m/s}}.$$

(b) As it passes over the wall, the ball is above the street by $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$$y_f = 0 + (18.1\text{ m/s})(\sin 53°)(2.2\text{ s}) + \frac{1}{2}\left(-9.8\text{ m/s}^2\right)(2.2\text{ s})^2 = 8.13\text{ m}.$$

So it clears the parapet by $8.13\text{ m} - 7\text{ m} = \boxed{1.13\text{ m}}$.

(c) Note that the highest point of the ball's trajectory is not directly above the wall. For the whole flight, we have from the trajectory equation

$$y_f = (\tan\theta_i)x_f - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x_f^2$$

or

$$6\text{ m} = (\tan 53°)x_f - \left(\frac{9.8\text{ m/s}^2}{2(18.1\text{ m/s})^2\cos^2 53°}\right)x_f^2.$$

Solving,

$$\left(0.041\,2\text{ m}^{-1}\right)x_f^2 - 1.33x_f + 6\text{ m} = 0$$

and

$$x_f = \frac{1.33 \pm \sqrt{1.33^2 - 4(0.0412)(6)}}{2\left(0.0412\text{ m}^{-1}\right)}.$$

This yields two results:

$$x_f = 26.8\text{ m or } 5.44\text{ m}$$

The ball passes twice through the level of the roof.

It hits the roof at distance from the wall

$$26.8\text{ m} - 24\text{ m} = \boxed{2.79\text{ m}}.$$

***P4.22** When the bomb has fallen a vertical distance 2.15 km, it has traveled a horizontal distance x_f given by

$$x_f = \sqrt{(3.25\text{ km})^2 - (2.15\text{ km})^2} = 2.437\text{ km}$$

$$y_f = x_f \tan\theta - \frac{g x_f^2}{2v_i^2 \cos^2\theta_i}$$

$$-2\,150\text{ m} = (2\,437\text{ m})\tan\theta_i - \frac{(9.8\text{ m/s}^2)(2\,437\text{ m})^2}{2(280\text{ m/s})^2\cos^2\theta_i}$$

$$\therefore -2\,150\text{ m} = (2\,437\text{ m})\tan\theta_i - (371.19\text{ m})(1+\tan^2\theta_i)$$

$$\therefore \tan^2\theta - 6.565\tan\theta_i - 4.792 = 0$$

$$\therefore \tan\theta_i = \frac{1}{2}\left(6.565 \pm \sqrt{(6.565)^2 - 4(1)(-4.792)}\right) = 3.283 \pm 3.945.$$

Select the negative solution, since θ_i is below the horizontal.

$$\therefore \tan\theta_i = -0.662,\ \boxed{\theta_i = -33.5°}$$

P4.23 The horizontal kick gives zero vertical velocity to the rock. Then its time of flight follows from

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$-40.0\text{ m} = 0 + 0 + \frac{1}{2}(-9.80\text{ m/s}^2)t^2$$

$$t = 2.86\text{ s}.$$

The extra time $3.00\text{ s} - 2.86\text{ s} = 0.143\text{ s}$ is the time required for the sound she hears to travel straight back to the player.

It covers distance

$$(343\text{ m/s})0.143\text{ s} = 49.0\text{ m} = \sqrt{x^2 + (40.0\text{ m})^2}$$

where x represents the horizontal distance the rock travels.

$$x = 28.3\text{ m} = v_{xi}t + 0t^2$$

$$\therefore v_{xi} = \frac{28.3\text{ m}}{2.86\text{ s}} = \boxed{9.91\text{ m/s}}$$

P4.24 From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2(-9.80 \text{ m/s}^2)(1.85 - 1.02) \text{ m}$. From this, $v_{yi} = 4.03 \text{ m/s}$. [Note that this is the answer to part (c) of this problem.]

For the downward part of the flight, the equation gives $v_{yf}^2 = 0 + 2(-9.80 \text{ m/s}^2)(0.900 - 1.85) \text{ m}$. Thus the vertical velocity just before he lands is

$$v_{yf} = -4.32 \text{ m/s}.$$

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$:

$$-4.32 \text{ m/s} = 4.03 \text{ m/s} + (-9.80 \text{ m/s}^2)t$$

or $t = \boxed{0.852 \text{ s}}$.

(b) Looking at the total horizontal displacement during the leap, $x = v_{xi}t$ becomes

$$2.80 \text{ m} = v_{xi}(0.852 \text{ s})$$

which yields $v_{xi} = \boxed{3.29 \text{ m/s}}$.

(c) $v_{yi} = \boxed{4.03 \text{ m/s}}$. See above for proof.

(d) The takeoff angle is: $\theta = \tan^{-1}\left(\frac{v_{yi}}{v_{xi}}\right) = \tan^{-1}\left(\frac{4.03 \text{ m/s}}{3.29 \text{ m/s}}\right) = \boxed{50.8°}$.

(e) Similarly for the deer, the upward part of the flight gives $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$:

$$0 = v_{yi}^2 + 2(-9.80 \text{ m/s}^2)(2.50 - 1.20) \text{ m}$$

so $v_{yi} = 5.04 \text{ m/s}$.

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ yields $v_{yf}^2 = 0 + 2(-9.80 \text{ m/s}^2)(0.700 - 2.50) \text{ m}$ and $v_{yf} = -5.94 \text{ m/s}$.

The hang time is then found as $v_{yf} = v_{yi} + a_y t$: $-5.94 \text{ m/s} = 5.04 \text{ m/s} + (-9.80 \text{ m/s}^2)t$ and

$$\boxed{t = 1.12 \text{ s}}.$$

***P4.25** The arrow's flight time to the collision point is

$$t=\frac{x_f-x_i}{v_{xi}}=\frac{150\text{ m}}{(45\text{ m/s})\cos 50°}=5.19\text{ s}.$$

The arrow's altitude at the collision is

$$\begin{aligned}y_f&=y_i+v_{yi}t+\frac{1}{2}a_yt^2\\&=0+(45\text{ m/s})(\sin 50°)5.19\text{ s}+\frac{1}{2}\left(-9.8\text{ m/s}^2\right)(5.19\text{ s})^2=47.0\text{ m}.\end{aligned}$$

(a) The required launch speed for the apple is given by

$$\begin{aligned}v_{yf}^2&=v_{yi}^2+2a_y\left(y_f-y_i\right)\\0&=v_{yi}^2+2\left(-9.8\text{ m/s}^2\right)(47\text{ m}-0)\\v_{yi}&=\boxed{30.3\text{ m/s}}.\end{aligned}$$

(b) The time of flight of the apple is given by

$$\begin{aligned}v_{yf}&=v_{yi}+a_yt\\0&=30.3\text{ m/s}-9.8\text{ m/s}^2\,t\\t&=3.10\text{ s}.\end{aligned}$$

So the apple should be launched after the arrow by $5.19\text{ s}-3.10\text{ s}=\boxed{2.09\text{ s}}$.

***P4.26** For the smallest impact angle

$$\theta=\tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right),$$

we want to minimize v_{yf} and maximize $v_{xf}=v_{xi}$. The final y-component of velocity is related to v_{yi} by $v_{yf}^2=v_{yi}^2+2gh$, so we want to minimize v_{yi} and maximize v_{xi}. Both are accomplished by making the initial velocity horizontal. Then $v_{xi}=v$, $v_{yi}=0$, and $v_{yf}=\sqrt{2gh}$. At last, the impact angle is

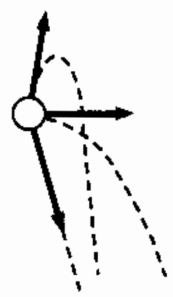


FIG. P4.26

$$\theta=\tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right)=\boxed{\tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)}.$$

Section 4.4 Uniform Circular Motion

P4.27 $a_c = \frac{v^2}{r} = \frac{(20.0\ \text{m/s})^2}{1.06\ \text{m}} = \boxed{377\ \text{m/s}^2}$

The mass is unnecessary information.

P4.28 $a = \frac{v^2}{R}$, $T = 24\ \text{h}(3\,600\ \text{s/h}) = 86\,400\ \text{s}$

$$v = \frac{2\pi R}{T} = \frac{2\pi(6.37\times10^6\ \text{m})}{86\,400\ \text{s}} = 463\ \text{m/s}$$

$$a = \frac{(463\ \text{m/s})^2}{6.37\times10^6\ \text{m}} = \boxed{0.033\,7\ \text{m/s}^2\ \text{directed toward the center of Earth}}.$$

P4.29 $r = 0.500\ \text{m}$;

$$v_t = \frac{2\pi r}{T} = \frac{2\pi(0.500\ \text{m})}{\frac{60.0\ \text{s}}{200\ \text{rev}}} = 10.47\ \text{m/s} = \boxed{10.5\ \text{m/s}}$$

$$a = \frac{v^2}{R} = \frac{(10.47)^2}{0.5} = \boxed{219\ \text{m/s}^2\ \text{inward}}$$

P4.30 $a_c = \frac{v^2}{r}$

$$v = \sqrt{a_c r} = \sqrt{3(9.8\ \text{m/s}^2)(9.45\ \text{m})} = 16.7\ \text{m/s}$$

Each revolution carries the astronaut over a distance of $2\pi r = 2\pi(9.45\ \text{m}) = 59.4\ \text{m}$. Then the rotation rate is

$$16.7\ \text{m/s}\left(\frac{1\ \text{rev}}{59.4\ \text{m}}\right) = \boxed{0.281\ \text{rev/s}}.$$

P4.31 (a) $v = r\omega$

At 8.00 rev/s, $v = (0.600\ \text{m})(8.00\ \text{rev/s})(2\pi\ \text{rad/rev}) = 30.2\ \text{m/s} = 9.60\pi\ \text{m/s}$.

At 6.00 rev/s, $v = (0.900\ \text{m})(6.00\ \text{rev/s})(2\pi\ \text{rad/rev}) = 33.9\ \text{m/s} = 10.8\pi\ \text{m/s}$.

$\boxed{6.00\ \text{rev/s}}$ gives the larger linear speed.

(b) Acceleration $= \frac{v^2}{r} = \frac{(9.60\pi\ \text{m/s})^2}{0.600\ \text{m}} = \boxed{1.52\times10^3\ \text{m/s}^2}$.

(c) At 6.00 rev/s, acceleration $= \frac{(10.8\pi\ \text{m/s})^2}{0.900\ \text{m}} = \boxed{1.28\times10^3\ \text{m/s}^2}$.

P4.32 The satellite is in free fall. Its acceleration is due to gravity and is by effect a centripetal acceleration.

$$a_c = g$$

so

$$\frac{v^2}{r} = g.$$

Solving for the velocity, $v = \sqrt{rg} = \sqrt{(6{,}400 + 600)(10^3 \text{ m})(8.21 \text{ m/s}^2)} = \boxed{7.58 \times 10^3 \text{ m/s}}$

$$v = \frac{2\pi r}{T}$$

and

$$T = \frac{2\pi r}{v} = \frac{2\pi(7{,}000 \times 10^3 \text{ m})}{7.58 \times 10^3 \text{ m/s}} = \boxed{5.80 \times 10^3 \text{ s}}$$

$$T = 5.80 \times 10^3 \text{ s}\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 96.7 \text{ min}.$$

Section 4.5 **Tangential and Radial Acceleration**

P4.33 We assume the train is still slowing down at the instant in question.

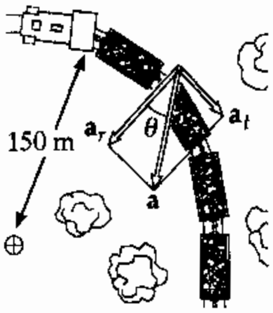


FIG. P4.33

$$a_c = \frac{v^2}{r} = 1.29 \text{ m/s}^2$$

$$a_t = \frac{\Delta v}{\Delta t} = \frac{(-40.0 \text{ km/h})(10^3 \text{ m/km})\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right)}{15.0 \text{ s}} = -0.741 \text{ m/s}^2$$

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(1.29 \text{ m/s}^2)^2 + (-0.741 \text{ m/s}^2)^2}$$

at an angle of $\tan^{-1}\left(\frac{|a_t|}{a_c}\right) = \tan^{-1}\left(\frac{0.741}{1.29}\right)$

$$\mathbf{a} = \boxed{1.48 \text{ m/s}^2 \text{ inward and } 29.9° \text{ backward}}$$

P4.34 (a) $a_t = \boxed{0.600 \text{ m/s}^2}$

(b) $a_r = \frac{v^2}{r} = \frac{(4.00 \text{ m/s})^2}{20.0 \text{ m}} = \boxed{0.800 \text{ m/s}^2}$

(c) $a = \sqrt{a_t^2 + a_r^2} = \boxed{1.00 \text{ m/s}^2}$

$\theta = \tan^{-1}\frac{a_r}{a_t} = \boxed{53.1° \text{ inward from path}}$

P4.35 $r = 2.50\text{ m},\ a = 15.0\ \text{m/s}^2$

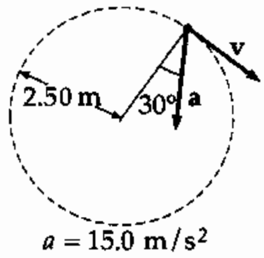


FIG. P4.35

(a) $a_c = a\cos 30.0° = \left(15.0\ \text{m/s}^2\right)(\cos 30°) = \boxed{13.0\ \text{m/s}^2}$

(b) $a_c = \dfrac{v^2}{r}$

so $v^2 = ra_c = 2.50\ \text{m}\left(13.0\ \text{m/s}^2\right) = 32.5\ \text{m}^2/\text{s}^2$

$v = \sqrt{32.5}\ \text{m/s} = \boxed{5.70\ \text{m/s}}$

(c) $a^2 = a_t^2 + a_r^2$

so $a_t = \sqrt{a^2 - a_r^2} = \sqrt{\left(15.0\ \text{m/s}^2\right)^2 - \left(13.0\text{m/s}^2\right)} = \boxed{7.50\ \text{m/s}^2}$

P4.36 (a) See figure to the right.

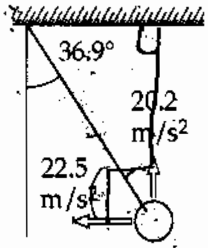


FIG. P4.36

(b) The components of the 20.2 and the 22.5 m/s^2 along the rope together constitute the centripetal acceleration:

$$a_c = \left(22.5\ \text{m/s}^2\right)\cos\left(90.0° - 36.9°\right) + \left(20.2\ \text{m/s}^2\right)\cos 36.9° = \boxed{29.7\ \text{m/s}^2}$$

(c) $a_c = \dfrac{v^2}{r}$ so $v = \sqrt{a_c r} = \sqrt{29.7\ \text{m/s}^2\,(1.50\ \text{m})} = 6.67\ \text{m/s}$ tangent to circle

$\mathbf{v} = \boxed{6.67\ \text{m/s at } 36.9° \text{ above the horizontal}}$

***P4.37** Let i be the starting point and f be one revolution later. The curvilinear motion with constant tangential acceleration is described by

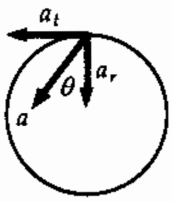


FIG. P4.37

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$

$$2\pi r = 0 + \frac{1}{2}a_t t^2$$

$$a_t = \frac{4\pi r}{t^2}$$

and $v_{xf} = v_{xi} + a_x t,\ v_f = 0 + a_t t = \dfrac{4\pi r}{t}$. The magnitude of the radial acceleration is $a_r = \dfrac{v_f^2}{r} = \dfrac{16\pi^2 r^2}{t^2 r}$.

Then $\tan\theta = \dfrac{a_t}{a_r} = \dfrac{4\pi r t^2}{t^2 16\pi^2 r} = \dfrac{1}{4\pi}$ $\qquad \theta = \boxed{4.55°}$.

Section 4.6 Relative Velocity and Relative Acceleration

P4.38 (a)

$$\mathbf{v}_H = 0 + \mathbf{a}_H t = \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right)\ \text{m/s}^2 (5.00\ \text{s})$$

$$\mathbf{v}_H = \left(15.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$\mathbf{v}_J = 0 + \mathbf{a}_J t = \left(1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\ \text{m/s}^2 (5.00\ \text{s})$$

$$\mathbf{v}_J = \left(5.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$\mathbf{v}_{HJ} = v_H - v_J = \left(15.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 5.00\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$\mathbf{v}_{HJ} = \left(10.0\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}}\right)\ \text{m/s}$$

$$\left|\mathbf{v}_{HJ}\right| = \sqrt{(10.0)^2 + (25.0)^2}\ \text{m/s} = \boxed{26.9\ \text{m/s}}$$

(b)

$$\mathbf{r}_H = 0 + 0 + \frac{1}{2}\mathbf{a}_H t^2 = \frac{1}{2}\left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right)\ \text{m/s}^2 (5.00\ \text{s})^2$$

$$\mathbf{r}_H = \left(37.5\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}}\right)\ \text{m}$$

$$\mathbf{r}_J = \frac{1}{2}\left(1.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\ \text{m/s}^2 (5.00\text{s})^2 = \left(12.5\hat{\mathbf{i}} + 37.5\hat{\mathbf{j}}\right)\ \text{m}$$

$$\mathbf{r}_{HJ} = \mathbf{r}_H - \mathbf{r}_J = \left(37.5\hat{\mathbf{i}} - 25.0\hat{\mathbf{j}} - 12.5\hat{\mathbf{i}} - 37.5\hat{\mathbf{j}}\right)\ \text{m}$$

$$\mathbf{r}_{HJ} = \left(25.0\hat{\mathbf{i}} - 62.5\hat{\mathbf{j}}\right)\ \text{m}$$

$$\left|\mathbf{r}_{HJ}\right| = \sqrt{(25.0)^2 + (62.5)^2}\ \text{m} = \boxed{67.3\ \text{m}}$$

(c)

$$\mathbf{a}_{HJ} = \mathbf{a}_H - \mathbf{a}_J = \left(3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}}\right)\ \text{m/s}^2$$

$$\mathbf{a}_{HJ} = \boxed{\left(2.00\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)\ \text{m/s}^2}$$

***P4.39** $\mathbf{v}_{ce}$ = the velocity of the car relative to the earth.
$\mathbf{v}_{wc}$ = the velocity of the water relative to the car.
$\mathbf{v}_{we}$ = the velocity of the water relative to the earth.

These velocities are related as shown in the diagram at the right.

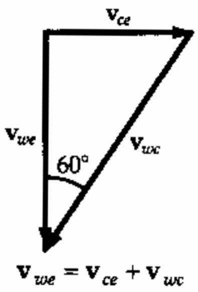


FIG. P4.39

(a) Since $\mathbf{v}_{we}$ is vertical, $v_{wc}\sin 60.0° = v_{ce} = 50.0$ km/h or $\mathbf{v}_{wc} = \boxed{57.7\ \text{km/h at } 60.0°\ \text{west of vertical}}$.

(b) Since $\mathbf{v}_{ce}$ has zero vertical component,

$$v_{we} = v_{wc}\cos 60.0° = (57.7\ \text{km/h})\cos 60.0° = \boxed{28.9\ \text{km/h downward}}.$$

P4.40 The bumpers are initially 100 m = 0.100 km apart. After time t the bumper of the leading car travels 40.0 t, while the bumper of the chasing car travels 60.0t.

Since the cars are side by side at time t, we have

$$0.100 + 40.0t = 60.0t,$$

yielding

$$t = 5.00 \times 10^{-3}\ \text{h} = \boxed{18.0\ \text{s}}.$$

P4.41 Total time in still water $t = \frac{d}{v} = \frac{2\,000}{1.20} = \boxed{1.67 \times 10^3\ \text{s}}$.

Total time = time upstream plus time downstream:

$$t_{\text{up}} = \frac{1\,000}{(1.20 - 0.500)} = 1.43 \times 10^3\ \text{s}$$

$$t_{\text{down}} = \frac{1\,000}{1.20 + 0.500} = 588\ \text{s}.$$

Therefore, $t_{\text{total}} = 1.43 \times 10^3 + 588 = \boxed{2.02 \times 10^3\ \text{s}}$.

P4.42 $v = \sqrt{150^2 + 30.0^2} = \boxed{153\ \text{km/h}}$

$\theta = \tan^{-1}\left(\frac{30.0}{150}\right) = \boxed{11.3°\ \text{north of west}}$

P4.43 For Alan, his speed downstream is $c + v$, while his speed upstream is $c - v$.

Therefore, the total time for Alan is

$$t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \boxed{\frac{\frac{2L}{c}}{1 - \frac{v^2}{c^2}}}.$$

For Beth, her cross-stream speed (both ways) is

$$\sqrt{c^2 - v^2}.$$

Thus, the total time for Beth is $t_2 = \frac{2L}{\sqrt{c^2 - v^2}} = \boxed{\frac{\frac{2L}{c}}{\sqrt{1 - \frac{v^2}{c^2}}}}$.

Since $1 - \frac{v^2}{c^2} < 1$, $t_1 > t_2$, or Beth, who swims cross-stream, returns first.

P4.44 (a) To an observer at rest in the train car, the bolt accelerates downward and toward the rear of the train.

$$a=\sqrt{(2.50\text{ m/s})^2+(9.80\text{ m/s})^2}=\boxed{10.1\text{ m/s}^2}$$

$$\tan\theta=\frac{2.50\text{ m/s}^2}{9.80\text{ m/s}^2}=0.255$$

$$\theta=\boxed{14.3°\text{ to the south from the vertical}}$$

(b) $a=\boxed{9.80\text{ m/s}^2\text{ vertically downward}}$

P4.45 Identify the student as the S' observer and the professor as the S observer. For the initial motion in S', we have

$$\frac{v'_y}{v'_x}=\tan 60.0°=\sqrt{3}.$$

Let u represent the speed of S' relative to S. Then because there is no x-motion in S, we can write $v_x=v'_x+u=0$ so that $v'_x=-u=-10.0\text{ m/s}$. Hence the ball is thrown backwards in S'. Then,

$$v_y=v'_y=\sqrt{3}|v'_x|=10.0\sqrt{3}\text{ m/s}.$$

Using $v_y^2=2gh$ we find

$$h=\frac{(10.0\sqrt{3}\text{ m/s})^2}{2(9.80\text{ m/s}^2)}=\boxed{15.3\text{ m}}.$$

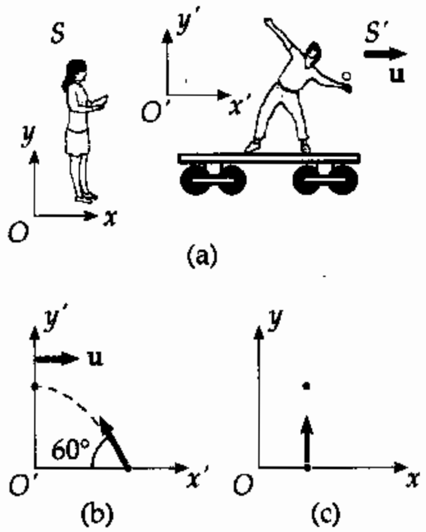


FIG. P4.45

The motion of the ball as seen by the student in S' is shown in diagram (b). The view of the professor in S is shown in diagram (c).

***P4.46** Choose the x-axis along the 20-km distance. The y-components of the displacements of the ship and the speedboat must agree:

$$(26\text{ km/h})t\sin(40°-15°)=(50\text{ km/h})t\sin\alpha$$

$$\alpha=\sin^{-1}\frac{11.0}{50}=12.7°.$$

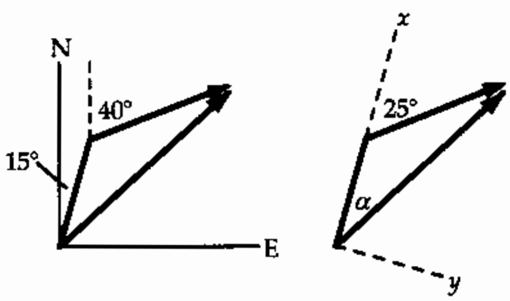


FIG. P4.46

The speedboat should head

$$15°+12.7°=\boxed{27.7°\text{ east of north}}.$$

Additional Problems

***P4.47** (a) The speed at the top is $v_x = v_i \cos\theta_i = (143 \text{ m/s})\cos 45° = \boxed{101 \text{ m/s}}$.

(b) In free fall the plane reaches altitude given by

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$

$$0 = (143 \text{ m/s} \sin 45°)^2 + 2(-9.8 \text{ m/s}^2)(y_f - 31\,000 \text{ ft})$$

$$y_f = 31\,000 \text{ ft} + 522 \text{ m}\left(\frac{3.28 \text{ ft}}{1 \text{ m}}\right) = \boxed{3.27 \times 10^3 \text{ ft}}.$$

(c) For the whole free fall motion $v_{yf} = v_{yi} + a_y t$

$$-101 \text{ m/s} = +101 \text{ m/s} - (9.8 \text{ m/s}^2)t$$

$$t = \boxed{20.6 \text{ s}}$$

(d) $a_c = \dfrac{v^2}{r}$

$$v = \sqrt{a_c r} = \sqrt{0.8(9.8 \text{ m/s}^2)4{,}130 \text{ m}} = \boxed{180 \text{ m/s}}$$

P4.48 At any time t, the two drops have identical y-coordinates. The distance between the two drops is then just twice the magnitude of the horizontal displacement either drop has undergone. Therefore,

$$d = 2|x(t)| = 2(v_{xi}t) = 2(v_i \cos\theta_i)t = \boxed{2v_i t \cos\theta_i}.$$

P4.49 After the string breaks the ball is a projectile, and reaches the ground at time t: $y_f = v_{yi}t + \frac{1}{2}a_y t^2$

$$-1.20 \text{ m} = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)t^2$$

so $t = 0.495$ s.

Its constant horizontal speed is $v_x = \dfrac{x}{t} = \dfrac{2.00 \text{ m}}{0.495 \text{ s}} = 4.04 \text{ m/s}$

so before the string breaks $a_c = \dfrac{v_x^2}{r} = \dfrac{(4.04 \text{ m/s})^2}{0.300 \text{ m}} = \boxed{54.4 \text{ m/s}^2}$.

P4.50 (a) $y_f = (\tan\theta_i)(x_f) - \dfrac{g}{2v_i^2\cos^2\theta_i}x_f^2$

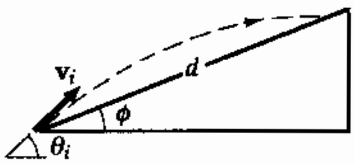


FIG. P4.50

Setting $x_f = d\cos\phi$, and $y_f = d\sin\phi$, we have

$$d\sin\phi = (\tan\theta_i)(d\cos\phi) - \frac{g}{2v_i^2\cos^2\theta_i}(d\cos\phi)^2.$$

Solving for d yields, $d = \dfrac{2v_i^2\cos\theta_i[\sin\theta_i\cos\phi - \sin\phi\cos\theta_i]}{g\cos^2\phi}$

or $\boxed{d = \dfrac{2v_i^2\cos\theta_i\sin(\theta_i - \phi)}{g\cos^2\phi}}$.

(b) Setting $\dfrac{dd}{d\theta_i} = 0$ leads to $\boxed{\theta_i = 45° + \dfrac{\phi}{2}}$ and $\boxed{d_{max} = \dfrac{v_i^2(1-\sin\phi)}{g\cos^2\phi}}$.

P4.51 Refer to the sketch:

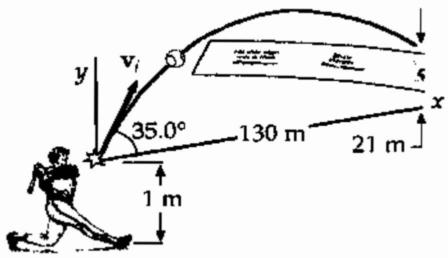


FIG. P4.51

(b) $\Delta x = v_{xi}t$; substitution yields $130 = (v_i\cos 35.0°)t$.

$\Delta y = v_{yi}t + \dfrac{1}{2}at^2$; substitution yields

$$20.0 = (v_i\sin 35.0°)t + \frac{1}{2}(-9.80)t^2.$$

Solving the above gives $t = \boxed{3.81\text{ s}}$.

(a) $v_i = \boxed{41.7\text{ m/s}}$

(c) $v_{yf} = v_i\sin\theta_i - gt$, $v_x = v_i\cos\theta_i$

At $t = 3.81$ s, $v_{yf} = 41.7\sin 35.0° - (9.80)(3.81) = \boxed{-13.4\text{ m/s}}$

$$v_x = (41.7\cos 35.0°) = \boxed{34.1\text{ m/s}}$$

$$v_f = \sqrt{v_x^2 + v_{yf}^2} = \boxed{36.7\text{ m/s}}.$$

P4.52 (a) The moon's gravitational acceleration is the probe's centripetal acceleration: (For the moon's radius, see end papers of text.)

$$a = \frac{v^2}{r}$$

$$\frac{1}{6}\left(9.80\ \text{m/s}^2\right) = \frac{v^2}{1.74 \times 10^6\ \text{m}}$$

$$v = \sqrt{2.84 \times 10^6\ \text{m}^2/\text{s}^2} = \boxed{1.69\ \text{km/s}}$$

(b)

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi(1.74 \times 10^6\ \text{m})}{1.69 \times 10^3\ \text{m/s}} = 6.47 \times 10^3\ \text{s} = \boxed{1.80\ \text{h}}$$

P4.53 (a)

$$a_c = \frac{v^2}{r} = \frac{(5.00\ \text{m/s})^2}{1.00\ \text{m}} = \boxed{25.0\ \text{m/s}^2}$$

$$a_t = g = \boxed{9.80\ \text{m/s}^2}$$

(b) See figure to the right.

(c)

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(25.0\ \text{m/s}^2\right)^2 + \left(9.80\ \text{m/s}^2\right)^2} = \boxed{26.8\ \text{m/s}^2}$$

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\frac{9.80\ \text{m/s}^2}{25.0\ \text{m/s}^2} = \boxed{21.4°}$$

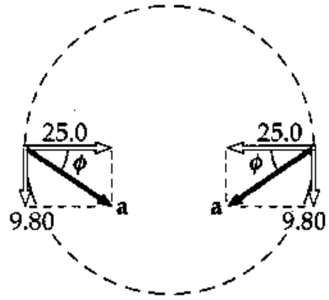


FIG. P4.53

P4.54 $x_f = v_{ix}t = v_i t \cos 40.0°$

Thus, when $x_f = 10.0\ \text{m}$, $t = \frac{10.0\ \text{m}}{v_i \cos 40.0°}$.

At this time, y_f should be $3.05\ \text{m} - 2.00\ \text{m} = 1.05\ \text{m}$.

Thus, $1.05\ \text{m} = \frac{(v_i \sin 40.0°)10.0\ \text{m}}{v_i \cos 40.0°} + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)\left[\frac{10.0\ \text{m}}{v_i \cos 40.0°}\right]^2$.

From this, $v_i = \boxed{10.7\ \text{m/s}}$.

P4.55 The special conditions allowing use of the horizontal range equation applies.
For the ball thrown at 45°,

$$D = R_{45} = \frac{v_i^2 \sin 90}{g}.$$

For the bouncing ball,

$$D = R_1 + R_2 = \frac{v_i^2 \sin 2\theta}{g} + \frac{\left(\frac{v_i}{2}\right)^2 \sin 2\theta}{g}$$

where θ is the angle it makes with the ground when thrown and when bouncing.

(a) We require:

$$\frac{v_i^2}{g} = \frac{v_i^2 \sin 2\theta}{g} + \frac{v_i^2 \sin 2\theta}{4g}$$

$$\sin 2\theta = \frac{4}{5}$$

$$\boxed{\theta = 26.6°}$$

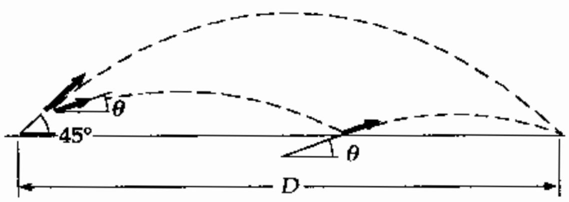


FIG. P4.55

(b) The time for any symmetric parabolic flight is given by

$$y_f = v_{yi}t - \frac{1}{2}gt^2$$

$$0 = v_i \sin\theta_i t - \frac{1}{2}gt^2.$$

If $t = 0$ is the time the ball is thrown, then $t = \frac{2v_i \sin\theta_i}{g}$ is the time at landing.

So for the ball thrown at 45.0°

$$t_{45} = \frac{2v_i \sin 45.0°}{g}.$$

For the bouncing ball,

$$t = t_1 + t_2 = \frac{2v_i \sin 26.6°}{g} + \frac{2\left(\frac{v_i}{2}\right)\sin 26.6°}{g} = \frac{3v_i \sin 26.6°}{g}.$$

The ratio of this time to that for no bounce is

$$\frac{\frac{3v_i \sin 26.6°}{g}}{\frac{2v_i \sin 45.0°}{g}} = \frac{1.34}{1.41} = \boxed{0.949}.$$

P4.56 Using the range equation (Equation 4.14)

$$R = \frac{v_i^2 \sin(2\theta_i)}{g}$$

the maximum range occurs when $\theta_i = 45°$, and has a value $R = \frac{v_i^2}{g}$. Given R, this yields $v_i = \sqrt{gR}$.

If the boy uses the same speed to throw the ball vertically upward, then

$$v_y = \sqrt{gR} - gt \text{ and } y = \sqrt{gR}\,t - \frac{gt^2}{2}$$

at any time, t.

At the maximum height, $v_y = 0$, giving $t = \sqrt{\frac{R}{g}}$, and so the maximum height reached is

$$y_{\max} = \sqrt{gR}\sqrt{\frac{R}{g}} - \frac{g}{2}\left(\sqrt{\frac{R}{g}}\right)^2 = R - \frac{R}{2} = \boxed{\frac{R}{2}}.$$

P4.57 Choose upward as the positive y-direction and leftward as the positive x-direction. The vertical height of the stone when released from A or B is

$$y_i = (1.50 + 1.20\sin 30.0°)\text{ m} = 2.10\text{ m}$$

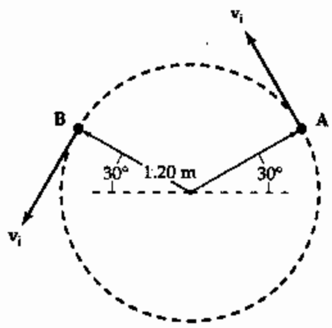


FIG. P4.57

(a) The equations of motion after release at A are

$$v_y = v_i \sin 60.0° - gt = (1.30 - 9.80t)\text{ m/s}$$
$$v_x = v_i \cos 60.0° = 0.750\text{ m/s}$$
$$y = \left(2.10 + 1.30t - 4.90t^2\right)\text{ m}$$
$$\Delta x_A = (0.750t)\text{ m}$$

When $y = 0$, $t = \frac{-1.30 \pm \sqrt{(1.30)^2 + 41.2}}{-9.80} = 0.800\text{ s}$. Then, $\Delta x_A = (0.750)(0.800)\text{ m} = \boxed{0.600\text{ m}}$.

(b) The equations of motion after release at point B are

$$v_y = v_i(-\sin 60.0°) - gt = (-1.30 - 9.80t)\text{ m/s}$$
$$v_x = v_i \cos 60.0 = 0.750\text{ m/s}$$
$$y_i = \left(2.10 - 1.30t - 4.90t^2\right)\text{ m.}$$

When $y = 0$, $t = \frac{+1.30 \pm \sqrt{(-1.30)^2 + 41.2}}{-9.80} = 0.536\text{ s}$. Then, $\Delta x_B = (0.750)(0.536)\text{ m} = \boxed{0.402\text{ m}}$.

(c) $a_r = \frac{v^2}{r} = \frac{(1.50\text{ m/s})^2}{1.20\text{ m}} = \boxed{1.87\text{ m/s}^2 \text{ toward the center}}$

(d) After release, $\mathbf{a} = -g\hat{\mathbf{j}} = \boxed{9.80\text{ m/s}^2\text{ downward}}$

P4.58 The football travels a horizontal distance

$$R = \frac{v_i^2 \sin(2\theta_i)}{g} = \frac{(20.0)^2 \sin(60.0°)}{9.80} = 35.3 \text{ m}.$$

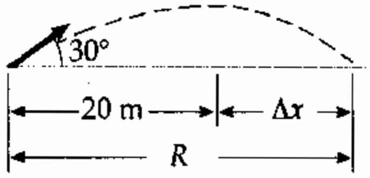


FIG. P4.58

Time of flight of ball is

$$t = \frac{2v_i \sin\theta_i}{g} = \frac{2(20.0)\sin 30.0°}{9.80} = 2.04 \text{ s}.$$

The receiver is Δx away from where the ball lands and $\Delta x = 35.3 - 20.0 = 15.3$ m. To cover this distance in 2.04 s, he travels with a velocity

$$v = \frac{15.3}{2.04} = \boxed{7.50 \text{ m/s in the direction the ball was thrown}}.$$

P4.59 (a) $\Delta y = -\frac{1}{2}gt^2$; $\Delta x = v_i t$

Combine the equations eliminating t:

$$\Delta y = -\frac{1}{2}g\left(\frac{\Delta x}{v_i}\right)^2.$$

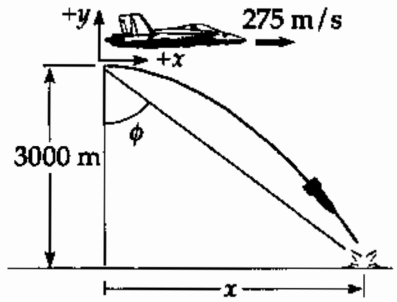


FIG. P4.59

From this, $(\Delta x)^2 = \left(\frac{-2\Delta y}{g}\right)v_i^2$

thus $\Delta x = v_i\sqrt{\frac{-2\Delta y}{g}} = 275\sqrt{\frac{-2(-300)}{9.80}} = 6.80 \times 10^3 = \boxed{6.80 \text{ km}}$.

(b) The plane has the same velocity as the bomb in the x direction. Therefore, the plane will be $\boxed{3\,000 \text{ m directly above the bomb}}$ when it hits the ground.

(c) When ϕ is measured from the vertical, $\tan\phi = \frac{\Delta x}{\Delta y}$

therefore, $\phi = \tan^{-1}\left(\frac{\Delta x}{\Delta y}\right) = \tan^{-1}\left(\frac{6\,800}{3\,000}\right) = \boxed{66.2°}$.

***P4.60** (a) We use the approximation mentioned in the problem. The time to travel 200 m horizontally is $t = \frac{\Delta x}{v_x} = \frac{200\text{ m}}{1{,}000\text{ m/s}} = 0.200\text{ s}$. The bullet falls by

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}\left(-9.8\text{ m/s}^2\right)(0.2\text{ s})^2 = \boxed{-0.196\text{ m}}.$$

(b) The telescope axis must point below the barrel axis by $\theta = \tan^{-1}\frac{0.196\text{ m}}{200\text{ m}} = \boxed{0.0561°}$.

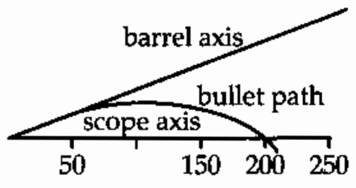


FIG. P4.60(b)

(c) $t = \frac{50.0\text{ m}}{1\,000\text{ m/s}} = 0.050\,0\text{ s}$. The bullet falls by only

$$\Delta y = \frac{1}{2}\left(-9.8\text{ m/s}^2\right)(0.05\text{ s})^2 = -0.0122\text{ m}.$$

At range $50\text{ m} = \frac{1}{4}(200\text{ m})$, the scope axis points to a location $\frac{1}{4}(19.6\text{ cm}) = 4.90\text{ cm}$ above the barrel axis, so the sharpshooter must $\boxed{\text{aim low}}$ by $4.90\text{ cm} - 1.22\text{ cm} = \boxed{3.68\text{ cm}}$.

(d) $t = \frac{150\text{ m}}{1\,000\text{ m/s}} = 0.150\text{ s}$

$$\Delta y = \frac{1}{2}\left(-9.8\text{ m/s}^2\right)(0.15\text{ s})^2 = 0.110\text{ m}$$

$\boxed{\text{Aim low}}$ by $\frac{150}{200}(19.6\text{ cm}) - 11.0\text{ cm} = \boxed{3.68\text{ cm}}$.

(e) $t = \frac{250\text{ m}}{1\,000\text{ m/s}} = 0.250\text{ s}$

$$\Delta y = \frac{1}{2}\left(-9.8\text{ m/s}^2\right)(0.25\text{ s})^2 = 0.306\text{ m}$$

$\boxed{\text{Aim high}}$ by $30.6\text{ cm} - \frac{250}{200}(19.6\text{ cm}) = \boxed{6.12\text{ cm}}$.

(f), (g) Many marksmen have a hard time believing it, but they should aim low in both cases. As in case (a) above, the time of flight is very nearly 0.200 s and the bullet falls below the barrel axis by 19.6 cm on its way. The 0.0561° angle would cut off a 19.6-cm distance on a vertical wall at a horizontal distance of 200 m, but on a vertical wall up at 30° it cuts off distance h as shown, where $\cos 30° = 19.6\text{ cm}/h$, $h = 22.6\text{ cm}$. The marksman must $\boxed{\text{aim low}}$ by $22.6\text{ cm} - 19.6\text{ cm} = 3.03\text{ cm}$. The answer can be obtained by considering limiting cases. Suppose the target is nearly straight above or below you. Then gravity will not cause deviation of the path of the bullet, and one must aim low as in part (c) to cancel out the sighting-in of the telescope.

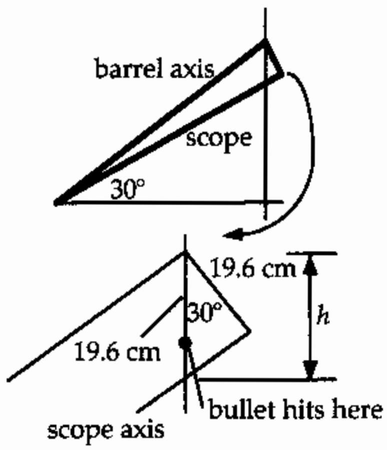


FIG. P4.60(f–g)

P4.61 (a) From Part (c), the raptor dives for $6.34-2.00=4.34$ s undergoing displacement 197 m downward and $(10.0)(4.34)=43.4$ m forward.

$$v=\frac{\Delta d}{\Delta t}=\frac{\sqrt{(197)^2+(43.4)^2}}{4.34}=\boxed{46.5\ \text{m/s}}$$

(b) $\alpha=\tan^{-1}\left(\frac{-197}{43.4}\right)=\boxed{-77.6°}$

(c) $197=\frac{1}{2}gt^2$, $\boxed{t=6.34\ \text{s}}$

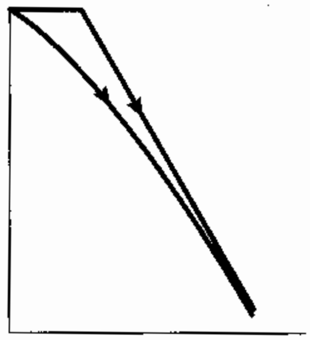


FIG. P4.61

P4.62 Measure heights above the level ground. The elevation y_b of the ball follows

$$y_b=R+0-\frac{1}{2}gt^2$$

with $x=v_i t$ so $y_b=R-\frac{gx^2}{2v_i^2}$.

(a) The elevation y_r of points on the rock is described by

$$y_r^2+x^2=R^2.$$

We will have $y_b=y_r$ at $x=0$, but for all other x we require the ball to be above the rock surface as in $y_b>y_r$. Then $y_b^2+x^2>R^2$

$$\left(R-\frac{gx^2}{2v_i^2}\right)^2+x^2>R^2$$

$$R^2-\frac{gx^2R}{v_i^2}+\frac{g^2x^4}{4v_i^4}+x^2>R^2$$

$$\frac{g^2x^4}{4v_i^4}+x^2>\frac{gx^2R}{v_i^2}.$$

If this inequality is satisfied for x approaching zero, it will be true for all x. If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock: $1>\frac{gR}{v_i^2}$

$$\boxed{v_i>\sqrt{gR}}.$$

(b) With $v_i=\sqrt{gR}$ and $y_b=0$, we have $0=R-\frac{gx^2}{2gR}$

or $x=R\sqrt{2}$.

The distance from the rock's base is

$$x-R=\boxed{\left(\sqrt{2}-1\right)R}.$$

P4.63 (a) While on the incline

$$v_f^2 - v_i^2 = 2a\Delta x$$
$$v_f - v_i = at$$
$$v_f^2 - 0 = 2(4.00)(50.0)$$
$$20.0 - 0 = 4.00t$$
$$v_f = \boxed{20.0 \text{ m/s}}$$
$$t = \boxed{5.00 \text{ s}}$$

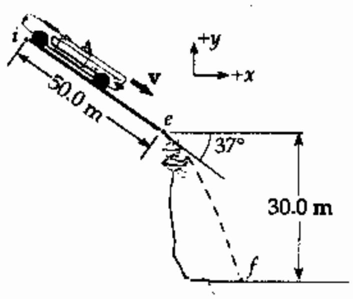


FIG. P4.63

(b) Initial free-flight conditions give us

$$v_{xi} = 20.0\cos 37.0° = 16.0 \text{ m/s}$$

and

$$v_{yi} = -20.0\sin 37.0° = -12.0 \text{ m/s}$$
$$v_{xf} = v_{xi} \text{ since } a_x = 0$$
$$v_{yf} = -\sqrt{2a_y\Delta y + v_{yi}{}^2} = -\sqrt{2(-9.80)(-30.0) + (-12.0)^2} = -27.1 \text{ m/s}$$
$$v_f = \sqrt{v_{xf}{}^2 + v_{yf}{}^2} = \sqrt{(16.0)^2 + (-27.1)^2} = \boxed{31.5 \text{ m/s at } 59.4° \text{ below the horizontal}}$$

(c) $$t_1 = 5 \text{ s};\ t_2 = \frac{v_{yf} - v_{yi}}{a_y} = \frac{-27.1 + 12.0}{-9.80} = 1.53 \text{ s}$$

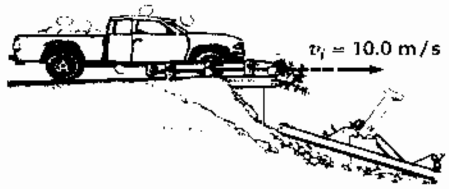
$$t = t_1 + t_2 = \boxed{6.53 \text{ s}}$$

(d) $$\Delta x = v_{xi}t_1 = 16.0(1.53) = \boxed{24.5 \text{ m}}$$

P4.64 Equation of bank: $y^2 = 16x$ (1)

Equations of motion: $x = v_i t$ (2)

$y = -\frac{1}{2}gt^2$ (3)



Substitute for t from (2) into (3) $y = -\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right)$. Equate y from the bank equation to y from the equations of motion:

FIG. P4.64

$$16x = \left[-\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right)\right]^2 \Rightarrow \frac{g^2x^4}{4v_i^4} - 16x = x\left(\frac{g^2x^3}{4v_i^4} - 16\right) = 0.$$

From this, $x = 0$ or $x^3 = \frac{64v_i^4}{g^2}$ and $x = 4\left(\frac{10^4}{9.80^2}\right)^{1/3} = \boxed{18.8 \text{ m}}$. Also,

$$y = -\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right) = -\frac{1}{2}\frac{(9.80)(18.8)^2}{(10.0)^2} = \boxed{-17.3 \text{ m}}.$$

P4.65 (a) Coyote: $\Delta x = \frac{1}{2}at^2$; $70.0 = \frac{1}{2}(15.0)t^2$

Roadrunner: $\Delta x = v_i t$; $70.0 = v_i t$

Solving the above, we get

$$v_i = \boxed{22.9 \text{ m/s}} \text{ and } t = 3.06 \text{ s}.$$

(b) At the edge of the cliff,

$$v_{xi} = at = (15.0)(3.06) = 45.8 \text{ m/s}.$$

Substituting into $\Delta y = \frac{1}{2}a_y t^2$, we find

$$-100 = \frac{1}{2}(-9.80)t^2$$
$$t = 4.52 \text{ s}$$
$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2 = (45.8)(4.52 \text{ s}) + \frac{1}{2}(15.0)(4.52 \text{ s})^2.$$

Solving,

$$\Delta x = \boxed{360 \text{ m}}.$$

(c) For the Coyote's motion through the air

$$v_{xf} = v_{xi} + a_x t = 45.8 + 15(4.52) = \boxed{114 \text{ m/s}}$$
$$v_{yf} = v_{yi} + a_y t = 0 - 9.80(4.52) = \boxed{-44.3 \text{ m/s}}.$$

P4.66 Think of shaking down the mercury in an old fever thermometer. Swing your hand through a circular arc, quickly reversing direction at the bottom end. Suppose your hand moves through one-quarter of a circle of radius 60 cm in 0.1 s. Its speed is

$$\frac{\frac{1}{4}(2\pi)(0.6 \text{ m})}{0.1 \text{ s}} \cong 9 \text{ m/s}$$

and its centripetal acceleration is $\frac{v^2}{r} \cong \frac{(9 \text{ m/s})^2}{0.6 \text{ m}} \boxed{\sim 10^2 \text{ m/s}^2}$.

The tangential acceleration of stopping and reversing the motion will make the total acceleration somewhat larger, but will not affect its order of magnitude.

P4.67 (a) $\Delta x = v_{xi}t,\ \Delta y = v_{yi}t + \frac{1}{2}gt^2$

$$d\cos 50.0° = (10.0\cos 15.0°)t$$

and

$$-d\sin 50.0° = (10.0\sin 15.0°)t + \frac{1}{2}(-9.80)t^2.$$

Solving, $d = \boxed{43.2\text{ m}}$ and $t = 2.88$ s.

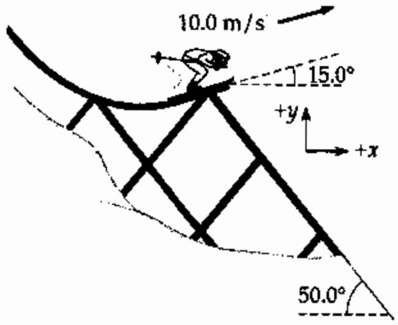


FIG. P4.67

(b) Since $a_x = 0$,

$$v_{xf} = v_{xi} = 10.0\cos 15.0° = \boxed{9.66\text{ m/s}}$$
$$v_{yf} = v_{yi} + a_y t = 10.0\sin 15.0° - 9.80(2.88) = \boxed{-25.6\text{ m/s}}.$$

Air resistance would decrease the values of the range and maximum height. As an airfoil, he can get some lift and increase his distance.

***P4.68** For one electron, we have

$$y = v_{iy}t,\ D = v_{ix}t + \frac{1}{2}a_x t^2 \cong \frac{1}{2}a_x t^2,\ v_{yf} = v_{yi},\text{ and } v_{xf} = v_{xi} + a_x t \cong a_x t.$$

The angle its direction makes with the x-axis is given by

$$\theta = \tan^{-1}\frac{v_{yf}}{v_{xf}} = \tan^{-1}\frac{v_{yi}}{a_x t} = \tan^{-1}\frac{v_{yi}t}{a_x t^2} = \tan^{-1}\frac{y}{2D}.$$

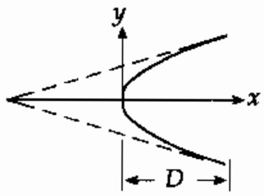


FIG. P4.68

Thus the horizontal distance from the aperture to the virtual source is $2D$. The source is at coordinate $\boxed{x = -D}$.

***P4.69** (a) The ice chest floats downstream 2 km in time t, so that $2\text{ km} = v_w t$. The upstream motion of the boat is described by $d = (v - v_w)15\text{ min}$. The downstream motion is described by $d + 2\text{ km} = (v + v_w)(t - 15\text{ min})$. We eliminate $t = \frac{2\text{ km}}{v_w}$ and d by substitution:

$$(v - v_w)15\text{ min} + 2\text{ km} = (v + v_w)\left(\frac{2\text{ km}}{v_w} - 15\text{ min}\right)$$

$$v(15\text{ min}) - v_w(15\text{ min}) + 2\text{ km} = \frac{v}{v_w}2\text{ km} + 2\text{ km} - v(15\text{ min}) - v_w(15\text{ min})$$

$$v(30\text{ min}) = \frac{v}{v_w}2\text{ km}$$

$$v_w = \frac{2\text{ km}}{30\text{ min}} = \boxed{4.00\text{ km/h}}.$$

(b) In the reference frame of the water, the chest is motionless. The boat travels upstream for 15 min at speed v, and then downstream at the same speed, to return to the same point. Thus it travels for 30 min. During this time, the falls approach the chest at speed v_w, traveling 2 km. Thus

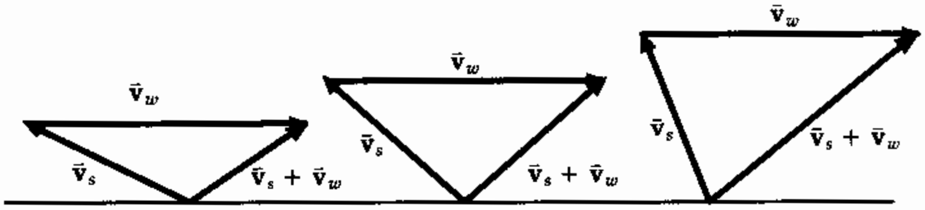
$$v_w = \frac{\Delta x}{\Delta t} = \frac{2\text{ km}}{30\text{ min}} = \boxed{4.00\text{ km/h}}.$$

***P4.70** Let the river flow in the x direction.

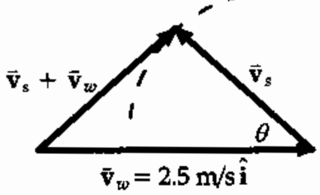
(a) To minimize time, swim perpendicular to the banks in the y direction. You are in the water for time t in $\Delta y = v_y t$, $t = \dfrac{80\text{ m}}{1.5\text{ m/s}} = 53.3\text{ s}$.

(b) The water carries you downstream by $\Delta x = v_x t = (2.50\text{ m/s})53.3\text{ s} = \boxed{133\text{ m}}$.

(c)



To minimize downstream drift, you should swim so that your resultant velocity $\vec{v}_s + \vec{v}_w$ is perpendicular to your swimming velocity $\vec{v}_s$ relative to the water. This condition is shown in the middle picture. It maximizes the angle between the resultant velocity and the shore. The angle between $\vec{v}_s$ and the shore is given by $\cos\theta = \dfrac{1.5\text{ m/s}}{2.5\text{ m/s}}$,



$\boxed{\theta = 53.1°}$.

(d) Now $v_y = v_s \sin\theta = 1.5\text{ m/s}\ \sin 53.1° = 1.20\text{ m/s}$

$$t = \frac{\Delta y}{v_y} = \frac{80\text{ m}}{1.2\text{ m/s}} = 66.7\text{ s}$$

$$\Delta x = v_x t = (2.5\text{ m/s} - 1.5\text{ m/s}\cos 53.1°)66.7\text{ s} = \boxed{107\text{ m}}.$$

***P4.71** Find the highest firing angle θ_H for which the projectile will clear the mountain peak; this will yield the range of the closest point of bombardment. Next find the lowest firing angle; this will yield the maximum range under these conditions if both θ_H and θ_L are $> 45°$; $x = 2500$ m, $y = 1800$ m, $v_i = 250$ m/s.

$$y_f = v_{yi}t - \frac{1}{2}gt^2 = v_i(\sin\theta)t - \frac{1}{2}gt^2$$
$$x_f = v_{xi}t = v_i(\cos\theta)t$$

Thus

$$t = \frac{x_f}{v_i \cos\theta}.$$

Substitute into the expression for y_f

$$y_f = v_i(\sin\theta)\frac{x_f}{v_i\cos\theta} - \frac{1}{2}g\left(\frac{x_f}{v_i\cos\theta}\right)^2 = x_f\tan\theta - \frac{gx_f^2}{2v_i^2\cos^2\theta}$$

but $\frac{1}{\cos^2\theta} = \tan^2\theta + 1$ so $y_f = x_f\tan\theta - \frac{gx_f^2}{2v_i^2}\left(\tan^2\theta + 1\right)$ and

$$0 = \frac{gx_f^2}{2v_i^2}\tan^2\theta - x_f\tan\theta + \frac{gx_f^2}{2v_i^2} + y_f.$$

Substitute values, use the quadratic formula and find

$$\tan\theta = 3.905 \text{ or } 1.197, \text{ which gives } \theta_H = 75.6° \text{ and } \theta_L = 50.1°.$$

Range (at θ_H) $= \frac{v_i^2\sin 2\theta_H}{g} = 3.07 \times 10^3$ m from enemy ship

$$3.07 \times 10^3 - 2\,500 - 300 = 270 \text{ m from shore.}$$

Range (at θ_L) $= \frac{v_i^2\sin 2\theta_L}{g} = 6.28 \times 10^3$ m from enemy ship

$$6.28 \times 10^3 - 2\,500 - 300 = 3.48 \times 10^3 \text{ from shore.}$$

Therefore, safe distance is $\boxed{< 270 \text{ m}}$ or $\boxed{> 3.48 \times 10^3 \text{ m}}$ from the shore.

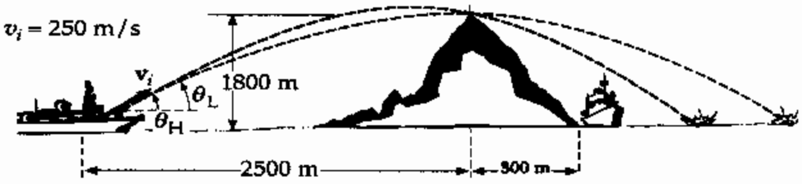


FIG. P4.71

***P4.72** We follow the steps outlined in Example 4.7, eliminating $t = \frac{d\cos\phi}{v_i\cos\theta}$ to find

$$\frac{v_i\sin\theta d\cos\phi}{v_i\cos\theta} - \frac{gd^2\cos^2\phi}{2v_i^2\cos^2\theta} = -d\sin\phi.$$

Clearing of fractions,

$$2v_i^2\cos\theta\sin\theta\cos\phi - gd\cos^2\phi = -2v_i^2\cos^2\theta\sin\phi.$$

To maximize d as a function of θ, we differentiate through with respect to θ and set $\frac{dd}{d\theta} = 0$:

$$2v_i^2\cos\theta\cos\theta\cos\phi + 2v_i^2\sin\theta(-\sin\theta)\cos\phi - g\frac{dd}{d\theta}\cos^2\phi = -2v_i^2 2\cos\theta(-\sin\theta)\sin\phi.$$

We use the trigonometric identities from Appendix B4 $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$ to find $\cos\phi\cos 2\theta = \sin 2\theta\sin\phi$. Next, $\frac{\sin\phi}{\cos\phi} = \tan\phi$ and $\cot 2\theta = \frac{1}{\tan 2\theta}$ give $\cot 2\phi = \tan\phi = \tan(90° - 2\theta)$ so $\phi = 90° - 2\theta$ and $\theta = 45° - \frac{\phi}{2}$.

ANSWERS TO EVEN PROBLEMS

P4.2 (a) $\mathbf{r} = 18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}$;
(b) $\mathbf{v} = 18.0\hat{\mathbf{i}} + (4.00 - 9.80t)\hat{\mathbf{j}}$;
(c) $\mathbf{a} = \left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}$;
(d) $(54.0\ \text{m})\hat{\mathbf{i}} - (32.1\ \text{m})\hat{\mathbf{j}}$;
(e) $(18.0\ \text{m/s})\hat{\mathbf{i}} - (25.4\ \text{m/s})\hat{\mathbf{j}}$;
(f) $\left(-9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}$

P4.4 (a) $\mathbf{v} = \left(-5.00\omega\hat{\mathbf{i}} + 0\hat{\mathbf{j}}\right)\ \text{m/s}$;
$\mathbf{a} = \left(0\hat{\mathbf{i}} + 5.00\omega^2\hat{\mathbf{j}}\right)\ \text{m/s}^2$;
(b) $\mathbf{r} = 4.00\ \text{m}\ \hat{\mathbf{j}} + 5.00\ \text{m}\left(-\sin\omega t\,\hat{\mathbf{i}} - \cos\omega t\,\hat{\mathbf{j}}\right)$;
$\mathbf{v} = 5.00\ \text{m}\ \omega\left(-\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}}\right)$;
$\mathbf{a} = 5.00\ \text{m}\ \omega^2\left(\sin\omega t\,\hat{\mathbf{i}} + \cos\omega t\,\hat{\mathbf{j}}\right)$;
(c) a circle of radius 5.00 m centered at (0, 4.00 m)

P4.6 (a) $\mathbf{v} = -12.0t\,\hat{\mathbf{j}}\ \text{m/s}$; $\mathbf{a} = -12.0\hat{\mathbf{j}}\ \text{m/s}^2$;
(b) $\mathbf{r} = \left(3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}\right)$ m; $\mathbf{v} = -12.0\hat{\mathbf{j}}\ \text{m/s}$

P4.8 (a) $\mathbf{r} = \left(5.00t\hat{\mathbf{i}} + 1.50t^2\hat{\mathbf{j}}\right)$ m;
$\mathbf{v} = \left(5.00\hat{\mathbf{i}} + 3.00t\hat{\mathbf{j}}\right)$ m/s;
(b) $\mathbf{r} = \left(10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}\right)$ m; 7.81 m/s

P4.10 $\left(7.23\times10^3\ \text{m},\ 1.68\times10^3\ \text{m}\right)$

P4.12 (a) $d\sqrt{\frac{g}{2h}}$ horizontally;
(b) $\tan^{-1}\left(\frac{2h}{d}\right)$ below the horizontal

P4.14 $0.600\ \text{m/s}^2$

P4.16 (a) 76.0°; (b) the same; (c) $\frac{17d}{8}$

P4.18 25.8 m/s

P4.20 $d\tan\theta_i - \frac{gd^2}{\left(2v_i^2\cos^2\theta_i\right)}$

P4.22 33.5° below the horizontal

P4.24 (a) 0.852 s; (b) 3.29 m/s; (c) 4.03 m/s; (d) 50.8°; (e) 1.12 s

P4.26 $\tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)$

P4.28 0.033 7 m/s^2 toward the center of the Earth

P4.30 0.281 rev/s

P4.32 7.58×10^3 m/s; 5.80×10^3 s

P4.34 (a) 0.600 m/s^2 forward;
(b) 0.800 m/s^2 inward;
(c) 1.00 m/s^2 forward and 53.1° inward

P4.36 (a) see the solution; (b) 29.7 m/s^2;
(c) 6.67 m/s at 36.9° above the horizontal

P4.38 (a) 26.9 m/s; (b) 67.3 m;
(c) $\left(2.00\hat{\mathbf{i}}-5.00\hat{\mathbf{j}}\right)$ m/s^2

P4.40 18.0 s

P4.42 153 km/h at 11.3° north of west

P4.44 (a) 10.1 m/s^2 at 14.3° south from the vertical; (b) 9.80 m/s^2 vertically downward

P4.46 27.7° east of north

P4.48 $2v_i t\cos\theta_i$

P4.50 (a) see the solution;
(b) $\theta_i = 45°+\frac{\phi}{2}$; $d_{\text{max}} = \frac{v_i^2(1-\sin\phi)}{g\cos^2\phi}$

P4.52 (a) 1.69 km/s; (b) 6.47×10^3 s

P4.54 10.7 m/s

P4.56 $\frac{R}{2}$

P4.58 7.50 m/s in the direction the ball was thrown

P4.60 (a) 19.6 cm; (b) 0.056 1°;
(c) aim low 3.68 cm; (d) aim low 3.68 cm;
(e) aim high 6.12 cm; (f) aim low;
(g) aim low

P4.62 (a) $\sqrt{gR}$; (b) $\left(\sqrt{2}-1\right)R$

P4.64 (18.8 m; −17.3 m)

P4.66 see the solution; $\sim10^2$ m/s^2

P4.68 $x=-D$

P4.70 (a) at 90° to the bank; (b) 133 m;
(c) upstream at 53.1° to the bank; (d) 107 m

P4.72 see the solution

5

The Laws of Motion

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q5.1 (a) The force due to gravity of the earth pulling down on the ball—the reaction force is the force due to gravity of the ball pulling up on the earth. The force of the hand pushing up on the ball—reaction force is ball pushing down on the hand.

(b) The only force acting on the ball in free-fall is the gravity due to the earth -the reaction force is the gravity due to the ball pulling on the earth.

Q5.2 The resultant force is zero, as the acceleration is zero.

Q5.3 Mistake one: The car might be momentarily at rest, in the process of (suddenly) reversing forward into backward motion. In this case, the forces on it add to a (large) backward resultant.

Mistake two: There are no cars in interstellar space. If the car is remaining at rest, there are some large forces on it, including its weight and some force or forces of support.

Mistake three: The statement reverses cause and effect, like a politician who thinks that his getting elected was the reason for people to vote for him.

Q5.4 When the bus starts moving, the mass of Claudette is accelerated by the force of the back of the seat on her body. Clark is standing, however, and the only force on him is the friction between his shoes and the floor of the bus. Thus, when the bus starts moving, his feet start accelerating forward, but the rest of his body experiences almost no accelerating force (only that due to his being attached to his accelerating feet!). As a consequence, his body tends to stay almost at rest, according to Newton's first law, relative to the ground. Relative to Claudette, however, he is moving toward her and falls into her lap. (Both performers won Academy Awards.)

Q5.5 First ask, "Was the bus moving forward or backing up?" If it was moving forward, the passenger is lying. A fast stop would make the suitcase fly toward the front of the bus, not toward the rear. If the bus was backing up at any reasonable speed, a sudden stop could not make a suitcase fly far. Fine her for malicious litigiousness.

Q5.6 It would be smart for the explorer to gently push the rock back into the storage compartment. Newton's 3rd law states that the rock will apply the same size force on her that she applies on it. The harder she pushes on the rock, the larger her resulting acceleration.

Q5.7 The molecules of the floor resist the ball on impact and push the ball back, upward. The actual force acting is due to the forces between molecules that allow the floor to keep its integrity and to prevent the ball from passing through. Notice that for a ball passing through a window, the molecular forces weren't strong enough.

Q5.8 While a football is in flight, the force of gravity and air resistance act on it. When a football is in the process of being kicked, the foot pushes forward on the ball and the ball pushes backward on the foot. At this time and while the ball is in flight, the Earth pulls down on the ball (gravity) and the ball pulls up on the Earth. The moving ball pushes forward on the air and the air backward on the ball.

Q5.9 It is impossible to string a horizontal cable without its sagging a bit. Since the cable has a mass, gravity pulls it downward. A vertical component of the tension must balance the weight for the cable to be in equilibrium. If the cable were completely horizontal, then there would be no vertical component of the tension to balance the weight.

Some physics teachers demonstrate this by asking a beefy student to pull on the ends of a cord supporting a can of soup at its center. Some get two burly young men to pull on opposite ends of a strong rope, while the smallest person in class gleefully mashes the center of the rope down to the table. Point out the beauty of sagging suspension-bridge cables. With a laser and an optical lever, demonstrate that the mayor makes the courtroom table sag when he sits on it, and the judge bends the bench. Give them "I make the floor sag" buttons, available to instructors using this manual. Estimate the cost of an infinitely strong cable, and the truth will always win.

Q5.10 As the barbell goes through the bottom of a cycle, the lifter exerts an upward force on it, and the scale reads the larger upward force that the floor exerts on them together. Around the top of the weight's motion, the scale reads less than average. If the iron is moving upward, the lifter can declare that she has thrown it, just by letting go of it for a moment, so our answer applies also to this case.

Q5.11 As the sand leaks out, the acceleration increases. With the same driving force, a decrease in the mass causes an increase in the acceleration.

Q5.12 As the rocket takes off, it burns fuel, pushing the gases from the combustion out the back of the rocket. Since the gases have mass, the total remaining mass of the rocket, fuel, and oxidizer decreases. With a constant thrust, a decrease in the mass results in an increasing acceleration.

Q5.13 The friction of the road pushing on the tires of a car causes an automobile to move. The push of the air on the propeller moves the airplane. The push of the water on the oars causes the rowboat to move.

Q5.14 As a man takes a step, the action is the force his foot exerts on the Earth; the reaction is the force of the Earth on his foot. In the second case, the action is the force exerted on the girl's back by the snowball; the reaction is the force exerted on the snowball by the girl's back. The third action is the force of the glove on the ball; the reaction is the force of the ball on the glove. The fourth action is the force exerted on the window by the air molecules; the reaction is the force on the air molecules exerted by the window. We could in each case interchange the terms 'action' and 'reaction.'

Q5.15 The tension in the rope must be 9 200 N. Since the rope is moving at a constant speed, then the resultant force on it must be zero. The 49ers are pulling with a force of 9 200 N. If the 49ers were winning with the rope steadily moving in their direction or if the contest was even, then the tension would still be 9 200 N. In all of these case, the acceleration is zero, and so must be the resultant force on the rope. To win the tug-of-war, a team must exert a larger force on the ground than their opponents do.

Q5.16 The tension in the rope when pulling the car is twice that in the tug-of-war. One could consider the car as behaving like another team of twenty more people.

Q5.17 This statement contradicts Newton's 3rd law. The force that the locomotive exerted on the wall is the same as that exerted by the wall on the locomotive. The wall temporarily exerted on the locomotive a force greater than the force that the wall could exert without breaking.

Q5.18 The sack of sand moves up with the athlete, regardless of how quickly the athlete climbs. Since the athlete and the sack of sand have the same weight, the acceleration of the system must be zero.

Q5.19 The resultant force doesn't always add to zero. If it did, nothing could ever accelerate. If we choose a single object as our system, action and reaction forces can never add to zero, as they act on different objects.

Q5.20 An object cannot exert a force on itself. If it could, then objects would be able to accelerate themselves, without interacting with the environment. You cannot lift yourself by tugging on your bootstraps.

Q5.21 To get the box to slide, you must push harder than the maximum static frictional force. Once the box is moving, you need to push with a force equal to the kinetic frictional force to maintain the box's motion.

Q5.22 The stopping distance will be the same if the mass of the truck is doubled. The stopping distance will decrease by a factor of four if the initial speed is cut in half.

Q5.23 If you slam on the brakes, your tires will skid on the road. The force of kinetic friction between the tires and the road is less than the maximum static friction force. Anti-lock brakes work by "pumping" the brakes (much more rapidly that you can) to minimize skidding of the tires on the road.

Q5.24 With friction, it takes longer to come down than to go up. On the way up, the frictional force and the component of the weight down the plane are in the same direction, giving a large acceleration. On the way down, the forces are in opposite directions, giving a relatively smaller acceleration. If the incline is frictionless, it takes the same amount of time to go up as it does to come down.

Q5.25 (a) The force of static friction between the crate and the bed of the truck causes the crate to accelerate. Note that the friction force on the crate is in the direction of its motion relative to the ground (but opposite to the direction of possible sliding motion of the crate relative to the truck bed).

(b) It is most likely that the crate would slide forward relative to the bed of the truck.

Q5.26 In Question 25, part (a) is an example of such a situation. Any situation in which friction is the force that accelerates an object from rest is an example. As you pull away from a stop light, friction is the force that accelerates forward a box of tissues on the level floor of the car. At the same time, friction of the ground on the tires of the car accelerates the car forward.

SOLUTIONS TO PROBLEMS

The following problems cover Sections 5.1–5.6.

Section 5.1 **The Concept of Force**

Section 5.2 **Newton's First Law and Inertial Frames**

Section 5.3 **Mass**

Section 5.4 **Newton's Second Law**

Section 5.5 **The Gravitational Force and Weight**

Section 5.6 **Newton's Third Law**

P5.1 For the same force F, acting on different masses

$$F = m_1 a_1$$

and

$$F = m_2 a_2$$

(a) $\dfrac{m_1}{m_2} = \dfrac{a_2}{a_1} = \boxed{\dfrac{1}{3}}$

(b) $F = (m_1 + m_2)a = 4m_1 a = m_1\left(3.00 \text{ m/s}^2\right)$

$a = \boxed{0.750 \text{ m/s}^2}$

***P5.2** $v_f = 880 \text{ m/s},\ m = 25.8 \text{ kg},\ x_f = 6 \text{ m}$

$v_f^2 = 2ax_f = 2x_f\left(\dfrac{F}{m}\right)$

$F = \dfrac{mv_f^2}{2x_f} = \boxed{1.66 \times 10^6 \text{ N forward}}$

P5.3 $m = 3.00 \text{ kg}$

$\mathbf{a} = \left(2.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right) \text{ m/s}^2$

$\sum \mathbf{F} = m\mathbf{a} = \boxed{\left(6.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right) \text{N}}$

$\left|\sum \mathbf{F}\right| = \sqrt{(6.00)^2 + (15.0)^2} \text{ N} = \boxed{16.2 \text{ N}}$

P5.4 $F_g = \text{weight of ball} = mg$

$v_{\text{release}} = v$ and time to accelerate $= t$:

$$\mathbf{a} = \frac{\Delta v}{\Delta t} = \frac{v}{t} = \frac{v}{t}\hat{\mathbf{i}}$$

(a) Distance $x = \bar{v}t$:

$$x = \left(\frac{v}{2}\right)t = \boxed{\frac{vt}{2}}$$

(b) $\mathbf{F}_p - F_g\hat{\mathbf{j}} = \frac{F_g v}{gt}\hat{\mathbf{i}}$

$$\mathbf{F}_p = \boxed{\frac{F_g v}{gt}\hat{\mathbf{i}} + F_g\hat{\mathbf{j}}}$$

P5.5 $m = 4.00 \text{ kg}$, $\mathbf{v}_i = 3.00\hat{\mathbf{i}} \text{ m/s}$, $\mathbf{v}_8 = \left(8.00\hat{\mathbf{i}} + 10.0\hat{\mathbf{j}}\right) \text{ m/s}$, $t = 8.00 \text{ s}$

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{t} = \frac{5.00\hat{\mathbf{i}} + 10.0\hat{\mathbf{j}}}{8.00} \text{ m/s}^2$$

$$\mathbf{F} = m\mathbf{a} = \boxed{\left(2.50\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right) \text{N}}$$

$$F = \sqrt{(2.50)^2 + (5.00)^2} = \boxed{5.59 \text{ N}}$$

P5.6 (a) Let the x-axis be in the original direction of the molecule's motion.

$$v_f = v_i + at: \quad -670 \text{ m/s} = 670 \text{ m/s} + a\left(3.00 \times 10^{-13} \text{ s}\right)$$

$$a = \boxed{-4.47 \times 10^{15} \text{ m/s}^2}$$

(b) For the molecule, $\sum \mathbf{F} = m\mathbf{a}$. Its weight is negligible.

$$\mathbf{F}_{\text{wall on molecule}} = 4.68 \times 10^{-26} \text{ kg}\left(-4.47 \times 10^{15} \text{ m/s}^2\right) = -2.09 \times 10^{-10} \text{ N}$$

$$\vec{\mathbf{F}}_{\text{molecule on wall}} = \boxed{+2.09 \times 10^{-10} \text{ N}}$$

P5.7 (a) $\sum F = ma$ and $v_f^2 = v_i^2 + 2ax_f$ or $a = \dfrac{v_f^2 - v_i^2}{2x_f}$.

Therefore,

$$\sum F = m\frac{\left(v_f^2 - v_i^2\right)}{2x_f}$$

$$\sum F = 9.11\times10^{-31}\text{ kg}\frac{\left[\left(7.00\times10^5\text{ m/s}^2\right)^2 - \left(3.00\times10^5\text{ m/s}^2\right)^2\right]}{2(0.050\,0\text{ m})} = \boxed{3.64\times10^{-18}\text{ N}}.$$

(b) The weight of the electron is

$$F_g = mg = \left(9.11\times10^{-31}\text{ kg}\right)\left(9.80\text{ m/s}^2\right) = 8.93\times10^{-30}\text{ N}$$

The accelerating force is $\boxed{4.08\times10^{11}\text{ times the weight of the electron.}}$

P5.8 (a) $F_g = mg = 120\text{ lb} = (4.448\text{ N/lb})(120\text{ lb}) = \boxed{534\text{ N}}$

(b) $m = \dfrac{F_g}{g} = \dfrac{534\text{ N}}{9.80\text{ m/s}^2} = \boxed{54.5\text{ kg}}$

P5.9 $F_g = mg = 900\text{ N}$, $m = \dfrac{900\text{ N}}{9.80\text{ m/s}^2} = 91.8\text{ kg}$

$$\left(F_g\right)_{\text{on Jupiter}} = 91.8\text{ kg}\left(25.9\text{ m/s}^2\right) = \boxed{2.38\text{ kN}}$$

P5.10 Imagine a quick trip by jet, on which you do not visit the rest room and your perspiration is just canceled out by a glass of tomato juice. By subtraction, $\left(F_g\right)_p = mg_p$ and $\left(F_g\right)_C = mg_C$ give

$$\Delta F_g = m\left(g_p - g_C\right).$$

For a person whose mass is 88.7 kg, the change in weight is

$$\Delta F_g = 88.7\text{ kg}(9.809\,5 - 9.780\,8) = \boxed{2.55\text{ N}}.$$

A precise balance scale, as in a doctor's office, reads the same in different locations because it compares you with the standard masses on its beams. A typical bathroom scale is not precise enough to reveal this difference.

P5.11 (a) $\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = (20.0\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}})\text{ N}$

$$\sum \mathbf{F} = m\mathbf{a}: \quad 20.0\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}} = 5.00\mathbf{a}$$

$$\mathbf{a} = (4.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}})\text{ m/s}^2$$

or

$$\boxed{a = 5.00\text{ m/s}^2 \text{ at } \theta = 36.9°}$$

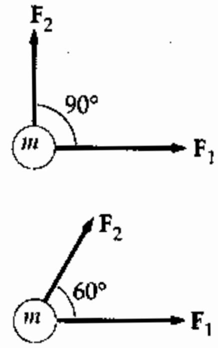


FIG. P5.11

(b) $F_{2x} = 15.0\cos 60.0° = 7.50\text{ N}$

$F_{2y} = 15.0\sin 60.0° = 13.0\text{ N}$

$\mathbf{F}_2 = (7.50\hat{\mathbf{i}} + 13.0\hat{\mathbf{j}})\text{ N}$

$\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = (27.5\hat{\mathbf{i}} + 13.0\hat{\mathbf{j}})\text{ N} = m\mathbf{a} = 5.00\mathbf{a}$

$\mathbf{a} = \boxed{(5.50\hat{\mathbf{i}} + 2.60\hat{\mathbf{j}})\text{ m/s}^2 = 6.08\text{ m/s}^2 \text{ at } 25.3°}$

P5.12 We find acceleration:

$$\mathbf{r}_f - \mathbf{r}_i = \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$$

$$4.20\text{ m}\hat{\mathbf{i}} - 3.30\text{ m}\hat{\mathbf{j}} = 0 + \frac{1}{2}\mathbf{a}(1.20\text{ s})^2 = 0.720\text{ s}^2\mathbf{a}$$

$$\mathbf{a} = (5.83\hat{\mathbf{i}} - 4.58\hat{\mathbf{j}})\text{ m/s}^2.$$

Now $\sum \mathbf{F} = m\mathbf{a}$ becomes

$$\mathbf{F}_g + \mathbf{F}_2 = m\mathbf{a}$$

$$\mathbf{F}_2 = 2.80\text{ kg}(5.83\hat{\mathbf{i}} - 4.58\hat{\mathbf{j}})\text{ m/s}^2 + (2.80\text{ kg})(9.80\text{ m/s}^2)\hat{\mathbf{j}}$$

$$\mathbf{F}_2 = \boxed{(16.3\hat{\mathbf{i}} + 14.6\hat{\mathbf{j}})\text{ N}}.$$

P5.13 (a) You and the earth exert equal forces on each other: $m_y g = M_e a_e$. If your mass is 70.0 kg,

$$a_e = \frac{(70.0\text{ kg})(9.80\text{ m/s}^2)}{5.98\times 10^{24}\text{ kg}} = \boxed{\sim 10^{-22}\text{ m/s}^2}.$$

(b) You and the planet move for equal times intervals according to $x = \frac{1}{2}at^2$. If the seat is 50.0 cm high,

$$\sqrt{\frac{2x_y}{a_y}} = \sqrt{\frac{2x_e}{a_e}}$$

$$x_e = \frac{a_e}{a_y}x_y = \frac{m_y}{m_e}x_y = \frac{70.0\text{ kg}(0.500\text{ m})}{5.98\times 10^{24}\text{ kg}}\boxed{\sim 10^{-23}\text{ m}}.$$

P5.14 $\sum \mathbf{F} = m\mathbf{a}$ reads

$$\left(-2.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} + 5.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}}\right)\text{ N} = m\left(3.75\text{ m/s}^2\right)\hat{\mathbf{a}}$$

where $\hat{\mathbf{a}}$ represents the direction of **a**

$$\left(-42.0\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)\text{ N} = m\left(3.75\text{ m/s}^2\right)\hat{\mathbf{a}}$$
$$\sum \mathbf{F} = \sqrt{(42.0)^2 + (1.00)^2}\text{ N at }\tan^{-1}\left(\frac{1.00}{42.0}\right)\text{ below the }-x\text{-axis}$$
$$\sum \mathbf{F} = 42.0\text{ N at }181° = m\left(3.75\text{ m/s}^2\right)\hat{\mathbf{a}}.$$

For the vectors to be equal, their magnitudes and their directions must be equal.

(a) $\therefore \boxed{\hat{\mathbf{a}}\text{ is at }181°}$ counterclockwise from the x-axis

(b) $m = \dfrac{42.0\text{ N}}{3.75\text{ m/s}^2} = \boxed{11.2\text{ kg}}$

(d) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = 0 + \left(3.75\text{ m/s}^2\text{ at }181°\right)10.0\text{ s so }\mathbf{v}_f = 37.5\text{ m/s at }181°$

$\mathbf{v}_f = 37.5\text{ m/s }\cos 181°\,\hat{\mathbf{i}} + 37.5\text{ m/s }\sin 181°\,\hat{\mathbf{j}}\text{ so }\mathbf{v}_f = \boxed{\left(-37.5\hat{\mathbf{i}} - 0.893\hat{\mathbf{j}}\right)\text{ m/s}}$

(c) $\left|\mathbf{v}_f\right| = \sqrt{37.5^2 + 0.893^2}\text{ m/s} = \boxed{37.5\text{ m/s}}$

P5.15 (a) $\boxed{15.0\text{ lb up}}$

(b) $\boxed{5.00\text{ lb up}}$

(c) $\boxed{0}$

Section 5.7 **Some Applications of Newton's Laws**

P5.16 $v_x = \dfrac{dx}{dt} = 10t,\ v_y = \dfrac{dy}{dt} = 9t^2$

$a_x = \dfrac{dv_x}{dt} = 10,\ a_y = \dfrac{dv_y}{dt} = 18t$

At $t = 2.00\text{ s}$, $a_x = 10.0\text{ m/s}^2$, $a_y = 36.0\text{ m/s}^2$

$\sum F_x = ma_x$: $3.00\text{ kg}\left(10.0\text{ m/s}^2\right) = 30.0\text{ N}$
$\sum F_y = ma_y$: $3.00\text{ kg}\left(36.0\text{ m/s}^2\right) = 108\text{ N}$

$$\sum F = \sqrt{F_x^2 + F_y^2} = \boxed{112\text{ N}}$$

P5.17

$$m = 1.00 \text{ kg}$$
$$mg = 9.80 \text{ N}$$
$$\tan\alpha = \frac{0.200 \text{ m}}{25.0 \text{ m}}$$
$$\alpha = 0.458°$$

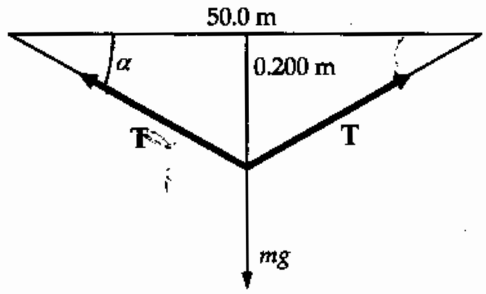


FIG. P5.17

Balance forces,

$$2T\sin\alpha = mg$$
$$T = \frac{9.80 \text{ N}}{2\sin\alpha} = \boxed{613 \text{ N}}$$

P5.18

$$T_3 = F_g \quad (1)$$

$$T_1\sin\theta_1 + T_2\sin\theta_2 = F_g \quad (2)$$

$$T_1\cos\theta_1 = T_2\cos\theta_2 \quad (3)$$

Eliminate T_2 and solve for T_1

$$T_1 = \frac{F_g\cos\theta_2}{(\sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2)} = \frac{F_g\cos\theta_2}{\sin(\theta_1+\theta_2)}$$
$$T_3 = F_g = \boxed{325 \text{ N}}$$
$$T_1 = F_g\left(\frac{\cos 25.0°}{\sin 85.0°}\right) = \boxed{296 \text{ N}}$$
$$T_2 = T_1\left(\frac{\cos\theta_1}{\cos\theta_2}\right) = 296 \text{ N}\left(\frac{\cos 60.0°}{\cos 25.0°}\right) = \boxed{163 \text{ N}}$$

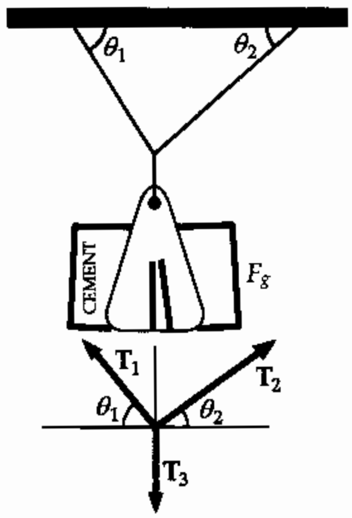


FIG. P5.18

P5.19 See the solution for T_1 in Problem 5.18.

P5.20 (a) An explanation proceeding from fundamental physical principles will be best for the parents and for you. Consider forces on the bit of string touching the weight hanger as shown in the free-body diagram:

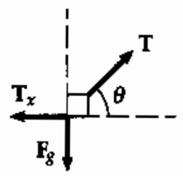


FIG. P5.20

Horizontal Forces: $\sum F_x = ma_x$: $-T_x + T\cos\theta = 0$
Vertical Forces: $\sum F_y = ma_y$: $-F_g + T\sin\theta = 0$

You need only the equation for the vertical forces to find that the tension in the string is given by $T = \boxed{\dfrac{F_g}{\sin\theta}}$. The force the child feels gets smaller, changing from T to $T\cos\theta$, while the counterweight hangs on the string. On the other hand, the kite does not notice what you are doing and the tension in the main part of the string stays constant. You do not need a level, since you learned in physics lab to sight to a horizontal line in a building. Share with the parents your estimate of the experimental uncertainty, which you make by thinking critically about the measurement, by repeating trials, practicing in advance and looking for variations and improvements in technique, including using other observers. You will then be glad to have the parents themselves repeat your measurements.

(b) $$T = \frac{F_g}{\sin\theta} = \frac{0.132\text{ kg}\left(9.80\text{ m/s}^2\right)}{\sin 46.3°} = \boxed{1.79\text{ N}}$$

P5.21 (a) Isolate either mass

$$T + mg = ma = 0$$
$$|T| = |mg|.$$

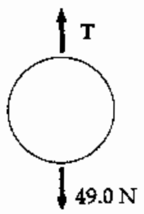


FIG. P5.21(a)

The scale reads the tension T,

so

$$T = mg = 5.00\text{ kg}\left(9.80\text{ m/s}^2\right) = \boxed{49.0\text{ N}}.$$

(b) Isolate the pulley

$$\mathbf{T}_2 + 2\mathbf{T}_1 = 0$$
$$T_2 = 2|T_1| = 2mg = \boxed{98.0\text{ N}}.$$

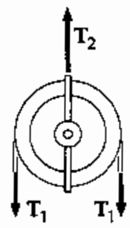


FIG. P5.21(b)

(c) $\sum \mathbf{F} = \mathbf{n} + \mathbf{T} + m\mathbf{g} = 0$

Take the component along the incline

$$\mathbf{n}_x + \mathbf{T}_x + m\mathbf{g}_x = 0$$

or

$$0 + T - mg\sin 30.0° = 0$$

$$T = mg\sin 30.0° = \frac{mg}{2} = \frac{5.00(9.80)}{2}$$
$$= \boxed{24.5\text{ N}}.$$

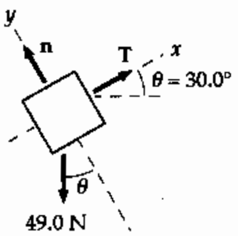


FIG. P5.21(c)

P5.22 The two forces acting on the block are the normal force, n, and the weight, mg. If the block is considered to be a point mass and the x-axis is chosen to be parallel to the plane, then the free body diagram will be as shown in the figure to the right. The angle θ is the angle of inclination of the plane. Applying Newton's second law for the accelerating system (and taking the direction up the plane as the positive x direction) we have

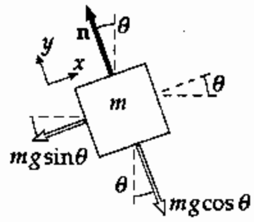


FIG. P5.22

$$\sum F_y = n - mg\cos\theta = 0:\ n = mg\cos\theta$$
$$\sum F_x = -mg\sin\theta = ma:\ a = -g\sin\theta$$

(a) When $\theta = 15.0°$

$$a = \boxed{-2.54\ \text{m/s}^2}$$

(b) Starting from rest

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = 2ax_f$$
$$|v_f| = \sqrt{2ax_f} = \sqrt{2(-2.54\ \text{m/s}^2)(-2.00\ \text{m})} = \boxed{3.18\ \text{m/s}}$$

P5.23 Choose a coordinate system with $\hat{\mathbf{i}}$ East and $\hat{\mathbf{j}}$ North.

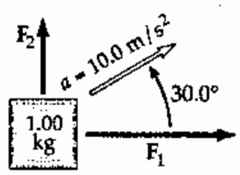


FIG. P5.23

$$\sum \mathbf{F} = m\mathbf{a} = 1.00\ \text{kg}(10.0\ \text{m/s}^2)\ \text{at}\ 30.0°$$
$$(5.00\ \text{N})\hat{\mathbf{j}} + \mathbf{F}_1 = (10.0\ \text{N})\angle 30.0° = (5.00\ \text{N})\hat{\mathbf{j}} + (8.66\ \text{N})\hat{\mathbf{i}}$$
$$\therefore F_1 = \boxed{8.66\ \text{N (East)}}$$

***P5.24** First, consider the block moving along the horizontal. The only force in the direction of movement is T. Thus, $\sum F_x = ma$

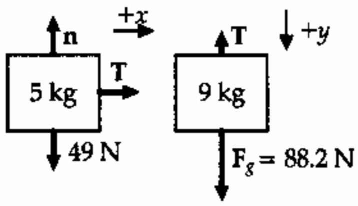


FIG. P5.24

$$T = (5\ \text{kg})a \qquad (1)$$

Next consider the block that moves vertically. The forces on it are the tension T and its weight, 88.2 N.

We have $\sum F_y = ma$

$$88.2\ \text{N} - T = (9\ \text{kg})a \qquad (2)$$

Note that both blocks must have the same magnitude of acceleration. Equations (1) and (2) can be added to give $88.2\ \text{N} = (14\ \text{kg})a$. Then

$$\boxed{a = 6.30\ \text{m/s}^2\ \text{and}\ T = 31.5\ \text{N}}.$$

P5.25 After it leaves your hand, the block's speed changes only because of one component of its weight:

$$\sum F_x = ma_x \quad -mg\sin 20.0° = ma$$
$$v_f^2 = v_i^2 + 2a(x_f - x_i).$$

Taking $v_f = 0$, $v_i = 5.00$ m/s, and $a = -g\sin(20.0°)$ gives

$$0 = (5.00)^2 - 2(9.80)\sin(20.0°)(x_f - 0)$$

or

$$x_f = \frac{25.0}{2(9.80)\sin(20.0°)} = \boxed{3.73 \text{ m}}.$$

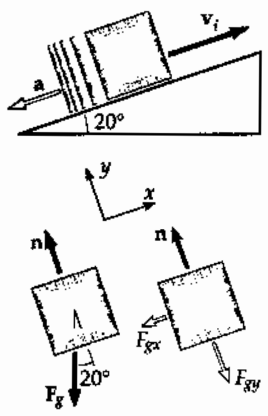


FIG. P5.25

P5.26 $m_1 = 2.00$ kg, $m_2 = 6.00$ kg, $\theta = 55.0°$

(a) $\sum F_x = m_2 g\sin\theta - T = m_2 a$

and

$$T - m_1 g = m_1 a$$
$$a = \frac{m_2 g\sin\theta - m_1 g}{m_1 + m_2} = \boxed{3.57 \text{ m/s}^2}$$

(b) $T = m_1(a+g) = \boxed{26.7 \text{ N}}$

(c) Since $v_i = 0$, $v_f = at = (3.57 \text{ m/s}^2)(2.00 \text{ s}) = \boxed{7.14 \text{ m/s}}$.

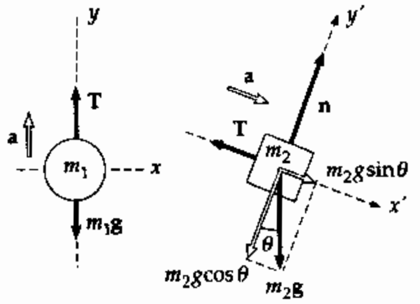


FIG. P5.26

***P5.27** We assume the vertical bar is in compression, pushing up on the pin with force A, and the tilted bar is in tension, exerting force B on the pin at $-50°$.

$$\sum F_x = 0: \quad -2\,500 \text{ N}\cos 30° + B\cos 50° = 0$$
$$\boxed{B = 3.37\times10^3 \text{ N}}$$
$$\sum F_y = 0: \quad -2\,500 \text{ N}\sin 30° + A - 3.37\times10^3 \text{ N}\ \sin 50° = 0$$
$$\boxed{A = 3.83\times10^3 \text{ N}}$$

Positive answers confirm that

$\boxed{B \text{ is in tension and } A \text{ is in compression.}}$

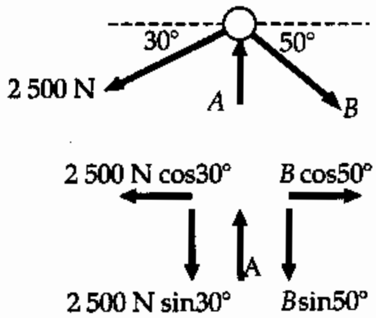


FIG. P5.27

P5.28 First, consider the 3.00 kg rising mass. The forces on it are the tension, T, and its weight, 29.4 N. With the upward direction as positive, the second law becomes

$$\sum F_y = ma_y: T - 29.4\ \text{N} = (3.00\ \text{kg})a \qquad (1)$$

The forces on the falling 5.00 kg mass are its weight and T, and its acceleration is the same as that of the rising mass. Calling the positive direction down for this mass, we have

$$\sum F_y = ma_y: 49\ \text{N} - T = (5.00\ \text{kg})a \qquad (2)$$

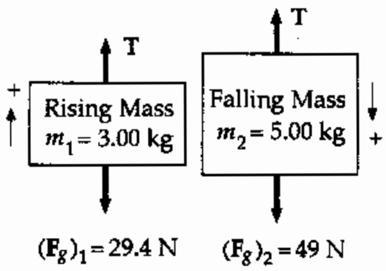


FIG. P5.28

Equations (1) and (2) can be solved simultaneously by adding them:

$$T - 29.4\ \text{N} + 49.0\ \text{N} - T = (3.00\ \text{kg})a + (5.00\ \text{kg})a$$

(b) This gives the acceleration as

$$a = \frac{19.6\ \text{N}}{8.00\ \text{kg}} = \boxed{2.45\ \text{m/s}^2}.$$

(a) Then

$$T - 29.4\ \text{N} = (3.00\ \text{kg})(2.45\ \text{m/s}^2) = 7.35\ \text{N}.$$

The tension is

$$T = \boxed{36.8\ \text{N}}.$$

(c) Consider either mass. We have

$$y = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(2.45\ \text{m/s}^2)(1.00\ \text{s})^2 = \boxed{1.23\ \text{m}}.$$

***P5.29** As the man rises steadily the pulley turns steadily and the tension in the rope is the same on both sides of the pulley. Choose man-pulley-and-platform as the system:

$$\sum F_y = ma_y$$
$$+T - 950\ \text{N} = 0$$
$$T = 950\ \text{N}.$$

The worker must pull on the rope with force $\boxed{950\ \text{N}}$.

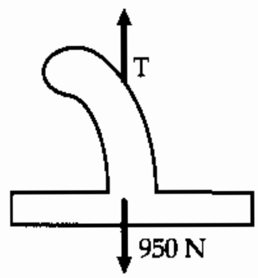


FIG. P5.29

***P5.30** Both blocks move with acceleration $a=\left(\dfrac{m_2-m_1}{m_2+m_1}\right)g$:

$$a=\left(\frac{7\text{ kg}-2\text{ kg}}{7\text{ kg}+2\text{ kg}}\right)9.8\text{ m/s}^2=5.44\text{ m/s}^2.$$

(a) Take the upward direction as positive for m_1.

$$v_{xf}^2=v_{xi}^2+2a_x\left(x_f-x_i\right):\quad 0=(-2.4\text{ m/s})^2+2\left(5.44\text{ m/s}^2\right)\left(x_f-0\right)$$

$$x_f=-\frac{5.76\text{ m}^2/\text{s}^2}{2\left(5.44\text{ m/s}^2\right)}=-0.529\text{ m}$$

$$x_f=\boxed{0.529\text{ m below its initial level}}$$

(b) $v_{xf}=v_{xi}+a_xt$: $v_{xf}=-2.40\text{ m/s}+\left(5.44\text{ m/s}^2\right)(1.80\text{ s})$

$$v_{xf}=\boxed{7.40\text{ m/s upward}}$$

P5.31 Forces acting on 2.00 kg block:

$$T-m_1g=m_1a \qquad (1)$$

Forces acting on 8.00 kg block:

$$F_x-T=m_2a \qquad (2)$$

(a) Eliminate T and solve for a:

$$a=\frac{F_x-m_1g}{m_1+m_2}$$

$$\boxed{a>0\text{ for }F_x>m_1g=19.6\text{ N}}.$$

(b) Eliminate a and solve for T:

$$T=\frac{m_1}{m_1+m_2}\left(F_x+m_2g\right)$$

$$\boxed{T=0\text{ for }F_x\le -m_2g=-78.4\text{ N}}.$$

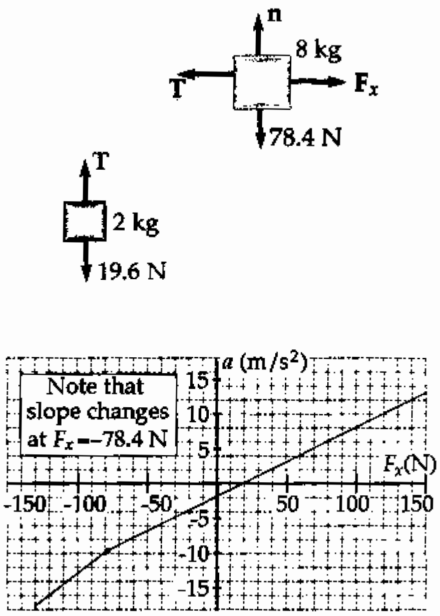


FIG. P5.31

(c)

F_x, N	–100	–78.4	–50.0	0	50.0	100
a_x, m/s^2	–12.5	–9.80	–6.96	–1.96	3.04	8.04

***P5.32** (a) For force components along the incline, with the upward direction taken as positive,

$$\sum F_x = ma_x: \quad -mg\sin\theta = ma_x$$

$$a_x = -g\sin\theta = -\left(9.8\ \text{m/s}^2\right)\sin 35° = -5.62\ \text{m/s}^2.$$

For the upward motion,

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

$$0 = (5\ \text{m/s})^2 + 2\left(-5.62\ \text{m/s}^2\right)\left(x_f - 0\right)$$

$$x_f = \frac{25\ \text{m}^2/\text{s}^2}{2\left(5.62\ \text{m/s}^2\right)} = \boxed{2.22\ \text{m}}.$$

(b) The time to slide down is given by

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$0 = 2.22\ \text{m} + 0 + \frac{1}{2}\left(-5.62\ \text{m/s}^2\right)t^2$$

$$t = \sqrt{\frac{2(2.22\ \text{m})}{5.62\ \text{m/s}^2}} = 0.890\ \text{s}.$$

For the second particle,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$0 = 10\ \text{m} + v_{xi}(0.890\ \text{s}) + \left(-5.62\ \text{m/s}^2\right)(0.890\ \text{s})^2$$

$$v_{xi} = \frac{-10\ \text{m} + 2.22\ \text{m}}{0.890\ \text{s}} = -8.74\ \text{m/s}$$

$$\text{speed} = \boxed{8.74\ \text{m/s}}.$$

P5.33 First, we will compute the needed accelerations:

(1) Before it starts to move: $a_y = 0$

(2) During the first 0.800 s: $a_y = \dfrac{v_{yf} - v_{yi}}{t} = \dfrac{1.20\ \text{m/s} - 0}{0.800\ \text{s}} = 1.50\ \text{m/s}^2$

(3) While moving at constant velocity: $a_y = 0$

(4) During the last 1.50 s: $a_y = \dfrac{v_{yf} - v_{yi}}{t} = \dfrac{0 - 1.20\ \text{m/s}}{1.50\ \text{s}} = -0.800\ \text{m/s}^2$

FIG. P5.33

Newton's second law is: $\sum F_y = ma_y$

$$+S - (72.0\ \text{kg})(9.80\ \text{m/s}^2) = (72.0\ \text{kg})a_y$$

$$S = 706\ \text{N} + (72.0\ \text{kg})a_y.$$

(a) When $a_y = 0$, $S = \boxed{706\ \text{N}}$.

(b) When $a_y = 1.50\ \text{m/s}^2$, $S = \boxed{814\ \text{N}}$.

(c) When $a_y = 0$, $S = \boxed{706\ \text{N}}$.

(d) When $a_y = -0.800\ \text{m/s}^2$, $S = \boxed{648\ \text{N}}$.

P5.34 (a) Pulley P_1 has acceleration a_2.
Since m_1 moves *twice* the distance P_1 moves in the same time, m_1 has twice the acceleration of P_1, i.e., $\boxed{a_1 = 2a_2}$.

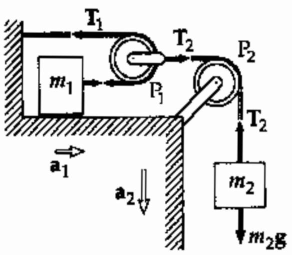


FIG. P5.34

(b) From the figure, and using

$$\sum F = ma: \quad m_2 g - T_2 = m_2 a_2 \quad (1)$$

$$T_1 = m_1 a_1 = 2m_1 a_2 \quad (2)$$

$$T_2 - 2T_1 = 0 \quad (3)$$

Equation (1) becomes $m_2 g - 2T_1 = m_2 a_2$. This equation combined with Equation (2) yields

$$\frac{T_1}{m_1}\left(2m_1 + \frac{m_2}{2}\right) = m_2 g$$

$$\boxed{T_1 = \frac{m_1 m_2}{2m_1 + \frac{1}{2}m_2} g} \text{ and } \boxed{T_2 = \frac{m_1 m_2}{m_1 + \frac{1}{4}m_2} g}.$$

(c) From the values of T_1 and T_2 we find that

$$a_1 = \frac{T_1}{m_1} = \boxed{\frac{m_2 g}{2m_1 + \frac{1}{2}m_2}} \text{ and } a_2 = \frac{1}{2}a_1 = \boxed{\frac{m_2 g}{4m_1 + m_2}}.$$

Section 5.8 **Forces of Friction**

***P5.35**

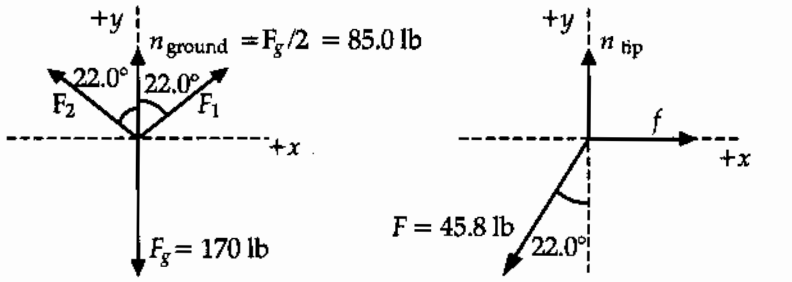


FIG. P5.35

From the free-body diagram of the person,

$$\sum F_x = F_1 \sin(22.0°) - F_2 \sin(22.0°) = 0,$$

which gives

$$F_1 = F_2 = F.$$

Then, $\sum F_y = 2F\cos 22.0° + 85.0 \text{ lbs} - 170 \text{ lbs} = 0$ yields $F = 45.8 \text{ lb}$.

(a) Now consider the free-body diagram of a crutch tip.

$$\sum F_x = f - (45.8 \text{ lb})\sin 22.0° = 0,$$

or

$$f = 17.2 \text{ lb}.$$

$$\sum F_y = n_{tip} - (45.8 \text{ lb})\cos 22.0° = 0,$$

which gives

$$n_{tip} = 42.5 \text{ lb}.$$

For minimum coefficient of friction, the crutch tip will be on the verge of slipping, so $f = (f_s)_{max} = \mu_s n_{tip}$ and $\mu_s = \dfrac{f}{n_{tip}} = \dfrac{17.2 \text{ lb}}{42.5 \text{ lb}} = \boxed{0.404}$.

(b) As found above, the compression force in each crutch is

$$F_1 = F_2 = F = \boxed{45.8 \text{ lb}}.$$

P5.36 For equilibrium: $f = F$ and $n = F_g$. Also, $f = \mu n$ i.e.,

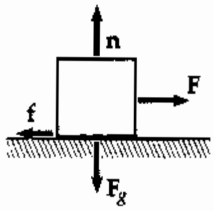


FIG. P5.36

$$\mu = \frac{f}{n} = \frac{F}{F_g}$$

$$\mu_s = \frac{75.0 \text{ N}}{25.0(9.80) \text{ N}} = \boxed{0.306}$$

and

$$\mu_k = \frac{60.0 \text{ N}}{25.0(9.80) \text{ N}} = \boxed{0.245}.$$

P5.37 $\sum F_y = ma_y$: $+n - mg = 0$

$$f_s \le \mu_s n = \mu_s mg$$

This maximum magnitude of static friction acts so long as the tires roll without skidding.

$$\sum F_x = ma_x: \quad -f_s = ma$$

The maximum acceleration is

$$a = -\mu_s g.$$

The initial and final conditions are: $x_i = 0$, $v_i = 50.0 \text{ mi/h} = 22.4 \text{ m/s}$, $v_f = 0$

$$v_f^2 = v_i^2 + 2a(x_f - x_i): \quad -v_i^2 = -2\mu_s g x_f$$

(a) $x_f = \dfrac{v_i^2}{2\mu g}$

$$x_f = \frac{(22.4 \text{ m/s})^2}{2(0.100)(9.80 \text{ m/s}^2)} = \boxed{256 \text{ m}}$$

(b) $x_f = \dfrac{v_i^2}{2\mu g}$

$$x_f = \frac{(22.4 \text{ m/s})^2}{2(0.600)(9.80 \text{ m/s}^2)} = \boxed{42.7 \text{ m}}$$

P5.38 If all the weight is on the rear wheels,

(a) $F = ma$: $\mu_s mg = ma$
But

$$\Delta x = \frac{at^2}{2} = \frac{\mu_s g t^2}{2}$$

so $\mu_s = \frac{2\Delta x}{gt^2}$:

$$\mu_s = \frac{2(0.250 \text{ mi})(1\,609 \text{ m/mi})}{(9.80 \text{ m/s}^2)(4.96 \text{ s})^2} = \boxed{3.34}.$$

(b) Time would increase, as the wheels would skid and only kinetic friction would act; or perhaps the car would flip over.

***P5.39** (a) The person pushes backward on the floor. The floor pushes forward on the person with a force of friction. This is the only horizontal force on the person. If the person's shoe is on the point of slipping the static friction force has its maximum value.

FIG. P5.39

$$\sum F_x = ma_x: \qquad f = \mu_s n = ma_x$$
$$\sum F_y = ma_y: \qquad n - mg = 0$$
$$ma_x = \mu_s mg \qquad a_x = \mu_s g = 0.5(9.8 \text{ m/s}^2) = 4.9 \text{ m/s}^2$$
$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \qquad 3 \text{ m} = 0 + 0 + \frac{1}{2}(4.9 \text{ m/s}^2)t^2$$
$$t = \boxed{1.11 \text{ s}}$$

(b) $x_f = \frac{1}{2}\mu_s g t^2$, $t = \sqrt{\frac{2x_f}{\mu_s g}} = \sqrt{\frac{2(3 \text{ m})}{(0.8)(9.8 \text{ m/s}^2)}} = \boxed{0.875 \text{ s}}$

P5.40 $m_{\text{suitcase}} = 20.0 \text{ kg}$, $F = 35.0 \text{ N}$

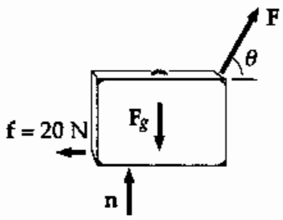


FIG. P5.40

$$\sum F_x = ma_x: \quad -20.0 \text{ N} + F\cos\theta = 0$$
$$\sum F_y = ma_y: \quad +n + F\sin\theta - F_g = 0$$

(a) $F\cos\theta = 20.0 \text{ N}$

$$\cos\theta = \frac{20.0 \text{ N}}{35.0 \text{ N}} = 0.571$$

$$\boxed{\theta = 55.2°}$$

(b) $n = F_g - F\sin\theta = [196 - 35.0(0.821)] \text{ N}$

$$\boxed{n = 167 \text{ N}}$$

P5.41 $m = 3.00\text{ kg}$, $\theta = 30.0°$, $x = 2.00\text{ m}$, $t = 1.50\text{ s}$

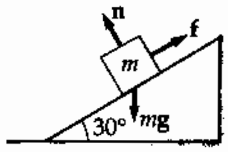


FIG. P5.41

(a) $x = \frac{1}{2}at^2$:

$$2.00\text{ m} = \frac{1}{2}a(1.50\text{ s})^2$$

$$a = \frac{4.00}{(1.50)^2} = \boxed{1.78\text{ m/s}^2}$$

$\sum \mathbf{F} = \mathbf{n} + \mathbf{f} + m\mathbf{g} = m\mathbf{a}$:

Along x: $0 - f + mg\sin 30.0° = ma$

$f = m(g\sin 30.0° - a)$

Along y: $n + 0 - mg\cos 30.0° = 0$

$n = mg\cos 30.0°$

(b) $\mu_k = \frac{f}{n} = \frac{m(g\sin 30.0° - a)}{mg\cos 30.0°}$, $\mu_k = \tan 30.0° - \frac{a}{g\cos 30.0°} = \boxed{0.368}$

(c) $f = m(g\sin 30.0° - a)$, $f = 3.00(9.80\sin 30.0° - 1.78) = \boxed{9.37\text{ N}}$

(d) $v_f^2 = v_i^2 + 2a(x_f - x_i)$

where

$$x_f - x_i = 2.00\text{ m}$$

$$v_f^2 = 0 + 2(1.78)(2.00) = 7.11\text{ m}^2/\text{s}^2$$

$$v_f = \sqrt{7.11\text{ m}^2/\text{s}^2} = \boxed{2.67\text{ m/s}}$$

***P5.42** First we find the coefficient of friction:

$$\sum F_y = 0: \qquad +n - mg = 0$$
$$f = \mu_s n = \mu_s mg$$
$$\sum F_x = ma_x: \qquad v_f^2 = v_i^2 + 2a_x \Delta x = 0$$

$$-\mu_s mg = -\frac{mv_i^2}{2\Delta x}$$

$$\mu_s = \frac{v_i^2}{2g\Delta x} = \frac{(88\ \text{ft/s})^2}{2(32.1\ \text{ft/s}^2)(123\ \text{ft})} = 0.981$$

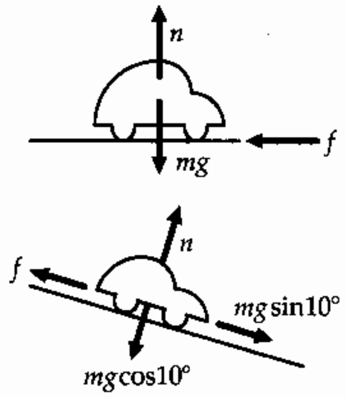


FIG. P5.42

Now on the slope

$$\sum F_y = 0: \qquad +n - mg\cos 10° = 0$$
$$f_s = \mu_s n = \mu_s mg\cos 10°$$
$$\sum F_x = ma_x: \quad -\mu_s mg\cos 10° + mg\sin 10° = -\frac{mv_i^2}{2\Delta x}$$
$$\Delta x = \frac{v_i^2}{2g(\mu_s\cos 10° - \sin 10°)}$$
$$= \frac{(88\ \text{ft/s})^2}{2(32.1\ \text{ft/s}^2)(0.981\cos 10° - \sin 10°)} = \boxed{152\ \text{ft}}.$$

P5.43 $T - f_k = 5.00a$ (for 5.00 kg mass)

$9.00g - T = 9.00a$ (for 9.00 kg mass)

Adding these two equations gives:

$$9.00(9.80) - 0.200(5.00)(9.80) = 14.0a$$
$$a = 5.60\ \text{m/s}^2$$
$$\therefore T = 5.00(5.60) + 0.200(5.00)(9.80)$$
$$= \boxed{37.8\ \text{N}}$$

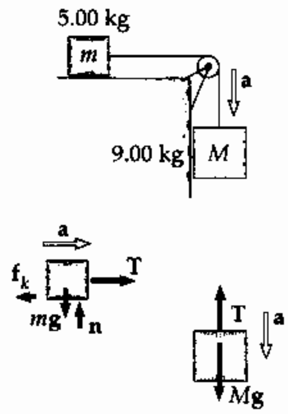


FIG. P5.43

P5.44 Let a represent the positive magnitude of the acceleration $-a\hat{j}$ of m_1, of the acceleration $-a\hat{i}$ of m_2, and of the acceleration $+a\hat{j}$ of m_3. Call T_{12} the tension in the left rope and T_{23} the tension in the cord on the right.

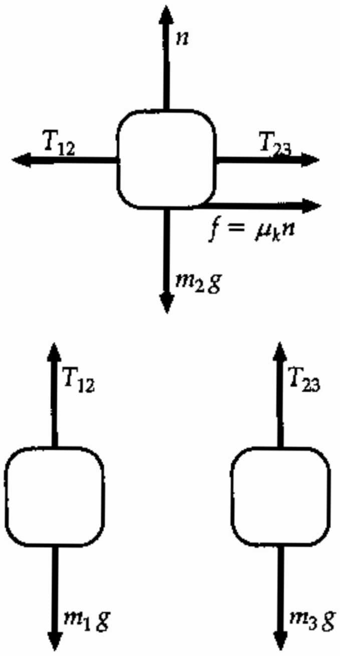


FIG. P5.44

For m_1, $\sum F_y = ma_y$ $\quad +T_{12} - m_1 g = -m_1 a$

For m_2, $\sum F_x = ma_x$ $\quad -T_{12} + \mu_k n + T_{23} = -m_2 a$

and $\sum F_y = ma_y$ $\quad n - m_2 g = 0$

for m_3, $\sum F_y = ma_y$ $\quad T_{23} - m_3 g = +m_3 a$

we have three simultaneous equations

$$-T_{12} + 39.2\text{ N} = (4.00\text{ kg})a$$
$$+T_{12} - 0.350(9.80\text{ N}) - T_{23} = (1.00\text{ kg})a$$
$$+T_{23} - 19.6\text{ N} = (2.00\text{ kg})a.$$

(a) Add them up:

$$+39.2\text{ N} - 3.43\text{ N} - 19.6\text{ N} = (7.00\text{ kg})a$$

$$a = \boxed{2.31\text{ m/s}^2\text{, down for } m_1\text{, left for } m_2\text{, and up for } m_3}.$$

(b) Now $-T_{12} + 39.2\text{ N} = (4.00\text{ kg})(2.31\text{ m/s}^2)$

$$\boxed{T_{12} = 30.0\text{ N}}$$

and $T_{23} - 19.6\text{ N} = (2.00\text{ kg})(2.31\text{ m/s}^2)$

$$\boxed{T_{23} = 24.2\text{ N}}.$$

P5.45 (a) See Figure to the right

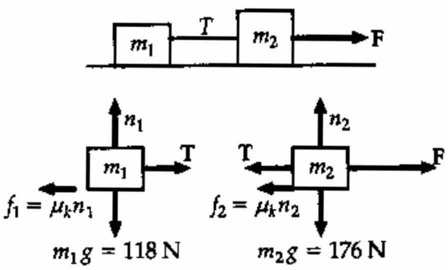


FIG. P5.45

(b) $68.0 - T - \mu m_2 g = m_2 a$ (Block #2)

$T - \mu m_1 g = m_1 a$ (Block #1)

Adding,

$$68.0 - \mu(m_1 + m_2)g = (m_1 + m_2)a$$

$$a = \frac{68.0}{(m_1 + m_2)} - \mu g = \boxed{1.29\text{ m/s}^2}$$

$$T = m_1 a + \mu m_1 g = \boxed{27.2\text{ N}}$$

P5.46 (Case 1, impending upward motion)
Setting

$$\sum F_x = 0: \quad P\cos 50.0° - n = 0$$
$$f_{s,\,max} = \mu_s n: \quad f_{s,\,max} = \mu_s P\cos 50.0° = 0.250(0.643)P = 0.161P$$

Setting

$$\sum F_y = 0: \quad P\sin 50.0° - 0.161P - 3.00(9.80) = 0$$
$$P_{max} = \boxed{48.6\text{ N}}$$

(Case 2, impending downward motion)
As in Case 1,

$$f_{s,\,max} = 0.161P$$

Setting

$$\sum F_y = 0: \quad P\sin 50.0° + 0.161P - 3.00(9.80) = 0$$
$$P_{min} = \boxed{31.7\text{ N}}$$

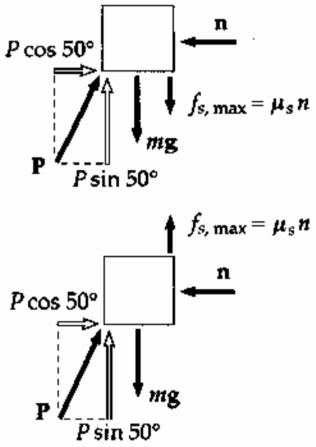


FIG. P5.46

***P5.47** When the sled is sliding uphill

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$f = \mu_k n = \mu_k mg\cos\theta$$
$$\sum F_x = ma_x: \quad +mg\sin\theta + \mu_k mg\cos\theta = ma_{up}$$
$$v_f = 0 = v_i + a_{up}t_{up}$$
$$v_i = -a_{up}t_{up}$$
$$\Delta x = \frac{1}{2}(v_i + v_f)t_{up}$$
$$\Delta x = \frac{1}{2}(a_{up}t_{up} + 0)t_{up} = \frac{1}{2}a_{up}t_{up}^2$$

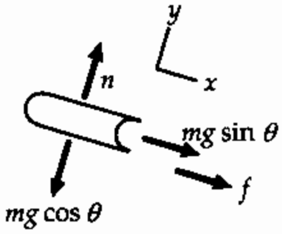


FIG. P5.47

When the sled is sliding down, the direction of the friction force is reversed:

$$mg\sin\theta - \mu_k mg\cos\theta = ma_{down}$$
$$\Delta x = \frac{1}{2}a_{down}t_{down}^2.$$

Now

$$t_{down} = 2t_{up}$$
$$\frac{1}{2}a_{up}t_{up}^2 = \frac{1}{2}a_{down}(2t_{up})^2$$
$$a_{up} = 4a_{down}$$
$$g\sin\theta + \mu_k g\cos\theta = 4(g\sin\theta - \mu_k g\cos\theta)$$
$$5\mu_k\cos\theta = 3\sin\theta$$
$$\boxed{\mu_k = \left(\frac{3}{5}\right)\tan\theta}$$

***P5.48** Since the board is in equilibrium, $\sum F_x = 0$ and we see that the normal forces must be the same on both sides of the board. Also, if the minimum normal forces (compression forces) are being applied, the board is on the verge of slipping and the friction force on each side is

$$f = (f_s)_{\max} = \mu_s n.$$

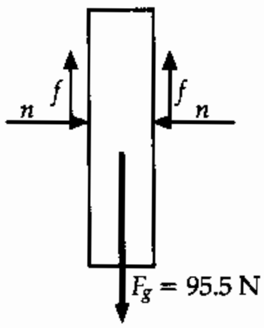


FIG. P5.48

The board is also in equilibrium in the vertical direction, so

$$\sum F_y = 2f - F_g = 0, \text{ or } f = \frac{F_g}{2}.$$

The minimum compression force needed is then

$$n = \frac{f}{\mu_s} = \frac{F_g}{2\mu_s} = \frac{95.5 \text{ N}}{2(0.663)} = \boxed{72.0 \text{ N}}.$$

***P5.49** (a) $n + F\sin 15° - (75 \text{ N})\cos 25° = 0$

$\therefore n = 67.97 - 0.259F$

$f_{s,\max} = \mu_s n = 24.67 - 0.094F$

For equilibrium: $F\cos 15° + 24.67 - 0.094F - 75\sin 25° = 0$.

This gives $\boxed{F = 8.05 \text{ N}}$.

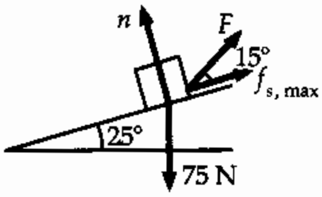


FIG. P5.49(a)

(b) $F\cos 15° - (24.67 - 0.094F) - 75\sin 25° = 0$.

This gives $\boxed{F = 53.2 \text{ N}}$.

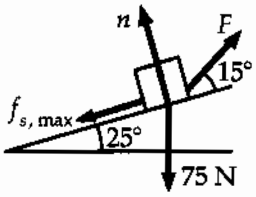


FIG. P5.49(b)

(c) $f_k = \mu_k n = 10.6 - 0.040F$. Since the velocity is constant, the net force is zero:

$$F\cos 15° - (10.6 - 0.040F) - 75\sin 25° = 0.$$

This gives $\boxed{F = 42.0 \text{ N}}$.

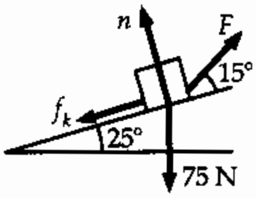


FIG. P5.49(c)

***P5.50** We must consider separately the disk when it is in contact with the roof and when it has gone over the top into free fall. In the first case, we take x and y as parallel and perpendicular to the surface of the roof:

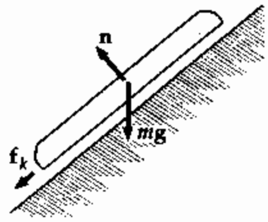


FIG. P5.50

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$n = mg\cos\theta$$

then friction is $f_k = \mu_k n = \mu_k mg\cos\theta$

$$\sum F_x = ma_x: \quad -f_k - mg\sin\theta = ma_x$$
$$a_x = -\mu_k g\cos\theta - g\sin\theta = (-0.4\cos 37° - \sin 37°)9.8\ \text{m/s}^2 = -9.03\ \text{m/s}^2$$

The Frisbee goes ballistic with speed given by

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) = (15\ \text{m/s})^2 + 2(-9.03\ \text{m/s}^2)(10\ \text{m} - 0) = 44.4\ \text{m}^2/\text{s}^2$$
$$v_{xf} = 6.67\ \text{m/s}$$

For the free fall, we take x and y horizontal and vertical:

$$v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$$
$$0 = (6.67\ \text{m/s}\ \sin 37°)^2 + 2(-9.8\ \text{m/s}^2)(y_f - 10\ \text{m}\ \sin 37°)$$
$$y_f = 6.02\ \text{m} + \frac{(4.01\ \text{m/s})^2}{19.6\ \text{m/s}^2} = \boxed{6.84\ \text{m}}$$

Additional Problems

P5.51 (a) see figure to the right

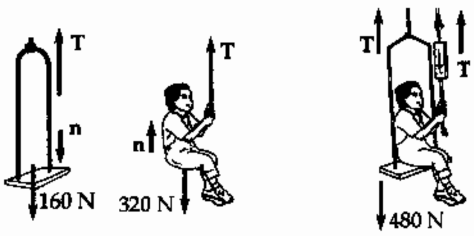


FIG. P5.51

(b) First consider Pat and the chair as the system. Note that *two* ropes support the system, and $T = 250$ N in each rope. Applying $\sum F = ma$

$$2T - 480 = ma, \text{ where } m = \frac{480}{9.80} = 49.0\ \text{kg}.$$

Solving for a gives

$$a = \frac{500 - 480}{49.0} = \boxed{0.408\ \text{m/s}^2}.$$

(c) $\sum F = ma$ on Pat:

$$\sum F = n + T - 320 = ma, \text{ where } m = \frac{320}{9.80} = 32.7\ \text{kg}$$

$$n = ma + 320 - T = 32.7(0.408) + 320 - 250 = \boxed{83.3\ \text{N}}.$$

P5.52 $\sum \mathbf{F} = m\mathbf{a}$ gives the object's acceleration

$$\mathbf{a} = \frac{\sum F}{m} = \frac{(8.00\hat{\mathbf{i}} - 4.00t\hat{\mathbf{j}})\text{ N}}{2.00\text{ kg}}$$

$$\mathbf{a} = (4.00\text{ m/s}^2)\hat{\mathbf{i}} - (2.00\text{ m/s}^3)t\hat{\mathbf{j}} = \frac{d\mathbf{v}}{dt}.$$

Its velocity is

$$\int_{v_i}^{v} d\mathbf{v} = \mathbf{v} - \mathbf{v}_i = \mathbf{v} - 0 = \int_0^t \mathbf{a}\,dt$$

$$\mathbf{v} = \int_0^t \left[(4.00\text{ m/s}^2)\hat{\mathbf{i}} - (2.00\text{ m/s}^3)t\hat{\mathbf{j}}\right]dt$$

$$\mathbf{v} = (4.00t\text{ m/s}^2)\hat{\mathbf{i}} - (1.00t^2\text{ m/s}^3)\hat{\mathbf{j}}.$$

(a) We require $|\mathbf{v}| = 15.0\text{ m/s}$, $|\mathbf{v}|^2 = 225\text{ m}^2/\text{s}^2$

$$16.0t^2\text{ m}^2/\text{s}^4 + 1.00t^4\text{ m}^2/\text{s}^6 = 225\text{ m}^2/\text{s}^2$$

$$1.00t^4 + 16.0\text{ s}^2t^2 - 225\text{ s}^4 = 0$$

$$t^2 = \frac{-16.0 \pm \sqrt{(16.0)^2 - 4(-225)}}{2.00} = 9.00\text{ s}^2$$

$$t = \boxed{3.00\text{ s}}.$$

Take $\mathbf{r}_i = 0$ at $t = 0$. The position is

$$\mathbf{r} = \int_0^t \mathbf{v}\,dt = \int_0^t \left[(4.00t\text{ m/s}^2)\hat{\mathbf{i}} - (1.00t^2\text{ m/s}^3)\hat{\mathbf{j}}\right]dt$$

$$\mathbf{r} = (4.00\text{ m/s}^2)\frac{t^2}{2}\hat{\mathbf{i}} - (1.00\text{ m/s}^3)\frac{t^3}{3}\hat{\mathbf{j}}$$

at $t = 3\text{ s}$ we evaluate.

(c) $\mathbf{r} = \boxed{(18.0\hat{\mathbf{i}} - 9.00\hat{\mathbf{j}})\text{ m}}$

(b) So $|\mathbf{r}| = \sqrt{(18.0)^2 + (9.00)^2}\text{ m} = \boxed{20.1\text{ m}}$

***P5.53** (a) Situation A

$$\sum F_x = ma_x: \quad F_A + \mu_s n - mg\sin\theta = 0$$
$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$

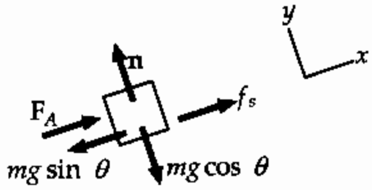


FIG. P5.53(a)

Eliminate $n = mg\cos\theta$ to solve for

$$\boxed{F_A = mg(\sin\theta - \mu_s\cos\theta)}.$$

(b) Situation B

$$\sum F_x = ma_x: \quad F_B\cos\theta + \mu_s n - mg\sin\theta = 0$$
$$\sum F_y = ma_y: \quad -F_B\sin\theta + n - mg\cos\theta = 0$$

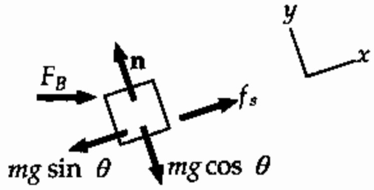


FIG. P5.53(b)

Substitute $n = mg\cos\theta + F_B\sin\theta$ to find

$$F_B\cos\theta + \mu_s mg\cos\theta + \mu_s F_B\sin\theta - mg\sin\theta = 0$$
$$\boxed{F_B = \frac{mg(\sin\theta - \mu_s\cos\theta)}{\cos\theta + \mu_s\sin\theta}}$$

(c) $F_A = 2\text{ kg } 9.8\text{ m/s}^2(\sin 25° - 0.16\cos 25°) = 5.44\text{ N}$

$$F_B = \frac{19.6\text{ N}(0.278)}{\cos 25° + 0.16\sin 25°} = 5.59\text{ N}$$

Student $\boxed{A}$ need exert less force.

(d) $$F_B = \frac{F_A}{\cos 25° + 0.38\sin 25°} = \frac{F_A}{1.07}$$

Student $\boxed{B}$ need exert less force.

P5.54

$$18\text{ N} - P = (2\text{ kg})a$$
$$P - Q = (3\text{ kg})a$$
$$Q = (4\text{ kg})a$$

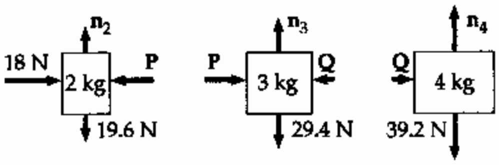


FIG. P5.54

Adding gives $18\text{ N} = (9\text{ kg})a$ so

$$a = \boxed{2.00\text{ m/s}^2}.$$

(b) $Q = 4\text{ kg}(2\text{ m/s}^2) = \boxed{8.00\text{ N net force on the 4 kg}}$

$P - 8\text{ N} = 3\text{ kg}(2\text{ m/s}^2) = \boxed{6.00\text{ N net force on the 3 kg}}$ and $P = 14\text{ N}$

$18\text{ N} - 14\text{ N} = 2\text{ kg}(2\text{ m/s}^2) = \boxed{4.00\text{ N net force on the 2 kg}}$

continued on next page

(c) From above, $Q = \boxed{8.00\text{ N}}$ and $P = \boxed{14.0\text{ N}}$.

(d) The 3-kg block models the heavy block of wood. The contact force on your back is represented by Q, which is much less than the force F. The difference between F and Q is the net force causing acceleration of the 5-kg pair of objects. The acceleration is real and nonzero, but lasts for so short a time that it never is associated with a large velocity. The frame of the building and your legs exert forces, small relative to the hammer blow, to bring the partition, block, and you to rest again over a time large relative to the hammer blow. This problem lends itself to interesting lecture demonstrations. One person can hold a lead brick in one hand while another hits the brick with a hammer.

P5.55 (a) First, we note that $F = T_1$. Next, we focus on the mass M and write $T_5 = Mg$. Next, we focus on the bottom pulley and write $T_5 = T_2 + T_3$. Finally, we focus on the top pulley and write $T_4 = T_1 + T_2 + T_3$.

Since the pulleys are not starting to rotate and are frictionless, $T_1 = T_3$, and $T_2 = T_3$. From this information, we have $T_5 = 2T_2$, so $T_2 = \frac{Mg}{2}$.

Then $\boxed{T_1 = T_2 = T_3 = \frac{Mg}{2}}$, and $\boxed{T_4 = \frac{3Mg}{2}}$, and $\boxed{T_5 = Mg}$.

(b) Since $F = T_1$, we have $\boxed{F = \frac{Mg}{2}}$.

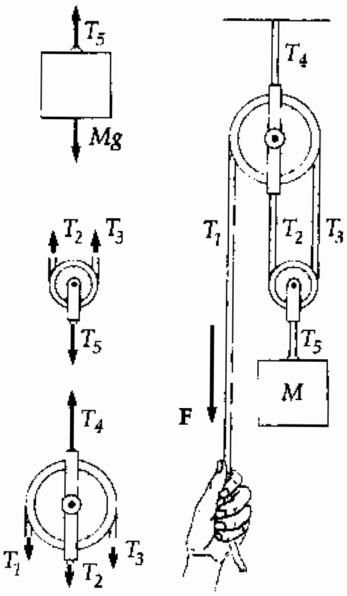


FIG. P5.55

P5.56 We find the diver's impact speed by analyzing his free-fall motion:

$$v_f^2 = v_i^2 + 2ax = 0 + 2(-9.80\text{ m/s}^2)(-10.0\text{ m}) \text{ so } v_f = -14.0\text{ m/s}.$$

Now for the 2.00 s of stopping, we have $v_f = v_i + at$:

$$0 = -14.0\text{ m/s} + a(2.00\text{ s})$$
$$a = +7.00\text{ m/s}^2.$$

Call the force exerted by the water on the diver R. Using $\sum F_y = ma$,

$$+R - 70.0\text{ kg}\left(9.80\text{ m/s}^2\right) = 70.0\text{ kg}\left(7.00\text{ m/s}^2\right)$$
$$R = \boxed{1.18\text{ kN}}.$$

P5.57 (a) The crate is in equilibrium, just before it starts to move. Let the normal force acting on it be n and the friction force, f_s.

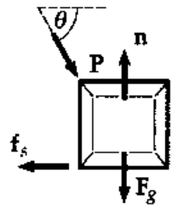


FIG. P5.57

Resolving vertically:

$$n = F_g + P\sin\theta$$

Horizontally:

$$P\cos\theta = f_s$$

But,

$$f_s \le \mu_s n$$

i.e.,

$$P\cos\theta \le \mu_s\left(F_g + P\sin\theta\right)$$

or

$$P(\cos\theta - \mu_s \sin\theta) \le \mu_s F_g .$$

Divide by $\cos\theta$:

$$P(1 - \mu_s \tan\theta) \le \mu_s F_g \sec\theta .$$

Then

$$\boxed{P_{\text{minimum}} = \frac{\mu_s F_g \sec\theta}{1 - \mu_s \tan\theta}} .$$

(b) $P = \dfrac{0.400(100\text{ N})\sec\theta}{1 - 0.400\tan\theta}$

θ(deg)	0.00	15.0	30.0	45.0	60.0
$P(N)$	40.0	46.4	60.1	94.3	260

If the angle were 68.2° or more, the expression for P would go to infinity and motion would become impossible.

P5.58 (a) Following the in-chapter Example about a block on a frictionless incline, we have

$$a = g\sin\theta = (9.80\ \text{m/s}^2)\sin 30.0°$$

$$\boxed{a = 4.90\ \text{m/s}^2}$$

(b) The block slides distance x on the incline, with $\sin 30.0° = \dfrac{0.500\ \text{m}}{x}$

$x = 1.00\ \text{m}$: $v_f^2 = v_i^2 + 2a(x_f - x_i) = 0 + 2(4.90\ \text{m/s}^2)(1.00\ \text{m})$

$v_f = \boxed{3.13\ \text{m/s}}$ after time $t_s = \dfrac{2x_f}{v_f} = \dfrac{2(1.00\ \text{m})}{3.13\ \text{m/s}} = 0.639\ \text{s}$.

(c) Now in free fall $y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2$:

$$-2.00 = (-3.13\ \text{m/s})\sin 30.0°\, t - \frac{1}{2}(9.80\ \text{m/s}^2)t^2$$

$$(4.90\ \text{m/s}^2)t^2 + (1.56\ \text{m/s})t - 2.00\ \text{m} = 0$$

$$t = \frac{-1.56\ \text{m/s} \pm \sqrt{(1.56\ \text{m/s})^2 - 4(4.90\ \text{m/s}^2)(-2.00\ \text{m})}}{9.80\ \text{m/s}^2}$$

Only one root is physical

$$t = 0.499\ \text{s}$$

$$x_f = v_x t = [(3.13\ \text{m/s})\cos 30.0°](0.499\ \text{s}) = \boxed{1.35\ \text{m}}$$

(d) total time $= t_s + t = 0.639\ \text{s} + 0.499\ \text{s} = \boxed{1.14\ \text{s}}$

(e) The mass of the block makes no difference.

P5.59 With motion impending,

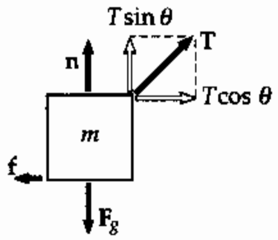


FIG. P5.59

$$n + T\sin\theta - mg = 0$$
$$f = \mu_s(mg - T\sin\theta)$$

and

$$T\cos\theta - \mu_s mg + \mu_s T\sin\theta = 0$$

so

$$T = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}.$$

To minimize T, we maximize $\cos\theta + \mu_s\sin\theta$

$$\frac{d}{d\theta}(\cos\theta + \mu_s\sin\theta) = 0 = -\sin\theta + \mu_s\cos\theta.$$

(a) $\theta = \tan^{-1}\mu_s = \tan^{-1}0.350 = \boxed{19.3°}$

(b) $T = \dfrac{0.350(1.30\text{ kg})(9.80\text{ m/s}^2)}{\cos 19.3° + 0.350\sin 19.3°} = \boxed{4.21\text{ N}}$

***P5.60** (a) See Figure (a) to the right.

(b) See Figure (b) to the right.

$mg = (36.4\text{ kg})(9.8\text{ m/s}^2) = 357\text{ N}$

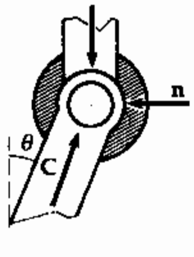


FIG. P5.60(a)

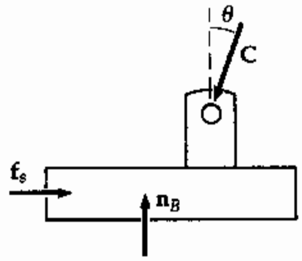


FIG. P5.60(b)

(c) For the pin,

$$\sum F_y = ma_y:\quad C\cos\theta - 357\text{ N} = 0$$
$$C = \frac{357\text{ N}}{\cos\theta}.$$

For the foot,

$$\sum F_y = ma_y:\quad +n_B - C\cos\theta = 0$$
$$n_B = \boxed{357\text{ N}}.$$

(d) For the foot with motion impending,

$$\sum F_x = ma_x:\quad +f_s - C\sin\theta_s = 0$$
$$\mu_s n_B = C\sin\theta_s$$
$$\mu_s = \frac{C\sin\theta_s}{n_B} = \frac{(357\text{ N}/\cos\theta_s)\sin\theta_s}{357\text{ N}} = \tan\theta_s.$$

(e) The maximum coefficient is

$$\mu_s = \tan\theta_s = \tan 50.2° = \boxed{1.20}.$$

P5.61 $\sum F = ma$

For m_1: $T = m_1 a$

For m_2: $T - m_2 g = 0$

Eliminating T,

$$a = \frac{m_2 g}{m_1}$$

For all 3 blocks:

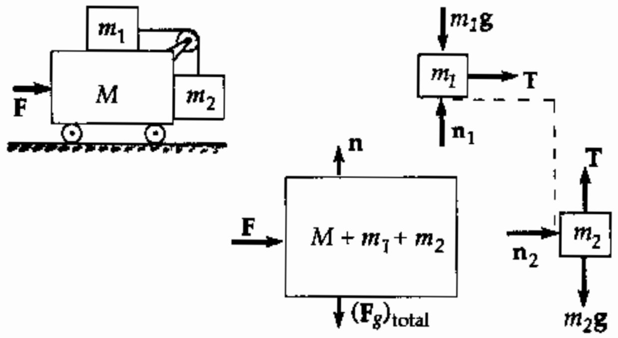


FIG. P5.61

$$F = (M + m_1 + m_2)a = \boxed{(M + m_1 + m_2)\left(\frac{m_2 g}{m_1}\right)}$$

P5.62

t(s)	t^2(s^2)	x(m)
0	0	0
1.02	1.040	0.100
1.53	2.341	0.200
2.01	4.040	0.350
2.64	6.970	0.500
3.30	10.89	0.750
3.75	14.06	1.00

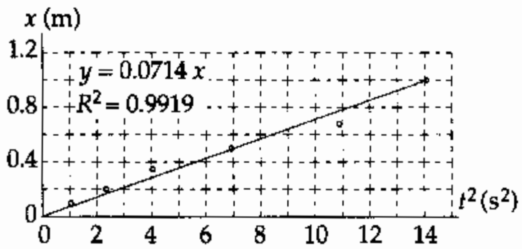


FIG. P5.62

From $x = \frac{1}{2}at^2$ the slope of a graph of x versus t^2 is $\frac{1}{2}a$, and

$$a = 2 \times \text{slope} = 2\left(0.0714 \text{ m/s}^2\right) = \boxed{0.143 \text{ m/s}^2}.$$

From $a' = g \sin\theta$,

$$a' = 9.80 \text{ m/s}^2\left(\frac{1.774}{127.1}\right) = 0.137 \text{ m/s}^2\text{, different by 4\%.}$$

The difference is accounted for by the uncertainty in the data, which we may estimate from the third point as

$$\frac{0.350 - (0.0714)(4.04)}{0.350} = 18\%.$$

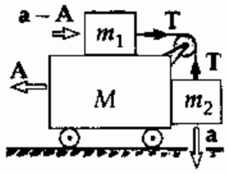


FIG. P5.63

P5.63 (1) $m_1(a-A)=T \Rightarrow a=\dfrac{T}{m_1}+A$

(2) $MA=R_x=T \Rightarrow A=\dfrac{T}{M}$

(3) $m_2 a=m_2 g-T \Rightarrow T=m_2(g-a)$

(a) Substitute the value for a from (1) into (3) and solve for T:

$$T=m_2\left[g-\left(\frac{T}{m_1}+A\right)\right].$$

Substitute for A from (2):

$$T=m_2\left[g-\left(\frac{T}{m_1}+\frac{T}{M}\right)\right]=\boxed{m_2 g\left[\frac{m_1 M}{m_1 M+m_2(m_1+M)}\right]}.$$

(b) Solve (3) for a and substitute value of T:

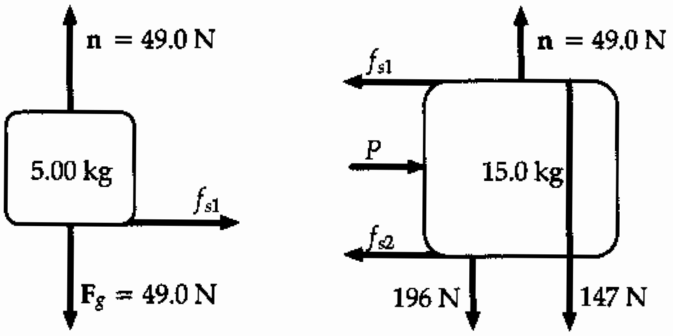
$$\boxed{a=\frac{m_2 g(m_1+M)}{m_1 M+m_2(M+m_1)}}.$$

(c) From (2), $A=\dfrac{T}{M}$, Substitute the value of T:

$$\boxed{A=\frac{m_1 m_2 g}{m_1 M+m_2(m_1+M)}}.$$

(d) $\boxed{a-A=\dfrac{M m_2 g}{m_1 M+m_2(m_1+M)}}$

P5.64 (a), (b) Motion impending



$f_{s1} = \mu n = 14.7$ N $\qquad f_{s2} = 0.500(196\text{ N}) = 98.0$ N

FIG. P5.64

$$P = f_{s1} + f_{s2} = 14.7\text{ N} + 98.0\text{ N} = \boxed{113\text{ N}}$$

(c) Once motion starts, kinetic friction acts.

$$112.7\text{ N} - 0.100(49.0\text{ N}) - 0.400(196\text{ N}) = (15.0\text{ kg})a_2$$

$$a_2 = \boxed{1.96\text{ m/s}^2}$$

$$0.100(49.0\text{ N}) = (5.00\text{ kg})a_1$$

$$a_1 = \boxed{0.980\text{ m/s}^2}$$

***P5.65** (a) Let x represent the position of the glider along the air track. Then $z^2 = x^2 + h_0^2$, $x = \left(z^2 - h_0^2\right)^{1/2}$, $v_x = \frac{dx}{dt} = \frac{1}{2}\left(z^2 - h_0^2\right)^{-1/2}(2z)\frac{dz}{dt}$. Now $\frac{dz}{dt}$ is the rate at which string passes over the pulley, so it is equal to v_y of the counterweight.

$$v_x = z\left(z^2 - h_0^2\right)^{-1/2} v_y = uv_y$$

(b) $a_x = \frac{dv_x}{dt} = \frac{d}{dt}uv_y = u\frac{dv_y}{dt} + v_y\frac{du}{dt}$ at release from rest, $v_y = 0$ and $a_x = ua_y$.

(c) $\sin 30.0° = \frac{80.0\text{ cm}}{z}$, $z = 1.60$ m, $u = \left(z^2 - h_0^2\right)^{-1/2} z = \left(1.6^2 - 0.8^2\right)^{-1/2}(1.6) = 1.15$.

For the counterweight

$$\sum F_y = ma_y: \quad T - 0.5\text{ kg } 9.8\text{ m/s}^2 = -0.5\text{ kg}a_y$$

$$a_y = -2T + 9.8$$

For the glider

$$\sum F_x = ma_x: \quad T\cos 30° = 1.00\text{ kg } a_x = 1.15a_y = 1.15(-2T + 9.8) = -2.31T + 11.3\text{ N}$$

$$3.18T = 11.3\text{ N}$$

$$T = \boxed{3.56\text{ N}}$$

***P5.66** The upward acceleration of the rod is described by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$1\times10^{-3}\text{ m} = 0 + 0 + \frac{1}{2}a_y\left(8\times10^{-3}\text{ s}\right)^2$$

$$a_y = 31.2\text{ m/s}^2$$

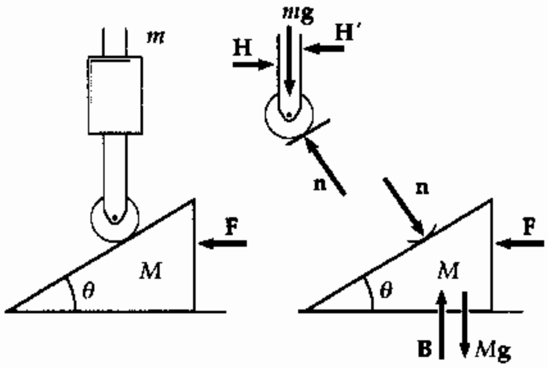


FIG. P5.66

The distance y moved by the rod and the distance x moved by the wedge in the same time are related by $\tan 15° = \frac{y}{x} \Rightarrow x = \frac{y}{\tan 15°}$. Then their speeds and accelerations are related by

$$\frac{dx}{dt} = \frac{1}{\tan 15°}\frac{dy}{dt}$$

and

$$\frac{d^2x}{dt^2} = \frac{1}{\tan 15°}\frac{d^2y}{dt^2} = \left(\frac{1}{\tan 15°}\right)31.2\text{ m/s}^2 = 117\text{ m/s}^2.$$

The free body diagram for the rod is shown. Here H and H' are forces exerted by the guide.

$$\sum F_y = ma_y: \qquad n\cos 15° - mg = ma_y$$

$$n\cos 15° - 0.250\text{ kg}\left(9.8\text{ m/s}^2\right) = 0.250\text{ kg}\left(31.2\text{ m/s}^2\right)$$

$$n = \frac{10.3\text{ N}}{\cos 15°} = 10.6\text{ N}$$

For the wedge,

$$\sum F_x = Ma_x: \quad -n\sin 15° + F = 0.5\text{ kg}\left(117\text{ m/s}^2\right)$$

$$F = (10.6\text{ N})\sin 15° + 58.3\text{ N} = \boxed{61.1\text{ N}}$$

***P5.67** (a) Consider forces on the midpoint of the rope. It is nearly in equilibrium just before the car begins to move. Take the y-axis in the direction of the force you exert:

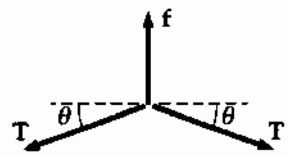


FIG. P5.67

$$\sum F_y = ma_y: \quad -T\sin\theta + f - T\sin\theta = 0$$

$$\boxed{T = \frac{f}{2\sin\theta}}.$$

(b) $T = \frac{100\text{ N}}{2\sin 7°} = \boxed{410\text{ N}}$

P5.68 Since it has a larger mass, we expect the 8.00-kg block to move down the plane. The acceleration for both blocks should have the same magnitude since they are joined together by a non-stretching string. Define up the left hand plane as positive for the 3.50-kg object and down the right hand plane as positive for the 8.00-kg object.

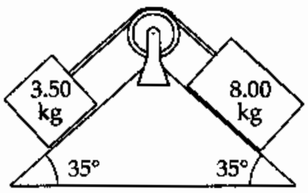


FIG. P5.68

$$\sum F_1 = m_1 a_1: \quad -m_1 g \sin 35.0° + T = m_1 a$$
$$\sum F_2 = m_2 a_2: \quad m_2 g \sin 35.0° - T = m_2 a$$

and

$$-(3.50)(9.80)\sin 35.0° + T = 3.50a$$
$$(8.00)(9.80)\sin 35.0° - T = 8.00a.$$

Adding, we obtain

$$+45.0\text{ N} - 19.7\text{ N} = (11.5\text{ kg})a.$$

(b) Thus the acceleration is

$$\boxed{a = 2.20\text{ m/s}^2}.$$

By substitution,

$$-19.7\text{ N} + T = (3.50\text{ kg})(2.20\text{ m/s}^2) = 7.70\text{ N}.$$

(a) The tension is

$$\boxed{T = 27.4\text{ N}}.$$

P5.69 Choose the x-axis pointing down the slope.

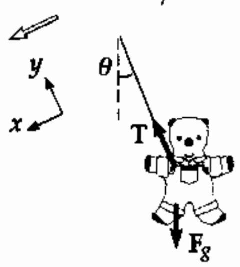


FIG. P5.69

$$v_f = v_i + at: \quad 30.0\text{ m/s} = 0 + a(6.00\text{ s})$$
$$a = 5.00\text{ m/s}^2.$$

Consider forces on the toy.

$$\sum F_x = ma_x: \quad mg\sin\theta = m(5.00\text{ m/s}^2)$$
$$\theta = \boxed{30.7°}$$
$$\sum F_y = ma_y: \quad -mg\cos\theta + T = 0$$
$$T = mg\cos\theta = (0.100)(9.80)\cos 30.7°$$
$$T = \boxed{0.843\text{ N}}$$

***P5.70** Throughout its up and down motion after release the block has

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$n = mg\cos\theta.$$

Let $\mathbf{R} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$ represent the force of table on incline. We have

$$\sum F_x = ma_x: \quad +R_x - n\sin\theta = 0$$
$$R_x = mg\cos\theta\sin\theta$$
$$\sum F_y = ma_y: \quad -Mg - n\cos\theta + R_y = 0$$
$$R_y = Mg + mg\cos^2\theta.$$

$$\boxed{\mathbf{R} = mg\cos\theta\sin\theta \text{ to the right} + \left(M + m\cos^2\theta\right)g \text{ upward}}$$

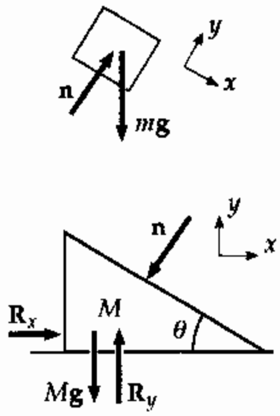


FIG. P5.70

***P5.71** Take $+x$ in the direction of motion of the tablecloth. For the mug:

$$\sum F_x = ma_x \quad 0.1\text{ N} = 0.2\text{ kg}\, a_x$$
$$a_x = 0.5\text{ m/s}^2.$$

Relative to the tablecloth, the acceleration of the mug is $0.5\text{ m/s}^2 - 3\text{ m/s}^2 = -2.5\text{ m/s}^2$. The mug reaches the edge of the tablecloth after time given by

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$
$$-0.3\text{ m} = 0 + \frac{1}{2}\left(-2.5\text{ m/s}^2\right)t^2$$
$$t = 0.490\text{ s}.$$

The motion of the mug relative to tabletop is over distance

$$\frac{1}{2}a_x t^2 = \frac{1}{2}\left(0.5\text{ m/s}^2\right)(0.490\text{ s})^2 = \boxed{0.060\,0\text{ m}}.$$

The tablecloth slides 36 cm over the table in this process.

P5.72 $\sum F_y = ma_y: n - mg\cos\theta = 0$

or

$$n = 8.40(9.80)\cos\theta$$
$$n = (82.3\text{ N})\cos\theta$$

$\sum F_x = ma_x: mg\sin\theta = ma$

or

$$a = g\sin\theta$$
$$a = \left(9.80\text{ m/s}^2\right)\sin\theta$$

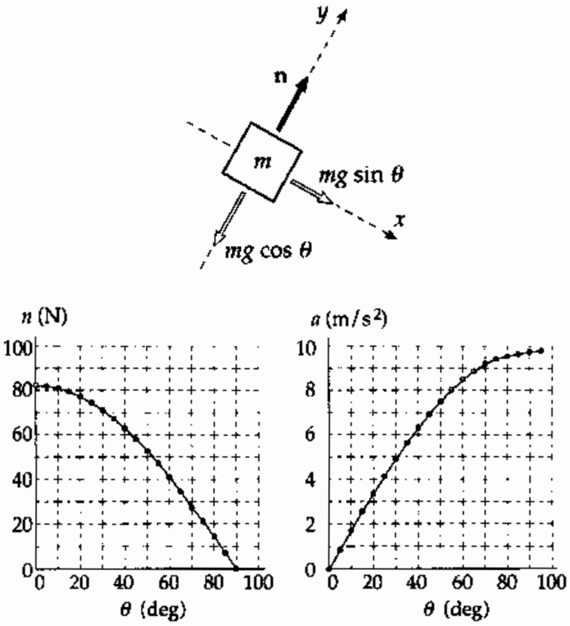


FIG. P5.72

θ, deg	n, N	a, m/s^2
0.00	82.3	0.00
5.00	82.0	0.854
10.0	81.1	1.70
15.0	79.5	2.54
20.0	77.4	3.35
25.0	74.6	4.14
30.0	71.3	4.90
35.0	67.4	5.62
40.0	63.1	6.30
45.0	58.2	6.93
50.0	52.9	7.51
55.0	47.2	8.03
60.0	41.2	8.49
65.0	34.8	8.88
70.0	28.2	9.21
75.0	21.3	9.47
80.0	14.3	9.65
85.0	7.17	9.76
90.0	0.00	9.80

At 0°, the normal force is the full weight and the acceleration is zero. At 90°, the mass is in free fall next to the vertical incline.

P5.73 (a) Apply Newton's second law to two points where butterflies are attached on either half of mobile (other half the same, by symmetry)

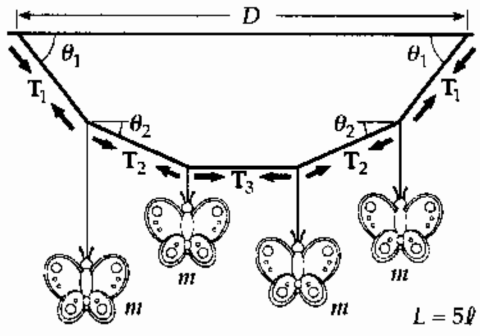


FIG. P5.69

(1) $T_2\cos\theta_2 - T_1\cos\theta_1 = 0$
(2) $T_1\sin\theta_1 - T_2\sin\theta_2 - mg = 0$
(3) $T_2\cos\theta_2 - T_3 = 0$
(4) $T_2\sin\theta_2 - mg = 0$

Substituting (4) into (2) for $T_2\sin\theta_2$,

$$T_1\sin\theta_1 - mg - mg = 0.$$

Then

$$\boxed{T_1 = \frac{2mg}{\sin\theta_1}}.$$

Substitute (3) into (1) for $T_2\cos\theta_2$:

$$T_3 - T_1\cos\theta_1 = 0,\ T_3 = T_1\cos\theta_1$$

Substitute value of T_1:

$$T_3 = 2mg\frac{\cos\theta_1}{\sin\theta_1} = \boxed{\frac{2mg}{\tan\theta_1} = T_3}.$$

From Equation (4),

$$T_2 = \frac{mg}{\sin\theta_2}.$$

(b) Divide (4) by (3):

$$\frac{T_2\sin\theta_2}{T_2\cos\theta_2} = \frac{mg}{T_3}.$$

Substitute value of T_3:

$$\tan\theta_2 = \frac{mg\tan\theta_1}{2mg},\ \boxed{\theta_2 = \tan^{-1}\left(\frac{\tan\theta_1}{2}\right)}.$$

Then we can finish answering part (a):

$$\boxed{T_2 = \frac{mg}{\sin\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right]}}.$$

(c) D is the horizontal distance between the points at which the two ends of the string are attached to the ceiling.

$$D = 2\ell\cos\theta_1 + 2\ell\cos\theta_2 + \ell \text{ and } L = 5\ell$$

$$D = \frac{L}{5}\left\{2\cos\theta_1 + 2\cos\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right] + 1\right\}$$

ANSWERS TO EVEN PROBLEMS

P5.2 1.66×10^{6} N forward

P5.4 (a) $\frac{vt}{2}$; (b) $\left(\frac{F_g v}{gt}\right)\hat{\mathbf{i}}+F_g\hat{\mathbf{j}}$

P5.6 (a) 4.47×10^{15} m/s^2 away from the wall; (b) 2.09×10^{-10} N toward the wall

P5.8 (a) 534 N down; (b) 54.5 kg

P5.10 2.55 N for an 88.7 kg person

P5.12 $(16.3\hat{\mathbf{i}}+14.6\hat{\mathbf{j}})$ N

P5.14 (a) 181°; (b) 11.2 kg; (c) 37.5 m/s; (d) $(-37.5\hat{\mathbf{i}}-0.893\hat{\mathbf{j}})$ m/s

P5.16 112 N

P5.18 $T_1=296$ N; $T_2=163$ N; $T_3=325$ N

P5.20 (a) see the solution; (b) 1.79 N

P5.22 (a) 2.54 m/s^2 down the incline; (b) 3.18 m/s

P5.24 see the solution; 6.30 m/s^2; 31.5 N

P5.26 (a) 3.57 m/s^2; (b) 26.7 N; (c) 7.14 m/s

P5.28 (a) 36.8 N; (b) 2.45 m/s^2; (c) 1.23 m

P5.30 (a) 0.529 m; (b) 7.40 m/s upward

P5.32 (a) 2.22 m; (b) 8.74 m/s

P5.34 (a) $a_1=2a_2$;
(b) $T_1=\frac{m_1m_2g}{2m_1+\frac{m_2}{2}}$; $T_2=\frac{m_1m_2g}{m_1+\frac{m_2}{4}}$;
(c) $a_1=\frac{m_2g}{2m_1+\frac{m_2}{2}}$; $a_2=\frac{m_2g}{4m_1+m_2}$

P5.36 $\mu_s=0.306$; $\mu_k=0.245$

P5.38 (a) 3.34; (b) Time would increase

P5.40 (a) 55.2°; (b) 167 N

P5.42 152 ft

P5.44 (a) 2.31 m/s^2 down for m_1, left for m_2 and up for m_3; (b) 30.0 N and 24.2 N

P5.46 Any value between 31.7 N and 48.6 N

P5.48 72.0 N

P5.50 6.84 m

P5.52 (a) 3.00 s; (b) 20.1 m; (c) $(18.0\hat{\mathbf{i}}-9.00\hat{\mathbf{j}})$ m

P5.54 (a) 2.00 m/s^2 to the right;
(b) 8.00 N right on 4 kg;
6.00 N right on 3 kg; 4 N right on 2 kg;
(c) 8.00 N between 4 kg and 3 kg;
14.0 N between 2 kg and 3 kg;
(d) see the solution

P5.56 1.18 kN

P5.58 (a) 4.90 m/s^2; (b) 3.13 m/s at 30.0° below the horizontal; (c) 1.35 m; (d) 1.14 s; (e) No

P5.60 (a) and (b) see the solution; (c) 357 N; (d) see the solution; (e) 1.20

P5.62 see the solution; 0.143 m/s^2 agrees with 0.137 m/s^2

P5.64 (a) see the solution;
(b) on block one:
$49.0\text{ N}\,\hat{\mathbf{j}}-49.0\text{ N}\,\hat{\mathbf{j}}+14.7\text{ N}\,\hat{\mathbf{i}}$;
on block two: $-49.0\text{ N}\,\hat{\mathbf{j}}-14.7\text{ N}\,\hat{\mathbf{i}}-147\text{ N}\,\hat{\mathbf{j}}$ $+196\text{ N}\,\hat{\mathbf{j}}-98.0\text{ N}\,\hat{\mathbf{i}}+113\text{ N}\,\hat{\mathbf{i}}$;
(c) for block one: $0.980\hat{\mathbf{i}}$ m/s^2;
for block two: 1.96 $\text{m/s}^2\,\hat{\mathbf{i}}$

P5.66 61.1 N

P5.68 (a) 2.20 m/s^2; (b) 27.4 N

P5.70 $mg\cos\theta\sin\theta$ to the right $+(M+m\cos^2\theta)g$ upward

P5.72 see the solution

Circular Motion and Other Applications of Newton's Laws

ANSWERS TO QUESTIONS

Q6.1 Mud flies off a rapidly spinning tire because the resultant force is not sufficient to keep it moving in a circular path. In this case, the force that plays a major role is the adhesion between the mud and the tire.

Q6.2 The spring will stretch. In order for the object to move in a circle, the force exerted on the object by the spring must have a size of $\frac{mv^2}{r}$. Newton's third law says that the force exerted on the object by the spring has the same size as the force exerted by the object on the spring. It is the force exerted on the spring that causes the spring to stretch.

Q6.3 Driving in a circle at a constant speed requires a centripetal acceleration but no tangential acceleration.

Q6.4 (a) The object will move in a circle at a constant speed.

(b) The object will move in a straight line at a changing speed.

Q6.5 The speed changes. The tangential force component causes tangential acceleration.

Q6.6 Consider the force required to keep a rock in the Earth's crust moving in a circle. The size of the force is proportional to the radius of the circle. If that rock is at the Equator, the radius of the circle through which it moves is about 6400 km. If the rock is at the north pole, the radius of the circle through which it moves is zero!

Q6.7 Consider standing on a bathroom scale. The resultant force on you is your actual weight minus the normal force. The scale reading shows the size of the normal force, and is your 'apparent weight.' If you are at the North or South Pole, it can be precisely equal to your actual weight. If you are at the equator, your apparent weight must be less, so that the resultant force on you can be a downward force large enough to cause your centripetal acceleration as the Earth rotates.

Q6.8 A torque is exerted by the thrust force of the water times the distance between the nozzles.

Q6.9 I would not accept that statement for two reasons. First, to be "beyond the pull of gravity," one would have to be infinitely far away from all other matter. Second, astronauts in orbit are moving in a circular path. It is the gravitational pull of Earth on the astronauts that keeps them in orbit. In the space shuttle, just above the atmosphere, gravity is only slightly weaker than at the Earth's surface. Gravity does its job most clearly on an orbiting spacecraft, because the craft feels no other forces and is in free fall.

Q6.10 This is the same principle as the centrifuge. All the material inside the cylinder tends to move along a straight-line path, but the walls of the cylinder exert an inward force to keep everything moving around in a circular path.

Q6.11 The ball would not behave as it would when dropped on the Earth. As the astronaut holds the ball, she and the ball are moving with the same angular velocity. The ball, however, being closer to the center of rotation, is moving with a slower tangential velocity. Once the ball is released, it acts according to Newton's first law, and simply drifts with constant velocity in the original direction of its velocity when released—it is no longer "attached" to the rotating space station. Since the ball follows a straight line and the astronaut follows a circular path, it will appear to the astronaut that the ball will "fall to the floor". But other dramatic effects will occur. Imagine that the ball is held so high that it is just slightly away from the center of rotation. Then, as the ball is released, it will move very slowly along a straight line. Thus, the astronaut may make several full rotations around the circular path before the ball strikes the floor. This will result in three obvious variations with the Earth drop. First, the time to fall will be much larger than that on the Earth, even though the feet of the astronaut are pressed into the floor with a force that suggests the same force of gravity as on Earth. Second, the ball may actually appear to bob up and down if several rotations are made while it "falls". As the ball moves in a straight line while the astronaut rotates, sometimes she is on the side of the circle on which the ball is moving toward her and other times she is on the other side, where the ball is moving away from her. The third effect is that the ball will not drop straight down to her feet. In the extreme case we have been imagining, it may actually strike the surface while she is on the opposite side, so it looks like it ended up "falling up". In the least extreme case, in which only a portion of a rotation is made before the ball strikes the surface, the ball will appear to move backward relative to the astronaut as it falls.

Q6.12 The water has inertia. The water tends to move along a straight line, but the bucket pulls it in and around in a circle.

Q6.13 There is no such force. If the passenger slides outward across the slippery car seat, it is because the passenger is moving forward in a straight line while the car is turning under him. If the passenger pushes hard against the outside door, the door is exerting an inward force on him. No object is exerting an outward force on him, but he should still buckle his seatbelt.

Q6.14 Blood pressure cannot supply the force necessary both to balance the gravitational force and to provide the centripetal acceleration, to keep blood flowing up to the pilot's brain.

Q6.15 The person in the elevator is in an accelerating reference frame. The apparent acceleration due to gravity, "g," is changed inside the elevator. "g" $= g \pm a$

Q6.16 When you are not accelerating, the normal force and your weight are equal in size. Your body interprets the force of the floor pushing up on you as your weight. When you accelerate in an elevator, this normal force changes so that you accelerate with the elevator. In free fall, you are never weightless since the Earth's gravity and your mass do not change. It is the normal force—your apparent weight—that is zero.

Q6.17 From the proportionality of the drag force to the speed squared and from Newton's second law, we derive the equation that describes the motion of the skydiver:

$$m\frac{dv_y}{dt}=mg-\frac{D\rho A}{2}v_y^2$$

where D is the coefficient of drag of the parachutist, and A is the projected area of the parachutist's body. At terminal speed,

$$a_y=\frac{dv_y}{dt}=0 \text{ and } V_T\left(\frac{2mg}{D\rho A}\right)^{1/2}.$$

When the parachute opens, the coefficient of drag D and the effective area A both increase, thus reducing the speed of the skydiver.

Modern parachutes also add a third term, lift, to change the equation to

$$m\frac{dv_y}{dt}=mg-\frac{D\rho A}{2}v_y^2-\frac{L\rho A}{2}v_x^2$$

where v_y is the vertical velocity, and v_x is the horizontal velocity. The effect of lift is clearly seen in the "paraplane," an ultralight airplane made from a fan, a chair, and a parachute.

Q6.18 The larger drop has higher terminal speed. In the case of spheres, the text demonstrates that terminal speed is proportional to the square root of radius. When moving with terminal speed, an object is in equilibrium and has zero acceleration.

Q6.19 Lower air density reduces air resistance, so a tank-truck-load of fuel takes you farther.

Q6.20 Suppose the rock is moving rapidly when it enters the water. The speed of the rock decreases until it reaches terminal velocity. The acceleration, which is upward, decreases to zero as the rock approaches terminal velocity.

Q6.21 The thesis is false. The moment of decay of a radioactive atomic nucleus (for example) cannot be predicted. Quantum mechanics implies that the future is indeterminate. On the other hand, our sense of free will, of being able to make choices for ourselves that can appear to be random, may be an illusion. It may have nothing to do with the subatomic randomness described by quantum mechanics.

Section 6.1 Newton's Second Law Applied to Uniform Circular Motion

P6.1 $m = 3.00\text{ kg}$, $r = 0.800\text{ m}$. The string will break if the tension exceeds the weight corresponding to 25.0 kg, so

$$T_{\max} = Mg = 25.0(9.80) = 245\text{ N}.$$

When the 3.00 kg mass rotates in a horizontal circle, the tension causes the centripetal acceleration,

so $$T = \frac{mv^2}{r} = \frac{(3.00)v^2}{0.800}.$$

Then $$v^2 = \frac{rT}{m} = \frac{(0.800)T}{3.00} \le \frac{(0.800)T_{\max}}{3.00} = \frac{0.800(245)}{3.00} = 65.3\text{ m}^2/\text{s}^2$$

and $$0 \le v \le \sqrt{65.3}$$

or $$\boxed{0 \le v \le 8.08\text{ m/s}}.$$

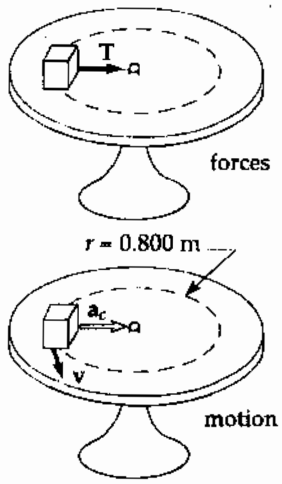


FIG. P6.1

P6.2 In $\sum F = m\frac{v^2}{r}$, both m and r are unknown but remain constant. Therefore, $\sum F$ is proportional to v^2 and increases by a factor of $\left(\frac{18.0}{14.0}\right)^2$ as v increases from 14.0 m/s to 18.0 m/s. The total force at the higher speed is then

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 (130\text{ N}) = 215\text{ N}.$$

Symbolically, write $\sum F_{\text{slow}} = \left(\frac{m}{r}\right)(14.0\text{ m/s})^2$ and $\sum F_{\text{fast}} = \left(\frac{m}{r}\right)(18.0\text{ m/s})^2$.

Dividing gives $\frac{\sum F_{\text{fast}}}{\sum F_{\text{slow}}} = \left(\frac{18.0}{14.0}\right)^2$, or

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 \sum F_{\text{slow}} = \left(\frac{18.0}{14.0}\right)^2 (130\text{ N}) = \boxed{215\text{ N}}.$$

This force must be $\boxed{\text{horizontally inward}}$ to produce the driver's centripetal acceleration.

P6.3 (a) $F=\frac{mv^2}{r}=\frac{(9.11\times10^{-31}\text{ kg})(2.20\times10^6\text{ m/s})^2}{0.530\times10^{-10}\text{ m}}=\boxed{8.32\times10^{-8}\text{ N inward}}$

(b) $a=\frac{v^2}{r}=\frac{(2.20\times10^6\text{ m/s})^2}{0.530\times10^{-10}\text{ m}}=\boxed{9.13\times10^{22}\text{ m/s}^2\text{ inward}}$

P6.4 Neglecting relativistic effects. $F=ma_c=\frac{mv^2}{r}$

$$F=(2\times1.661\times10^{-27}\text{ kg})\frac{(2.998\times10^7\text{ m/s})^2}{(0.480\text{ m})}=\boxed{6.22\times10^{-12}\text{ N}}$$

P6.5 (a) $\boxed{\text{static friction}}$

(b) $ma\hat{\mathbf{i}}=f\hat{\mathbf{i}}+n\hat{\mathbf{j}}+mg(-\hat{\mathbf{j}})$

$\sum F_y=0=n-mg$

thus $n=mg$ and $\sum F_r=m\frac{v^2}{r}=f=\mu n=\mu mg$.

Then $\mu=\frac{v^2}{rg}=\frac{(50.0\text{ cm/s})^2}{(30.0\text{ cm})(980\text{ cm/s}^2)}=\boxed{0.085\,0}$.

P6.6 (a) $\sum F_y=ma_y$, mg_{moon} down $=\frac{mv^2}{r}$ down

$v=\sqrt{g_{\text{moon}}r}=\sqrt{(1.52\text{ m/s}^2)(1.7\times10^6\text{ m}+100\times10^3\text{ m})}=\boxed{1.65\times10^3\text{ m/s}}$

(b) $v=\frac{2\pi r}{T}$, $T=\frac{2\pi(1.8\times10^6\text{ m})}{1.65\times10^3\text{ m/s}}=\boxed{6.84\times10^3\text{ s}}=1.90\text{ h}$

P6.7 $n=mg$ since $a_y=0$

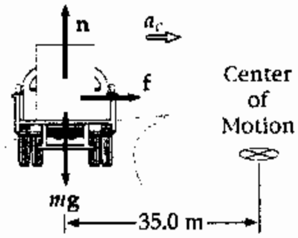


FIG. P6.7

The force causing the centripetal acceleration is the frictional force f.

From Newton's second law $f=ma_c=\frac{mv^2}{r}$.

But the friction condition is $f\le\mu_s n$

i.e., $\frac{mv^2}{r}\le\mu_s mg$

$v\le\sqrt{\mu_s rg}=\sqrt{0.600(35.0\text{ m})(9.80\text{ m/s}^2)}$ $\quad v\le\boxed{14.3\text{ m/s}}$

P6.8 $$a=\frac{v^2}{r}=\frac{\left[(86.5\ \text{km/h})\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1\,000\ \text{m}}{1\ \text{km}}\right)\right]^2}{61.0\ \text{m}}\left(\frac{1\ g}{9.80\ \text{m/s}^2}\right)=\boxed{0.966g}$$

P6.9 $$T\cos 5.00°=mg=(80.0\ \text{kg})(9.80\ \text{m/s}^2)$$

(a) $T=787\ \text{N}$: $\mathbf{T}=\boxed{(68.6\ \text{N})\hat{\mathbf{i}}+(784\ \text{N})\hat{\mathbf{j}}}$

(b) $T\sin 5.00°=ma_c$: $\boxed{a_c=0.857\ \text{m/s}^2}$ toward the center of the circle.

The length of the wire is unnecessary information. We could, on the other hand, use it to find the radius of the circle, the speed of the bob, and the period of the motion.

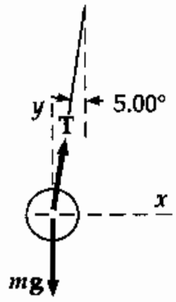


FIG. P6.9

P6.10 (b) $$v=\frac{235\ \text{m}}{36.0\ \text{s}}=\boxed{6.53\ \text{m/s}}$$

The radius is given by $\frac{1}{4}2\pi r=235\ \text{m}$

$$r=150\ \text{m}$$

(a) $$\mathbf{a}_r=\left(\frac{v^2}{r}\right)\text{ toward center}$$

$$=\frac{(6.53\ \text{m/s})^2}{150\ \text{m}}\text{ at }35.0°\text{ north of west}$$

$$=(0.285\ \text{m/s}^2)\left(\cos 35.0°(-\hat{\mathbf{i}})+\sin 35.0°\,\hat{\mathbf{j}}\right)$$

$$=\boxed{-0.233\ \text{m/s}^2\,\hat{\mathbf{i}}+0.163\ \text{m/s}^2\,\hat{\mathbf{j}}}$$

(c) $$\bar{\mathbf{a}}=\frac{(\mathbf{v}_f-\mathbf{v}_i)}{t}$$

$$=\frac{(6.53\ \text{m/s}\,\hat{\mathbf{j}}-6.53\ \text{m/s}\,\hat{\mathbf{i}})}{36.0\ \text{s}}$$

$$=\boxed{-0.181\ \text{m/s}^2\,\hat{\mathbf{i}}+0.181\ \text{m/s}^2\,\hat{\mathbf{j}}}$$

***P6.11** $F_g = mg = (4\text{ kg})(9.8\text{ m/s}^2) = 39.2\text{ N}$

$\sin\theta = \dfrac{1.5\text{ m}}{2\text{ m}}$

$\theta = 48.6°$

$r = (2\text{ m})\cos 48.6° = 1.32\text{ m}$

$\sum F_x = ma_x = \dfrac{mv^2}{r}$

$T_a \cos 48.6° + T_b \cos 48.6° = \dfrac{(4\text{ kg})(6\text{ m/s})^2}{1.32\text{ m}}$

$T_a + T_b = \dfrac{109\text{ N}}{\cos 48.6°} = 165\text{ N}$

$\sum F_y = ma_y$

$+T_a \sin 48.6° - T_b \sin 48.6° - 39.2\text{ N} = 0$

$T_a - T_b = \dfrac{39.2\text{ N}}{\sin 48.6°} = 52.3\text{ N}$

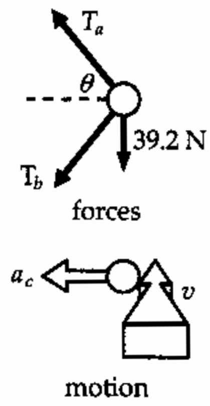


FIG. P6.11

(a) To solve simultaneously, we add the equations in T_a and T_b:

$$T_a + T_b + T_a - T_b = 165\text{ N} + 52.3\text{ N}$$

$$T_a = \frac{217\text{ N}}{2} = \boxed{108\text{ N}}$$

(b) $T_b = 165\text{ N} - T_a = 165\text{ N} - 108\text{ N} = \boxed{56.2\text{ N}}$

***P6.12** $a_c = \dfrac{v^2}{r}$. Let f represent the rotation rate. Each revolution carries each bit of metal through distance $2\pi r$, so $v = 2\pi r f$ and

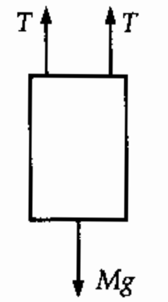
$$a_c = \frac{v^2}{r} = 4\pi^2 r f^2 = 100\,g.$$

A smaller radius implies smaller acceleration. To meet the criterion for each bit of metal we consider the minimum radius:

$$f = \left(\frac{100\,g}{4\pi^2 r}\right)^{1/2} = \left(\frac{100 \cdot 9.8\text{ m/s}^2}{4\pi^2(0.021\text{ m})}\right)^{1/2} = 34.4\ \frac{1}{\text{s}}\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{2.06 \times 10^3\text{ rev/min}}.$$

Section 6.2 **Nonuniform Circular Motion**

P6.13 $M = 40.0\text{ kg},\ R = 3.00\text{ m},\ T = 350\text{ N}$



child + seat

FIG. P6.13(a)

child alone

FIG. P6.13(b)

(a) $$\sum F = 2T - Mg = \frac{Mv^2}{R}$$

$$v^2 = (2T - Mg)\left(\frac{R}{M}\right)$$

$$v^2 = [700 - (40.0)(9.80)]\left(\frac{3.00}{40.0}\right) = 23.1\ (\text{m}^2/\text{s}^2)$$

$$\boxed{v = 4.81\ \text{m/s}}$$

(b) $$n - Mg = F = \frac{Mv^2}{R}$$

$$n = Mg + \frac{Mv^2}{R} = 40.0\left(9.80 + \frac{23.1}{3.00}\right) = \boxed{700\text{ N}}$$

P6.14 (a) Consider the forces acting on the system consisting of the child *and* the seat:

$$\sum F_y = ma_y \Rightarrow 2T - mg = m\frac{v^2}{R}$$

$$v^2 = R\left(\frac{2T}{m} - g\right)$$

$$v = \boxed{\sqrt{R\left(\frac{2T}{m} - g\right)}}$$

(b) Consider the forces acting on the child alone:

$$\sum F_y = ma_y \Rightarrow n = m\left(g + \frac{v^2}{R}\right)$$

and from above, $v^2 = R\left(\frac{2T}{m} - g\right)$, so

$$n = m\left(g + \frac{2T}{m} - g\right) = \boxed{2T}.$$

P6.15 Let the tension at the lowest point be T.

$$\sum F = ma:\quad T - mg = ma_c = \frac{mv^2}{r}$$

$$T = m\left(g + \frac{v^2}{r}\right)$$

$$T = (85.0\text{ kg})\left[9.80\ \text{m/s}^2 + \frac{(8.00\ \text{m/s})^2}{10.0\text{ m}}\right] = 1.38\text{ kN} > 1\,000\text{ N}$$

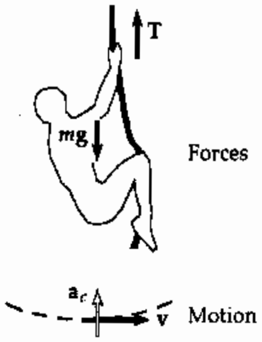


FIG. P6.15

He doesn't make it across the river because the vine breaks.

P6.16 (a) $a_c = \frac{v^2}{r} = \frac{(4.00\ \text{m/s})^2}{12.0\ \text{m}} = \boxed{1.33\ \text{m/s}^2}$

(b) $a = \sqrt{a_c^2 + a_t^2}$

$a = \sqrt{(1.33)^2 + (1.20)^2} = \boxed{1.79\ \text{m/s}^2}$

at an angle $\theta = \tan^{-1}\left(\frac{a_c}{a_t}\right) = \boxed{48.0^\circ\ \text{inward}}$

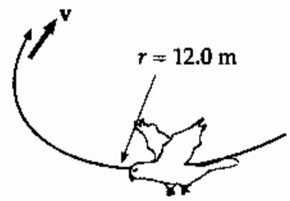


FIG. P6.16

P6.17 $\sum F_y = \frac{mv^2}{r} = mg + n$

But $n = 0$ at this minimum speed condition, so

$\frac{mv^2}{r} = mg \Rightarrow v = \sqrt{gr} = \sqrt{(9.80\ \text{m/s}^2)(1.00\ \text{m})} = \boxed{3.13\ \text{m/s}}$.

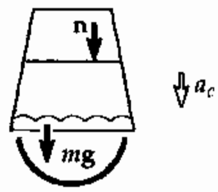


FIG. P6.17

P6.18 At the top of the vertical circle,

$$T = m\frac{v^2}{R} - mg$$

or $T = (0.400)\frac{(4.00)^2}{0.500} - (0.400)(9.80) = \boxed{8.88\ \text{N}}$

P6.19 (a) $v = 20.0\ \text{m/s}$,

$n =$ force of track on roller coaster, and

$R = 10.0\ \text{m}$.

$$\sum F = \frac{Mv^2}{R} = n - Mg$$

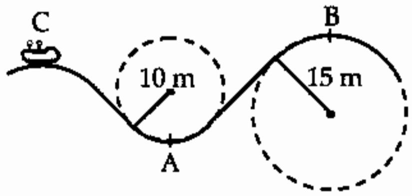


FIG. P6.19

From this we find

$$n = Mg + \frac{Mv^2}{R} = (500\ \text{kg})(9.80\ \text{m/s}^2) + \frac{(500\ \text{kg})(20.0\ \text{m/s}^2)}{10.0\ \text{m}}$$

$$n = 4\,900\ \text{N} + 20\,000\ \text{N} = \boxed{2.49 \times 10^4\ \text{N}}$$

(b) At B, $n - Mg = -\frac{Mv^2}{R}$

The max speed at B corresponds to

$$n = 0$$

$$-Mg = -\frac{Mv_{\text{max}}^2}{R} \Rightarrow v_{\text{max}} = \sqrt{Rg} = \sqrt{15.0(9.80)} = \boxed{12.1\ \text{m/s}}$$

P6.20 (a) $a_c = \frac{v^2}{r}$ $\quad r = \frac{v^2}{a_c} = \frac{(13.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{8.62 \text{ m}}$

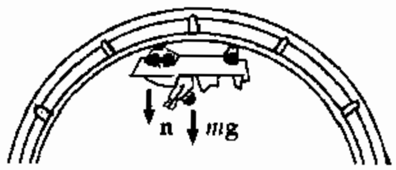


FIG. P6.20

(b) Let n be the force exerted by the rail.

Newton's law gives

$$Mg + n = \frac{Mv^2}{r}$$

$$n = M\left(\frac{v^2}{r} - g\right) = M(2g - g) = \boxed{Mg, \text{ downward}}$$

(c) $a_c = \frac{v^2}{r}$ $\quad a_c = \frac{(13.0 \text{ m/s})^2}{20.0 \text{ m}} = \boxed{8.45 \text{ m/s}^2}$

If the force exerted by the rail is n_1

then $\quad n_1 + Mg = \frac{Mv^2}{r} = Ma_c$

$$n_1 = M(a_c - g) \text{ which is } < 0 \text{ since } a_c = 8.45 \text{ m/s}^2$$

Thus, the normal force would have to point away from the center of the curve. Unless they have belts, the riders will fall from the cars. To be safe we must require n_1 to be positive. Then $a_c > g$. We need

$$\frac{v^2}{r} > g \text{ or } v > \sqrt{rg} = \sqrt{(20.0 \text{ m})(9.80 \text{ m/s}^2)},\ v > 14.0 \text{ m/s}.$$

Section 6.3 Motion in Accelerated Frames

P6.21 (a) $\sum F_x = Ma$, $a = \frac{T}{M} = \frac{18.0 \text{ N}}{5.00 \text{ kg}} = \boxed{3.60 \text{ m/s}^2}$ to the right.

(b) If $v =$ const, $a = 0$, so $\boxed{T = 0}$ (This is also an equilibrium situation.)

(c) Someone in the car (noninertial observer) claims that the forces on the mass along x are T and a fictitious force ($-Ma$). Someone at rest outside the car (inertial observer) claims that T is the only force on M in the x-direction.

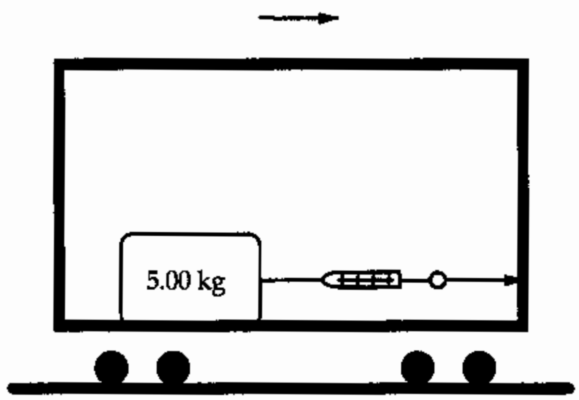


FIG. P6.21

***P6.22** We adopt the view of an inertial observer. If it is on the verge of sliding, the cup is moving on a circle with its centripetal acceleration caused by friction.

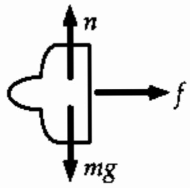


FIG. P6.22

$$\sum F_y = ma_y: \quad +n - mg = 0$$

$$\sum F_x = ma_x: \quad f = \frac{mv^2}{r} = \mu_s n = \mu_s mg$$

$$v = \sqrt{\mu_s g r} = \sqrt{0.8\left(9.8\ \text{m/s}^2\right)(30\ \text{m})} = \boxed{15.3\ \text{m/s}}$$

If you go too fast the cup will begin sliding $\boxed{\text{straight across the dashboard to the left.}}$

P6.23 The only forces acting on the suspended object are the force of gravity mg and the force of tension **T**, as shown in the free-body diagram. Applying Newton's second law in the x and y directions,

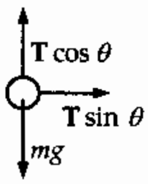


FIG. P6.23

$$\sum F_x = T\sin\theta = ma \qquad (1)$$

$$\sum F_y = T\cos\theta - mg = 0$$

or $\quad T\cos\theta = mg \qquad (2)$

(a) Dividing equation (1) by (2) gives

$$\tan\theta = \frac{a}{g} = \frac{3.00\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 0.306.$$

Solving for θ, $\theta = \boxed{17.0°}$

(b) From Equation (1),

$$T = \frac{ma}{\sin\theta} = \frac{(0.500\ \text{kg})\left(3.00\ \text{m/s}^2\right)}{\sin(17.0°)} = \boxed{5.12\ \text{N}}.$$

***P6.24** The water moves at speed

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.12\ \text{m})}{7.25\ \text{s}} = 0.104\ \text{m/s}.$$

The top layer of water feels a downward force of gravity mg and an outward fictitious force in the turntable frame of reference,

$$\frac{mv^2}{r} = \frac{m(0.104\ \text{m/s})^2}{0.12\ \text{m}} = m9.01\times10^{-2}\ \text{m/s}^2.$$

It behaves as if it were stationary in a gravity field pointing downward and outward at

$$\tan^{-1}\frac{0.0901\ \text{m/s}^2}{9.8\ \text{m/s}^2} = \boxed{0.527°}.$$

Its surface slopes upward toward the outside, making this angle with the horizontal.

P6.25 $F_{max} = F_g + ma = 591\text{ N}$

$F_{min} = F_g - ma = 391\text{ N}$

(a) Adding, $2F_g = 982\text{ N}$, $F_g = \boxed{491\text{ N}}$

(b) Since $F_g = mg$, $m = \dfrac{491\text{ N}}{9.80\text{ m/s}^2} = \boxed{50.1\text{ kg}}$

(c) Subtracting the above equations,

$2ma = 200\text{ N}$ $\qquad \therefore a = \boxed{2.00\text{ m/s}^2}$

P6.26 (a) $\sum F_r = ma_r$

$$mg = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$$

$$g = \frac{4\pi^2 R}{T^2}$$

$$T = \sqrt{\frac{4\pi^2 R}{g}} = 2\pi\sqrt{\frac{6.37\times 10^6\text{ m}}{9.80\text{ m/s}^2}} = 5.07\times 10^3\text{ s} = \boxed{1.41\text{ h}}$$

(b) speed increase factor $= \dfrac{v_{new}}{v_{current}} = \dfrac{2\pi R}{T_{new}}\left(\dfrac{T_{current}}{2\pi R}\right) = \dfrac{T_{current}}{T_{new}} = \dfrac{24.0\text{ h}}{1.41\text{ h}} = \boxed{17.1}$

***P6.27** The car moves to the right with acceleration a. We find the acceleration of a_b of the block relative to the Earth. The block moves to the right also.

$$\sum F_y = ma_y: \quad +n - mg = 0,\ n = mg,\ f = \mu_k mg$$
$$\sum F_x = ma_x: \quad +\mu_k mg = ma_b,\ a_b = \mu_k g$$

The acceleration of the block relative to the car is $a_b - a = \mu_k g - a$. In this frame the block starts from rest and undergoes displacement $-\ell$ and gains speed according to

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$
$$v_{xf}^2 = 0 + 2(\mu_k g - a)(-\ell - 0) = 2\ell(a - \mu_k g).$$

(a) $\boxed{v = \left(2\ell(a - \mu_k g)\right)^{1/2}}$ to the left

continued on next page

(b) The time for which the box slides is given by

$$\Delta x = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t$$
$$-\ell = \frac{1}{2}\left[0 - \left(2\ell\left(a - \mu_k g\right)\right)^{1/2}\right]t$$
$$t = \left(\frac{2\ell}{a - \mu_k g}\right)^{1/2}.$$

The car in the Earth frame acquires finals speed $v_{xf} = v_{xi} + at = 0 + a\left(\frac{2\ell}{a - \mu_k g}\right)^{1/2}$. The speed of the box in the Earth frame is then

$$v_{be} = v_{bc} + v_{ce} = -\left[2\ell\left(a - \mu_k g\right)\right]^{1/2} + a\left(\frac{2\ell}{a - \mu_k g}\right)^{1/2}$$
$$= \frac{-(2\ell)^{1/2}\left(a - \mu_k g\right) + (2\ell)^{1/2} a}{\left(a - \mu_k g\right)^{1/2}} = \boxed{\frac{\mu_k g (2\ell)^{1/2}}{\left(a - \mu_k g\right)^{1/2}}}$$
$$= \frac{\mu_k g 2\ell}{\left[2\ell\left(a - \mu_k g\right)\right]^{1/2}} = \frac{2\mu_k g \ell}{v}.$$

***P6.28** Consider forces on the backpack as it slides in the Earth frame of reference.

$$\sum F_y = ma_y: \quad +n - mg = ma,\ n = m(g + a),\ f_k = \mu_k m(g + a)$$
$$\sum F_x = ma_x: \quad -\mu_k m(g + a) = ma_x$$

The motion across the floor is described by $L = vt + \frac{1}{2}a_x t^2 = vt - \frac{1}{2}\mu_k (g + a)t^2$.

We solve for μ_k: $vt - L = \frac{1}{2}\mu_k (g + a)t^2$, $\boxed{\frac{2(vt - L)}{(g + a)t^2} = \mu_k}$.

P6.29 In an inertial reference frame, the girl is accelerating horizontally inward at

$$\frac{v^2}{r} = \frac{(5.70\ \text{m/s})^2}{2.40\ \text{m}} = 13.5\ \text{m/s}^2$$

In her own non-inertial frame, her head feels a horizontally outward fictitious force equal to its mass times this acceleration. Together this force and the weight of her head add to have a magnitude equal to the mass of her head times an acceleration of

$$\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2} = \sqrt{(9.80)^2 + (13.5)^2}\ \text{m/s}^2 = 16.7\ \text{m/s}^2$$

This is larger than g by a factor of $\frac{16.7}{9.80} = 1.71$.

Thus, the force required to lift her head is larger by this factor, or the required force is

$$F = 1.71(55.0\ \text{N}) = \boxed{93.8\ \text{N}}.$$

***P6.30** (a) The chunk is at radius $r = \dfrac{0.137\text{ m} + 0.080\text{ m}}{4} = 0.054\,2\text{ m}$. Its speed is

$$v = \frac{2\pi r}{T} = 2\pi(0.054\,2\text{ m})\frac{20\,000}{60\text{ s}} = 114\text{ m/s}$$

and its acceleration

$$a_c = \frac{v^2}{r} = \frac{(114\text{ m/s})^2}{0.054\,2\text{ m}} = \boxed{2.38\times10^5\text{ m/s}^2\text{ horizontally inward}}$$

$$= 2.38\times10^5\text{ m/s}^2\left(\frac{g}{9.8\text{ m/s}^2}\right) = \boxed{2.43\times10^4 g}.$$

(b) In the frame of the turning cone, the chunk feels a horizontally outward force of $\dfrac{mv^2}{r}$. In this frame its acceleration is up along the cone, at $\tan^{-1}\dfrac{3.3\text{ cm}}{\frac{(13.7-8)\text{ cm}}{2}} = 49.2°$.

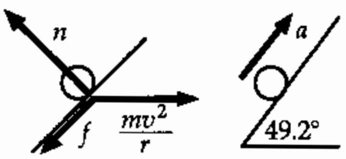


FIG. P6.30(b)

Take the y axis perpendicular to the cone:

$$\sum F_y = ma_y:\quad +n - \frac{mv^2}{r}\sin 49.2° = 0$$

$$n = (2\times10^{-3}\text{ kg})(2.38\times10^5\text{ m/s}^2)\sin 49.2° = \boxed{360\text{ N}}$$

(c) $f = \mu_k n = 0.6(360\text{ N}) = 216\text{ N}$

$$\sum F_x = ma_x:\quad \frac{mv^2}{r}\cos 49.2° - f = ma_x$$

$$(2\times10^{-3}\text{ kg})(2.38\times10^5\text{ m/s}^2)\cos 49.2° - 216\text{ N} = (2\times10^{-3}\text{ kg})a_x$$

$$a_x = \boxed{47.5\times10^4\text{ m/s}^2\text{ radially up the wall of the cone}}$$

P6.31 $a_r = \left(\dfrac{4\pi^2 R_e}{T^2}\right)\cos 35.0° = 0.027\,6\text{ m/s}^2$

We take the y axis along the local vertical.

$(a_{\text{net}})_y = 9.80 - (a_r)_y = 9.78\text{ m/s}^2$

$(a_{\text{net}})_x = 0.015\,8\text{ m/s}^2$

$\theta = \arctan\dfrac{a_x}{a_y} = \boxed{0.092\,8°}$

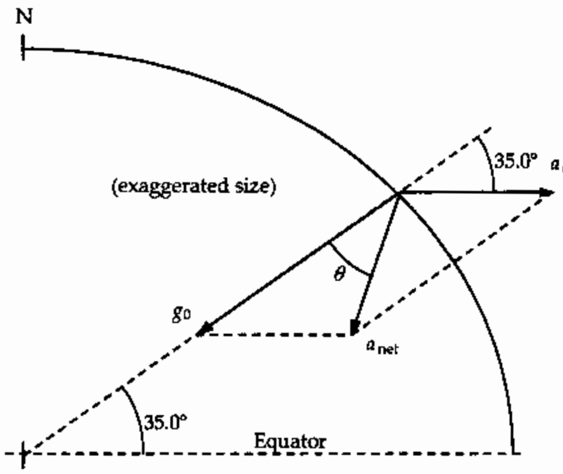


FIG. P6.31

Section 6.4 Motion in the Presence of Resistive Forces

P6.32 $m = 80.0\text{ kg}$, $v_T = 50.0\text{ m/s}$, $mg = \frac{D\rho A v_T^2}{2} \therefore \frac{D\rho A}{2} = \frac{mg}{v_T^2} = 0.314\text{ kg/m}$

(a) At $v = 30.0\text{ m/s}$

$$a = g - \frac{\frac{D\rho A v^2}{2}}{m} = 9.80 - \frac{(0.314)(30.0)^2}{80.0} = \boxed{6.27\text{ m/s}^2\text{ downward}}$$

(b) At $v = 50.0\text{ m/s}$, terminal velocity has been reached.

$$\sum F_y = 0 = mg - R$$

$$\Rightarrow R = mg = (80.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{784\text{ N directed up}}$$

(c) At $v = 30.0\text{ m/s}$

$$\frac{D\rho A v^2}{2} = (0.314)(30.0)^2 = \boxed{283\text{ N}}\text{ upward}$$

P6.33 (a) $a = g - bv$

When $v = v_T$, $a = 0$ and $g = bv_T$ $\qquad b = \frac{g}{v_T}$

The Styrofoam falls 1.50 m at constant speed v_T in 5.00 s.

Thus, $\qquad v_T = \frac{y}{t} = \frac{1.50\text{ m}}{5.00\text{ s}} = 0.300\text{ m/s}$

Then $\qquad b = \frac{9.80\text{ m/s}^2}{0.300\text{ m/s}} = \boxed{32.7\text{ s}^{-1}}$

(b) At $t = 0$, $v = 0$ and $\qquad a = g = \boxed{9.80\text{ m/s}^2}$ down

(c) When $v = 0.150\text{ m/s}$, $\quad a = g - bv = 9.80\text{ m/s}^2 - (32.7\text{ s}^{-1})(0.150\text{ m/s}) = \boxed{4.90\text{ m/s}^2}$ down

P6.34 (a) $\rho = \frac{m}{V}$, $A = 0.0201\text{ m}^2$, $R = \frac{1}{2}\rho_{\text{air}} A D v_T^2 = mg$

$$m = \rho_{\text{bead}} V = 0.830\text{ g/cm}^3 \left[\frac{4}{3}\pi(8.00\text{ cm})^3\right] = 1.78\text{ kg}$$

Assuming a drag coefficient of $D = 0.500$ for this spherical object, and taking the density of air at 20°C from the endpapers, we have

$$v_T = \sqrt{\frac{2(1.78\text{ kg})(9.80\text{ m/s}^2)}{0.500(1.20\text{ kg/m}^3)(0.0201\text{ m}^2)}} = \boxed{53.8\text{ m/s}}$$

(b) $v_f^2 = v_i^2 + 2gh = 0 + 2gh$: $h = \frac{v_f^2}{2g} = \frac{(53.8\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{148\text{ m}}$

P6.35 Since the upward velocity is constant, the resultant force on the ball is zero. Thus, the upward applied force equals the sum of the gravitational and drag forces (both downward): $F = mg + bv$.

The mass of the copper ball is

$$m = \frac{4\pi\rho r^3}{3} = \left(\frac{4}{3}\right)\pi\left(8.92\times10^3\ \text{kg/m}^3\right)\left(2.00\times10^{-2}\ \text{m}\right)^3 = 0.299\ \text{kg}.$$

The applied force is then

$$F = mg + bv = (0.299)(9.80) + (0.950)\left(9.00\times10^{-2}\right) = \boxed{3.01\ \text{N}}.$$

P6.36 $\sum F_y = ma_y$

$+T\cos 40.0° - mg = 0$

$T = \dfrac{(620\ \text{kg})\left(9.80\ \text{m/s}^2\right)}{\cos 40.0°} = 7.93\times10^3\ \text{N}$

$\sum F_x = ma_x$

$-R + T\sin 40.0° = 0$

$R = \left(7.93\times10^3\ \text{N}\right)\sin 40.0° = 5.10\times10^3\ \text{N} = \frac{1}{2}D\rho Av^2$

$D = \dfrac{2R}{\rho Av^2} = \dfrac{2\left(5.10\times10^3\ \text{N}\right)\left(\frac{\text{kg m/s}^2}{\text{N}}\right)}{\left(1.20\ \text{kg/m}^3\right)\left(3.80\ \text{m}^2\right)(40.0\ \text{m/s})^2} = \boxed{1.40}$

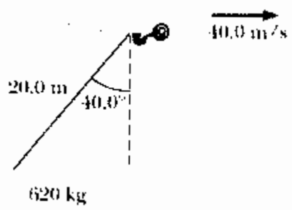


FIG. P6.36

P6.37 (a) At terminal velocity, $R = v_T b = mg$

$$\therefore b = \frac{mg}{v_T} = \frac{\left(3.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{2.00\times10^{-2}\ \text{m/s}} = \boxed{1.47\ \text{N}\cdot\text{s/m}}$$

(b) In the equation describing the time variation of the velocity, we have

$v = v_T\left(1 - e^{-bt/m}\right)$ $\qquad v = 0.632v_T$ when $e^{-bt/m} = 0.368$

or at time $\qquad t = -\left(\dfrac{m}{b}\right)\ln(0.368) = \boxed{2.04\times10^{-3}\ \text{s}}$

(c) At terminal velocity, $\qquad R = v_T b = mg = \boxed{2.94\times10^{-2}\ \text{N}}$

P6.38 The resistive force is

$$R = \frac{1}{2}D\rho Av^2 = \frac{1}{2}(0.250)\left(1.20\ \text{kg/m}^3\right)\left(2.20\ \text{m}^2\right)(27.8\ \text{m/s})^2$$

$$R = 255\ \text{N}$$

$$a = -\frac{R}{m} = -\frac{255\ \text{N}}{1200\ \text{kg}} = \boxed{-0.212\ \text{m/s}^2}$$

P6.39 (a) $v(t) = v_i e^{-ct}$ $v(20.0\text{ s}) = 5.00 = v_i e^{-20.0c}$, $v_i = 10.0\text{ m/s}$.

So $5.00 = 10.0e^{-20.0c}$ and $-20.0c = \ln\left(\frac{1}{2}\right)$ $c = -\frac{\ln\left(\frac{1}{2}\right)}{20.0} = \boxed{3.47 \times 10^{-2}\text{ s}^{-1}}$

(b) At $t = 40.0\text{ s}$ $v = (10.0\text{ m/s})e^{-40.0c} = (10.0\text{ m/s})(0.250) = \boxed{2.50\text{ m/s}}$

(c) $v = v_i e^{-ct}$ $s = \frac{dv}{dt} = -cv_i e^{-ct} = \boxed{-cv}$

P6.40 $\sum F = ma$

$$-kmv^2 = m\frac{dv}{dt}$$

$$-kdt = \frac{dv}{v^2}$$

$$-k\int_0^t dt = \int_{v_0}^{v} v^{-2}dv$$

$$-k(t-0) = \left.\frac{v^{-1}}{-1}\right|_{v_0}^{v} = -\frac{1}{v} + \frac{1}{v_0}$$

$$\frac{1}{v} = \frac{1}{v_0} + kt = \frac{1 + v_0 kt}{v_0}$$

$$v = \frac{v_0}{1 + v_0 kt}$$

***P6.41** (a) From Problem 40,

$$v = \frac{dx}{dt} = \frac{v_0}{1 + v_0 kt}$$

$$\int_0^x dx = \int_0^t v_0 \frac{dt}{1 + v_0 kt} = \frac{1}{k}\int_0^t \frac{v_0 kdt}{1 + v_0 kt}$$

$$x\Big|_0^x = \frac{1}{k}\ln(1 + v_0 kt)\Big|_0^t$$

$$x - 0 = \frac{1}{k}\left[\ln(1 + v_0 kt) - \ln 1\right]$$

$$\boxed{x = \frac{1}{k}\ln(1 + v_0 kt)}$$

(b) We have $\ln(1 + v_0 kt) = kx$

$1 + v_0 kt = e^{kx}$ so $v = \frac{v_0}{1 + v_0 kt} = \frac{v_0}{e^{kx}} = \boxed{v_0 e^{-kx} = v}$

***P6.42** We write $-kmv^2 = -\frac{1}{2}D\rho A v^2$ so

$$k = \frac{D\rho A}{2m} = \frac{0.305\left(1.20\text{ kg/m}^3\right)\left(4.2 \times 10^{-3}\text{ m}^2\right)}{2(0.145\text{ kg})} = 5.3 \times 10^{-3}/\text{m}$$

$$v = v_0 e^{-kx} = (40.2\text{ m/s})e^{-\left(5.3 \times 10^{-3}/\text{m}\right)(18.3\text{ m})} = \boxed{36.5\text{ m/s}}$$

P6.43 In $R=\frac{1}{2}D\rho Av^2$, we estimate that $D=1.00$, $\rho=1.20\ \text{kg/m}^3$, $A=(0.100\ \text{m})(0.160\ \text{m})=1.60\times10^{-2}\ \text{m}^2$ and $v=27.0\ \text{m/s}$. The resistance force is then

$$R=\frac{1}{2}(1.00)\left(1.20\ \text{kg/m}^3\right)\left(1.60\times10^{-2}\ \text{m}^2\right)(27.0\ \text{m/s})^2=7.00\ \text{N}$$

or

$$R\sim\boxed{10^1\ \text{N}}$$

Section 6.5 Numerical Modeling in Particle Dynamics

Note: In some problems we compute each new position as $x(t+\Delta t)=x(t)+v(t+\Delta t)\Delta t$, rather than $x(t+\Delta t)=x(t)+v(t)\Delta t$ as quoted in the text. This method has the same theoretical validity as that presented in the text, and in practice can give quicker convergence.

P6.44 (a) At $v=v_T$, $a=0$, $-mg+bv_T=0$ $\quad v_T=\frac{mg}{b}=\frac{\left(3.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{3.00\times10^{-2}\ \text{kg/s}}=\boxed{0.980\ \text{m/s}}$

(b)

t(s)	x(m)	v(m/s)	F(mN)	$a\left(\text{m/s}^2\right)$
0	2	0	−29.4	−9.8
0.005	2	−0.049	−27.93	−9.31
0.01	1.999 755	−0.095 55	−26.534	−8.844 5
0.015	1.999 3	−0.139 77	−25.2	−8.40

. . . we list the result after each tenth iteration

0.05	1.990	−0.393	−17.6	−5.87
0.1	1.965	−0.629	−10.5	−3.51
0.15	1.930	−0.770	−6.31	−2.10
0.2	1.889	−0.854	−3.78	−1.26
0.25	1.845	−0.904	−2.26	−0.754
0.3	1.799	−0.935	−1.35	−0.451
0.35	1.752	−0.953	−0.811	−0.270
0.4	1.704	−0.964	−0.486	−0.162
0.45	1.65	−0.970	−0.291	−0.096 9
0.5	1.61	−0.974	−0.174	−0.058 0
0.55	1.56	−0.977	−0.110	−0.034 7
0.6	1.51	−0.978	−0.062 4	−0.020 8
0.65	1.46	−0.979	−0.037 4	−0.012 5

Terminal velocity is never reached. The leaf is at 99.9% of v_T after 0.67 s. The fall to the ground takes about 2.14 s. Repeating with $\Delta t=0.001$ s, we find the fall takes 2.14 s.

P6.45 (a) When $v = v_T$, $a = 0$, $\sum F = -mg + Cv_T^2 = 0$

$$v_T = -\sqrt{\frac{mg}{C}} = -\sqrt{\frac{(4.80\times10^{-4}\ \text{kg})(9.80\ \text{m/s}^2)}{2.50\times10^{-5}\ \text{kg/m}}} = \boxed{-13.7\ \text{m/s}}$$

(b)

t(s)	x(m)	v(m/s)	F(mN)	a(m/s^2)
0	0	0	– 4.704	–9.8
0.2	0	–1.96	– 4.608	–9.599 9
0.4	–0.392	–3.88	– 4.327 6	–9.015 9
0.6	–1.168	–5.683 2	–3.896 5	–8.117 8
0.8	–2.30	–7.306 8	–3.369 3	–7.019 3
1.0	–3.77	–8.710 7	–2.807 1	–5.848 1
1.2	–5.51	–9.880 3	–2.263 5	–4.715 6
1.4	–7.48	–10.823	–1.775 3	–3.698 6
1.6	–9.65	–11.563	–1.361 6	–2.836 6
1.8	–11.96	–12.13	–1.03	–2.14
2	–14.4	–12.56	–0.762	–1.59
. . . listing results after each fifth step				
3	–27.4	–13.49	–0.154	–0.321
4	–41.0	–13.67	–0.029 1	–0.060 6
5	–54.7	–13.71	–0.005 42	–0.011 3

The hailstone reaches 99% of v_T after 3.3 s, 99.95% of v_T after 5.0 s, 99.99% of v_T after 6.0 s, 99.999% of v_T after 7.4 s.

P6.46 (a) At terminal velocity, $\sum F = 0 = -mg + Cv_T^2$

$$C = \frac{mg}{v_T^2} = \frac{(0.142\ \text{kg})(9.80\ \text{m/s}^2)}{(42.5\ \text{m/s})^2} = \boxed{7.70\times10^{-4}\ \text{kg/m}}$$

(b) $Cv^2 = (7.70\times10^{-4}\ \text{kg/m})(36.0\ \text{m/s})^2 = \boxed{0.998\ \text{N}}$

(c)

Elapsed Time (s)	Altitude (m)	Speed (m/s)	Resistance Force (N)	Net Force (N)	Acceleration (m/s^2)
0.000 00	0.000 00	36.000 00	–0.998 49	–2.390 09	–16.831 58
0.050 00	1.757 92	35.158 42	–0.952 35	–2.343 95	–16.506 67
...					
2.950 00	48.623 27	0.824 94	–0.000 52	–1.392 12	–9.803 69
3.000 00	48.640 00	0.334 76	–0.000 09	–1.391 69	–9.800 61
3.050 00	48.632 24	–0.155 27	0.000 02	–1.391 58	–9.799 87
...					
6.250 00	1.250 85	–26.852 97	0.555 55	–0.836 05	–5.887 69
6.300 00	–0.106 52	–27.147 36	0.567 80	–0.823 80	–5.801 44

Maximum height is about $\boxed{49\ \text{m}}$. It returns to the ground after about $\boxed{6.3\ \text{s}}$ with a speed of approximately $\boxed{27\ \text{m/s}}$.

P6.47 (a) At constant velocity $\sum F = 0 = -mg + Cv_T^2$

$$v_T = -\sqrt{\frac{mg}{C}} = -\sqrt{\frac{(50.0\text{ kg})(9.80\text{ m/s}^2)}{0.200\text{ kg/m}}} = \boxed{-49.5\text{ m/s}}$$ with chute closed and

$$v_T = -\sqrt{\frac{(50.0\text{ kg})(9.80\text{ m/s})}{20.0\text{ kg/m}}} = \boxed{-4.95\text{ m/s}}$$ with chute open.

(b) We use time increments of 0.1 s for $0 < t < 10$ s, then 0.01 s for $10\text{ s} < t < 12$ s, and then 0.1 s again.

time(s)	height(m)	velocity(m/s)
0	1000	0
1	995	–9.7
2	980	–18.6
4	929	–32.7
7	812	–43.7
10	674	–47.7
10.1	671	–16.7
10.3	669	–8.02
11	665	–5.09
12	659	–4.95
50	471	–4.95
100	224	–4.95
145	0	–4.95

6.48 (a) We use a time increment of 0.01 s.

time(s)	x(m)	y(m)
0	0	0
0.100	7.81	5.43
0.200	14.9	10.2
0.400	27.1	18.3
1.00	51.9	32.7
1.92	70.0	38.5
2.00	70.9	38.5
4.00	80.4	26.7
5.00	81.4	17.7
6.85	81.8	0

with θ	we find range
30.0°	86.410 m
35.0°	81.8 m
25.0°	90.181 m
20.0°	92.874 m
15.0°	93.812 m
10.0°	90.965 m
17.0°	93.732 m
16.0°	93.839 8 m
15.5°	93.829 m
15.8°	93.839 m
16.1°	93.838 m
15.9°	93.840 2 m

(b) range = $\boxed{81.8\text{ m}}$

(c) So we have maximum range at $\theta = \boxed{15.9°}$

P6.49 (a) At terminal speed, $\sum F = -mg + Cv^2 = 0$. Thus,

$$C = \frac{mg}{v^2} = \frac{(0.046\,0\ \text{kg})(9.80\ \text{m/s}^2)}{(44.0\ \text{m/s})^2} = \boxed{2.33 \times 10^{-4}\ \text{kg/m}}$$

(b) We set up a spreadsheet to calculate the motion, try different initial speeds, and home in on $\boxed{53\ \text{m/s}}$ as that required for horizontal range of 155 m, thus:

Time t (s)	x (m)	v_x (m/s)	a_x (m/s^2)	y (m)	v_y (m/s)	a_y (m/s^2)	$v = \sqrt{v_x^2 + v_y^2}$ (m/s)	$\tan^{-1}\left(\frac{v_y}{v_x}\right)$ (deg)
0.000 0	0.000 0	45.687 0	–10.565 9	0.000 0	27.451 5	–13.614 6	53.300 0	31.000 0
0.002 7	0.121 1	45.659 0	–10.552 9	0.072 7	27.415 5	–13.604 6	53.257 4	30.982 2
…								
2.501 6	90.194 6	28.937 5	–4.238 8	32.502 4	0.023 5	–9.800 0	28.937 5	0.046 6
2.504 3	90.271 3	28.926 3	–4.235 5	32.502 4	–0.002 4	–9.800 0	28.926 3	–0.004 8
2.506 9	90.348 0	28.915 0	–4.232 2	32.502 4	–0.028 4	–9.800 0	28.915 1	–0.056 3
…								
3.423 8	115.229 8	25.492 6	–3.289 6	28.397 2	–8.890 5	–9.399 9	26.998 4	–19.226 2
3.426 5	115.297 4	25.483 9	–3.287 4	28.373 6	–8.915 4	–9.397 7	26.998 4	–19.282 2
3.429 1	115.364 9	25.475 1	–3.285 1	28.350 0	–8.940 3	–9.395 4	26.998 4	–19.338 2
…								
5.151 6	154.996 8	20.843 8	–2.199 2	0.005 9	–23.308 7	–7.049 8	31.269 2	–48.195 4
5.154 3	155.052 0	20.838 0	–2.198 0	–0.055 9	–23.327 4	–7.045 4	31.279 2	–48.226 2

(c) Similarly, the initial speed is $\boxed{42\ \text{m/s}}$. The motion proceeds thus:

Time t (s)	x (m)	v_x (m/s)	a_x (m/s^2)	y (m)	v_y (m/s)	a_y (m/s^2)	$v = \sqrt{v_x^2 + v_y^2}$ (m/s)	$\tan^{-1}\left(\frac{v_y}{v_x}\right)$ (deg)
0.000 0	0.000 0	28.746 2	–4.182 9	0.000 0	30.826 6	–14.610 3	42.150 0	47.000 0
0.003 5	0.100 6	28.731 6	–4.178 7	0.107 9	30.775 4	–14.594 3	42.102 6	46.967 1
…								
2.740 5	66.307 8	20.548 4	–2.137 4	39.485 4	0.026 0	–9.800 0	20.548 5	0.072 5
2.744 0	66.379 7	20.541 0	–2.135 8	39.485 5	–0.008 3	–9.800 0	20.541 0	–0.023 1
2.747 5	66.451 6	20.533 5	–2.134 3	39.485 5	–0.042 6	–9.800 0	20.533 5	–0.118 8
…								
3.146 5	74.480 5	19.715 6	–1.967 6	38.696 3	–3.942 3	–9.721 3	20.105 8	–11.307 7
3.150 0	74.549 5	19.708 7	–1.966 2	38.682 5	–3.976 4	–9.720 0	20.105 8	–11.406 7
3.153 5	74.618 5	19.701 8	–1.964 9	38.668 6	–4.010 4	–9.718 6	20.105 8	–11.505 6
…								
5.677 0	118.969 7	15.739 4	–1.254 0	0.046 5	–25.260 0	–6.570 1	29.762 3	–58.073 1
5.680 5	119.024 8	15.735 0	–1.253 3	–0.041 9	–25.283 0	–6.564 2	29.779 5	–58.103 7

The trajectory in (c) reaches maximum height 39 m, as opposed to 33 m in (b). In both, the ball reaches maximum height when it has covered about 57% of its range. Its speed is a minimum somewhat later. The impact speeds are both about 30 m/s.

Additional Problems

***P6.50** When the cloth is at a lower angle θ, the radial component of $\sum F = ma$ reads

$$n + mg\sin\theta = \frac{mv^2}{r}.$$

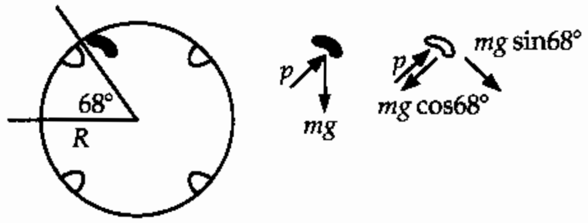


FIG. P6.50

At $\theta = 68.0°$, the normal force drops to zero and $g\sin 68° = \frac{v^2}{r}$.

$$v = \sqrt{rg\sin 68°} = \sqrt{(0.33\ \text{m})(9.8\ \text{m/s}^2)\sin 68°} = 1.73\ \text{m/s}$$

The rate of revolution is

$$\text{angular speed} = (1.73\ \text{m/s})\left(\frac{1\ \text{rev}}{2\pi r}\right)\left(\frac{2\pi r}{2\pi(0.33\ \text{m})}\right) = \boxed{0.835\ \text{rev/s}} = 50.1\ \text{rev/min}.$$

***P6.51** (a) $v = (30\ \text{km/h})\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1\,000\ \text{m}}{1\ \text{km}}\right) = 8.33\ \text{m/s}$

$\sum F_y = ma_y$: $+n - mg = -\frac{mv^2}{r}$

$$n = m\left(g - \frac{v^2}{r}\right) = 1\,800\ \text{kg}\left[9.8\ \text{m/s}^2 - \frac{(8.33\ \text{m/s})^2}{20.4\ \text{m}}\right]$$

$$= \boxed{1.15\times 10^4\ \text{N up}}$$

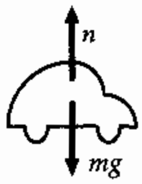


FIG. P6.51

(b) Take $n = 0$. Then $mg = \frac{mv^2}{r}$.

$$v = \sqrt{gr} = \sqrt{(9.8\ \text{m/s}^2)(20.4\ \text{m})} = \boxed{14.1\ \text{m/s}} = 50.9\ \text{km/h}$$

P6.52 (a) $\sum F_y = ma_y = \frac{mv^2}{R}$

$mg - n = \frac{mv^2}{R}$ $\quad n = \boxed{mg - \frac{mv^2}{R}}$

(b) When $n = 0$, $\quad mg = \frac{mv^2}{R}$

Then, $\quad v = \boxed{\sqrt{gR}}$.

***P6.53** (a) slope $= \dfrac{0.160\text{ N} - 0}{9.9\text{ m}^2/\text{s}^2} = \boxed{0.016\,2\text{ kg/m}}$

(b) slope $= \dfrac{R}{v^2} = \dfrac{\frac{1}{2}D\rho A v^2}{v^2} = \boxed{\frac{1}{2}D\rho A}$

(c) $\frac{1}{2}D\rho A = 0.016\,2\text{ kg/m}$

$$D = \frac{2(0.016\,2\text{ kg/m})}{(1.20\text{ kg/m}^3)\pi(0.105\text{ m})^2} = \boxed{0.778}$$

(d) From the table, the eighth point is at force $mg = 8(1.64\times10^{-3}\text{ kg})(9.8\text{ m/s}^2) = 0.129\text{ N}$ and horizontal coordinate $(2.80\text{ m/s})^2$. The vertical coordinate of the line is here $(0.016\,2\text{ kg/m})(2.8\text{ m/s})^2 = 0.127\text{ N}$. The scatter percentage is $\dfrac{0.129\text{ N} - 0.127\text{ N}}{0.127\text{ N}} = 1.5\%$.

(e) The interpretation of the graph can be stated thus: For stacked coffee filters falling at terminal speed, a graph of air resistance force as a function of squared speed demonstrates that the force is proportional to the speed squared within the experimental uncertainty estimated as 2%. This proportionality agrees with that described by the theoretical equation $R = \frac{1}{2}D\rho A v^2$. The value of the constant slope of the graph implies that the drag coefficient for coffee filters is $D = 0.78 \pm 2\%$.

P6.54 (a) While the car negotiates the curve, the accelerometer is at the angle θ.

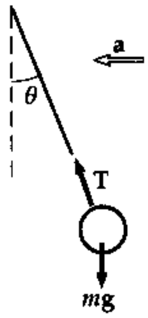


FIG. P6.54

Horizontally: $T\sin\theta = \dfrac{mv^2}{r}$

Vertically: $T\cos\theta = mg$

where r is the radius of the curve, and v is the speed of the car.

By division, $\tan\theta = \dfrac{v^2}{rg}$

Then $a_c = \dfrac{v^2}{r} = g\tan\theta$: $a_c = (9.80\text{ m/s}^2)\tan 15.0°$

$a_c = \boxed{2.63\text{ m/s}^2}$

(b) $r = \dfrac{v^2}{a_c}$ $\quad r = \dfrac{(23.0\text{ m/s})^2}{2.63\text{ m/s}^2} = \boxed{201\text{ m}}$

(c) $v^2 = rg\tan\theta = (201\text{ m})(9.80\text{ m/s}^2)\tan 9.00°$ $\quad v = \boxed{17.7\text{ m/s}}$

P6.55 Take x-axis up the hill

$$\sum F_x = ma_x: \quad +T\sin\theta - mg\sin\phi = ma$$
$$a = \frac{T}{m}\sin\theta - g\sin\phi$$
$$\sum F_y = ma_y: \quad +T\cos\theta - mg\cos\phi = 0$$
$$T = \frac{mg\cos\phi}{\cos\theta}$$
$$a = \frac{g\cos\phi\sin\theta}{\cos\theta} - g\sin\phi$$
$$a = \boxed{g(\cos\phi\tan\theta - \sin\phi)}$$

***P6.56** (a) The speed of the bag is $\frac{2\pi(7.46\text{ m})}{38\text{ s}} = 1.23\text{ m/s}$. The total force on it must add to

$$ma_c = \frac{(30\text{ kg})(1.23\text{ m/s})^2}{7.46\text{ m}} = 6.12\text{ N}$$

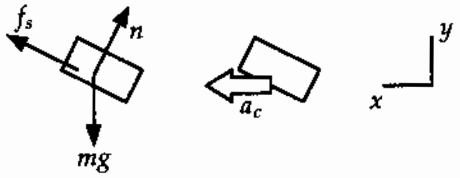


FIG. P6.56

$$\sum F_x = ma_x: \quad f_s\cos 20 - n\sin 20 = 6.12\text{ N}$$
$$\sum F_y = ma_y: \quad f_s\sin 20 + n\cos 20 - (30\text{ kg})\left(9.8\text{ m/s}^2\right) = 0$$
$$n = \frac{f_s\cos 20 - 6.12\text{ N}}{\sin 20}$$

Substitute:

$$f_s\sin 20 + f_s\frac{\cos^2 20}{\sin 20} - (6.12\text{ N})\frac{\cos 20}{\sin 20} = 294\text{ N}$$
$$f_s(2.92) = 294\text{ N} + 16.8\text{ N}$$
$$f_s = \boxed{106\text{ N}}$$

(b)

$$v = \frac{2\pi(7.94\text{ m})}{34\text{ s}} = 1.47\text{ m/s}$$
$$ma_c = \frac{(30\text{ kg})(1.47\text{ m/s})^2}{7.94\text{ m}} = 8.13\text{ N}$$
$$f_s\cos 20 - n\sin 20 = 8.13\text{ N}$$
$$f_s\sin 20 + n\cos 20 = 294\text{ N}$$
$$n = \frac{f_s\cos 20 - 8.13\text{ N}}{\sin 20}$$
$$f_s\sin 20 + f_s\frac{\cos^2 20}{\sin 20} - (8.13\text{ N})\frac{\cos 20}{\sin 20} = 294\text{ N}$$
$$f_s(2.92) = 294\text{ N} + 22.4\text{ N}$$
$$f_s = 108\text{ N}$$
$$n = \frac{(108\text{ N})\cos 20 - 8.13\text{ N}}{\sin 20} = 273\text{ N}$$
$$\mu_s = \frac{f_s}{n} = \frac{108\text{ N}}{273\text{ N}} = \boxed{0.396}$$

P6.57 (a) Since the centripetal acceleration of a person is downward (toward the axis of the earth), it is equivalent to the effect of a falling elevator. Therefore,

$$F_g' = F_g - \frac{mv^2}{r} \text{ or } \boxed{F_g > F_g'}$$

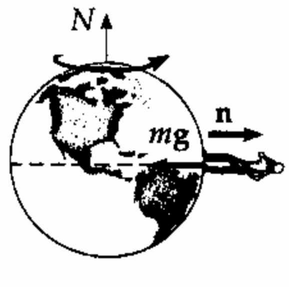


FIG. P6.57

(b) At the poles $v = 0$ and $F_g' = F_g = mg = 75.0(9.80) = \boxed{735\text{ N}}$ down.

At the equator, $F_g' = F_g - ma_c = 735\text{ N} - 75.0(0.033\,7)\text{ N} = \boxed{732\text{ N}}$ down.

P6.58 (a) Since the object of mass m_2 is in equilibrium, $\sum F_y = T - m_2 g = 0$

or $T = \boxed{m_2 g}$.

(b) The tension in the string provides the required centripetal acceleration of the puck.

Thus, $F_c = T = \boxed{m_2 g}$.

(c) From $F_c = \frac{m_1 v^2}{R}$

we have $v = \sqrt{\frac{RF_c}{m_1}} = \boxed{\sqrt{\left(\frac{m_2}{m_1}\right) gR}}$.

P6.59 (a) $v = (300\text{ mi/h})\left(\frac{88.0\text{ ft/s}}{60.0\text{ mi/h}}\right) = 440\text{ ft/s}$

At the lowest point, his seat exerts an upward force; therefore, his weight seems to increase. His apparent weight is

$$F_g' = mg + m\frac{v^2}{r} = 160 + \left(\frac{160}{32.0}\right)\frac{(440)^2}{1\,200} = \boxed{967\text{ lb}}.$$

(b) At the highest point, the force of the seat on the pilot is directed down and

$$F_g' = mg - m\frac{v^2}{r} = \boxed{-647\text{ lb}}.$$

Since the plane is upside down, the seat exerts this downward force.

(c) When $F_g' = 0$, then $mg = \frac{mv^2}{R}$. If we vary the aircraft's R and v such that the above is true, then the pilot feels weightless.

P6.60 For the block to remain stationary, $\sum F_y = 0$ and $\sum F_x = ma_r$.

$$n_1 = \left(m_p + m_b\right)g \text{ so } f \le \mu_{s1} n_1 = \mu_{s1}\left(m_p + m_b\right)g.$$

At the point of slipping, the required centripetal force equals the maximum friction force:

$$\therefore \left(m_p + m_b\right)\frac{v_{\max}^2}{r} = \mu_{s1}\left(m_p + m_b\right)g$$

or $v_{\max} = \sqrt{\mu_{s1} r g} = \sqrt{(0.750)(0.120)(9.80)} = 0.939 \text{ m/s}$.

For the penny to remain stationary on the block:

$$\sum F_y = 0 \Rightarrow n_2 - m_p g = 0 \text{ or } n_2 = m_p g$$

and $\sum F_x = ma_r \Rightarrow f_p = m_p \dfrac{v^2}{r}$.

When the penny is about to slip on the block, $f_p = f_{p,\max} = \mu_{s2} n_2$

or $\mu_{s2} m_p g = m_p \dfrac{v_{\max}^2}{r}$

$$v_{\max} = \sqrt{\mu_{s2} r g} = \sqrt{(0.520)(0.120)(9.80)} = 0.782 \text{ m/s}$$

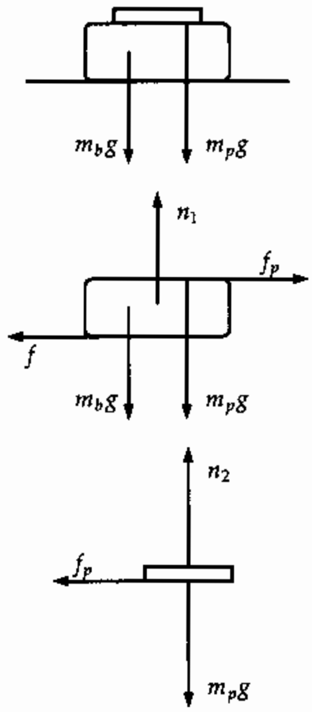


FIG. P6.60

This is less than the maximum speed for the block, so the penny slips before the block starts to slip. The maximum rotation frequency is

$$\text{Max rpm} = \frac{v_{\max}}{2\pi r} = (0.782 \text{ m/s})\left[\frac{1 \text{ rev}}{2\pi(0.120 \text{ m})}\right]\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{62.2 \text{ rev/min}}.$$

P6.61 $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(9.00 \text{ m})}{(15.0 \text{ s})} = 3.77 \text{ m/s}$

(a) $a_r = \dfrac{v^2}{r} = \boxed{1.58 \text{ m/s}^2}$

(b) $F_{\text{low}} = m(g + a_r) = \boxed{455 \text{ N}}$

(c) $F_{\text{high}} = m(g - a_r) = \boxed{328 \text{ N}}$

(d) $F_{\text{mid}} = m\sqrt{g^2 + a_r^2} = \boxed{397 \text{ N upward and}}$ at $\theta = \tan^{-1}\dfrac{a_r}{g} = \tan^{-1}\dfrac{1.58}{9.8} = \boxed{9.15° \text{ inward}}$.

P6.62 Standing on the inner surface of the rim, and moving with it, each person will feel a normal force exerted by the rim. This inward force causes the 3.00 m/s^2 centripetal acceleration:

$a_c = \dfrac{v^2}{r}$: $\quad v = \sqrt{a_c r} = \sqrt{\left(3.00 \text{ m/s}^2\right)(60.0 \text{ m})} = 13.4 \text{ m/s}$

The period of rotation comes from $v = \dfrac{2\pi r}{T}$: $\quad T = \dfrac{2\pi r}{v} = \dfrac{2\pi(60.0 \text{ m})}{13.4 \text{ m/s}} = 28.1 \text{ s}$

so the frequency of rotation is $\quad f = \dfrac{1}{T} = \dfrac{1}{28.1 \text{ s}} = \dfrac{1}{28.1 \text{ s}}\left(\dfrac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{2.14 \text{ rev/min}}$.

P6.63 (a) The mass at the end of the chain is in vertical equilibrium. Thus $T\cos\theta = mg$.

Horizontally $T\sin\theta = ma_r = \frac{mv^2}{r}$

$$r = (2.50\sin\theta + 4.00)\text{ m}$$
$$r = (2.50\sin 28.0° + 4.00)\text{ m} = 5.17\text{ m}$$

Then $a_r = \frac{v^2}{5.17\text{ m}}$.

By division $\tan\theta = \frac{a_r}{g} = \frac{v^2}{5.17g}$

$$v^2 = 5.17g\tan\theta = (5.17)(9.80)(\tan 28.0°)\text{ m}^2/\text{s}^2$$
$$v = \boxed{5.19\text{ m/s}}$$

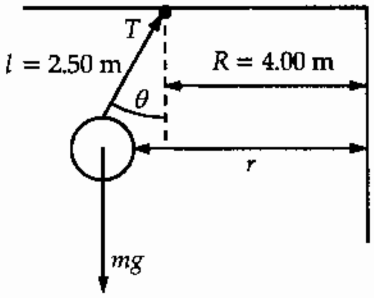


FIG. P6.63

(b) $T\cos\theta = mg$

$$T = \frac{mg}{\cos\theta} = \frac{(50.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 28.0°} = \boxed{555\text{ N}}$$

P6.64 (a) The putty, when dislodged, rises and returns to the original level in time t. To find t, we use $v_f = v_i + at$: i.e., $-v = +v - gt$ or $t = \frac{2v}{g}$ where v is the speed of a point on the rim of the wheel.

If R is the radius of the wheel, $v = \frac{2\pi R}{t}$, so $t = \frac{2v}{g} = \frac{2\pi R}{v}$.

Thus, $v^2 = \pi Rg$ and $\boxed{v = \sqrt{\pi Rg}}$.

(b) The putty is dislodged when F, the force holding it to the wheel is

$$F = \frac{mv^2}{R} = \boxed{m\pi g}.$$

P6.65 (a) $n = \frac{mv^2}{R}$ $\quad f - mg = 0$

$f = \mu_s n$ $\quad v = \frac{2\pi R}{T}$

$$T = \boxed{\sqrt{\frac{4\pi^2 R\mu_s}{g}}}$$

(b) $T = \boxed{2.54\text{ s}}$

$$\#\frac{\text{rev}}{\text{min}} = \frac{1\text{ rev}}{2.54\text{ s}}\left(\frac{60\text{ s}}{\text{min}}\right) = \boxed{23.6\ \frac{\text{rev}}{\text{min}}}$$

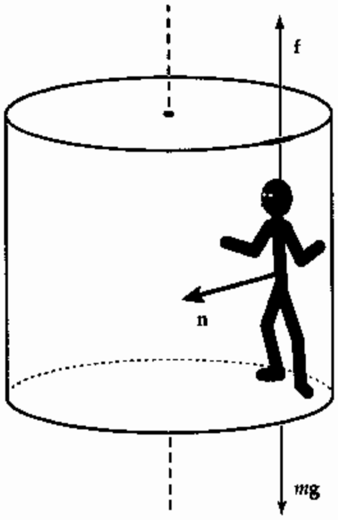


FIG. P6.65

P6.66 Let the x–axis point eastward, the y-axis upward, and the z-axis point southward.

(a) The range is $Z = \dfrac{v_i^2 \sin 2\theta_i}{g}$

The initial speed of the ball is therefore

$$v_i = \sqrt{\frac{gZ}{\sin 2\theta_i}} = \sqrt{\frac{(9.80)(285)}{\sin 96.0°}} = 53.0 \text{ m/s}$$

The time the ball is in the air is found from $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ as

$$0 = (53.0 \text{ m/s})(\sin 48.0°)t - \left(4.90 \text{ m/s}^2\right)t^2$$

giving $t = \boxed{8.04 \text{ s}}$.

(b) $v_{ix} = \dfrac{2\pi R_e \cos\phi_i}{86\,400 \text{ s}} = \dfrac{2\pi\left(6.37\times10^6 \text{ m}\right)\cos 35.0°}{86\,400 \text{ s}} = \boxed{379 \text{ m/s}}$

(c) 360° of latitude corresponds to a distance of $2\pi R_e$, so 285 m is a change in latitude of

$$\Delta\phi = \left(\frac{S}{2\pi R_e}\right)(360°) = \left(\frac{285 \text{ m}}{2\pi\left(6.37\times10^6 \text{ m}\right)}\right)(360°) = 2.56\times10^{-3} \text{ degrees}$$

The final latitude is then $\phi_f = \phi_i - \Delta\phi = 35.0° - 0.002\,56° = 34.997\,4°$.

The cup is moving eastward at a speed $v_{fx} = \dfrac{2\pi R_e \cos\phi_f}{86\,400 \text{ s}}$, which is larger than the eastward velocity of the tee by

$$\Delta v_x = v_{fx} - v_{fi} = \frac{2\pi R_e}{86\,400 \text{ s}}\left[\cos\phi_f - \cos\phi_i\right] = \frac{2\pi R_e}{86\,400 \text{ s}}\left[\cos(\phi_i - \Delta\phi) - \cos\phi_i\right]$$

$$= \frac{2\pi R_e}{86\,400 \text{ s}}\left[\cos\phi_i \cos\Delta\phi + \sin\phi_i \sin\Delta\phi - \cos\phi_i\right]$$

Since $\Delta\phi$ is such a small angle, $\cos\Delta\phi \approx 1$ and $\Delta v_x \approx \dfrac{2\pi R_e}{86\,400 \text{ s}}\sin\phi_i \sin\Delta\phi$.

$$\Delta v_x \approx \frac{2\pi\left(6.37\times10^6 \text{ m}\right)}{86\,400 \text{ s}}\sin 35.0° \sin 0.002\,56° = \boxed{1.19\times10^{-2} \text{ m/s}}$$

(d) $\Delta x = (\Delta v_x)t = \left(1.19\times10^{-2} \text{ m/s}\right)(8.04 \text{ s}) = 0.095\,5 \text{ m} = \boxed{9.55 \text{ cm}}$

P6.67 (a) If the car is about to slip *down* the incline, f is directed up the incline.

$\sum F_y = n\cos\theta + f\sin\theta - mg = 0$ where $f = \mu_s n$ gives

$$n = \frac{mg}{\cos\theta(1+\mu_s\tan\theta)} \text{ and } f = \frac{\mu_s mg}{\cos\theta(1+\mu_s\tan\theta)}.$$

Then, $\sum F_x = n\sin\theta - f\cos\theta = m\frac{v_{\min}^2}{R}$ yields

$$\boxed{v_{\min} = \sqrt{\frac{Rg(\tan\theta-\mu_s)}{1+\mu_s\tan\theta}}}.$$

When the car is about to slip *up* the incline, f is directed down the incline. Then, $\sum F_y = n\cos\theta - f\sin\theta - mg = 0$ with $f = \mu_s n$ yields

$$n = \frac{mg}{\cos\theta(1-\mu_s\tan\theta)} \text{ and } f = \frac{\mu_s mg}{\cos\theta(1-\mu_s\tan\theta)}.$$

In this case, $\sum F_x = n\sin\theta + f\cos\theta = m\frac{v_{\max}^2}{R}$, which gives

$$\boxed{v_{\max} = \sqrt{\frac{Rg(\tan\theta+\mu_s)}{1-\mu_s\tan\theta}}}.$$

(b) If $v_{\min} = \sqrt{\frac{Rg(\tan\theta-\mu_s)}{1+\mu_s\tan\theta}} = 0$, then $\boxed{\mu_s = \tan\theta}$.

(c)

$$v_{\min} = \sqrt{\frac{(100\text{ m})(9.80\text{ m/s}^2)(\tan 10.0° - 0.100)}{1+(0.100)\tan 10.0°}} = \boxed{8.57\text{ m/s}}$$

$$v_{\max} = \sqrt{\frac{(100\text{ m})(9.80\text{ m/s}^2)(\tan 10.0° + 0.100)}{1-(0.100)\tan 10.0°}} = \boxed{16.6\text{ m/s}}$$

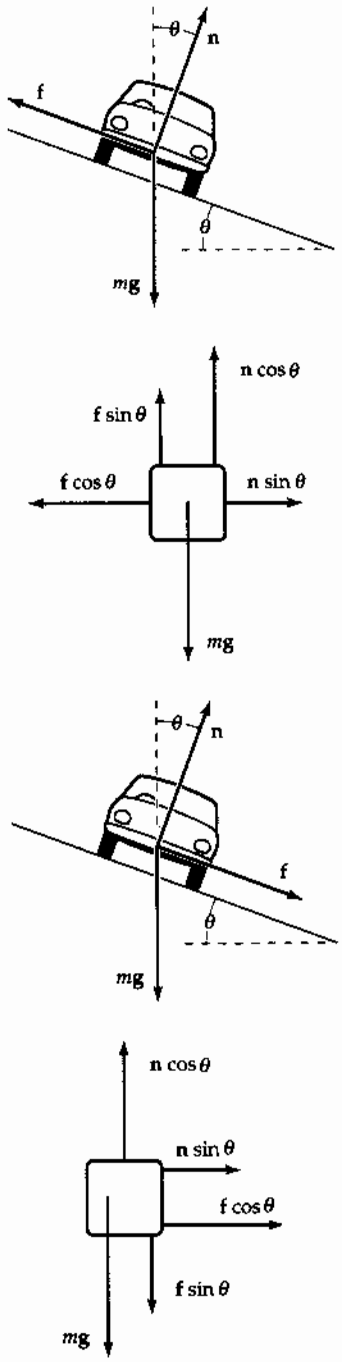


FIG. P6.67

P6.68 (a) The bead moves in a circle with radius $v = R\sin\theta$ at a speed of

$$v = \frac{2\pi r}{T} = \frac{2\pi R\sin\theta}{T}$$

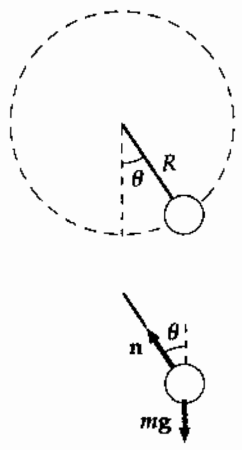


FIG. P6.68(a)

The normal force has
an inward radial component of $n\sin\theta$
and an upward component of $n\cos\theta$

$$\sum F_y = ma_y: \quad n\cos\theta - mg = 0$$

or

$$n = \frac{mg}{\cos\theta}$$

Then $\sum F_x = n\sin\theta = m\frac{v^2}{r}$ becomes $\left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{m}{R\sin\theta}\left(\frac{2\pi R\sin\theta}{T}\right)^2$

which reduces to $\frac{g\sin\theta}{\cos\theta} = \frac{4\pi^2 R\sin\theta}{T^2}$

This has two solutions: $\sin\theta = 0 \Rightarrow \theta = 0°$ (1)

and $\cos\theta = \frac{gT^2}{4\pi^2 R}$ (2)

If $R = 15.0$ cm and $T = 0.450$ s, the second solution yields

$$\cos\theta = \frac{(9.80\ \text{m/s}^2)(0.450\ \text{s})^2}{4\pi^2(0.150\ \text{m})} = 0.335 \text{ and } \theta = 70.4°$$

Thus, in this case, the bead can ride at two positions $\boxed{\theta = 70.4°}$ and $\boxed{\theta = 0°}$.

(b) At this slower rotation, solution (2) above becomes

$$\cos\theta = \frac{(9.80\ \text{m/s}^2)(0.850\ \text{s})^2}{4\pi^2(0.150\ \text{m})} = 1.20, \text{ which is impossible.}$$

In this case, the bead can ride only at the bottom of the loop, $\boxed{\theta = 0°}$. The loop's rotation must be faster than a certain threshold value in order for the bead to move away from the lowest position.

P6.69 At terminal velocity, the accelerating force of gravity is balanced by frictional drag: $mg = arv + br^2v^2$

(a) $mg = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$

For water, $m = \rho V = 1\,000\ \text{kg/m}^3\left[\frac{4}{3}\pi\left(10^{-5}\ \text{m}\right)^3\right]$

$$4.11\times10^{-11} = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$$

Assuming v is small, ignore the second term on the right hand side: $\boxed{v = 0.013\,2\ \text{m/s}}$.

(b) $mg = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$

Here we cannot ignore the second term because the coefficients are of nearly equal magnitude.

$$4.11\times10^{-8} = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$$

$$v = \frac{-3.10 \pm \sqrt{(3.10)^2 + 4(0.870)(4.11)}}{2(0.870)} = \boxed{1.03\ \text{m/s}}$$

(c) $mg = \left(3.10\times10^{-7}\right)v + \left(0.870\times10^{-6}\right)v^2$

Assuming $v > 1\ \text{m/s}$, and ignoring the first term:

$4.11\times10^{-5} = \left(0.870\times10^{-6}\right)v^2$ $\quad v = \boxed{6.87\ \text{m/s}}$

P6.70 $v = \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right]$ where $\exp(x) = e^x$ is the exponential function.

At $t \to \infty$, $\quad v \to v_T = \frac{mg}{b}$

At $t = 5.54$ s $\quad 0.500v_T = v_T\left[1 - \exp\left(\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right)\right]$

$$\exp\left(\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right) = 0.500;$$

$$\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}} = \ln 0.500 = -0.693;$$

$$b = \frac{(9.00\ \text{kg})(0.693)}{5.54\ \text{s}} = 1.13\ \text{m/s}$$

(a) $v_T = \frac{mg}{b}$ $\quad v_T = \frac{(9.00\ \text{kg})\left(9.80\ \text{m/s}^2\right)}{1.13\ \text{kg/s}} = \boxed{78.3\ \text{m/s}}$

(b) $0.750v_T = v_T\left[1 - \exp\left(\frac{-1.13t}{9.00\ \text{s}}\right)\right]$ $\quad \exp\left(\frac{-1.13t}{9.00\ \text{s}}\right) = 0.250$

$$t = \frac{9.00(\ln 0.250)}{-1.13}\ \text{s} = \boxed{11.1\ \text{s}}$$

continued on next page

(c) $\frac{dx}{dt}=\left(\frac{mg}{b}\right)\left[1-\exp\left(-\frac{bt}{m}\right)\right]$; $\int_{x_0}^{x} dx=\int_0^t\left(\frac{mg}{b}\right)\left[1-\exp\left(\frac{-bt}{m}\right)\right]dt$

$$x-x_0=\frac{mgt}{b}+\left(\frac{m^2g}{b^2}\right)\exp\left(\frac{-bt}{m}\right)\Big|_0^t=\frac{mgt}{b}+\left(\frac{m^2g}{b^2}\right)\left[\exp\left(\frac{-bt}{m}\right)-1\right]$$

At $t=5.54$ s, $x=9.00\text{ kg}\left(9.80\text{ m/s}^2\right)\frac{5.54\text{ s}}{1.13\text{ kg/s}}+\left(\frac{(9.00\text{ kg})^2\left(9.80\text{ m/s}^2\right)}{(1.13\text{ m/s})^2}\right)\left[\exp(-0.693)-1\right]$

$$x=434\text{ m}+626\text{ m}(-0.500)=\boxed{121\text{ m}}$$

P6.71 $\sum F_y=L_y-T_y-mg=L\cos 20.0°-T\sin 20.0°-7.35\text{ N}=ma_y=0$

$\sum F_x=L_x+T_x=L\sin 20.0°+T\cos 20.0°=m\frac{v^2}{r}$

$m\frac{v^2}{r}=0.750\text{ kg}\frac{(35.0\text{ m/s})^2}{(60.0\text{ m})\cos 20.0°}=16.3\text{ N}$

$\therefore L\sin 20.0°+T\cos 20.0°=16.3\text{ N}$

$L\cos 20.0°-T\sin 20.0°=7.35\text{ N}$

$L+T\frac{\cos 20.0°}{\sin 20.0°}=\frac{16.3\text{ N}}{\sin 20.0°}$

$L-T\frac{\sin 20.0°}{\cos 20.0°}=\frac{7.35\text{ N}}{\cos 20.0°}$

$T(\cot 20.0°+\tan 20.0°)=\frac{16.3\text{ N}}{\sin 20.0°}-\frac{7.35\text{ N}}{\cos 20.0°}$

$T(3.11)=39.8\text{ N}$

$T=\boxed{12.8\text{ N}}$

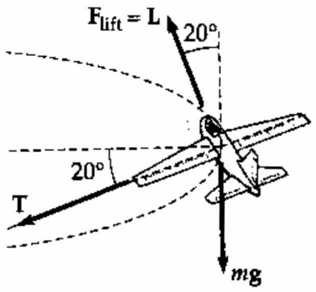
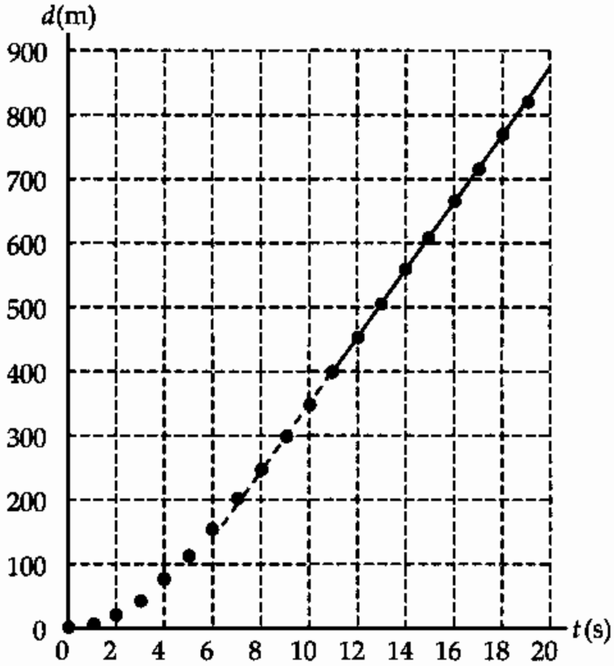


FIG. P6.71

P6.72 (a)

t(s)	d(m)
1.00	4.88
2.00	18.9
3.00	42.1
4.00	73.8
5.00	112
6.00	154
7.00	199
8.00	246
9.00	296
10.0	347
11.0	399
12.0	452
13.0	505
14.0	558
15.0	611
16.0	664
17.0	717
18.0	770
19.0	823
20.0	876

(b)



(c) A straight line fits the points from $t = 11.0$ s to 20.0 s quite precisely. Its slope is the terminal speed.

$$v_T = \text{slope} = \frac{876\text{ m} - 399\text{ m}}{20.0\text{ s} - 11.0\text{ s}} = \boxed{53.0\text{ m/s}}$$

***P6.73** $v = v_i - kx$ implies the acceleration is $a = \frac{dv}{dt} = 0 - k\frac{dx}{dt} = -kv$

Then the total force is $\sum F = ma = m(-kv)$

The resistive force is opposite to the velocity: $\boxed{\sum \mathbf{F} = -km\mathbf{v}}$.

ANSWERS TO EVEN PROBLEMS

P6.2 215 N horizontally inward

P6.4 6.22×10^{-12} N

P6.6 (a) 1.65 km/s; (b) 6.84×10^3 s

P6.8 $0.966\,g$

P6.10 (a) $(-0.233\,\hat{\mathbf{i}} + 0.163\,\hat{\mathbf{j}})\text{m/s}^2$; (b) 6.53 m/s; (c) $(-0.181\,\hat{\mathbf{i}} + 0.181\,\hat{\mathbf{j}})\text{m/s}^2$

P6.12 2.06×10^3 rev/min

P6.14 (a) $\sqrt{R\left(\frac{2T}{m} - g\right)}$; (b) $2T$ upward

P6.16 (a) 1.33 m/s^2; (b) 1.79 m/s^2 forward and 48.0° inward

P6.18 8.88 N

P6.20 (a) 8.62 m; (b) Mg downward; (c) 8.45 m/s^2, Unless they are belted in, the riders will fall from the cars.

P6.22 15.3 m/s Straight across the dashboard to the left

P6.24 0.527°

P6.26 (a) 1.41 h; (b) 17.1

P6.28 $\mu_k = \frac{2(vt-L)}{(g+a)t^2}$

P6.30 (a) $2.38 \times 10^5\ \text{m/s}^2$ horizontally inward $= 2.43 \times 10^4 g$; (b) 360 N inward perpendicular to the cone; (c) $47.5 \times 10^4\ \text{m/s}^2$

P6.32 (a) 6.27 m/s^2 downward; (b) 784 N up; (c) 283 N up

P6.34 (a) 53.8 m/s; (b) 148 m

P6.36 1.40

P6.38 $-0.212\ \text{m/s}^2$

P6.40 see the solution

P6.42 36.5 m/s

P6.44 (a) 0.980 m/s; (b) see the solution

P6.46 (a) 7.70×10^{-4} kg/m; (b) 0.998 N; (c) The ball reaches maximum height 49 m. Its flight lasts 6.3 s and its impact speed is 27 m/s.

P6.48 (a) see the solution; (b) 81.8 m; (c) 15.9°

P6.50 0.835 rev/s

P6.52 (a) $mg - \frac{mv^2}{R}$; (b) $v = \sqrt{gR}$

P6.54 (a) 2.63 m/s^2; (b) 201 m; (c) 17.7 m/s

P6.56 (a) 106 N; (b) 0.396

P6.58 (a) $m_2 g$; (b) $m_2 g$; (c) $\sqrt{\left(\frac{m_2}{m_1}\right) gR}$

P6.60 62.2 rev/min

P6.62 2.14 rev/min

P6.64 (a) $v = \sqrt{\pi R g}$; (b) $m\pi g$

P6.66 (a) 8.04 s; (b) 379 m/s; (c) 1.19 cm/s; (d) 9.55 cm

P6.68 (a) either 70.4° or 0°; (b) 0°

P6.70 (a) 78.3 m/s; (b) 11.1 s; (c) 121 m

P6.72 (a) and (b) see the solution; (c) 53.0 m/s

7

Energy and Energy Transfer

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q7.1 The force is perpendicular to every increment of displacement. Therefore, $\mathbf{F} \cdot \Delta \mathbf{r} = 0$.

Q7.2
(a) Positive work is done by the chicken on the dirt.

(b) No work is done, although it may seem like there is.

(c) Positive work is done on the bucket.

(d) Negative work is done on the bucket.

(e) Negative work is done on the person's torso.

Q7.3 Yes. Force times distance over which the toe is in contact with the ball. No, he is no longer applying a force. Yes, both air friction and gravity do work.

Q7.4 Force of tension on a ball rotating on the end of a string. Normal force and gravitational force on an object at rest or moving across a level floor.

Q7.5
(a) Tension
(b) Air resistance

(c) Positive in increasing velocity on the downswing.
Negative in decreasing velocity on the upswing.

Q7.6 No. The vectors might be in the third and fourth quadrants, but if the angle between them is less than 90° their dot product is positive.

Q7.7 The scalar product of two vectors is positive if the angle between them is between 0 and 90°. The scalar product is negative when $90° < \theta < 180°$.

Q7.8 If the coils of the spring are initially in contact with one another, as the load increases from zero, the graph would be an upwardly curved arc. After the load increases sufficiently, the graph will be linear, described by Hooke's Law. This linear region will be quite large compared to the first region. The graph will then be a downward curved arc as the coiled spring becomes a completely straight wire. As the load increases with a straight wire, the graph will become a straight line again, with a significantly smaller slope. Eventually, the wire would break.

Q7.9 $k' = 2k$. To stretch the smaller piece one meter, each coil would have to stretch twice as much as one coil in the original long spring, since there would be half as many coils. Assuming that the spring is ideal, twice the stretch requires twice the force.

Q7.10 Kinetic energy is always positive. Mass and squared speed are both positive. A moving object can always do positive work in striking another object and causing it to move along the same direction of motion.

Q7.11 Work is only done in accelerating the ball from rest. The work is done over the effective length of the pitcher's arm—the distance his hand moves through windup and until release.

Q7.12 Kinetic energy is proportional to mass. The first bullet has twice as much kinetic energy.

Q7.13 The longer barrel will have the higher muzzle speed. Since the accelerating force acts over a longer distance, the change in kinetic energy will be larger.

Q7.14 (a) Kinetic energy is proportional to squared speed. Doubling the speed makes an object's kinetic energy four times larger.

(b) If the total work on an object is zero in some process, its speed must be the same at the final point as it was at the initial point.

Q7.15 The larger engine is unnecessary. Consider a 30 minute commute. If you travel the same speed in each car, it will take the same amount of time, expending the same amount of energy. The extra power available from the larger engine isn't used.

Q7.16 If the instantaneous power output by some agent changes continuously, its average power in a process must be equal to its instantaneous power at least one instant. If its power output is constant, its instantaneous power is always equal to its average power.

Q7.17 It decreases, as the force required to lift the car decreases.

Q7.18 As you ride an express subway train, a backpack at your feet has no kinetic energy as measured by you since, according to you, the backpack is not moving. In the frame of reference of someone on the side of the tracks as the train rolls by, the backpack is moving and has mass, and thus has kinetic energy.

Q7.19 The rock increases in speed. The farther it has fallen, the more force it might exert on the sand at the bottom; but it might instead make a deeper crater with an equal-size average force. The farther it falls, the more work it will do in stopping. Its kinetic energy is increasing due to the work that the gravitational force does on it.

Q7.20 The normal force does no work because the angle between the normal force and the direction of motion is usually 90°. Static friction usually does no work because there is no distance through which the force is applied.

Q7.21 An argument for: As a glider moves along an airtrack, the only force that the track applies on the glider is the normal force. Since the angle between the direction of motion and the normal force is 90°, the work done must be zero, even if the track is not level.
Against: An airtrack has bumpers. When a glider bounces from the bumper at the end of the airtrack, it loses a bit of energy, as evidenced by a decreased speed. The airtrack does negative work.

Q7.22 Gaspard de Coriolis first stated the work-kinetic energy theorem. Jean Victor Poncelet, an engineer who invaded Russia with Napoleon, is most responsible for demonstrating its wide practical applicability, in his 1829 book *Industrial Mechanics*. Their work came remarkably late compared to the elucidation of momentum conservation in collisions by Descartes and to Newton's *Mathematical Principles of the Philosophy of Nature*, both in the 1600's.

SOLUTIONS TO PROBLEMS

Section 7.1 Systems and Environments

Section 7.2 Work Done by a Constant Force

P7.1 (a) $W = F\Delta r\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$

(b), (c) The normal force and the weight are both at 90° to the displacement in any time interval. Both do $\boxed{0}$ work.

(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

P7.2 The component of force along the direction of motion is

$$F\cos\theta = (35.0\text{ N})\cos 25.0° = 31.7\text{ N}.$$

The work done by this force is

$$W = (F\cos\theta)\Delta r = (31.7\text{ N})(50.0\text{ m}) = \boxed{1.59\times 10^3\text{ J}}.$$

P7.3 **Method One.**

Let ϕ represent the instantaneous angle the rope makes with the vertical as it is swinging up from $\phi_i = 0$ to $\phi_f = 60°$. In an incremental bit of motion from angle ϕ to $\phi + d\phi$, the definition of radian measure implies that $\Delta r = (12\text{ m})d\phi$. The angle θ between the incremental displacement and the force of gravity is $\theta = 90° + \phi$. Then $\cos\theta = \cos(90° + \phi) = -\sin\phi$.
The work done by the gravitational force on Batman is

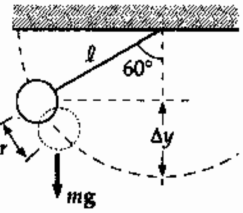


FIG. P7.3

$$W = \int_i^f F\cos\theta dr = \int_{\phi=0}^{\phi=60°} mg(-\sin\phi)(12\text{ m})d\phi$$
$$= -mg(12\text{ m})\int_0^{60°}\sin\phi\, d\phi = (-80\text{ kg})\left(9.8\text{ m/s}^2\right)(12\text{ m})(-\cos\phi)\Big|_0^{60°}$$
$$= (-784\text{ N})(12\text{ m})(-\cos 60° + 1) = \boxed{-4.70\times 10^3\text{ J}}$$

Method Two.

The force of gravity on Batman is $mg = (80\text{ kg})\left(9.8\text{ m/s}^2\right) = 784\text{ N}$ down. Only his vertical displacement contributes to the work gravity does. His original y-coordinate below the tree limb is -12 m. His final y-coordinate is $(-12\text{ m})\cos 60° = -6\text{ m}$. His change in elevation is $-6\text{ m} - (-12\text{ m}) = 6\text{ m}$. The work done by gravity is

$$W = F\Delta r\cos\theta = (784\text{ N})(6\text{ m})\cos 180° = \boxed{-4.70\text{ kJ}}.$$

P7.4 (a) $W = mgh = \left(3.35\times10^{-5}\right)(9.80)(100)\text{ J} = \boxed{3.28\times10^{-2}\text{ J}}$

(b) Since $R = mg$, $W_{\text{air resistance}} = \boxed{-3.28\times10^{-2}\text{ J}}$

Section 7.3 The Scalar Product of Two Vectors

P7.5 $A = 5.00$; $B = 9.00$; $\theta = 50.0°$

$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = (5.00)(9.00)\cos 50.0° = \boxed{28.9}$

P7.6
$$\mathbf{A}\cdot\mathbf{B} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right)\cdot\left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$
$$\begin{aligned}\mathbf{A}\cdot\mathbf{B} = {} & A_xB_x\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}\right) + A_xB_y\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{j}}\right) + A_xB_z\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{k}}\right)\\ & + A_yB_x\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}\right) + A_yB_y\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}\right) + A_yB_z\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{k}}\right)\\ & + A_zB_x\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{i}}\right) + A_zB_y\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{j}}\right) + A_zB_z\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}\right)\end{aligned}$$
$$\mathbf{A}\cdot\mathbf{B} = \boxed{A_xB_x + A_yB_y + A_zB_z}$$

P7.7 (a) $W = \mathbf{F}\cdot\Delta\mathbf{r} = F_x x + F_y y = (6.00)(3.00)\text{ N}\cdot\text{m} + (-2.00)(1.00)\text{ N}\cdot\text{m} = \boxed{16.0\text{ J}}$

(b) $$\theta = \cos^{-1}\left(\frac{\mathbf{F}\cdot\Delta\mathbf{r}}{F\Delta r}\right) = \cos^{-1}\frac{16}{\sqrt{\left((6.00)^2+(-2.00)^2\right)\left((3.00)^2+(1.00)^2\right)}} = \boxed{36.9°}$$

P7.8 We must first find the angle between the two vectors. It is:

$$\theta = 360° - 118° - 90.0° - 132° = 20.0°$$

Then

$$\mathbf{F}\cdot\mathbf{v} = Fv\cos\theta = (32.8\text{ N})(0.173\text{ m/s})\cos 20.0°$$

or $\mathbf{F}\cdot\mathbf{v} = 5.33\dfrac{\text{N}\cdot\text{m}}{\text{s}} = 5.33\dfrac{\text{J}}{\text{s}} = \boxed{5.33\text{ W}}$

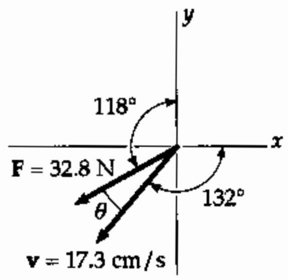


FIG. P7.8

P7.9 (a) $\mathbf{A} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$

$\mathbf{B} = 4.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}$ $\qquad \theta = \cos^{-1}\dfrac{\mathbf{A}\cdot\mathbf{B}}{AB} = \cos^{-1}\dfrac{12.0+8.00}{\sqrt{(13.0)(32.0)}} = \boxed{11.3°}$

(b) $\mathbf{B} = 3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\mathbf{A} = -2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$ $\qquad \cos\theta = \dfrac{\mathbf{A}\cdot\mathbf{B}}{AB} = \dfrac{-6.00-16.0}{\sqrt{(20.0)(29.0)}}$ $\qquad \theta = \boxed{156°}$

(c) $\mathbf{A} = \hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\mathbf{B} = 3.00\hat{\mathbf{j}} + 4.00\hat{\mathbf{k}}$ $\qquad \theta = \cos^{-1}\left(\dfrac{\mathbf{A}\cdot\mathbf{B}}{AB}\right) = \cos^{-1}\left(\dfrac{-6.00+8.00}{\sqrt{9.00}\cdot\sqrt{25.0}}\right) = \boxed{82.3°}$

P7.10 $\mathbf{A}-\mathbf{B}=\left(3.00\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)-\left(-\hat{\mathbf{i}}+2.00\hat{\mathbf{j}}+5.00\hat{\mathbf{k}}\right)$

$\mathbf{A}-\mathbf{B}=4.00\hat{\mathbf{i}}-\hat{\mathbf{j}}-6.00\hat{\mathbf{k}}$

$\mathbf{C}\cdot(\mathbf{A}-\mathbf{B})=\left(2.00\hat{\mathbf{j}}-3.00\hat{\mathbf{k}}\right)\cdot\left(4.00\hat{\mathbf{i}}-\hat{\mathbf{j}}-6.00\hat{\mathbf{k}}\right)=0+(-2.00)+(+18.0)=\boxed{16.0}$

Section 7.4 Work Done by a Varying Force

P7.11 $W=\int_i^f F\,dx=$ area under curve from x_i to x_f

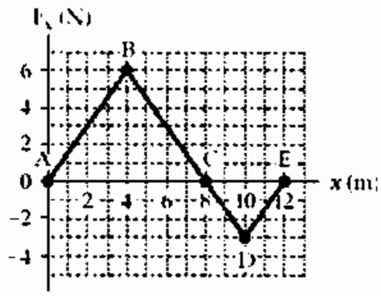


FIG. P7.11

(a) $x_i=0$ $\quad x_f=8.00\text{ m}$

$W=$ area of triangle $ABC=\left(\frac{1}{2}\right)AC\times$ altitude,

$W_{0\to 8}=\left(\frac{1}{2}\right)\times 8.00\text{ m}\times 6.00\text{ N}=\boxed{24.0\text{ J}}$

(b) $x_i=8.00\text{ m}$ $\quad x_f=10.0\text{ m}$

$W=$ area of $\Delta CDE=\left(\frac{1}{2}\right)CE\times$ altitude,

$W_{8\to 10}=\left(\frac{1}{2}\right)\times(2.00\text{ m})\times(-3.00\text{ N})=\boxed{-3.00\text{ J}}$

(c) $W_{0\to 10}=W_{0\to 8}+W_{8\to 10}=24.0+(-3.00)=\boxed{21.0\text{ J}}$

P7.12 $F_x=(8x-16)\text{ N}$

(a) See figure to the right

(b) $W_{\text{net}}=\dfrac{-(2.00\text{ m})(16.0\text{ N})}{2}+\dfrac{(1.00\text{ m})(8.00\text{ N})}{2}=\boxed{-12.0\text{ J}}$

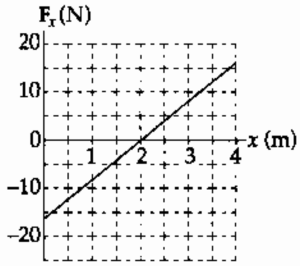


FIG. P7.12

P7.13 $W = \int F_x dx$

and W equals the area under the Force-Displacement curve

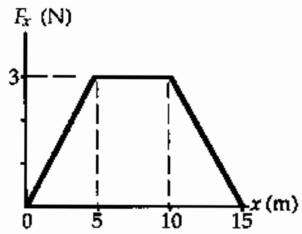


FIG. P7.13

(a) For the region $0 \le x \le 5.00$ m,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(b) For the region $5.00 \le x \le 10.0$,

$$W = (3.00\text{ N})(5.00\text{ m}) = \boxed{15.0\text{ J}}$$

(c) For the region $10.0 \le x \le 15.0$,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

(d) For the region $0 \le x \le 15.0$

$$W = (7.50 + 7.50 + 15.0)\text{ J} = \boxed{30.0\text{ J}}$$

P7.14 $W = \int_i^f \mathbf{F}\cdot d\mathbf{r} = \int_0^{5\text{ m}} \left(4x\hat{\mathbf{i}} + 3y\hat{\mathbf{j}}\right)\text{ N}\cdot dx\hat{\mathbf{i}}$

$$\int_0^{5\text{ m}} (4\text{ N/m})x\,dx + 0 = (4\text{ N/m})\left.\frac{x^2}{2}\right|_0^{5\text{ m}} = \boxed{50.0\text{ J}}$$

P7.15 $k = \frac{F}{y} = \frac{Mg}{y} = \frac{(4.00)(9.80)\text{ N}}{2.50\times 10^{-2}\text{ m}} = 1.57\times 10^3\text{ N/m}$

(a) For 1.50 kg mass $y = \frac{mg}{k} = \frac{(1.50)(9.80)}{1.57\times 10^3} = \boxed{0.938\text{ cm}}$

(b) Work $= \frac{1}{2}ky^2$

Work $= \frac{1}{2}\left(1.57\times 10^3\text{ N}\cdot\text{m}\right)\left(4.00\times 10^{-2}\text{ m}\right)^2 = \boxed{1.25\text{ J}}$

P7.16 (a) Spring constant is given by $F = kx$

$$k = \frac{F}{x} = \frac{(230\text{ N})}{(0.400\text{ m})} = \boxed{575\text{ N/m}}$$

(b) Work $= F_{\text{avg}}x = \frac{1}{2}(230\text{ N})(0.400\text{ m}) = \boxed{46.0\text{ J}}$

***P7.17** (a)

$$F_{\text{applied}} = k_{\text{leaf}}x_\ell + k_{\text{helper}}x_h = k_\ell x_\ell + k_h(x_\ell - y_0)$$

$$5\times10^5\ \text{N} = 5.25\times10^5\ \frac{\text{N}}{\text{m}}x_\ell + 3.60\times10^5\ \frac{\text{N}}{\text{m}}(x_\ell - 0.5\ \text{m})$$

$$x_\ell = \frac{6.8\times10^5\ \text{N}}{8.85\times10^5\ \text{N/m}} = \boxed{0.768\ \text{m}}$$

(b)

$$W = \frac{1}{2}k_\ell x_\ell^2 + \frac{1}{2}k_h x_h^2 = \frac{1}{2}\left(5.25\times10^5\ \frac{\text{N}}{\text{m}}\right)(0.768\ \text{m})^2 + \frac{1}{2}3.60\times10^5\ \frac{\text{N}}{\text{m}}(0.268\ \text{m})^2$$

$$= \boxed{1.68\times10^5\ \text{J}}$$

P7.18 (a)

$$W = \int_i^f \mathbf{F}\cdot d\mathbf{r}$$

$$W = \int_0^{0.600\ \text{m}} \left(15\,000\ \text{N} + 10\,000x\ \text{N/m} - 25\,000x^2\ \text{N/m}^2\right)dx\cos 0°$$

$$W = 15\,000x + \frac{10\,000x^2}{2} - \frac{25\,000x^3}{3}\Bigg|_0^{0.600\ \text{m}}$$

$$W = 9.00\ \text{kJ} + 1.80\ \text{kJ} - 1.80\ \text{kJ} = \boxed{9.00\ \text{kJ}}$$

(b) Similarly,

$$W = (15.0\ \text{kN})(1.00\ \text{m}) + \frac{(10.0\ \text{kN/m})(1.00\ \text{m})^2}{2} - \frac{(25.0\ \text{kN/m}^2)(1.00\ \text{m})^3}{3}$$

$W = \boxed{11.7\ \text{kJ}}$, larger by 29.6%

P7.19 $4.00\ \text{J} = \frac{1}{2}k(0.100\ \text{m})^2$

$\therefore k = 800\ \text{N/m}$ and to stretch the spring to 0.200 m requires

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00\ \text{J} = \boxed{12.0\ \text{J}}$$

P7.20 (a) The radius to the object makes angle θ with the horizontal, so its weight makes angle θ with the negative side of the x-axis, when we take the x–axis in the direction of motion tangent to the cylinder.

$$\sum F_x = ma_x$$

$$F - mg\cos\theta = 0$$

$$F = \boxed{mg\cos\theta}$$

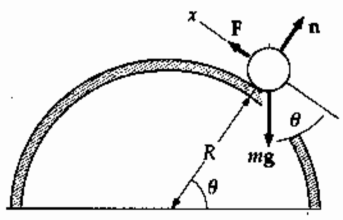


FIG. P7.20

(b)

$$W = \int_i^f \mathbf{F}\cdot d\mathbf{r}$$

We use radian measure to express the next bit of displacement as $dr = Rd\theta$ in terms of the next bit of angle moved through:

$$W = \int_0^{\pi/2} mg\cos\theta R d\theta = mgR\sin\theta\Big|_0^{\pi/2}$$

$$W = mgR(1-0) = \boxed{mgR}$$

***P7.21** The same force makes both light springs stretch.

(a) The hanging mass moves down by

$$x = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$
$$= 1.5 \text{ kg } 9.8 \text{ m/s}^2\left(\frac{1 \text{ m}}{1\,200 \text{ N}} + \frac{1 \text{ m}}{1\,800 \text{ N}}\right) = \boxed{2.04 \times 10^{-2} \text{ m}}$$

(b) We define the effective spring constant as

$$k = \frac{F}{x} = \frac{mg}{mg(1/k_1 + 1/k_2)} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$
$$= \left(\frac{1 \text{ m}}{1\,200 \text{ N}} + \frac{1 \text{ m}}{1\,800 \text{ N}}\right)^{-1} = \boxed{720 \text{ N/m}}$$

***P7.22** See the solution to problem 7.21.

(a) $x = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$

(b) $k = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$

P7.23 $[k] = \left[\frac{F}{x}\right] = \frac{\text{N}}{\text{m}} = \frac{\text{kg} \cdot \text{m/s}^2}{\text{m}} = \boxed{\frac{\text{kg}}{\text{s}^2}}$

Section 7.5 **Kinetic Energy and the Work-Kinetic Energy Theorem**

Section 7.6 **The Non-Isolated System—Conservation of Energy**

P7.24 (a) $K_A = \frac{1}{2}(0.600 \text{ kg})(2.00 \text{ m/s})^2 = \boxed{1.20 \text{ J}}$

(b) $\frac{1}{2}mv_B^2 = K_B$: $v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{(2)(7.50)}{0.600}} = \boxed{5.00 \text{ m/s}}$

(c) $\sum W = \Delta K = K_B - K_A = \frac{1}{2}m\left(v_B^2 - v_A^2\right) = 7.50 \text{ J} - 1.20 \text{ J} = \boxed{6.30 \text{ J}}$

P7.25 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.300 \text{ kg})(15.0 \text{ m/s})^2 = \boxed{33.8 \text{ J}}$

(b) $K = \frac{1}{2}(0.300)(30.0)^2 = \frac{1}{2}(0.300)(15.0)^2(4) = 4(33.8) = \boxed{135 \text{ J}}$

P7.26 $\mathbf{v}_i = \left(6.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}\right) = \text{ m/s}$

(a) $v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{40.0} \text{ m/s}$

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3.00 \text{ kg})\left(40.0 \text{ m}^2/\text{s}^2\right) = \boxed{60.0 \text{ J}}$$

(b) $\mathbf{v}_f = 8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$

$$v_f^2 = \mathbf{v}_f \cdot \mathbf{v}_f = 64.0 + 16.0 = 80.0 \text{ m}^2/\text{s}^2$$

$$\Delta K = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{3.00}{2}(80.0) - 60.0 = \boxed{60.0 \text{ J}}$$

P7.27 Consider the work done on the pile driver from the time it starts from rest until it comes to rest at the end of the fall. Let $d = 5.00$ m represent the distance over which the driver falls freely, and $h = 0.12$ m the distance it moves the piling.

$$\sum W = \Delta K: \quad W_{\text{gravity}} + W_{\text{beam}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

so $\quad (mg)(h+d)\cos 0° + \left(\bar{F}\right)(d)\cos 180° = 0 - 0.$

Thus, $\quad \bar{F} = \dfrac{(mg)(h+d)}{d} = \dfrac{(2\,100 \text{ kg})\left(9.80 \text{ m/s}^2\right)(5.12 \text{ m})}{0.120 \text{ m}} = \boxed{8.78 \times 10^5 \text{ N}}$. The force on the pile driver is $\boxed{\text{upward}}$.

P7.28 (a) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 5.00$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(7.50 \text{ J})}{4.00 \text{ kg}}} = \boxed{1.94 \text{ m/s}}$$

(b) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 10.0$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(22.5 \text{ J})}{4.00 \text{ kg}}} = \boxed{3.35 \text{ m/s}}$$

(c) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 15.0$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(30.0 \text{ J})}{4.00 \text{ kg}}} = \boxed{3.87 \text{ m/s}}$$

P7.29 (a) $K_i + \sum W = K_f = \frac{1}{2}mv_f^2$

$$0 + \sum W = \frac{1}{2}\left(15.0 \times 10^{-3} \text{ kg}\right)(780 \text{ m/s})^2 = \boxed{4.56 \text{ kJ}}$$

(b) $$F = \frac{W}{\Delta r \cos\theta} = \frac{4.56 \times 10^3 \text{ J}}{(0.720 \text{ m})\cos 0°} = \boxed{6.34 \text{ kN}}$$

(c) $$a = \frac{v_f^2 - v_i^2}{2x_f} = \frac{(780 \text{ m/s})^2 - 0}{2(0.720 \text{ m})} = \boxed{422 \text{ km/s}^2}$$

(d) $$\sum F = ma = \left(15 \times 10^{-3} \text{ kg}\right)\left(422 \times 10^3 \text{ m/s}^2\right) = \boxed{6.34 \text{ kN}}$$

P7.30 (a) $v_f = 0.096\left(3\times10^8\ \text{m/s}\right) = 2.88\times10^7\ \text{m/s}$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}\left(9.11\times10^{-31}\ \text{kg}\right)\left(2.88\times10^7\ \text{m/s}\right)^2 = \boxed{3.78\times10^{-16}\ \text{J}}$$

(b) $K_i + W = K_f$: $\quad 0 + F\Delta r\cos\theta = K_f$

$$F(0.028\ \text{m})\cos 0° = 3.78\times10^{-16}\ \text{J}$$

$$F = \boxed{1.35\times10^{-14}\ \text{N}}$$

(c) $\sum F = ma$; $\quad a = \frac{\sum F}{m} = \frac{1.35\times10^{-14}\ \text{N}}{9.11\times10^{-31}\ \text{kg}} = \boxed{1.48\times10^{+16}\ \text{m/s}^2}$

(d) $v_{xf} = v_{xi} + a_x t \quad 2.88\times10^7\ \text{m/s} = 0 + \left(1.48\times10^{16}\ \text{m/s}^2\right)t$

$$t = \boxed{1.94\times10^{-9}\ \text{s}}$$

Check: $x_f = x_i + \frac{1}{2}\left(v_{xi} + v_{xf}\right)t$

$$0.028\ \text{m} = 0 + \frac{1}{2}\left(0 + 2.88\times10^7\ \text{m/s}\right)t$$

$$t = 1.94\times10^{-9}\ \text{s}$$

Section 7.7 Situations Involving Kinetic Friction

P7.31 $\sum F_y = ma_y$: $\quad n - 392\ \text{N} = 0$

$$n = 392\ \text{N}$$

$$f_k = \mu_k n = (0.300)(392\ \text{N}) = 118\ \text{N}$$

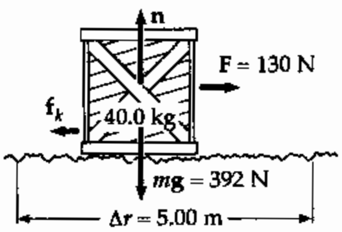


FIG. P7.31

(a) $W_F = F\Delta r\cos\theta = (130)(5.00)\cos 0° = \boxed{650\ \text{J}}$

(b) $\Delta E_{int} = f_k\Delta x = (118)(5.00) = \boxed{588\ \text{J}}$

(c) $W_n = n\Delta r\cos\theta = (392)(5.00)\cos 90° = \boxed{0}$

(d) $W_g = mg\Delta r\cos\theta = (392)(5.00)\cos(-90°) = \boxed{0}$

(e) $\Delta K = K_f - K_i = \sum W_{other} - \Delta E_{int}$

$$\frac{1}{2}mv_f^2 - 0 = 650\ \text{J} - 588\ \text{J} + 0 + 0 = \boxed{62.0\ \text{J}}$$

(f) $v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(62.0\ \text{J})}{40.0\ \text{kg}}} = \boxed{1.76\ \text{m/s}}$

P7.32 (a) $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}(500)(5.00 \times 10^{-2})^2 - 0 = 0.625 \text{ J}$

$W_s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 - 0$

so $v_f = \sqrt{\frac{2(\sum W)}{m}} = \sqrt{\frac{2(0.625)}{2.00}} \text{ m/s} = \boxed{0.791 \text{ m/s}}$

(b) $\frac{1}{2}mv_i^2 - f_k\Delta x + W_s = \frac{1}{2}mv_f^2$

$0 - (0.350)(2.00)(9.80)(0.050\,0) \text{ J} + 0.625 \text{ J} = \frac{1}{2}mv_f^2$

$0.282 \text{ J} = \frac{1}{2}(2.00 \text{ kg})v_f^2$

$v_f = \sqrt{\frac{2(0.282)}{2.00}} \text{ m/s} = \boxed{0.531 \text{ m/s}}$

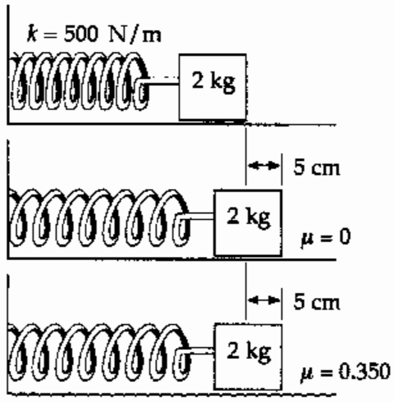


FIG. P7.32

P7.33 (a) $W_g = mg\ell\cos(90.0° + \theta)$

$W_g = (10.0 \text{ kg})(9.80 \text{ m/s}^2)(5.00 \text{ m})\cos 110° = \boxed{-168 \text{ J}}$

(b) $f_k = \mu_k n = \mu_k mg\cos\theta$

$\Delta E_{\text{int}} = \ell f_k = \ell\mu_k mg\cos\theta$

$\Delta E_{\text{int}} = (5.00 \text{ m})(0.400)(10.0)(9.80)\cos 20.0° = \boxed{184 \text{ J}}$

(c) $W_F = F\ell = (100)(5.00) = \boxed{500 \text{ J}}$

(d) $\Delta K = \sum W_{\text{other}} - \Delta E_{\text{int}} = W_F + W_g - \Delta E_{\text{int}} = \boxed{148 \text{ J}}$

(e) $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$v_f = \sqrt{\frac{2(\Delta K)}{m} + v_i^2} = \sqrt{\frac{2(148)}{10.0} + (1.50)^2} = \boxed{5.65 \text{ m/s}}$

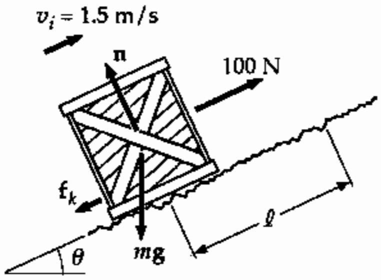


FIG. P7.33

P7.34 $\sum F_y = ma_y$: $n + (70.0 \text{ N})\sin 20.0° - 147 \text{ N} = 0$

$n = 123 \text{ N}$

$f_k = \mu_k n = 0.300(123 \text{ N}) = 36.9 \text{ N}$

(a) $W = F\Delta r\cos\theta = (70.0 \text{ N})(5.00 \text{ m})\cos 20.0° = \boxed{329 \text{ J}}$

(b) $W = F\Delta r\cos\theta = (123 \text{ N})(5.00 \text{ m})\cos 90.0° = \boxed{0 \text{ J}}$

(c) $W = F\Delta r\cos\theta = (147 \text{ N})(5.00 \text{ m})\cos 90.0° = \boxed{0}$

(d) $\Delta E_{\text{int}} = F\Delta x = (36.9 \text{ N})(5.00 \text{ m}) = \boxed{185 \text{ J}}$

(e) $\Delta K = K_f - K_i = \sum W - \Delta E_{\text{int}} = 329 \text{ J} - 185 \text{ J} = \boxed{+144 \text{ J}}$

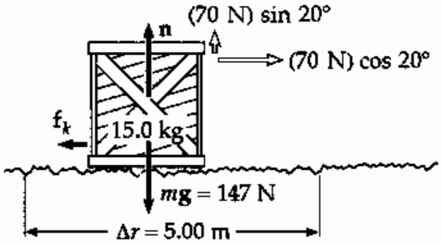


FIG. P7.34

P7.35 $v_i = 2.00\ \text{m/s}$ $\qquad \mu_k = 0.100$

$$K_i - f_k\Delta x + W_{\text{other}} = K_f: \qquad \frac{1}{2}mv_i^2 - f_k\Delta x = 0$$

$$\frac{1}{2}mv_i^2 = \mu_k mg\Delta x \qquad \Delta x = \frac{v_i^2}{2\mu_k g} = \frac{(2.00\ \text{m/s})^2}{2(0.100)(9.80)} = \boxed{2.04\ \text{m}}$$

Section 7.8 **Power**

***P7.36** $$\mathscr{P}_{\text{av}} = \frac{W}{\Delta t} = \frac{K_f}{\Delta t} = \frac{mv^2}{2\Delta t} = \frac{0.875\ \text{kg}(0.620\ \text{m/s})^2}{2\left(21\times10^{-3}\ \text{s}\right)} = \boxed{8.01\ \text{W}}$$

P7.37 $$\text{Power} = \frac{W}{t} \qquad \mathscr{P} = \frac{mgh}{t} = \frac{(700\ \text{N})(10.0\ \text{m})}{8.00\ \text{s}} = \boxed{875\ \text{W}}$$

P7.38 A 1 300-kg car speeds up from rest to 55.0 mi/h = 24.6 m/s in 15.0 s. The output work of the engine is equal to its final kinetic energy,

$$\frac{1}{2}(1\,300\ \text{kg})(24.6\ \text{m/s})^2 = 390\ \text{kJ}$$

with power $\mathscr{P} = \dfrac{390\,000\ \text{J}}{15.0\ \text{s}} \boxed{\sim 10^4\ \text{W}}$ around 30 horsepower.

P7.39 (a) $\sum W = \Delta K$, but $\Delta K = 0$ because he moves at constant speed. The skier rises a vertical distance of $(60.0\ \text{m})\sin 30.0° = 30.0\ \text{m}$. Thus,

$$W_{\text{in}} = -W_g = (70.0\ \text{kg})\left(9.8\ \text{m/s}^2\right)(30.0\ \text{m}) = \boxed{2.06\times10^4\ \text{J}} = \boxed{20.6\ \text{kJ}}.$$

(b) The time to travel 60.0 m at a constant speed of 2.00 m/s is 30.0 s. Thus,

$$\mathscr{P}_{\text{input}} = \frac{W}{\Delta t} = \frac{2.06\times10^4\ \text{J}}{30.0\ \text{s}} = \boxed{686\ \text{W}} = 0.919\ \text{hp}.$$

P7.40 (a) The distance moved upward in the first 3.00 s is

$$\Delta y = \bar{v}t = \left[\frac{0+1.75\ \text{m/s}}{2}\right](3.00\ \text{s}) = 2.63\ \text{m}.$$

The motor and the earth's gravity do work on the elevator car:

$$\frac{1}{2}mv_i^2 + W_{\text{motor}} + mg\Delta y\cos 180° = \frac{1}{2}mv_f^2$$

$$W_{\text{motor}} = \frac{1}{2}(650\ \text{kg})(1.75\ \text{m/s})^2 - 0 + (650\ \text{kg})g(2.63\ \text{m}) = 1.77\times10^4\ \text{J}$$

Also, $W = \bar{\mathscr{P}}t$ so $\bar{\mathscr{P}} = \dfrac{W}{t} = \dfrac{1.77\times10^4\ \text{J}}{3.00\ \text{s}} = \boxed{5.91\times10^3\ \text{W}} = 7.92\ \text{hp}.$

(b) When moving upward at constant speed $(v = 1.75\ \text{m/s})$ the applied force equals the weight $= (650\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 6.37\times10^3\ \text{N}$. Therefore,

$$\mathscr{P} = Fv = \left(6.37\times10^3\ \text{N}\right)(1.75\ \text{m/s}) = \boxed{1.11\times10^4\ \text{W}} = 14.9\ \text{hp}.$$

P7.41 *energy = power × time*

For the 28.0 W bulb:

$$\text{Energy used} = (28.0\text{ W})\left(1.00\times10^4\text{ h}\right) = 280\text{ kilowatt}\cdot\text{hrs}$$
$$\text{total cost} = \$17.00 + (280\text{ kWh})(\$0.080/\text{kWh}) = \$39.40$$

For the 100 W bulb:

$$\text{Energy used} = (100\text{ W})\left(1.00\times10^4\text{ h}\right) = 1.00\times10^3\text{ kilowatt}\cdot\text{hrs}$$
$$\#\text{ bulb used} = \frac{1.00\times10^4\text{ h}}{750\text{ h/bulb}} = 13.3$$
$$\text{total cost} = 13.3(\$0.420) + \left(1.00\times10^3\text{ kWh}\right)(\$0.080/\text{kWh}) = \$85.60$$

Savings with energy-efficient bulb $= \$85.60 - \$39.40 = \boxed{\$46.20}$

***P7.42** (a) Burning 1 lb of fat releases energy $1\text{ lb}\left(\frac{454\text{ g}}{1\text{ lb}}\right)\left(\frac{9\text{ kcal}}{1\text{ g}}\right)\left(\frac{4\,186\text{ J}}{1\text{ kcal}}\right) = 1.71\times10^7\text{ J}$.

The mechanical energy output is $\left(1.71\times10^7\text{ J}\right)(0.20) = nF\Delta r\cos\theta$.

Then

$$3.42\times10^6\text{ J} = nmg\Delta y\cos 0°$$
$$3.42\times10^6\text{ J} = n(50\text{ kg})\left(9.8\text{ m/s}^2\right)(80\text{ steps})(0.150\text{ m})$$
$$3.42\times10^6\text{ J} = n\left(5.88\times10^3\text{ J}\right)$$

where the number of times she must climb the steps is $n = \frac{3.42\times10^6\text{ J}}{5.88\times10^3\text{ J}} = \boxed{582}$.

This method is impractical compared to limiting food intake.

(b) Her mechanical power output is

$$\mathcal{P} = \frac{W}{t} = \frac{5.88\times10^3\text{ J}}{65\text{ s}} = \boxed{90.5\text{ W}} = 90.5\text{ W}\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{0.121\text{ hp}}.$$

***P7.43** (a) The fuel economy for walking is $\frac{1\text{ h}}{220\text{ kcal}}\left(\frac{3\text{ mi}}{\text{h}}\right)\left(\frac{1\text{ kcal}}{4186\text{ J}}\right)\left(\frac{1.30\times10^8\text{ J}}{1\text{ gal}}\right) = \boxed{423\text{ mi/gal}}$.

(b) For bicycling $\frac{1\text{ h}}{400\text{ kcal}}\left(\frac{10\text{ mi}}{\text{h}}\right)\left(\frac{1\text{ kcal}}{4186\text{ J}}\right)\left(\frac{1.30\times10^8\text{ J}}{1\text{ gal}}\right) = \boxed{776\text{ mi/gal}}$.

Section 7.9 Energy and the Automobile

P7.44 At a speed of 26.8 m/s (60.0 mph), the car described in Table 7.2 delivers a power of $\mathscr{P}_1 = 18.3 \text{ kW}$ to the wheels. If an additional load of 350 kg is added to the car, a larger output power of

$$\mathscr{P}_2 = \mathscr{P}_1 + (\text{power input to move 350 kg at speed } v)$$

will be required. The additional power output needed to move 350 kg at speed v is:

$$\Delta\mathscr{P}_{\text{out}} = (\Delta f)v = (\mu_r mg)v.$$

Assuming a coefficient of rolling friction of $\mu_r = 0.016\,0$, the power output now needed from the engine is

$$\mathscr{P}_2 = \mathscr{P}_1 + (0.016\,0)(350 \text{ kg})(9.80 \text{ m/s}^2)(26.8 \text{ m/s}) = 18.3 \text{ kW} + 1.47 \text{ kW}.$$

With the assumption of constant efficiency of the engine, the input power must increase by the same factor as the output power. Thus, the fuel economy must decrease by this factor:

$$(\text{fuel economy})_2 = \left(\frac{\mathscr{P}_1}{\mathscr{P}_2}\right)(\text{fuel economy})_1 = \left(\frac{18.3}{18.3+1.47}\right)(6.40 \text{ km/L})$$

or $(\text{fuel economy})_2 = \boxed{5.92 \text{ km/L}}$.

P7.45 (a) $$\text{fuel needed} = \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{\text{useful energy per gallon}} = \frac{\frac{1}{2}mv_f^2 - 0}{\text{eff.}\times(\text{energy content of fuel})}$$

$$= \frac{\frac{1}{2}(900 \text{ kg})(24.6 \text{ m/s})^2}{(0.150)(1.34\times10^8 \text{ J/gal})} = \boxed{1.35\times10^{-2} \text{ gal}}$$

(b) $\boxed{73.8}$

(c) $$\text{power} = \left(\frac{1 \text{ gal}}{38.0 \text{ mi}}\right)\left(\frac{55.0 \text{ mi}}{1.00 \text{ h}}\right)\left(\frac{1.00 \text{ h}}{3\,600 \text{ s}}\right)\left(\frac{1.34\times10^8 \text{ J}}{1 \text{ gal}}\right)(0.150) = \boxed{8.08 \text{ kW}}$$

Additional Problems

P7.46 At start, $\mathbf{v} = (40.0 \text{ m/s})\cos 30.0°\,\hat{\mathbf{i}} + (40.0 \text{ m/s})\sin 30.0°\,\hat{\mathbf{j}}$

At apex, $\mathbf{v} = (40.0 \text{ m/s})\cos 30.0°\,\hat{\mathbf{i}} + 0\hat{\mathbf{j}} = (34.6 \text{ m/s})\hat{\mathbf{i}}$

And $K = \frac{1}{2}mv^2 = \frac{1}{2}(0.150 \text{ kg})(34.6 \text{ m/s})^2 = \boxed{90.0 \text{ J}}$

P7.47 Concentration of Energy output $= (0.600 \text{ J/kg}\cdot\text{step})(60.0 \text{ kg})\left(\frac{1 \text{ step}}{1.50 \text{ m}}\right) = 24.0 \text{ J/m}$

$$F = (24.0 \text{ J/m})(1 \text{ N}\cdot\text{m/J}) = 24.0 \text{ N}$$
$$\mathcal{P} = Fv$$
$$70.0 \text{ W} = (24.0 \text{ N})v$$
$$v = \boxed{2.92 \text{ m/s}}$$

P7.48 (a) $\mathbf{A}\cdot\hat{\mathbf{i}} = (A)(1)\cos\alpha$. But also, $\mathbf{A}\cdot\hat{\mathbf{i}} = A_x$.

Thus, $(A)(1)\cos\alpha = A_x$ or $\boxed{\cos\alpha = \frac{A_x}{A}}$.

Similarly, $\boxed{\cos\beta = \frac{A_y}{A}}$

and $\boxed{\cos\gamma = \frac{A_z}{A}}$

where $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$.

(b) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \left(\frac{A_x}{A}\right)^2 + \left(\frac{A_y}{A}\right)^2 + \left(\frac{A_z}{A}\right)^2 = \frac{A^2}{A^2} = 1$

P7.49 (a) $x = t + 2.00t^3$

Therefore,

$$v = \frac{dx}{dt} = 1 + 6.00t^2$$
$$K = \frac{1}{2}mv^2 = \frac{1}{2}(4.00)(1 + 6.00t^2)^2 = \boxed{(2.00 + 24.0t^2 + 72.0t^4) \text{ J}}$$

(b) $a = \frac{dv}{dt} = \boxed{(12.0t) \text{ m/s}^2}$

$F = ma = 4.00(12.0t) = \boxed{(48.0t) \text{ N}}$

(c) $\mathcal{P} = Fv = (48.0t)(1 + 6.00t^2) = \boxed{(48.0t + 288t^3) \text{ W}}$

(d) $W = \int_0^{2.00} \mathcal{P}dt = \int_0^{2.00} (48.0t + 288t^3)dt = \boxed{1\,250 \text{ J}}$

***P7.50** (a) We write

$$F = ax^b$$
$$1\,000\text{ N} = a(0.129\text{ m})^b$$
$$5\,000\text{ N} = a(0.315\text{ m})^b$$
$$5 = \left(\frac{0.315}{0.129}\right)^b = 2.44^b$$
$$\ln 5 = b\ln 2.44$$
$$b = \frac{\ln 5}{\ln 2.44} = \boxed{1.80 = b}$$
$$a = \frac{1\,000\text{ N}}{(0.129\text{ m})^{1.80}} = \boxed{4.01\times 10^4\ \text{N/m}^{1.8} = a}$$

(b) $$W = \int_0^{0.25\text{ m}} F\,dx = \int_0^{0.25\text{ m}} 4.01\times 10^4\ \frac{\text{N}}{\text{m}^{1.8}}x^{1.8}dx$$
$$= 4.01\times 10^4\ \frac{\text{N}}{\text{m}^{1.8}}\frac{x^{2.8}}{2.8}\bigg|_0^{0.25\text{ m}} = 4.01\times 10^4\ \frac{\text{N}}{\text{m}^{1.8}}\frac{(0.25\text{ m})^{2.8}}{2.8}$$
$$= \boxed{294\text{ J}}$$

***P7.51** The work done by the applied force is

$$W = \int_i^f F_{\text{applied}}dx = \int_0^{x_{\text{max}}} -\left[-\left(k_1x + k_2x^2\right)\right]dx$$
$$= \int_0^{x_{\text{max}}} k_1x\,dx + \int_0^{x_{\text{max}}} k_2x^2dx = k_1\frac{x^2}{2}\bigg|_0^{x_{\text{max}}} + k_2\frac{x^3}{3}\bigg|_0^{x_{\text{max}}}$$
$$= \boxed{k_1\frac{x_{\text{max}}^2}{2} + k_2\frac{x_{\text{max}}^3}{3}}$$

P7.52 (a) The work done by the traveler is mgh_sN where N is the number of steps he climbs during the ride.

$$N = (\text{time on escalator})(n)$$

where $$(\text{time on escalator}) = \frac{h}{\text{vertical velocity of person}}$$

and $$\text{vertical velocity of person} = v + nh_s$$

Then, $$N = \frac{nh}{v + nh_s}$$

and the work done by the person becomes $W_{\text{person}} = \boxed{\dfrac{mgnhh_s}{v + nh_s}}$

continued on next page

(b) The work done by the escalator is

$$W_e = (\text{power})(\text{time}) = \left[(\text{force exerted})(\text{speed})(\text{time})\right] = mgvt$$

where $t = \frac{h}{v + nh_s}$ as above.

Thus, $W_e = \boxed{\frac{mgvh}{v + nh_s}}$.

As a check, the total work done on the person's body must add up to mgh, the work an elevator would do in lifting him.

It does add up as follows: $\sum W = W_{\text{person}} + W_e = \frac{mgnhh_s}{v + nh_s} + \frac{mgvh}{v + nh_s} = \frac{mgh(nh_s + v)}{v + nh_s} = mgh$

P7.53 (a) $\Delta K = \frac{1}{2}mv^2 - 0 = \sum W$, so

$v^2 = \frac{2W}{m}$ and $v = \boxed{\sqrt{\frac{2W}{m}}}$

(b) $W = \mathbf{F} \cdot \mathbf{d} = F_x d \Rightarrow F_x = \boxed{\frac{W}{d}}$

***P7.54** During its whole motion from $y = 10.0$ m to $y = -3.20$ mm, the force of gravity and the force of the plate do work on the ball. It starts and ends at rest

$$K_i + \sum W = K_f$$

$$0 + F_g \Delta y \cos 0° + F_p \Delta x \cos 180° = 0$$

$$mg(10.003\,2 \text{ m}) - F_p(0.003\,20 \text{ m}) = 0$$

$$F_p = \frac{5 \text{ kg}(9.8 \text{ m/s}^2)(10 \text{ m})}{3.2 \times 10^{-3} \text{ m}} = \boxed{1.53 \times 10^5 \text{ N upward}}$$

P7.55 (a) $\mathcal{P} = Fv = F(v_i + at) = F\left(0 + \frac{F}{m}t\right) = \boxed{\left(\frac{F^2}{m}\right)t}$

(b) $\mathcal{P} = \left[\frac{(20.0 \text{ N})^2}{5.00 \text{ kg}}\right](3.00 \text{ s}) = \boxed{240 \text{ W}}$

***P7.56** (a) $$W_1 = \int_i^f F_1 dx = \int_{x_{i1}}^{x_{i1}+x_a} k_1 x\,dx = \frac{1}{2}k_1\left[(x_{i1}+x_a)^2 - x_{i1}^2\right] = \frac{1}{2}k_1\left(x_a^2 + 2x_a x_{i1}\right)$$

(b) $$W_2 = \int_{-x_{i2}}^{-x_{i2}+x_a} k_2 x\,dx = \frac{1}{2}k_2\left[(-x_{i2}+x_a)^2 - x_{i2}^2\right] = \frac{1}{2}k_2\left(x_a^2 - 2x_a x_{i2}\right)$$

(c) Before the horizontal force is applied, the springs exert equal forces: $k_1 x_{i1} = k_2 x_{i2}$

$$x_{i2} = \frac{k_1 x_{i1}}{k_2}$$

(d) $$W_1 + W_2 = \frac{1}{2}k_1 x_a^2 + k_1 x_a x_{i1} + \frac{1}{2}k_2 x_a^2 - k_2 x_a x_{i2}$$
$$= \frac{1}{2}k_1 x_a^2 + \frac{1}{2}k_2 x_a^2 + k_1 x_a x_{i1} - k_2 x_a \frac{k_1 x_{i1}}{k_2}$$
$$= \frac{1}{2}(k_1 + k_2)x_a^2$$

***P7.57** (a) $$v = \int_0^t a\,dt = \int_0^t \left(1.16t - 0.21t^2 + 0.24t^3\right)dt$$
$$= 1.16\frac{t^2}{2} - 0.21\frac{t^3}{3} + 0.24\frac{t^4}{4}\Big|_0^t = 0.58t^2 - 0.07t^3 + 0.06t^4$$

At $t = 0$, $v_i = 0$. At $t = 2.5$ s,

$$v_f = \left(0.58\ \text{m/s}^3\right)(2.5\ \text{s})^2 - \left(0.07\ \text{m/s}^4\right)(2.5\ \text{s})^3 + \left(0.06\ \text{m/s}^5\right)(2.5\ \text{s})^4 = 4.88\ \text{m/s}$$
$$K_i + W = K_f$$
$$0 + W = \frac{1}{2}mv_f^2 = \frac{1}{2}1\,160\ \text{kg}(4.88\ \text{m/s})^2 = \boxed{1.38\times10^4\ \text{J}}$$

(b) At $t = 2.5$ s,

$$a = \left(1.16\ \text{m/s}^3\right)2.5\ \text{s} - \left(0.210\ \text{m/s}^4\right)(2.5\ \text{s})^2 + \left(0.240\ \text{m/s}^5\right)(2.5\ \text{s})^3 = 5.34\ \text{m/s}^2.$$

Through the axles the wheels exert on the chassis force

$$\sum F = ma = 1\,160\ \text{kg}\ 5.34\ \text{m/s}^2 = 6.19\times10^3\ \text{N}$$

and inject power

$$\mathcal{P} = Fv = 6.19\times10^3\ \text{N}(4.88\ \text{m/s}) = \boxed{3.02\times10^4\ \text{W}}.$$

P7.58 (a) The new length of each spring is $\sqrt{x^2+L^2}$, so its extension is $\sqrt{x^2+L^2}-L$ and the force it exerts is $k\left(\sqrt{x^2+L^2}-L\right)$ toward its fixed end. The y components of the two spring forces add to zero. Their x components add to

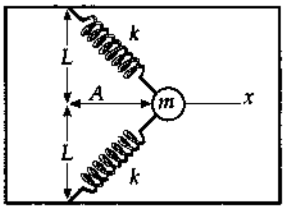


FIG. P7.58

$$\mathbf{F}=-2\hat{\mathbf{i}}k\left(\sqrt{x^2+L^2}-L\right)\frac{x}{\sqrt{x^2+L^2}}=\boxed{-2kx\hat{\mathbf{i}}\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)}.$$

(b) $$W=\int_i^f F_x dx \qquad W=\int_A^0 -2kx\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)dx$$

$$W=-2k\int_A^0 x\,dx+kL\int_A^0\left(x^2+L^2\right)^{-1/2}2x\,dx \qquad W=-2k\frac{x^2}{2}\bigg|_A^0+kL\frac{\left(x^2+L^2\right)^{1/2}}{(1/2)}\bigg|_A^0$$

$$W=-0+kA^2+2kL^2-2kL\sqrt{A^2+L^2} \qquad W=\boxed{2kL^2+kA^2-2kL\sqrt{A^2+L^2}}$$

***P7.59** For the rocket falling at terminal speed we have

$$\sum F=ma$$
$$+R-Mg=0$$
$$Mg=\frac{1}{2}D\rho Av_T^2$$

(a) For the rocket with engine exerting thrust T and flying up at the same speed,

$$\sum F=ma$$
$$+T-Mg-R=0$$
$$T=2Mg$$

The engine power is $\mathcal{P}=Fv=Tv_T=\boxed{2Mgv_T}$.

(b) For the rocket with engine exerting thrust T_b and flying down steadily at $3v_T$,
$R_b=\frac{1}{2}D\rho A(3v_T)^2=9Mg$

$$\sum F=ma$$
$$-T_b-Mg+9Mg=0$$
$$T_b=8Mg$$

The engine power is $\mathcal{P}=Tv=8Mg3v_T=\boxed{24Mgv_T}$.

P7.60 (a) $\mathbf{F}_1 = (25.0\text{ N})\left(\cos 35.0°\,\hat{\mathbf{i}} + \sin 35.0°\,\hat{\mathbf{j}}\right) = \boxed{\left(20.5\hat{\mathbf{i}} + 14.3\hat{\mathbf{j}}\right)\text{ N}}$

$\mathbf{F}_2 = (42.0\text{ N})\left(\cos 150°\,\hat{\mathbf{i}} + \sin 150°\,\hat{\mathbf{j}}\right) = \boxed{\left(-36.4\hat{\mathbf{i}} + 21.0\hat{\mathbf{j}}\right)\text{ N}}$

(b) $\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \boxed{\left(-15.9\hat{\mathbf{i}} + 35.3\hat{\mathbf{j}}\right)\text{ N}}$

(c) $\mathbf{a} = \dfrac{\sum \mathbf{F}}{m} = \boxed{\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\text{ m/s}^2}$

(d) $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t = \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)\text{ m/s} + \left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\left(\text{m/s}^2\right)(3.00\text{ s})$

$\mathbf{v}_f = \boxed{\left(-5.54\hat{\mathbf{i}} + 23.7\hat{\mathbf{j}}\right)\text{ m/s}}$

(e) $\mathbf{r}_f = \mathbf{r}_i + \mathbf{v}_i t + \dfrac{1}{2}\mathbf{a}t^2$

$\mathbf{r}_f = 0 + \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)(\text{m/s})(3.00\text{ s}) + \dfrac{1}{2}\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\left(\text{m/s}^2\right)(3.00\text{ s})^2$

$\Delta\mathbf{r} = \mathbf{r}_f = \boxed{\left(-2.30\hat{\mathbf{i}} + 39.3\hat{\mathbf{j}}\right)\text{ m}}$

(f) $K_f = \dfrac{1}{2}mv_f^2 = \dfrac{1}{2}(5.00\text{ kg})\left[(5.54)^2 + (23.7)^2\right]\left(\text{m/s}^2\right) = \boxed{1.48\text{ kJ}}$

(g) $K_f = \dfrac{1}{2}mv_i^2 + \sum \mathbf{F}\cdot\Delta\mathbf{r}$

$K_f = \dfrac{1}{2}(5.00\text{ kg})\left[(4.00)^2 + (2.50)^2\right](\text{m/s})^2 + \left[(-15.9\text{ N})(-2.30\text{ m}) + (35.3\text{ N})(39.3\text{ m})\right]$

$K_f = 55.6\text{ J} + 1\,426\text{ J} = \boxed{1.48\text{ kJ}}$

P7.61 (a) $\sum W = \Delta K$: $\quad W_s + W_g = 0$

$\dfrac{1}{2}kx_i^2 - 0 + mg\Delta x\cos(90° + 60°) = 0$

$\dfrac{1}{2}\left(1.40\times 10^3\text{ N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x = 0$

$\Delta x = \boxed{4.12\text{ m}}$

(b) $\sum W = \Delta K + \Delta E_{\text{int}}$: $\quad W_s + W_g - \Delta E_{\text{int}} = 0$

$\dfrac{1}{2}kx_i^2 + mg\Delta x\cos 150° - \mu_k mg\cos 60°\,\Delta x = 0$

$\dfrac{1}{2}\left(1.40\times 10^3\text{ N/m}\right)\times(0.100)^2 - (0.200)(9.80)(\sin 60.0°)\Delta x - (0.200)(9.80)(0.400)(\cos 60.0°)\Delta x = 0$

$\Delta x = \boxed{3.35\text{ m}}$

P7.62 (a)

F(N)	L(mm)	F(N)	L(mm)
2.00	15.0	14.0	112
4.00	32.0	16.0	126
6.00	49.0	18.0	149
8.00	64.0	20.0	175
10.0	79.0	22.0	190
12.0	98.0		

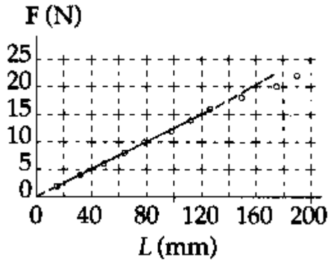


FIG. P7.62

(b) A straight line fits the first eight points, together with the origin. By least-square fitting, its slope is

$$0.125\ \text{N/mm} \pm 2\% = \boxed{125\ \text{N/m}} \pm 2\%$$

In $F = kx$, the spring constant is $k = \frac{F}{x}$, the same as the slope of the F-versus-x graph.

(c) $F = kx = (125\ \text{N/m})(0.105\ \text{m}) = \boxed{13.1\ \text{N}}$

P7.63

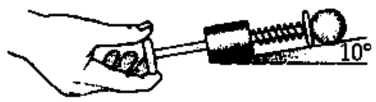


FIG. P7.63

$$K_i + W_s + W_g = K_f$$

$$\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 + mg\Delta x\cos\theta = \frac{1}{2}mv_f^2$$

$$0 + \frac{1}{2}kx_i^2 - 0 + mgx_i\cos 100° = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}(1.20\ \text{N/cm})(5.00\ \text{cm})(0.050\,0\ \text{m}) - (0.100\ \text{kg})(9.80\ \text{m/s}^2)(0.050\,0\ \text{m})\sin 10.0° = \frac{1}{2}(0.100\ \text{kg})v^2$$

$$0.150\ \text{J} - 8.51\times10^{-3}\ \text{J} = (0.050\,0\ \text{kg})v^2$$

$$v = \sqrt{\frac{0.141}{0.050\,0}} = \boxed{1.68\ \text{m/s}}$$

P7.64 (a) $\Delta E_{int} = -\Delta K = -\frac{1}{2}m\left(v_f^2 - v_i^2\right)$: $\Delta E_{int} = -\frac{1}{2}(0.400\ \text{kg})\left((6.00)^2 - (8.00)^2\right)(\text{m/s})^2 = \boxed{5.60\ \text{J}}$

(b) $\Delta E_{int} = f\Delta r = \mu_k mg(2\pi r)$: $5.60\ \text{J} = \mu_k(0.400\ \text{kg})(9.80\ \text{m/s}^2)2\pi(1.50\ \text{m})$

Thus, $\mu_k = \boxed{0.152}$.

(c) After N revolutions, the object comes to rest and $K_f = 0$.

Thus, $\Delta E_{int} = -\Delta K = -0 + K_i = \frac{1}{2}mv_i^2$

or $\mu_k mg[N(2\pi r)] = \frac{1}{2}mv_i^2$.

This gives $N = \dfrac{\frac{1}{2}mv_i^2}{\mu_k mg(2\pi r)} = \dfrac{\frac{1}{2}(8.00\ \text{m/s})^2}{(0.152)(9.80\ \text{m/s}^2)2\pi(1.50\ \text{m})} = \boxed{2.28\ \text{rev}}$.

P7.65 If positive F represents an outward force, (same as direction as r), then

$$W=\int_i^f \mathbf{F}\cdot d\mathbf{r}=\int_{r_i}^{r_f}\left(2F_0\sigma^{13}r^{-13}-F_0\sigma^7 r^{-7}\right)dr$$

$$W=\frac{2F_0\sigma^{13}r^{-12}}{-12}-\frac{F_0\sigma^7 r^{-6}}{-6}\Bigg|_{r_i}^{r_f}$$

$$W=\frac{-F_0\sigma^{13}\left(r_f^{-12}-r_i^{-12}\right)}{6}+\frac{F_0\sigma^7\left(r_f^{-6}-r_i^{-6}\right)}{6}=\frac{F_0\sigma^7}{6}\left[r_f^{-6}-r_i^{-6}\right]-\frac{F_0\sigma^{13}}{6}\left[r_f^{-12}-r_i^{-12}\right]$$

$$W=1.03\times10^{-77}\left[r_f^{-6}-r_i^{-6}\right]-1.89\times10^{-134}\left[r_f^{-12}-r_i^{-12}\right]$$

$$W=1.03\times10^{-77}\left[1.88\times10^{-6}-2.44\times10^{-6}\right]10^{60}-1.89\times10^{-134}\left[3.54\times10^{-12}-5.96\times10^{-8}\right]10^{120}$$

$$W=-2.49\times10^{-21}\text{ J}+1.12\times10^{-21}\text{ J}=\boxed{-1.37\times10^{-21}\text{ J}}$$

P7.66 $\mathcal{P}\Delta t=W=\Delta K=\dfrac{(\Delta m)v^2}{2}$

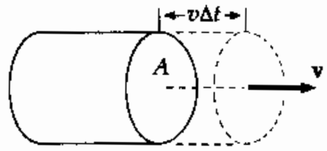


FIG. P7.66

The density is $\rho=\dfrac{\Delta m}{\text{vol}}=\dfrac{\Delta m}{A\Delta x}$.

Substituting this into the first equation and solving for $\mathcal{P}$, since $\dfrac{\Delta x}{\Delta t}=v$,

for a constant speed, we get $\boxed{\mathcal{P}=\dfrac{\rho A v^3}{2}}$.

Also, since $\mathcal{P}=Fv$, $\boxed{F=\dfrac{\rho A v^2}{2}}$.

Our model predicts the same proportionalities as the empirical equation, and gives $D=1$ for the drag coefficient. Air actually slips around the moving object, instead of accumulating in front of it. For this reason, the drag coefficient is not necessarily unity. It is typically less than one for a streamlined object and can be greater than one if the airflow around the object is complicated.

P7.67 We evaluate $\int_{12.8}^{23.7}\dfrac{375dx}{x^3+3.75x}$ by calculating

$$\frac{375(0.100)}{(12.8)^3+3.75(12.8)}+\frac{375(0.100)}{(12.9)^3+3.75(12.9)}+\ldots\frac{375(0.100)}{(23.6)^3+3.75(23.6)}=0.806$$

and

$$\frac{375(0.100)}{(12.9)^3+3.75(12.9)}+\frac{375(0.100)}{(13.0)^3+3.75(13.0)}+\ldots\frac{375(0.100)}{(23.7)^3+3.75(23.7)}=0.791.$$

The answer must be between these two values. We may find it more precisely by using a value for Δx smaller than 0.100. Thus, we find the integral to be $\boxed{0.799\text{ N}\cdot\text{m}}$.

*P7.68 $\mathscr{P} = \frac{1}{2} D\rho\pi r^2 v^3$

(a) $\mathscr{P}_a = \frac{1}{2} 1\left(1.20 \text{ kg/m}^3\right)\pi(1.5 \text{ m})^2 (8 \text{ m/s})^3 = \boxed{2.17 \times 10^3 \text{ W}}$

(b) $\frac{\mathscr{P}_b}{\mathscr{P}_a} = \frac{v_b^3}{v_a^3} = \left(\frac{24 \text{ m/s}}{8 \text{ m/s}}\right)^3 = 3^3 = 27$

$\mathscr{P}_b = 27\left(2.17 \times 10^3 \text{ W}\right) = \boxed{5.86 \times 10^4 \text{ W}}$

P7.69 (a) The suggested equation $\mathscr{P}\Delta t = bwd$ implies all of the following cases:

(1) $\mathscr{P}\Delta t = b\left(\frac{w}{2}\right)(2d)$ (2) $\mathscr{P}\left(\frac{\Delta t}{2}\right) = b\left(\frac{w}{2}\right)d$

(3) $\mathscr{P}\left(\frac{\Delta t}{2}\right) = bw\left(\frac{d}{2}\right)$ and (4) $\left(\frac{\mathscr{P}}{2}\right)\Delta t = b\left(\frac{w}{2}\right)d$

These are all of the proportionalities Aristotle lists.

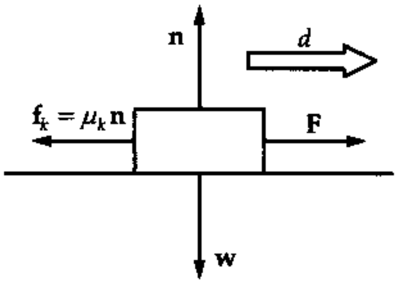


FIG. P7.69

(b) For one example, consider a horizontal force F pushing an object of weight w at constant velocity across a horizontal floor with which the object has coefficient of friction μ_k.

$\sum \mathbf{F} = m\mathbf{a}$ implies that:

$$+n - w = 0 \text{ and } F - \mu_k n = 0$$

so that $F = \mu_k w$

As the object moves a distance d, the agent exerting the force does work

$$W = Fd\cos\theta = Fd\cos 0° = \mu_k wd \text{ and puts out power } \mathscr{P} = \frac{W}{\Delta t}$$

This yields the equation $\mathscr{P}\Delta t = \mu_k wd$ which represents Aristotle's theory with $b = \mu_k$.

Our theory is more general than Aristotle's. Ours can also describe accelerated motion.

*P7.70 (a) So long as the spring force is greater than the friction force, the block will be gaining speed. The block slows down when the friction force becomes the greater. It has maximum speed when $kx_a - f_k = ma = 0$.

$\left(1.0 \times 10^3 \text{ N/m}\right)x_a - 4.0 \text{ N} = 0$ $\boxed{x = -4.0 \times 10^{-3} \text{ m}}$

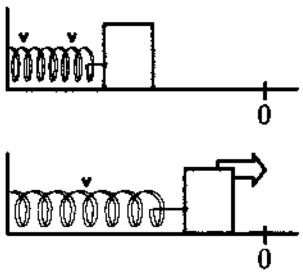


FIG. P7.70

(b) By the same logic,

$\left(1.0 \times 10^3 \text{ N/m}\right)x_b - 10.0 \text{ N} = 0$ $\boxed{x = -1.0 \times 10^{-2} \text{ m}}$

ANSWERS TO EVEN PROBLEMS

P7.2 1.59×10^3 J

P7.4 (a) 3.28×10^{-2} J; (b) -3.28×10^{-2} J

P7.6 see the solution

P7.8 5.33 W

P7.10 16.0

P7.12 (a) see the solution; (b) -12.0 J

P7.14 50.0 J

P7.16 (a) 575 N/m; (b) 46.0 J

P7.18 (a) 9.00 kJ; (b) 11.7 kJ, larger by 29.6%

P7.20 (a) see the solution; (b) mgR

P7.22 (a) $\frac{mg}{k_1} + \frac{mg}{k_2}$; (b) $\left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$

P7.24 (a) 1.20 J; (b) 5.00 m/s; (c) 6.30 J

P7.26 (a) 60.0 J; (b) 60.0 J

P7.28 (a) 1.94 m/s; (b) 3.35 m/s; (c) 3.87 m/s

P7.30 (a) 3.78×10^{-16} J; (b) 1.35×10^{-14} N; (c) $1.48 \times 10^{+16}$ m/s^2; (d) 1.94 ns

P7.32 (a) 0.791 m/s; (b) 0.531 m/s

P7.34 (a) 329 J; (b) 0; (c) 0; (d) 185 J; (e) 144 J

P7.36 8.01 W

P7.38 $\sim 10^4$ W

P7.40 (a) 5.91 kW; (b) 11.1 kW

P7.42 No. (a) 582; (b) 90.5 W = 0.121 hp

P7.44 5.92 km/L

P7.46 90.0 J

P7.48 (a) $\cos\alpha = \frac{A_x}{A}$; $\cos\beta = \frac{A_y}{A}$; $\cos\gamma = \frac{A_z}{A}$; (b) see the solution

P7.50 (a) $a = \frac{40.1 \text{ kN}}{m^{1.8}}$; $b = 1.80$; (b) 294 J

P7.52 (a) $\frac{mgnhh_s}{v + nh_s}$; (b) $\frac{mgvh}{v + nh_s}$

P7.54 1.53×10^5 N upward

P7.56 see the solution

P7.58 (a) see the solution; (b) $2kL^2 + kA^2 - 2kL\sqrt{A^2 + L^2}$

P7.60 (a) $\mathbf{F}_1 = (20.5\hat{\mathbf{i}} + 14.3\hat{\mathbf{j}})$ N; $\mathbf{F}_2 = (-36.4\hat{\mathbf{i}} + 21.0\hat{\mathbf{j}})$ N; (b) $(-15.9\hat{\mathbf{i}} + 35.3\hat{\mathbf{j}})$ N; (c) $(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}})$ m/s^2; (d) $(-5.54\hat{\mathbf{i}} + 23.7\hat{\mathbf{j}})$ m/s; (e) $(-2.30\hat{\mathbf{i}} + 39.3\hat{\mathbf{j}})$ m; (f) 1.48 kJ; (g) 1.48 kJ

P7.62 (a) see the solution; (b) 125 N/m ± 2%; (c) 13.1 N

P7.64 (a) 5.60 J; (b) 0.152; (c) 2.28 rev

P7.66 see the solution

P7.68 (a) 2.17 kW; (b) 58.6 kW

P7.70 (a) $x = -4.0$ mm; (b) -1.0 cm

8

Potential Energy

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q8.1 The final speed of the children will not depend on the slide length or the presence of bumps if there is no friction. If there is friction, a longer slide will result in a lower final speed. Bumps will have the same effect as they effectively lengthen the distance over which friction can do work, to decrease the total mechanical energy of the children.

Q8.2 Total energy is the sum of kinetic and potential energies. Potential energy can be negative, so the sum of kinetic plus potential can also be negative.

Q8.3 Both agree on the *change* in potential energy, and the kinetic energy. They may disagree on the value of gravitational potential energy, depending on their choice of a zero point.

Q8.4 (a) mgh is provided by the muscles.

(b) No further energy is supplied to the object-Earth system, but some chemical energy must be supplied to the muscles as they keep the weight aloft.

(c) The object loses energy mgh, giving it back to the muscles, where most of it becomes internal energy.

Q8.5 Lift a book from a low shelf to place it on a high shelf. The net change in its kinetic energy is zero, but the book-Earth system increases in gravitational potential energy. Stretch a rubber band to encompass the ends of a ruler. It increases in elastic energy. Rub your hands together or let a pearl drift down at constant speed in a bottle of shampoo. Each system (two hands; pearl and shampoo) increases in internal energy.

Q8.6 Three potential energy terms will appear in the expression of total mechanical energy, one for each conservative force. If you write an equation with initial energy on one side and final energy on the other, the equation contains six potential-energy terms.

Q8.7 (a) It does if it makes the object's speed change, but not if it only makes the direction of the velocity change.

(b) Yes, according to Newton's second law.

Q8.8 The original kinetic energy of the skidding can be degraded into kinetic energy of random molecular motion in the tires and the road: it is internal energy. If the brakes are used properly, the same energy appears as internal energy in the brake shoes and drums.

Q8.9 All the energy is supplied by foodstuffs that gained their energy from the sun.

Q8.10 Elastic potential energy of plates under stress plus gravitational energy is released when the plates "slip". It is carried away by mechanical waves.

Q8.11 The total energy of the ball-Earth system is conserved. Since the system initially has gravitational energy mgh and no kinetic energy, the ball will again have zero kinetic energy when it returns to its original position. Air resistance will cause the ball to come back to a point slightly below its initial position. On the other hand, if anyone gives a forward push to the ball anywhere along its path, the demonstrator will have to duck.

Q8.12 Using switchbacks requires no less work, as it does not change the *change* in potential energy from top to bottom. It does, however, require less force (of static friction on the rolling drive wheels of a car) to propel the car up the gentler slope. Less power is required if the work can be done over a longer period of time.

Q8.13 There is no work done since there is no change in kinetic energy. In this case, air resistance must be negligible since the acceleration is zero.

Q8.14 There is no violation. Choose the book as the system. You did work and the earth did work on the book. The average force you exerted just counterbalanced the weight of the book. The total work on the book is zero, and is equal to its overall change in kinetic energy.

Q8.15 Kinetic energy is greatest at the starting point. Gravitational energy is a maximum at the top of the flight of the ball.

Q8.16 Gravitational energy is proportional to mass, so it doubles.

Q8.17 In stirring cake batter and in weightlifting, your body returns to the same conformation after each stroke. During each stroke chemical energy is irreversibly converted into output work (and internal energy). This observation proves that muscular forces are nonconservative.

Q8.18 Let the gravitational energy be zero at the lowest point in the motion. If you start the vibration by pushing down on the block (2), its kinetic energy becomes extra elastic potential energy in the spring (U_s). After the block starts moving up at its lower turning point (3), this energy becomes both kinetic energy (K) and gravitational potential energy (U_g), and then just gravitational energy when the block is at its greatest height (1). The energy then turns back into kinetic and elastic potential energy, and the cycle repeats.

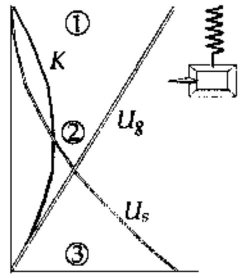


FIG. Q8.18

Q8.19 (a) Kinetic energy of the running athlete is transformed into elastic potential energy of the bent pole. This potential energy is transformed to a combination of kinetic energy and gravitational potential energy of the athlete and pole as the athlete approaches the bar. The energy is then all gravitational potential of the pole and the athlete as the athlete hopefully clears the bar. This potential energy then turns to kinetic energy as the athlete and pole fall to the ground. It immediately becomes internal energy as their macroscopic motion stops.

(b) Rotational kinetic energy of the athlete and shot is transformed into translational kinetic energy of the shot. As the shot goes through its trajectory as a projectile, the kinetic energy turns to a mix of kinetic and gravitational potential. The energy becomes internal energy as the shot comes to rest.

(c) Kinetic energy of the running athlete is transformed to a mix of kinetic and gravitational potential as the athlete becomes projectile going over a bar. This energy turns back into kinetic as the athlete falls down, and becomes internal energy as he stops on the ground.

The ultimate source of energy for all of these sports is the sun. See question 9.

Q8.20 Chemical energy in the fuel turns into internal energy as the fuel burns. Most of this leaves the car by heat through the walls of the engine and by matter transfer in the exhaust gases. Some leaves the system of fuel by work done to push down the piston. Of this work, a little results in internal energy in the bearings and gears, but most becomes work done on the air to push it aside. The work on the air immediately turns into internal energy in the air. If you use the windshield wipers, you take energy from the crankshaft and turn it into extra internal energy in the glass and wiper blades and wiper-motor coils. If you turn on the air conditioner, your end effect is to put extra energy out into the surroundings. You must apply the brakes at the end of your trip. As soon as the sound of the engine has died away, all you have to show for it is thermal pollution.

Q8.21 A graph of potential energy versus position is a straight horizontal line for a particle in neutral equilibrium. The graph represents a constant function.

Q8.22 The ball is in neutral equilibrium.

Q8.23 The ball is in stable equilibrium when it is directly below the pivot point. The ball is in unstable equilibrium when it is vertically above the pivot.

SOLUTIONS TO PROBLEMS

Section 8.1 Potential Energy of a System

P8.1 (a) With our choice for the zero level for potential energy when the car is at point B,

$$\boxed{U_B = 0}.$$

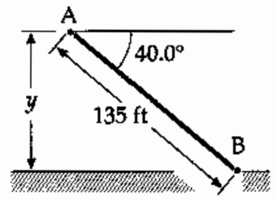


FIG. P8.1

When the car is at point A, the potential energy of the car-Earth system is given by

$$U_A = mgy$$

where y is the vertical height above zero level. With $135 \text{ ft} = 41.1 \text{ m}$, this height is found as:

$$y = (41.1 \text{ m})\sin 40.0° = 26.4 \text{ m}.$$

Thus,

$$U_A = (1\,000 \text{ kg})(9.80 \text{ m/s}^2)(26.4 \text{ m}) = \boxed{2.59 \times 10^5 \text{ J}}.$$

The change in potential energy as the car moves from A to B is

$$U_B - U_A = 0 - 2.59 \times 10^5 \text{ J} = \boxed{-2.59 \times 10^5 \text{ J}}.$$

(b) With our choice of the zero level when the car is at point A, we have $\boxed{U_A = 0}$. The potential energy when the car is at point B is given by $U_B = mgy$ where y is the vertical distance of point B below point A. In part (a), we found the magnitude of this distance to be 26.5 m. Because this distance is now below the zero reference level, it is a negative number.

Thus,

$$U_B = (1\,000 \text{ kg})(9.80 \text{ m/s}^2)(-26.5 \text{ m}) = \boxed{-2.59 \times 10^5 \text{ J}}.$$

The change in potential energy when the car moves from A to B is

$$U_B - U_A = -2.59 \times 10^5 \text{ J} - 0 = \boxed{-2.59 \times 10^5 \text{ J}}.$$

P8.2 (a) We take the zero configuration of system potential energy with the child at the lowest point of the arc. When the string is held horizontal initially, the initial position is 2.00 m above the zero level. Thus,

$$U_g = mgy = (400\text{ N})(2.00\text{ m}) = \boxed{800\text{ J}}.$$

(b) From the sketch, we see that at an angle of 30.0° the child is at a vertical height of $(2.00\text{ m})(1-\cos 30.0°)$ above the lowest point of the arc. Thus,

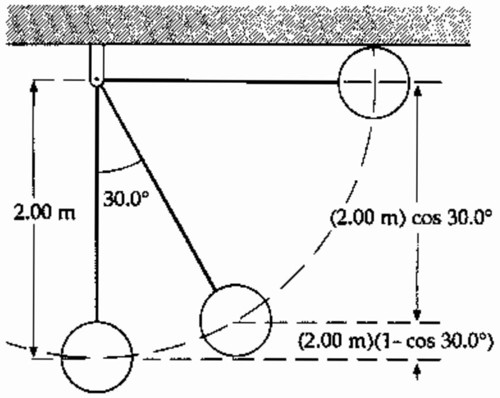


FIG. P8.2

$$U_g = mgy = (400\text{ N})(2.00\text{ m})(1-\cos 30.0°) = \boxed{107\text{ J}}.$$

(c) The zero level has been selected at the lowest point of the arc. Therefore, $\boxed{U_g = 0}$ at this location.

***P8.3** The volume flow rate is the volume of water going over the falls each second:

$$3\text{ m}(0.5\text{ m})(1.2\text{ m/s}) = 1.8\text{ m}^3/\text{s}$$

The mass flow rate is $\frac{m}{t} = \rho\frac{V}{t} = (1\,000\text{ kg/m}^3)(1.8\text{ m}^3/\text{s}) = 1\,800\text{ kg/s}$

If the stream has uniform width and depth, the speed of the water below the falls is the same as the speed above the falls. Then no kinetic energy, but only gravitational energy is available for conversion into internal and electric energy.

The input power is $\mathscr{P}_{\text{in}} = \frac{\text{energy}}{t} = \frac{mgy}{t} = \frac{m}{t}gy = (1\,800\text{ kg/s})(9.8\text{ m/s}^2)(5\text{ m}) = 8.82\times10^4\text{ J/s}$

The output power is $\mathscr{P}_{\text{useful}} = (\text{efficiency})\mathscr{P}_{\text{in}} = 0.25(8.82\times10^4\text{ W}) = \boxed{2.20\times10^4\text{ W}}$

The efficiency of electric generation at Hoover Dam is about 85%, with a head of water (vertical drop) of 174 m. Intensive research is underway to improve the efficiency of low head generators.

Section 8.2 The Isolated System—Conservation of Mechanical Energy

***P8.4** (a) One child in one jump converts chemical energy into mechanical energy in the amount that her body has as gravitational energy at the top of her jump:
$mgy = 36\text{ kg}(9.81\text{ m/s}^2)(0.25\text{ m}) = 88.3\text{ J}$. For all of the jumps of the children the energy is $12(1.05\times10^6)88.3\text{ J} = \boxed{1.11\times10^9\text{ J}}$.

(b) The seismic energy is modeled as $E = \frac{0.01}{100}1.11\times10^9\text{ J} = 1.11\times10^5\text{ J}$, making the Richter magnitude $\frac{\log E - 4.8}{1.5} = \frac{\log 1.11\times10^5 - 4.8}{1.5} = \frac{5.05-4.8}{1.5} = \boxed{0.2}$.

P8.5 $U_i + K_i = U_f + K_f$: $mgh + 0 = mg(2R) + \frac{1}{2}mv^2$

$$g(3.50R) = 2g(R) + \frac{1}{2}v^2$$

$$\boxed{v = \sqrt{3.00gR}}$$

$\sum F = m\frac{v^2}{R}$: $n + mg = m\frac{v^2}{R}$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00gR}{R} - g\right] = 2.00mg$$

$$n = 2.00\left(5.00 \times 10^{-3}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)$$

$$= \boxed{0.098\,0\text{ N downward}}$$

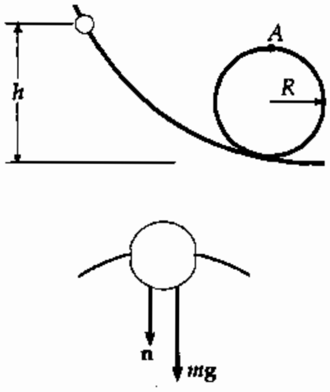


FIG. P8.5

P8.6 From leaving ground to the highest point, $K_i + U_i = K_f + U_f$

$$\frac{1}{2}m(6.00\text{ m/s})^2 + 0 = 0 + m\left(9.80\text{ m/s}^2\right)y$$

The mass makes no difference: $\therefore y = \frac{(6.00\text{ m/s})^2}{(2)\left(9.80\text{ m/s}^2\right)} = \boxed{1.84\text{ m}}$

***P8.7** (a) $\frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2$

$$0 + \frac{1}{2}(10\text{ N/m})(-0.18\text{ m})^2 = \frac{1}{2}(0.15\text{ kg})v_f^2 + 0$$

$$v_f = (0.18\text{ m})\sqrt{\left(\frac{10\text{ N}}{0.15\text{ kg}\cdot\text{m}}\right)\left(\frac{1\text{ kg}\cdot\text{m}}{1\text{ N}\cdot\text{s}^2}\right)} = \boxed{1.47\text{ m/s}}$$

(b) $K_i + U_{si} = K_f + U_{sf}$

$$0 + \frac{1}{2}(10\text{ N/m})(-0.18\text{ m})^2 = \frac{1}{2}(0.15\text{ kg})v_f^2 + \frac{1}{2}(10\text{ N/m})(0.25\text{ m} - 0.18\text{ m})^2$$

$$0.162\text{ J} = \frac{1}{2}(0.15\text{ kg})v_f^2 + 0.024\,5\text{ J}$$

$$v_f = \sqrt{\frac{2(0.138\text{ J})}{0.15\text{ kg}}} = \boxed{1.35\text{ m/s}}$$

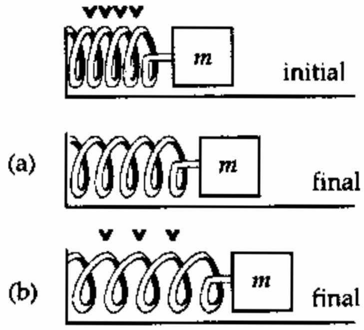


FIG. P8.7

***P8.8** The energy of the car is $E = \frac{1}{2}mv^2 + mgy$

$E = \frac{1}{2}mv^2 + mgd\sin\theta$ where d is the distance it has moved along the track.

$\mathcal{P} = \frac{dE}{dt} = mv\frac{dv}{dt} + mgv\sin\theta$

(a) When speed is constant,

$\mathcal{P} = mgv\sin\theta = 950\text{ kg}\left(9.80\text{ m/s}^2\right)(2.20\text{ m/s})\sin 30° = \boxed{1.02\times10^4\text{ W}}$

(b) $\frac{dv}{dt} = a = \frac{2.2\text{ m/s} - 0}{12\text{ s}} = 0.183\text{ m/s}^2$

Maximum power is injected just before maximum speed is attained:

$\mathcal{P} = mva + mgv\sin\theta = 950\text{ kg}(2.2\text{ m/s})\left(0.183\text{ m/s}^2\right) + 1.02\times10^4\text{ W} = \boxed{1.06\times10^4\text{ W}}$

(c) At the top end,

$\frac{1}{2}mv^2 + mgd\sin\theta = 950\text{ kg}\left(\frac{1}{2}(2.20\text{ m/s})^2 + \left(9.80\text{ m/s}^2\right)1\,250\text{ m}\sin 30°\right) = \boxed{5.82\times10^6\text{ J}}$

***P8.9** (a) Energy of the object-Earth system is conserved as the object moves between the release point and the lowest point. We choose to measure heights from $y = 0$ at the top end of the string.

$$\left(K+U_g\right)_i = \left(K+U_g\right)_f: \qquad 0 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

$$\left(9.8\text{ m/s}^2\right)(-2\text{ m}\cos 30°) = \frac{1}{2}v_f^2 + \left(9.8\text{ m/s}^2\right)(-2\text{ m})$$

$$v_f = \sqrt{2\left(9.8\text{ m/s}^2\right)(2\text{ m})(1-\cos 30°)} = \boxed{2.29\text{ m/s}}$$

(b) Choose the initial point at $\theta = 30°$ and the final point at $\theta = 15°$:

$$0 + mg(-L\cos 30°) = \frac{1}{2}mv_f^2 + mg(-L\cos 15°)$$

$$v_f = \sqrt{2gL(\cos 15° - \cos 30°)} = \sqrt{2\left(9.8\text{ m/s}^2\right)(2\text{ m})(\cos 15° - \cos 30°)} = \boxed{1.98\text{ m/s}}$$

P8.10 Choose the zero point of gravitational potential energy of the object-spring-Earth system as the configuration in which the object comes to rest. Then because the incline is frictionless, we have $E_B = E_A$:

$$K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$$

or

$$0 + mg(d+x)\sin\theta + 0 = 0 + 0 + \frac{1}{2}kx^2.$$

Solving for d gives

$$d = \boxed{\frac{kx^2}{2mg\sin\theta} - x}.$$

P8.11 From conservation of energy for the block-spring-Earth system,

$$U_{gt} = U_{si},$$

or

$$(0.250 \text{ kg})(9.80 \text{ m/s}^2)h = \left(\frac{1}{2}\right)(5\,000 \text{ N/m})(0.100 \text{ m})^2$$

This gives a maximum height $h = \boxed{10.2 \text{ m}}$.

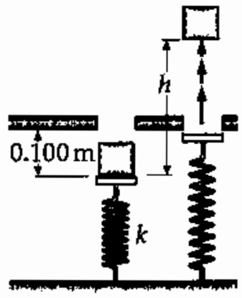


FIG. P8.11

P8.12 (a) The force needed to hang on is equal to the force F the trapeze bar exerts on the performer.

From the free-body diagram for the performer's body, as shown,

$$F - mg\cos\theta = m\frac{v^2}{\ell}$$

or

$$F = mg\cos\theta + m\frac{v^2}{\ell}$$

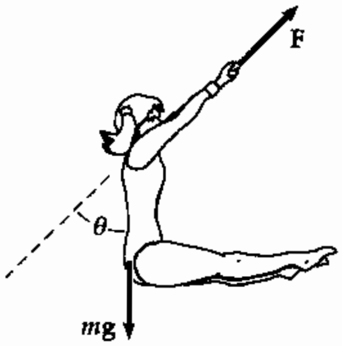


FIG. P8.12

Apply conservation of mechanical energy of the performer-Earth system as the performer moves between the starting point and any later point:

$$mg(\ell - \ell\cos\theta_i) = mg(\ell - \ell\cos\theta) + \frac{1}{2}mv^2$$

Solve for $\frac{mv^2}{\ell}$ and substitute into the force equation to obtain $F = \boxed{mg(3\cos\theta - 2\cos\theta_i)}$.

(b) At the bottom of the swing, $\theta = 0°$ so

$$F = mg(3 - 2\cos\theta_i)$$
$$F = 2mg = mg(3 - 2\cos\theta_i)$$

which gives

$$\theta_i = \boxed{60.0°}.$$

P8.13 Using conservation of energy for the system of the Earth and the two objects

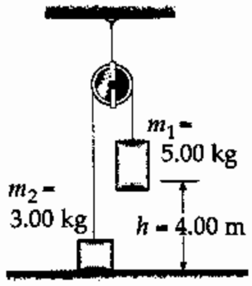


FIG. P8.13

(a) $(5.00\text{ kg})g(4.00\text{ m}) = (3.00\text{ kg})g(4.00\text{ m}) + \frac{1}{2}(5.00+3.00)v^2$

$v = \sqrt{19.6} = \boxed{4.43\text{ m/s}}$

(b) Now we apply conservation of energy for the system of the 3.00 kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00 kg object reaches its highest position in its free fall.

$$\frac{1}{2}(3.00)v^2 = mg\,\Delta y = 3.00g\Delta y$$
$$\Delta y = 1.00\text{ m}$$
$$y_{max} = 4.00\text{ m} + \Delta y = \boxed{5.00\text{ m}}$$

P8.14 $m_1 > m_2$

(a) $m_1gh = \frac{1}{2}(m_1+m_2)v^2 + m_2gh$

$v = \boxed{\sqrt{\frac{2(m_1-m_2)gh}{(m_1+m_2)}}}$

(b) Since m_2 has kinetic energy $\frac{1}{2}m_2v^2$, it will rise an additional height Δh determined from

$$m_2g\,\Delta h = \frac{1}{2}m_2v^2$$

or from (a),

$$\Delta h = \frac{v^2}{2g} = \frac{(m_1-m_2)h}{(m_1+m_2)}$$

The total height m_2 reaches is $h + \Delta h = \boxed{\frac{2m_1h}{m_1+m_2}}$.

P8.15 The force of tension and subsequent force of compression in the rod do no work on the ball, since they are perpendicular to each step of displacement. Consider energy conservation of the ball-Earth system between the instant just after you strike the ball and the instant when it reaches the top. The speed at the top is zero if you hit it just hard enough to get it there.

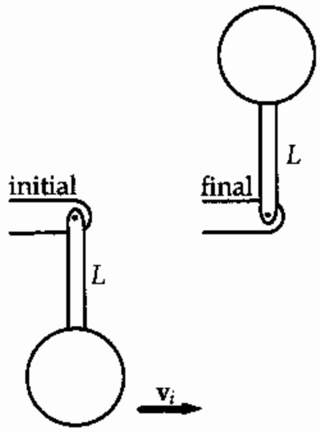


FIG. P8.15

$K_i + U_{gi} = K_f + U_{gf}$: $\quad \frac{1}{2}mv_i^2 + 0 = 0 + mg(2L)$

$$v_i = \sqrt{4gL} = \sqrt{4(9.80)(0.770)}$$
$$v_i = \boxed{5.49\text{ m/s}}$$

***P8.16** efficiency $= \dfrac{\text{useful output energy}}{\text{total input energy}} = \dfrac{\text{useful output power}}{\text{total input power}}$

$$e = \frac{m_{\text{water}} gy/t}{(1/2)m_{\text{air}}\left(v^2/t\right)} = \frac{2\rho_{\text{water}}\left(v_{\text{water}}/t\right)gy}{\rho_{\text{air}}\pi r^2\left(\ell v^2/t\right)} = \frac{2\rho_{\text{w}}\left(v_{\text{w}}/t\right)gy}{\rho_{\text{a}}\pi r^2 v^3}$$

where ℓ is the length of a cylinder of air passing through the mill and v_{w} is the volume of water pumped in time t. We need inject negligible kinetic energy into the water because it starts and ends at rest.

$$\frac{v_{\text{w}}}{t} = \frac{e\rho_{\text{a}}\pi r^2 v^3}{2\rho_{\text{w}} gy} = \frac{0.275\left(1.20\ \text{kg/m}^3\right)\pi(1.15\ \text{m})^2(11\ \text{m/s})^3}{2\left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)35\ \text{m}}$$

$$= 2.66\times10^{-3}\ \text{m}^3/\text{s}\left(\frac{1\,000\ \text{L}}{1\ \text{m}^3}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \boxed{160\ \text{L/min}}$$

P8.17 (a) $K_i + U_{gi} = K_f + U_{gf}$

$$\frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 + mgy_f$$

$$\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2 = \frac{1}{2}mv_{xf}^2 + mgy_f$$

But $v_{xi} = v_{xf}$, so for the first ball

$$y_f = \frac{v_{yi}^2}{2g} = \frac{(1\,000\sin 37.0°)^2}{2(9.80)} = \boxed{1.85\times10^4\ \text{m}}$$

and for the second

$$y_f = \frac{(1\,000)^2}{2(9.80)} = \boxed{5.10\times10^4\ \text{m}}$$

(b) The total energy of each is constant with value

$$\frac{1}{2}(20.0\ \text{kg})(1\,000\ \text{m/s})^2 = \boxed{1.00\times10^7\ \text{J}}.$$

P8.18 In the swing down to the breaking point, energy is conserved:

$$mgr\cos\theta = \frac{1}{2}mv^2$$

at the breaking point consider radial forces

$$\sum F_r = ma_r$$

$$+T_{max} - mg\cos\theta = m\frac{v^2}{r}$$

Eliminate $\frac{v^2}{r} = 2g\cos\theta$

$$T_{max} - mg\cos\theta = 2mg\cos\theta$$

$$T_{max} = 3mg\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{T_{max}}{3mg}\right) = \cos^{-1}\left(\frac{44.5\text{ N}}{3(2.00\text{ kg})(9.80\text{ m/s}^2)}\right)$$

$$\theta = \boxed{40.8°}$$

***P8.19** (a) For a 5-m cord the spring constant is described by $F = kx$, $mg = k(1.5\text{ m})$. For a longer cord of length L the stretch distance is longer so the spring constant is smaller in inverse proportion:

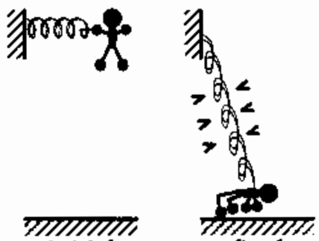


FIG. P8.19(a)

$$k = \frac{5\text{ m}}{L}\frac{mg}{1.5\text{ m}} = 3.33\,mg/L$$

$$(K + U_g + U_s)_i = (K + U_g + U_s)_f$$

$$0 + mgy_i + 0 = 0 + mgy_f + \frac{1}{2}kx_f^2$$

$$mg(y_i - y_f) = \frac{1}{2}kx_f^2 = \frac{1}{2}3.33\frac{mg}{L}x_f^2$$

here $y_i - y_f = 55\text{ m} = L + x_f$

$$55.0\text{ m}L = \frac{1}{2}3.33(55.0\text{ m} - L)^2$$

$$55.0\text{ m}L = 5.04\times10^3\text{ m}^2 - 183\text{ m}L + 1.67L^2$$

$$0 = 1.67L^2 - 238L + 5.04\times10^3 = 0$$

$$L = \frac{238 \pm \sqrt{238^2 - 4(1.67)(5.04\times10^3)}}{2(1.67)} = \frac{238\pm152}{3.33} = \boxed{25.8\text{ m}}$$

only the value of L less than 55 m is physical.

(b) $k = 3.33\frac{mg}{25.8\text{ m}}$ $\quad x_{max} = x_f = 55.0\text{ m} - 25.8\text{ m} = 29.2\text{ m}$

$\sum F = ma$ $\quad +kx_{max} - mg = ma$

$$3.33\frac{mg}{25.8\text{ m}}29.2\text{ m} - mg = ma$$

$$a = 2.77g = \boxed{27.1\text{ m/s}^2}$$

***P8.20** When block B moves up by 1 cm, block A moves down by 2 cm and the separation becomes 3 cm. We then choose the final point to be when B has moved up by $\frac{h}{3}$ and has speed $\frac{v_A}{2}$. Then A has moved down $\frac{2h}{3}$ and has speed v_A:

$$(K_A + K_B + U_g)_i = (K_A + K_B + U_g)_f$$

$$0+0+0 = \frac{1}{2}mv_A^2 + \frac{1}{2}m\left(\frac{v_A}{2}\right)^2 + \frac{mgh}{3} - \frac{mg2h}{3}$$

$$\frac{mgh}{3} = \frac{5}{8}mv_A^2$$

$$\boxed{v_A = \sqrt{\frac{8gh}{15}}}$$

Section 8.3 Conservative and Nonconservative Forces

P8.21 $F_g = mg = (4.00\text{ kg})(9.80\text{ m/s}^2) = 39.2\text{ N}$

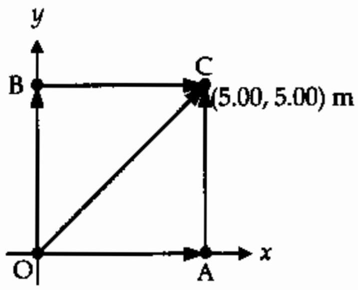


FIG. P8.21

(a) Work along OAC = work along OA + work along AC
$= F_g(\text{OA})\cos 90.0° + F_g(\text{AC})\cos 180°$
$= (39.2\text{ N})(5.00\text{ m}) + (39.2\text{ N})(5.00\text{ m})(-1)$
$= \boxed{-196\text{ J}}$

(b) W along OBC = W along OB + W along BC
$= (39.2\text{ N})(5.00\text{ m})\cos 180° + (39.2\text{ N})(5.00\text{ m})\cos 90.0°$
$= \boxed{-196\text{ J}}$

(c) Work along OC $= F_g(\text{OC})\cos 135°$
$= (39.2\text{ N})\left(5.00 \times \sqrt{2}\text{ m}\right)\left(-\frac{1}{\sqrt{2}}\right) = \boxed{-196\text{ J}}$

The results should all be the same, since gravitational forces are conservative.

P8.22 (a) $W = \int \mathbf{F} \cdot d\mathbf{r}$ and if the force is constant, this can be written as
$W = \mathbf{F} \cdot \int d\mathbf{r} = \boxed{\mathbf{F} \cdot (\mathbf{r}_f - \mathbf{r}_i)\text{, which depends only on end points, not path.}}$

(b) $W = \int \mathbf{F} \cdot d\mathbf{r} = \int (3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \cdot (dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}) = (3.00\text{ N})\int_0^{5.00\text{ m}} dx + (4.00\text{ N})\int_0^{5.00\text{ m}} dy$
$W = (3.00\text{ N})x\Big|_0^{5.00\text{ m}} + (4.00\text{ N})y\Big|_0^{5.00\text{ m}} = 15.0\text{ J} + 20.0\text{ J} = \boxed{35.0\text{ J}}$

The same calculation applies for all paths.

P8.23 (a)

$$W_{OA}=\int_0^{5.00\text{ m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} 2y\,dx$$

and since along this path, $y=0$ $\quad W_{OA}=0$

$$W_{AC}=\int_0^{5.00\text{ m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} x^2\,dy$$

For $x=5.00$ m, $\quad W_{AC}=125\text{ J}$

and $\quad W_{OAC}=0+125=\boxed{125\text{ J}}$

(b)

$$W_{OB}=\int_0^{5.00\text{ m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} x^2\,dy$$

since along this path, $x=0$, $\quad W_{OB}=0$

$$W_{BC}=\int_0^{5.00\text{ m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} 2y\,dx$$

since $y=5.00$ m, $\quad W_{BC}=50.0\text{ J}$

$$W_{OBC}=0+50.0=\boxed{50.0\text{ J}}$$

(c)

$$W_{OC}=\int\left(dx\hat{\mathbf{i}}+dy\hat{\mathbf{j}}\right)\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int\left(2y\,dx+x^2\,dy\right)$$

Since $x=y$ along OC, $\quad W_{OC}=\int_0^{5.00\text{ m}}\left(2x+x^2\right)dx=\boxed{66.7\text{ J}}$

(d) F is $\boxed{\text{nonconservative}}$ since the work done is path dependent.

P8.24 (a)

$$(\Delta K)_{A\to B}=\sum W=W_g=mg\Delta h=mg(5.00-3.20)$$

$$\frac{1}{2}mv_B^2-\frac{1}{2}mv_A^2=m(9.80)(1.80)$$

$$v_B=\boxed{5.94\text{ m/s}}$$

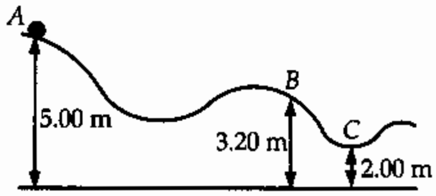


FIG. P8.24

Similarly, $v_C=\sqrt{v_A^2+2g(5.00-2.00)}=\boxed{7.67\text{ m/s}}$

(b) $W_g\big|_{A\to C}=mg(3.00\text{ m})=\boxed{147\text{ J}}$

P8.25 (a) $\mathbf{F} = \left(3.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right)$ N

$m = 4.00$ kg

$\mathbf{r} = \left(2.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}}\right)$ m

$W = 3.00(2.00) + 5.00(-3.00) = \boxed{-9.00 \text{ J}}$

The result does not depend on the path since the force is conservative.

(b) $W = \Delta K$

$$-9.00 = \frac{4.00v^2}{2} - 4.00\left(\frac{(4.00)^2}{2}\right)$$

so $v = \sqrt{\dfrac{32.0 - 9.00}{2.00}} = \boxed{3.39 \text{ m/s}}$

(c) $\Delta U = -W = \boxed{9.00 \text{ J}}$

Section 8.4 **Changes in Mechanical Energy for Nonconservative Forces**

P8.26 (a) $U_f = K_i - K_f + U_i$ $\qquad U_f = 30.0 - 18.0 + 10.0 = \boxed{22.0 \text{ J}}$

$\boxed{E = 40.0 \text{ J}}$

(b) Yes, $\Delta E_{\text{mech}} = \Delta K + \Delta U$ is not equal to zero. For conservative forces $\Delta K + \Delta U = 0$.

P8.27 The distance traveled by the ball from the top of the arc to the bottom is πR. The work done by the non-conservative force, the force exerted by the pitcher,

is $\qquad \Delta E = F\Delta r \cos 0° = F(\pi R)$.

We shall assign the gravitational energy of the ball-Earth system to be zero with the ball at the bottom of the arc.

Then $\qquad \Delta E_{\text{mech}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i$

becomes $\qquad \frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgy_i + F(\pi R)$

or $\qquad v_f = \sqrt{v_i^2 + 2gy_i + \dfrac{2F(\pi R)}{m}} = \sqrt{(15.0)^2 + 2(9.80)(1.20) + \dfrac{2(30.0)\pi(0.600)}{0.250}}$

$v_f = \boxed{26.5 \text{ m/s}}$

***P8.28** The useful output energy is

$$120 \text{ Wh}(1 - 0.60) = mg\left(y_f - y_i\right) = F_g \Delta y$$

$$\Delta y = \frac{120 \text{ W}(3\,600 \text{ s})0.40}{890 \text{ N}}\left(\frac{\text{J}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right) = \boxed{194 \text{ m}}$$

***P8.29** As the locomotive moves up the hill at constant speed, its output power goes into internal energy plus gravitational energy of the locomotive-Earth system:

$\mathcal{P}t = mgy + f\Delta r = mg\Delta r\sin\theta + f\Delta r$ $\qquad$ $\mathcal{P} = mgv_f \sin\theta + fv_f$

As the locomotive moves on level track,

$\mathcal{P} = fv_i$ $\qquad$ $1\,000\text{ hp}\left(\dfrac{746\text{ W}}{1\text{ hp}}\right) = f(27\text{ m/s})$ $\qquad$ $f = 2.76\times10^4\text{ N}$

Then also $746\,000\text{ W} = (160\,000\text{ kg})(9.8\text{ m/s}^2)v_f\left(\dfrac{5\text{ m}}{100\text{ m}}\right) + (2.76\times10^4\text{ N})v_f$

$$v_f = \frac{746\,000\text{ W}}{1.06\times10^5\text{ N}} = \boxed{7.04\text{ m/s}}$$

P8.30 We shall take the zero level of gravitational potential energy to be at the lowest level reached by the diver under the water, and consider the energy change from when the diver started to fall until he came to rest.

$$\Delta E = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i = f_k d\cos180°$$

$$0 - 0 - mg(y_i - y_f) = -f_k d$$

$$f_k = \frac{mg(y_i - y_f)}{d} = \frac{(70.0\text{ kg})(9.80\text{ m/s}^2)(10.0\text{ m} + 5.00\text{ m})}{5.00\text{ m}} = \boxed{2.06\text{ kN}}$$

P8.31 $U_i + K_i + \Delta E_{\text{mech}} = U_f + K_f$: $\qquad m_2gh - fh = \dfrac{1}{2}m_1v^2 + \dfrac{1}{2}m_2v^2$

$$f = \mu n = \mu m_1 g$$

$$m_2gh - \mu m_1 gh = \frac{1}{2}(m_1 + m_2)v^2$$

$$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$$

$$v = \sqrt{\frac{2(9.80\text{ m/s}^2)(1.50\text{ m})[5.00\text{ kg} - 0.400(3.00\text{ kg})]}{8.00\text{ kg}}} = \boxed{3.74\text{ m/s}}$$

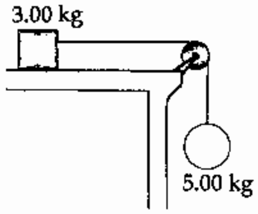


FIG. P8.31

P8.32 $\Delta E_{\text{mech}} = (K_f - K_i) + (U_{gf} - U_{gi})$

But $\Delta E_{\text{mech}} = W_{\text{app}} - f\Delta x$, where W_{app} is the work the boy did pushing forward on the wheels.

Thus, $\quad W_{\text{app}} = (K_f - K_i) + (U_{gf} - U_{gi}) + f\Delta x$

or $\quad W_{\text{app}} = \dfrac{1}{2}m(v_f^2 - v_i^2) + mg(-h) + f\Delta x$

$$W_{\text{app}} = \frac{1}{2}(47.0)\left[(6.20)^2 - (1.40)^2\right] - (47.0)(9.80)(2.60) + (41.0)(12.4)$$

$$W_{\text{app}} = \boxed{168\text{ J}}$$

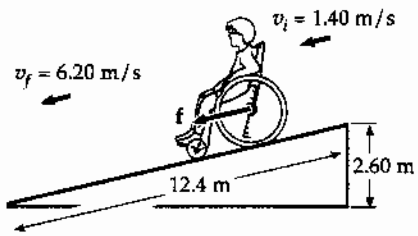


FIG. P8.32

P8.33 (a) $\Delta K = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = -\frac{1}{2}mv_i^2 = \boxed{-160\text{ J}}$

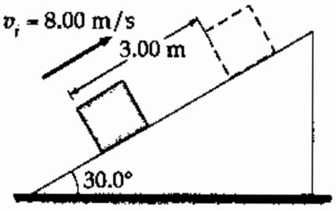


FIG. P8.33

(b) $\Delta U = mg(3.00\text{ m})\sin 30.0° = \boxed{73.5\text{ J}}$

(c) The mechanical energy converted due to friction is 86.5 J

$$f = \frac{86.5\text{ J}}{3.00\text{ m}} = \boxed{28.8\text{ N}}$$

(d) $f = \mu_k n = \mu_k mg\cos 30.0° = 28.8\text{ N}$

$$\mu_k = \frac{28.8\text{ N}}{(5.00\text{ kg})\left(9.80\text{ m/s}^2\right)\cos 30.0°} = \boxed{0.679}$$

P8.34 Consider the whole motion: $K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$

(a) $0 + mgy_i - f_1\Delta x_1 - f_2\Delta x_2 = \frac{1}{2}mv_f^2 + 0$

$$(80.0\text{ kg})\left(9.80\text{ m/s}^2\right)1\,000\text{ m} - (50.0\text{ N})(800\text{ m}) - (3\,600\text{ N})(200\text{ m}) = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$784\,000\text{ J} - 40\,000\text{ J} - 720\,000\text{ J} = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(24\,000\text{ J})}{80.0\text{ kg}}} = \boxed{24.5\text{ m/s}}$$

(b) $\boxed{\text{Yes}}$ this is too fast for safety.

(c) Now in the same energy equation as in part (a), Δx_2 is unknown, and $\Delta x_1 = 1\,000\text{ m} - \Delta x_2$:

$$784\,000\text{ J} - (50.0\text{ N})(1\,000\text{ m} - \Delta x_2) - (3\,600\text{ N})\Delta x_2 = \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2$$

$$784\,000\text{ J} - 50\,000\text{ J} - (3\,550\text{ N})\Delta x_2 = 1\,000\text{ J}$$

$$\Delta x_2 = \frac{733\,000\text{ J}}{3\,550\text{ N}} = \boxed{206\text{ m}}$$

(d) Really the air drag will depend on the skydiver's speed. It will be larger than her 784 N weight only after the chute is opened. It will be nearly equal to 784 N before she opens the chute and again before she touches down, whenever she moves near terminal speed.

P8.35 (a) $(K+U)_i + \Delta E_{\text{mech}} = (K+U)_f$:

$$0+\frac{1}{2}kx^2 - f\Delta x = \frac{1}{2}mv^2 + 0$$

$$\frac{1}{2}(8.00\text{ N/m})(5.00\times10^{-2}\text{ m})^2 - (3.20\times10^{-2}\text{ N})(0.150\text{ m}) = \frac{1}{2}(5.30\times10^{-3}\text{ kg})v^2$$

$$v=\sqrt{\frac{2(5.20\times10^{-3}\text{ J})}{5.30\times10^{-3}\text{ kg}}} = \boxed{1.40\text{ m/s}}$$

(b) When the spring force just equals the friction force, the ball will stop speeding up. Here $|\mathbf{F}_s| = kx$; the spring is compressed by

$$\frac{3.20\times10^{-2}\text{ N}}{8.00\text{ N/m}} = 0.400\text{ cm}$$

and the ball has moved

$$5.00\text{ cm} - 0.400\text{ cm} = \boxed{4.60\text{ cm from the start.}}$$

(c) Between start and maximum speed points,

$$\frac{1}{2}kx_i^2 - f\Delta x = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}8.00(5.00\times10^{-2})^2 - (3.20\times10^{-2})(4.60\times10^{-2}) = \frac{1}{2}(5.30\times10^{-3})v^2 + \frac{1}{2}8.00(4.00\times10^{-3})^2$$

$$v = \boxed{1.79\text{ m/s}}$$

P8.36 $\sum F_y = n - mg\cos 37.0° = 0$

$\therefore n = mg\cos 37.0° = 400\text{ N}$

$f = \mu n = 0.250(400\text{ N}) = 100\text{ N}$

$-f\Delta x = \Delta E_{\text{mech}}$

$(-100)(20.0) = \Delta U_A + \Delta U_B + \Delta K_A + \Delta K_B$

$\Delta U_A = m_A g(h_f - h_i) = (50.0)(9.80)(20.0\sin 37.0°) = 5.90\times10^3$

$\Delta U_B = m_B g(h_f - h_i) = (100)(9.80)(-20.0) = -1.96\times10^4$

$\Delta K_A = \frac{1}{2}m_A(v_f^2 - v_i^2)$

$\Delta K_B = \frac{1}{2}m_B(v_f^2 - v_i^2) = \frac{m_B}{m_A}\Delta K_A = 2\Delta K_A$

Adding and solving, $\Delta K_A = \boxed{3.92\text{ kJ}}$.

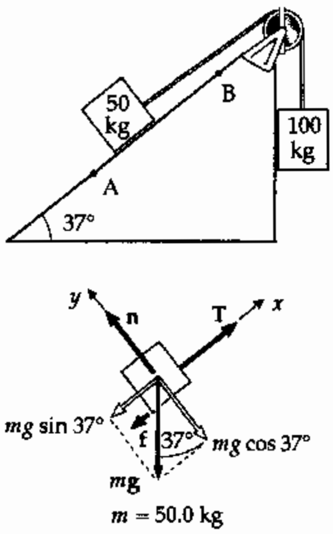


FIG. P8.36

P8.37 (a) The object moved down distance $1.20\text{ m} + x$. Choose $y = 0$ at its lower point.

$$K_i + U_{gi} + U_{si} + \Delta E_{\text{mech}} = K_f + U_{gf} + U_{sf}$$

$$0 + mgy_i + 0 + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$(1.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m} + x) = \frac{1}{2}(320\text{ N/m})x^2$$

$$0 = (160\text{ N/m})x^2 - (14.7\text{ N})x - 17.6\text{ J}$$

$$x = \frac{14.7\text{ N} \pm \sqrt{(-14.7\text{ N})^2 - 4(160\text{ N/m})(-17.6\text{ N}\cdot\text{m})}}{2(160\text{ N/m})}$$

$$x = \frac{14.7\text{ N} \pm 107\text{ N}}{320\text{ N/m}}$$

The negative root tells how high the object will rebound if it is instantly glued to the spring. We want

$$x = \boxed{0.381\text{ m}}$$

(b) From the same equation,

$$(1.50\text{ kg})(1.63\text{ m/s}^2)(1.20\text{ m} + x) = \frac{1}{2}(320\text{ N/m})x^2$$

$$0 = 160x^2 - 2.44x - 2.93$$

The positive root is $x = \boxed{0.143\text{ m}}$.

(c) The equation expressing the energy version of the nonisolated system model has one more term:

$$mgy_i - f\Delta x = \frac{1}{2}kx^2$$

$$(1.50\text{ kg})(9.80\text{ m/s}^2)(1.20\text{ m} + x) - 0.700\text{ N}(1.20\text{ m} + x) = \frac{1}{2}(320\text{ N/m})x^2$$

$$17.6\text{ J} + 14.7\text{ N}x - 0.840\text{ J} - 0.700\text{ N}x = 160\text{ N/m}\,x^2$$

$$160x^2 - 14.0x - 16.8 = 0$$

$$x = \frac{14.0 \pm \sqrt{(14.0)^2 - 4(160)(-16.8)}}{320}$$

$$x = \boxed{0.371\text{ m}}$$

P8.38 The total mechanical energy of the skysurfer-Earth system is

$$E_{\text{mech}} = K + U_g = \frac{1}{2}mv^2 + mgh.$$

Since the skysurfer has constant speed,

$$\frac{dE_{\text{mech}}}{dt} = mv\frac{dv}{dt} + mg\frac{dh}{dt} = 0 + mg(-v) = -mgv.$$

The rate the system is losing mechanical energy is then

$$\left|\frac{dE_{\text{mech}}}{dt}\right| = mgv = (75.0\text{ kg})(9.80\text{ m/s}^2)(60.0\text{ m/s}) = \boxed{44.1\text{ kW}}.$$

***P8.39** (a) Let m be the mass of the whole board. The portion on the rough surface has mass $\frac{mx}{L}$. The normal force supporting it is $\frac{mxg}{L}$ and the frictional force is $\frac{\mu_k mgx}{L} = ma$. Then $\boxed{a = \frac{\mu_k gx}{L}\text{ opposite to the motion.}}$

(b) In an incremental bit of forward motion dx, the kinetic energy converted into internal energy is $f_k dx = \frac{\mu_k mgx}{L}dx$. The whole energy converted is

$$\frac{1}{2}mv^2 = \int_0^L \frac{\mu_k mgx}{L}dx = \frac{\mu_k mg}{L}\frac{x^2}{2}\Big|_0^L = \frac{\mu_k mgL}{2}$$

$$\boxed{v = \sqrt{\mu_k gL}}$$

Section 8.5 Relationship Between Conservative Forces and Potential Energy

P8.40 (a) $U = -\int_0^x \left(-Ax + Bx^2\right)dx = \boxed{\frac{Ax^2}{2} - \frac{Bx^3}{3}}$

(b) $\Delta U = -\int_{2.00\text{ m}}^{3.00\text{ m}} F dx = \frac{A\left[\left(3.00^2\right) - (2.00)^2\right]}{2} - \frac{B\left[(3.00)^3 - (2.00)^3\right]}{3} = \boxed{\frac{5.00}{2}A - \frac{19.0}{3}B}$

$\Delta K = \boxed{\left(-\frac{5.00}{2}A + \frac{19.0}{3}B\right)}$

P8.41 (a) $W = \int F_x dx = \int_1^{5.00\text{ m}} (2x+4)dx = \left(\frac{2x^2}{2} + 4x\right)\Big|_1^{5.00\text{ m}} = 25.0 + 20.0 - 1.00 - 4.00 = \boxed{40.0\text{ J}}$

(b) $\Delta K + \Delta U = 0$ $\quad \Delta U = -\Delta K = -W = \boxed{-40.0\text{ J}}$

(c) $\Delta K = K_f - \frac{mv_1^2}{2}$ $\quad K_f = \Delta K + \frac{mv_1^2}{2} = \boxed{62.5\text{ J}}$

P8.42 $F_x = -\frac{\partial U}{\partial x} = -\frac{\partial(3x^3y - 7x)}{\partial x} = -(9x^2y - 7) = 7 - 9x^2y$

$$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial(3x^3y - 7x)}{\partial y} = -(3x^3 - 0) = -3x^3$$

Thus, the force acting at the point (x, y) is $\mathbf{F} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} = \boxed{(7 - 9x^2y)\hat{\mathbf{i}} - 3x^3\hat{\mathbf{j}}}$.

P8.43 $U(r) = \frac{A}{r}$

$F_r = -\frac{\partial U}{\partial r} = -\frac{d}{dr}\left(\frac{A}{r}\right) = \boxed{\frac{A}{r^2}}$. The positive value indicates a force of repulsion.

Section 8.6 Energy Diagrams and the Equilibrium of a System

P8.44

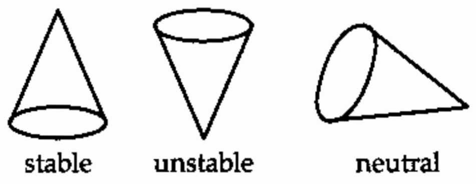


FIG. P8.44

P8.45 (a) F_x is zero at points A, C and E; F_x is positive at point B and negative at point D.

(b) A and E are unstable, and C is stable.

(c)

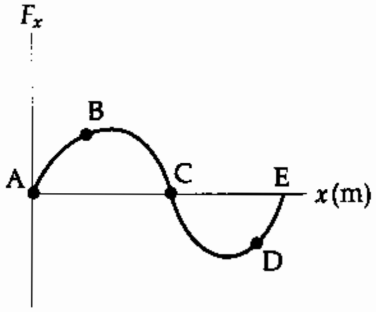


FIG. P8.45

P8.46 (a) There is an equilibrium point wherever the graph of potential energy is horizontal:

At $r = 1.5$ mm and 3.2 mm, the equilibrium is stable.
At $r = 2.3$ mm, the equilibrium is unstable.
A particle moving out toward $r \to \infty$ approaches neutral equilibrium.

(b) The system energy E cannot be less than –5.6 J. The particle is bound if $\boxed{-5.6\text{ J} \le E < 1\text{ J}}$.

(c) If the system energy is –3 J, its potential energy must be less than or equal to –3 J. Thus, the particle's position is limited to $\boxed{0.6\text{ mm} \le r \le 3.6\text{ mm}}$.

(d) $K + U = E$. Thus, $K_{max} = E - U_{min} = -3.0\text{ J} - (-5.6\text{ J}) = \boxed{2.6\text{ J}}$.

(e) Kinetic energy is a maximum when the potential energy is a minimum, at $\boxed{r = 1.5\text{ mm}}$.

(f) $-3\text{ J} + W = 1\text{ J}$. Hence, the binding energy is $W = \boxed{4\text{ J}}$.

P8.47 (a) When the mass moves distance x, the length of each spring changes from L to $\sqrt{x^2 + L^2}$, so each exerts force $k\left(\sqrt{x^2 + L^2} - L\right)$ towards its fixed end. The y-components cancel out and the x components add to:

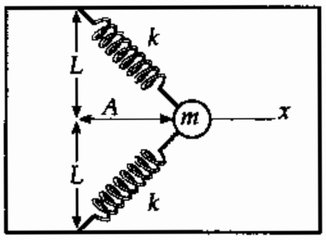


FIG. P8.47(a)

$$F_x = -2k\left(\sqrt{x^2 + L^2} - L\right)\left(\frac{x}{\sqrt{x^2 + L^2}}\right) = -2kx + \frac{2kLx}{\sqrt{x^2 + L^2}}$$

Choose $U = 0$ at $x = 0$. Then at any point the potential energy of the system is

$$U(x) = -\int_0^x F_x dx = -\int_0^x \left(-2kx + \frac{2kLx}{\sqrt{x^2 + L^2}}\right)dx = 2k\int_0^x x dx - 2kL\int_0^x \frac{x}{\sqrt{x^2 + L^2}}dx$$

$$U(x) = \boxed{kx^2 + 2kL\left(L - \sqrt{x^2 + L^2}\right)}$$

(b) $U(x) = 40.0x^2 + 96.0\left(1.20 - \sqrt{x^2 + 1.44}\right)$

For negative x, $U(x)$ has the same value as for positive x. The only equilibrium point (i.e., where $F_x = 0$) is $\boxed{x = 0}$.

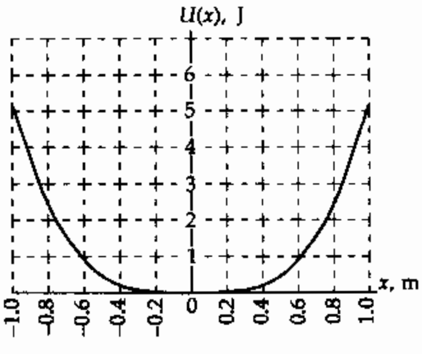


FIG. P8.47(b)

(c) $K_i + U_i + \Delta E_{mech} = K_f + U_f$

$0 + 0.400\text{ J} + 0 = \frac{1}{2}(1.18\text{ kg})v_f^2 + 0$

$v_f = \boxed{0.823\text{ m/s}}$

Additional Problems

P8.48 The potential energy of the block-Earth system is mgh.

An amount of energy $\mu_k mgd\cos\theta$ is converted into internal energy due to friction on the incline.

Therefore the final height y_{max} is found from

$$mgy_{max} = mgh - \mu_k mgd\cos\theta$$

where

$$d = \frac{y_{max}}{\sin\theta}$$

$$\therefore mgy_{max} = mgh - \mu_k mgy_{max}\cot\theta$$

Solving,

$$y_{max} = \frac{h}{1+\mu_k\cot\theta}.$$

FIG. P8.48

P8.49 At a pace I could keep up for a half-hour exercise period, I climb two stories up, traversing forty steps each 18 cm high, in 20 s. My output work becomes the final gravitational energy of the system of the Earth and me,

$$mgy = (85\text{ kg})(9.80\text{ m/s}^2)(40\times 0.18\text{ m}) = 6\,000\text{ J}$$

making my sustainable power $\dfrac{6\,000\text{ J}}{20\text{ s}} = \boxed{\sim 10^2\text{ W}}$.

P8.50 $v = 100\text{ km/h} = 27.8\text{ m/s}$

The retarding force due to air resistance is

$$R = \frac{1}{2}D\rho Av^2 = \frac{1}{2}(0.330)(1.20\text{ kg/m}^3)(2.50\text{ m}^2)(27.8\text{ m/s})^2 = 382\text{ N}$$

Comparing the energy of the car at two points along the hill,

$$K_i + U_{gi} + \Delta E = K_f + U_{gf}$$

or $$K_i + U_{gi} + \Delta W_e - R(\Delta s) = K_f + U_{gf}$$

where ΔW_e is the work input from the engine. Thus,

$$\Delta W_e = R(\Delta s) + (K_f - K_i) + (U_{gf} - U_{gi})$$

Recognizing that $K_f = K_i$ and dividing by the travel time Δt gives the required power input from the engine as

$$\mathcal{P} = \left(\frac{\Delta W_e}{\Delta t}\right) = R\left(\frac{\Delta s}{\Delta t}\right) + mg\left(\frac{\Delta y}{\Delta t}\right) = Rv + mgv\sin\theta$$

$$\mathcal{P} = (382\text{ N})(27.8\text{ m/s}) + (1\,500\text{ kg})(9.80\text{ m/s}^2)(27.8\text{ m/s})\sin 3.20°$$

$$\mathcal{P} = \boxed{33.4\text{ kW} = 44.8\text{ hp}}$$

P8.51 m = mass of pumpkin

R = radius of silo top

$\sum F_r = ma_r \Rightarrow n - mg\cos\theta = -m\dfrac{v^2}{R}$

When the pumpkin first loses contact with the surface, $n = 0$.
Thus, at the point where it leaves the surface: $v^2 = Rg\cos\theta$.

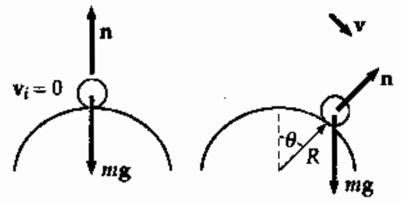


FIG. P8.51

Choose $U_g = 0$ in the $\theta = 90.0°$ plane. Then applying conservation of energy for the pumpkin-Earth system between the starting point and the point where the pumpkin leaves the surface gives

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv^2 + mgR\cos\theta = 0 + mgR$$

Using the result from the force analysis, this becomes

$\dfrac{1}{2}mRg\cos\theta + mgR\cos\theta = mgR$, which reduces to

$\cos\theta = \dfrac{2}{3}$, and gives $\theta = \cos^{-1}(2/3) = \boxed{48.2°}$

as the angle at which the pumpkin will lose contact with the surface.

P8.52 (a) $U_A = mgR = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m}) = \boxed{0.588\text{ J}}$

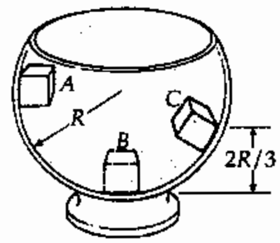


FIG. P8.52

(b) $K_A + U_A = K_B + U_B$

$K_B = K_A + U_A - U_B = mgR = \boxed{0.588\text{ J}}$

(c) $v_B = \sqrt{\dfrac{2K_B}{m}} = \sqrt{\dfrac{2(0.588\text{ J})}{0.200\text{ kg}}} = \boxed{2.42\text{ m/s}}$

(d) $U_C = mgh_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) = \boxed{0.392\text{ J}}$

$K_C = K_A + U_A - U_C = mg(h_A - h_C)$

$K_C = (0.200\text{ kg})(9.80\text{ m/s}^2)(0.300 - 0.200)\text{ m} = \boxed{0.196\text{ J}}$

P8.53 (a) $K_B = \dfrac{1}{2}mv_B^2 = \dfrac{1}{2}(0.200\text{ kg})(1.50\text{ m/s})^2 = \boxed{0.225\text{ J}}$

(b) $\Delta E_{\text{mech}} = \Delta K + \Delta U = K_B - K_A + U_B - U_A$

$= K_B + mg(h_B - h_A)$

$= 0.225\text{ J} + (0.200\text{ kg})(9.80\text{ m/s}^2)(0 - 0.300\text{ m})$

$= 0.225\text{ J} - 0.588\text{ J} = \boxed{-0.363\text{ J}}$

(c) It's possible to find an effective coefficient of friction, but not the actual value of μ since n and f vary with position.

P8.54 The gain in internal energy due to friction represents a loss in mechanical energy that must be equal to the change in the kinetic energy plus the change in the potential energy.

Therefore,

$$-\mu_k mgx\cos\theta = \Delta K + \frac{1}{2}kx^2 - mgx\sin\theta$$

and since $v_i = v_f = 0$, $\Delta K = 0$.

Thus,

$$-\mu_k(2.00)(9.80)(\cos 37.0°)(0.200) = \frac{(100)(0.200)^2}{2} - (2.00)(9.80)(\sin 37.0°)(0.200)$$

and we find $\mu_k = \boxed{0.115}$. Note that in the above we had a *gain* in elastic potential energy for the spring and a *loss* in gravitational potential energy.

P8.55 (a) Since no nonconservative work is done, $\Delta E = 0$

Also $\Delta K = 0$

therefore, $U_i = U_f$

where $U_i = (mg\sin\theta)x$

and $U_f = \frac{1}{2}kx^2$

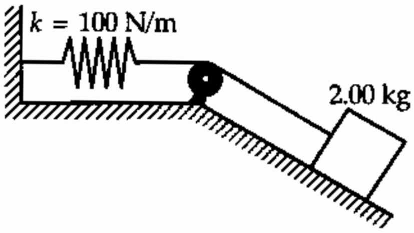


FIG. P8.55

Substituting values yields $(2.00)(9.80)\sin 37.0° = (100)\frac{x}{2}$ and solving we find

$$x = \boxed{0.236\text{ m}}$$

(b) $\sum F = ma$. Only gravity and the spring force act on the block, so

$$-kx + mg\sin\theta = ma$$

For $x = 0.236$ m,

$a = \boxed{-5.90\text{ m/s}^2}$. The negative sign indicates a is up the incline.

The $\boxed{\text{acceleration depends on position}}$.

(c) U(gravity) decreases monotonically as the height decreases.

U(spring) increases monotonically as the spring is stretched.

K initially increases, but then goes back to zero.

P8.56 $k = 2.50 \times 10^4\ \text{N/m}$, $m = 25.0\ \text{kg}$

$x_A = -0.100\ \text{m}$, $U_g\big|_{x=0} = U_s\big|_{x=0} = 0$

(a) $E_{\text{mech}} = K_A + U_{gA} + U_{sA}$

$$E_{\text{mech}} = 0 + mgx_A + \frac{1}{2}kx_A^2$$
$$E_{\text{mech}} = (25.0\ \text{kg})(9.80\ \text{m/s}^2)(-0.100\ \text{m}) + \frac{1}{2}(2.50 \times 10^4\ \text{N/m})(-0.100\ \text{m})^2$$
$$E_{\text{mech}} = -24.5\ \text{J} + 125\ \text{J} = \boxed{100\ \text{J}}$$

(b) Since only conservative forces are involved, the total energy of the child-pogo-stick-Earth system at point C is the same as that at point A.

$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$: $\quad 0 + (25.0\ \text{kg})(9.80\ \text{m/s}^2)x_C + 0 = 0 - 24.5\ \text{J} + 125\ \text{J}$

$$x_C = \boxed{0.410\ \text{m}}$$

(c) $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$: $\quad \frac{1}{2}(25.0\ \text{kg})v_B^2 + 0 + 0 = 0 + (-24.5\ \text{J}) + 125\ \text{J}$

$$v_B = \boxed{2.84\ \text{m/s}}$$

(d) K and v are at a maximum when $a = \sum F/m = 0$ (i.e., when the magnitude of the upward spring force equals the magnitude of the downward gravitational force).

This occurs at $x < 0$ where $k|x| = mg$

or
$$|x| = \frac{(25.0\ \text{kg})(9.8\ \text{m/s}^2)}{2.50 \times 10^4\ \text{N/m}} = 9.80 \times 10^{-3}\ \text{m}$$

Thus, $K = K_{\text{max}}$ at $x = \boxed{-9.80\ \text{mm}}$

(e) $K_{\text{max}} = K_A + \left(U_{gA} - U_g\big|_{x=-9.80\ \text{mm}}\right) + \left(U_{sA} - U_s\big|_{x=-9.80\ \text{mm}}\right)$

or
$$\frac{1}{2}(25.0\ \text{kg})v_{\text{max}}^2 = (25.0\ \text{kg})(9.80\ \text{m/s}^2)\left[(-0.100\ \text{m}) - (-0.009\,8\ \text{m})\right] + \frac{1}{2}(2.50 \times 10^4\ \text{N/m})\left[(-0.100\ \text{m})^2 - (-0.009\,8\ \text{m})^2\right]$$

yielding $v_{\text{max}} = \boxed{2.85\ \text{m/s}}$

P8.57
$$\Delta E_{\text{mech}} = -f\Delta x$$
$$E_f - E_i = -f \cdot d_{BC}$$
$$\frac{1}{2}kx^2 - mgh = -\mu mgd_{BC}$$
$$\mu = \frac{mgh - \frac{1}{2}kx^2}{mgd_{BC}} = \boxed{0.328}$$

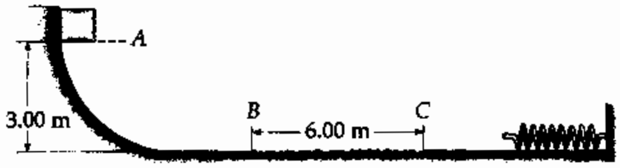


FIG. P8.57

P8.58 (a) $\mathbf{F} = -\frac{d}{dx}\left(-x^3 + 2x^2 + 3x\right)\hat{\mathbf{i}} = \boxed{\left(3x^2 - 4x - 3\right)\hat{\mathbf{i}}}$

(b) $F = 0$

when $x = \boxed{1.87 \text{ and } -0.535}$

(c) The stable point is at

$$x = -0.535 \text{ point of minimum } U(x).$$

The unstable point is at

$$x = 1.87 \text{ maximum in } U(x).$$

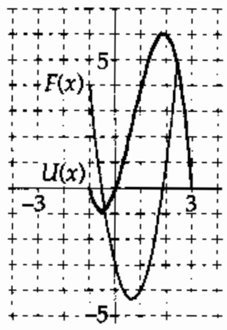


FIG. P8.58

P8.59 $(K+U)_i = (K+U)_f$

$$0 + (30.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) + \frac{1}{2}(250\text{ N/m})(0.200\text{ m})^2$$

$$= \frac{1}{2}(50.0\text{ kg})v^2 + (20.0\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})\sin 40.0°$$

$$58.8\text{ J} + 5.00\text{ J} = (25.0\text{ kg})v^2 + 25.2\text{ J}$$

$\boxed{v = 1.24\text{ m/s}}$

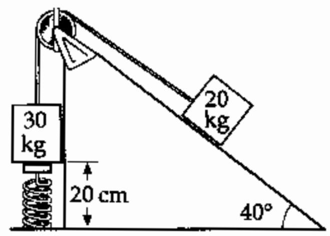


FIG. P8.59

P8.60 (a) Between the second and the third picture, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-\mu mgd = -\frac{1}{2}mv_i^2 + \frac{1}{2}kd^2$$

$$\frac{1}{2}(50.0\text{ N/m})d^2 + 0.250(1.00\text{ kg})(9.80\text{ m/s}^2)d - \frac{1}{2}(1.00\text{ kg})(3.00\text{ m/s}^2) = 0$$

$$d = \frac{[-2.45 \pm 21.25]\text{ N}}{50.0\text{ N/m}} = \boxed{0.378\text{ m}}$$

(b) Between picture two and picture four, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-f(2d) = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2$$

$$v = \sqrt{(3.00\text{ m/s})^2 - \frac{2}{(1.00\text{ kg})}(2.45\text{ N})(2)(0.378\text{ m})}$$

$$= \boxed{2.30\text{ m/s}}$$

(c) For the motion from picture two to picture five, $\Delta E_{\text{mech}} = \Delta K + \Delta U$

$$-f(D + 2d) = -\frac{1}{2}(1.00\text{ kg})(3.00\text{ m/s})^2$$

$$D = \frac{9.00\text{ J}}{2(0.250)(1.00\text{ kg})(9.80\text{ m/s}^2)} - 2(0.378\text{ m}) = \boxed{1.08\text{ m}}$$

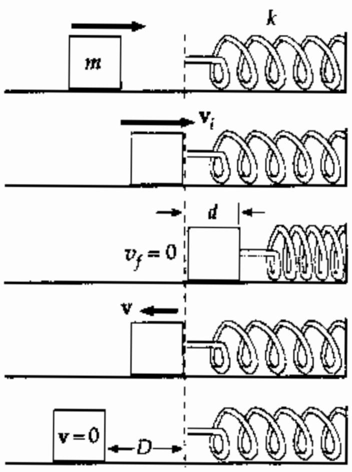


FIG. P8.60

P8.61 (a) Initial compression of spring: $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$$\frac{1}{2}(450\ \text{N/m})(\Delta x)^2 = \frac{1}{2}(0.500\ \text{kg})(12.0\ \text{m/s})^2$$

$$\therefore \Delta x = \boxed{0.400\ \text{m}}$$

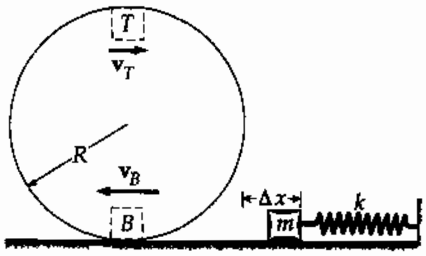


FIG. P8.61

(b) Speed of block at top of track: $\Delta E_{\text{mech}} = -f\Delta x$

$$\left(mgh_T + \frac{1}{2}mv_T^2\right) - \left(mgh_B + \frac{1}{2}mv_B^2\right) = -f(\pi R)$$

$$(0.500\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m}) + \frac{1}{2}(0.500\ \text{kg})v_T^2 - \frac{1}{2}(0.500\ \text{kg})(12.0\ \text{m/s})^2 = -(7.00\ \text{N})(\pi)(1.00\ \text{m})$$

$$0.250v_T^2 = 4.21$$

$$\therefore v_T = \boxed{4.10\ \text{m/s}}$$

(c) Does block fall off at or before top of track? Block falls if $a_c < g$

$$a_c = \frac{v_T^2}{R} = \frac{(4.10)^2}{1.00} = 16.8\ \text{m/s}^2$$

Therefore $a_c > g$ and the $\boxed{\text{block stays on the track}}$.

P8.62 Let λ represent the mass of each one meter of the chain and T represent the tension in the chain at the table edge. We imagine the edge to act like a frictionless and massless pulley.

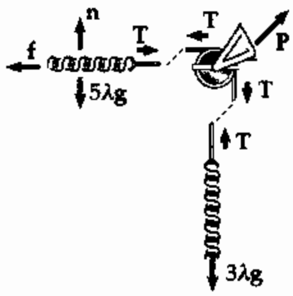


FIG. P8.62

(a) For the five meters on the table with motion impending,

$\sum F_y = 0:\quad +n - 5\lambda g = 0 \qquad n = 5\lambda g$

$f_s \le \mu_s n = 0.6(5\lambda g) = 3\lambda g$

$\sum F_x = 0:\quad +T - f_s = 0 \qquad T = f_s \qquad T \le 3\lambda g$

The maximum value is barely enough to support the hanging segment according to

$\sum F_y = 0:\quad +T - 3\lambda g = 0 \qquad T = 3\lambda g$

so it is at this point that the chain starts to slide.

continued on next page

(b) Let x represent the variable distance the chain has slipped since the start.

Then length $(5-x)$ remains on the table, with now

$$\sum F_y = 0: \quad +n-(5-x)\lambda g = 0 \qquad n=(5-x)\lambda g$$
$$f_k = \mu_k n = 0.4(5-x)\lambda g = 2\lambda g - 0.4x\lambda g$$

Consider energies of the chain-Earth system at the initial moment when the chain starts to slip, and a final moment when $x=5$, when the last link goes over the brink. Measure heights above the final position of the leading end of the chain. At the moment the final link slips off, the center of the chain is at $y_f = 4$ meters.

Originally, 5 meters of chain is at height 8 m and the middle of the dangling segment is at height $8-\frac{3}{2}=6.5$ m.

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f: \qquad 0+(m_1gy_1+m_2gy_2)_i - \int_i^f f_k dx = \left(\frac{1}{2}mv^2 + mgy\right)_f$$

$$(5\lambda g)8 + (3\lambda g)6.5 - \int_0^5 (2\lambda g - 0.4x\lambda g)dx = \frac{1}{2}(8\lambda)v^2 + (8\lambda g)4$$

$$40.0g + 19.5g - 2.00g\int_0^5 dx + 0.400g\int_0^5 x\,dx = 4.00v^2 + 32.0g$$

$$27.5g - 2.00gx\Big|_0^5 + 0.400g\frac{x^2}{2}\Big|_0^5 = 4.00v^2$$

$$27.5g - 2.00g(5.00) + 0.400g(12.5) = 4.00v^2$$

$$22.5g = 4.00v^2$$

$$v = \sqrt{\frac{(22.5\text{ m})(9.80\text{ m/s}^2)}{4.00}} = \boxed{7.42\text{ m/s}}$$

P8.63 Launch speed is found from

$$mg\left(\frac{4}{5}h\right) = \frac{1}{2}mv^2: \qquad v = \sqrt{2g\left(\frac{4}{5}\right)h}$$
$$v_y = v\sin\theta$$

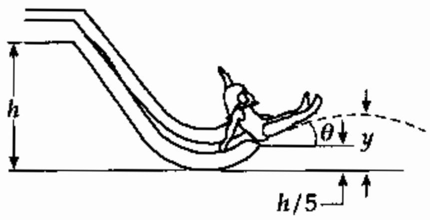


FIG. P8.63

The height y above the water (by conservation of energy for the child-Earth system) is found from

$$mgy = \frac{1}{2}mv_y^2 + mg\frac{h}{5} \qquad \left(\text{since } \frac{1}{2}mv_x^2 \text{ is constant in projectile motion}\right)$$

$$y = \frac{1}{2g}v_y^2 + \frac{h}{5} = \frac{1}{2g}v^2\sin^2\theta + \frac{h}{5}$$

$$y = \frac{1}{2g}\left[2g\left(\frac{4}{5}h\right)\right]\sin^2\theta + \frac{h}{5} = \boxed{\frac{4}{5}h\sin^2\theta + \frac{h}{5}}$$

***P8.64** (a) The length of string between glider and pulley is given by $\ell^2 = x^2 + h_0^2$. Then $2\ell\frac{d\ell}{dt} = 2x\frac{dx}{dt} + 0$.

Now $\frac{d\ell}{dt}$ is the rate at which string goes over the pulley: $\frac{d\ell}{dt} = v_y = \frac{x}{\ell}v_x = (\cos\theta)v_x$.

(b) $\left(K_A + K_B + U_g\right)_i = \left(K_A + K_B + U_g\right)_f$

$0 + 0 + m_B g(y_{30} - y_{45}) = \frac{1}{2}m_A v_x^2 + \frac{1}{2}m_B v_y^2$

Now $y_{30} - y_{45}$ is the amount of string that has gone over the pulley, $\ell_{30} - \ell_{45}$. We have $\sin 30° = \frac{h_0}{\ell_{30}}$ and $\sin 45° = \frac{h_0}{\ell_{45}}$, so $\ell_{30} - \ell_{45} = \frac{h_0}{\sin 30°} - \frac{h_0}{\sin 45°} = 0.40\text{ m}\left(2 - \sqrt{2}\right) = 0.234\text{ m}$.

From the energy equation

$$0.5\text{ kg }9.8\text{ m/s}^2\ 0.234\text{ m} = \frac{1}{2}1.00\text{ kg }v_x^2 + \frac{1}{2}0.500\text{ kg }v_x^2\cos^2 45°$$

$$v_x = \sqrt{\frac{1.15\text{ J}}{0.625\text{ kg}}} = \boxed{1.35\text{ m/s}}$$

(c) $v_y = v_x\cos\theta = (1.35\text{ m/s})\cos 45° = \boxed{0.958\text{ m/s}}$

(d) The acceleration of neither glider is constant, so knowing distance and acceleration at one point is not sufficient to find speed at another point.

P8.65 The geometry reveals $D = L\sin\theta + L\sin\phi$, $50.0\text{ m} = 40.0\text{ m}(\sin 50° + \sin\phi)$, $\phi = 28.9°$

(a) From takeoff to alighting for the Jane-Earth system

$\left(K + U_g\right)_i + W_{\text{wind}} = \left(K + U_g\right)_f$

$\frac{1}{2}mv_i^2 + mg(-L\cos\theta) + FD(-1) = 0 + mg(-L\cos\phi)$

$\frac{1}{2}50\text{ kg }v_i^2 + 50\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 50°) - 110\text{ N}(50\text{ m}) = 50\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 28.9°)$

$\frac{1}{2}50\text{ kg }v_i^2 - 1.26\times10^4\text{ J} - 5.5\times10^3\text{ J} = -1.72\times10^4\text{ J}$

$$v_i = \sqrt{\frac{2(947\text{ J})}{50\text{ kg}}} = \boxed{6.15\text{ m/s}}$$

(b) For the swing back

$\frac{1}{2}mv_i^2 + mg(-L\cos\phi) + FD(+1) = 0 + mg(-L\cos\theta)$

$\frac{1}{2}130\text{ kg }v_i^2 + 130\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 28.9°) + 110\text{ N}(50\text{ m})$

$= 130\text{ kg}\left(9.8\text{ m/s}^2\right)(-40\text{ m}\cos 50°)$

$\frac{1}{2}130\text{ kg }v_i^2 - 4.46\times10^4\text{ J} + 5\,500\text{ J} = -3.28\times10^4\text{ J}$

$$v_i = \sqrt{\frac{2(6\,340\text{ J})}{130\text{ kg}}} = \boxed{9.87\text{ m/s}}$$

P8.66 Case I: Surface is frictionless

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$k = \frac{mv^2}{x^2} = \frac{(5.00\text{ kg})(1.20\text{ m/s})^2}{10^{-2}\text{ m}^2} = 7.20\times10^2\text{ N/m}$$

Case II: Surface is rough,

$$\mu_k = 0.300$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 - \mu_k mgx$$

$$\frac{5.00\text{ kg}}{2}v^2 = \frac{1}{2}\left(7.20\times10^2\text{ N/m}\right)\left(10^{-1}\text{ m}\right)^2 - (0.300)(5.00\text{ kg})\left(9.80\text{ m/s}^2\right)\left(10^{-1}\text{ m}\right)$$

$$\boxed{v = 0.923\text{ m/s}}$$

***P8.67** (a) $(K+U_g)_A = (K+U_g)_B$

$$0 + mgy_A = \frac{1}{2}mv_B^2 + 0 \qquad v_B = \sqrt{2gy_A} = \sqrt{2\left(9.8\text{ m/s}^2\right)6.3\text{ m}} = \boxed{11.1\text{ m/s}}$$

(b) $$a_c = \frac{v^2}{r} = \frac{(11.1\text{ m/s})^2}{6.3\text{ m}} = \boxed{19.6\text{ m/s}^2\text{ up}}$$

(c) $\sum F_y = ma_y \qquad +n_B - mg = ma_c$

$$n_B = 76\text{ kg}\left(9.8\text{ m/s}^2 + 19.6\text{ m/s}^2\right) = \boxed{2.23\times10^3\text{ N up}}$$

(d) $$W = F\Delta r\cos\theta = 2.23\times10^3\text{ N}(0.450\text{ m})\cos0° = \boxed{1.01\times10^3\text{ J}}$$

(e) $(K+U_g)_B + W = (K+U_g)_D$

$$\frac{1}{2}mv_B^2 + 0 + 1.01\times10^3\text{ J} = \frac{1}{2}mv_D^2 + mg(y_D - y_B)$$

$$\frac{1}{2}76\text{ kg}(11.1\text{ m/s})^2 + 1.01\times10^3\text{ J} = \frac{1}{2}76\text{ kg }v_D^2 + 76\text{ kg}\left(9.8\text{ m/s}^2\right)6.3\text{ m}$$

$$\sqrt{\frac{\left(5.70\times10^3\text{ J} - 4.69\times10^3\text{ J}\right)2}{76\text{ kg}}} = v_D = \boxed{5.14\text{ m/s}}$$

(f) $(K+U_g)_D = (K+U_g)_E$ where E is the apex of his motion

$$\frac{1}{2}mv_D^2 + 0 = 0 + mg(y_E - y_D) \qquad y_E - y_D = \frac{v_D^2}{2g} = \frac{(5.14\text{ m/s})^2}{2\left(9.8\text{ m/s}^2\right)} = \boxed{1.35\text{ m}}$$

(g) Consider the motion with constant acceleration between takeoff and touchdown. The time is the positive root of

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$-2.34\text{ m} = 0 + 5.14\text{ m/s }t + \frac{1}{2}\left(-9.8\text{ m/s}^2\right)t^2$$

$$4.9t^2 - 5.14t - 2.34 = 0$$

$$t = \frac{5.14 \pm \sqrt{5.14^2 - 4(4.9)(-2.34)}}{9.8} = \boxed{1.39\text{ s}}$$

***P8.68** If the spring is just barely able to lift the lower block from the table, the spring lifts it through no noticeable distance, but exerts on the block a force equal to its weight Mg. The extension of the spring, from $|\mathbf{F}_s| = kx$, must be Mg/k. Between an initial point at release and a final point when the moving block first comes to rest, we have

$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$:

$$0 + mg\left(-\frac{4mg}{k}\right) + \frac{1}{2}k\left(\frac{4mg}{k}\right)^2 = 0 + mg\left(\frac{Mg}{k}\right) + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$-\frac{4m^2g^2}{k} + \frac{8m^2g^2}{k} = \frac{mMg^2}{k} + \frac{M^2g^2}{2k}$$

$$4m^2 = mM + \frac{M^2}{2}$$

$$\frac{M^2}{2} + mM - 4m^2 = 0$$

$$M = \frac{-m \pm \sqrt{m^2 - 4\left(\frac{1}{2}\right)\left(-4m^2\right)}}{2\left(\frac{1}{2}\right)} = -m \pm \sqrt{9m^2}$$

Only a positive mass is physical, so we take $M = m(3-1) = \boxed{2m}$.

P8.69 (a) Take the original point where the ball is released and the final point where its upward swing stops at height H and horizontal displacement

$$x = \sqrt{L^2 - (L-H)^2} = \sqrt{2LH - H^2}$$

Since the wind force is purely horizontal, it does work

$$W_{\text{wind}} = \int \mathbf{F} \cdot d\mathbf{s} = F\int dx = F\sqrt{2LH - H^2}$$

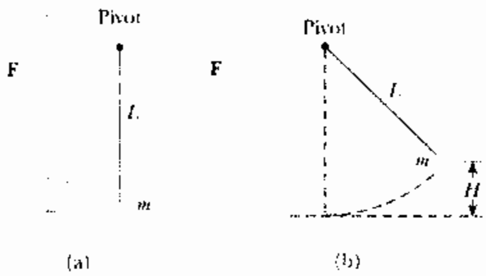


FIG. P8.69

The work-energy theorem can be written:

$K_i + U_{gi} + W_{\text{wind}} = K_f + U_{gf}$, or

$0 + 0 + F\sqrt{2LH - H^2} = 0 + mgH$ giving $F^2 2LH - F^2H^2 = m^2g^2H^2$

Here $H = 0$ represents the lower turning point of the ball's oscillation, and the upper limit is at $F^2(2L) = \left(F^2 + m^2g^2\right)H$. Solving for H yields

$$H = \frac{2LF^2}{F^2 + m^2g^2} = \boxed{\frac{2L}{1 + (mg/F)^2}}$$

As $F \to 0$, $H \to 0$ as is reasonable.
As $F \to \infty$, $H \to 2L$, which would be hard to approach experimentally.

(b) $$H = \frac{2(2.00\text{ m})}{1 + \left[(2.00\text{ kg})\left(9.80\text{ m/s}^2\right)/14.7\text{ N}\right]^2} = \boxed{1.44\text{ m}}$$

continued on next page

(c) Call θ the equilibrium angle with the vertical.

$$\sum F_x = 0 \Rightarrow T\sin\theta = F, \text{ and}$$
$$\sum F_y = 0 \Rightarrow T\cos\theta = mg$$

Dividing: $\tan\theta = \frac{F}{mg} = \frac{14.7\text{ N}}{19.6\text{ N}} = 0.750$, or $\theta = 36.9°$

Therefore, $H_{eq} = L(1-\cos\theta) = (2.00\text{ m})(1-\cos 36.9°) = \boxed{0.400\text{ m}}$

(d) As $F \to \infty$, $\tan\theta \to \infty$, $\theta \to 90.0°$ and $H_{eq} \to L$

A very strong wind pulls the string out horizontal, parallel to the ground. Thus, $\boxed{\left(H_{eq}\right)_{max} = L}$.

P8.70 Call $\phi = 180° - \theta$ the angle between the upward vertical and the radius to the release point. Call v_r the speed here. By conservation of energy

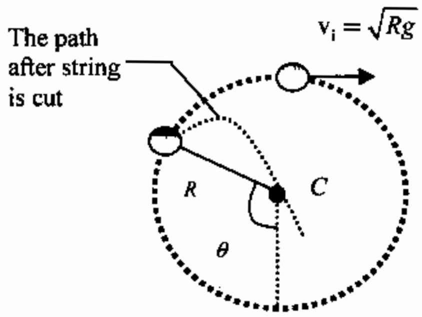


FIG. P8.70

$$K_i + U_i + \Delta E = K_r + U_r$$
$$\frac{1}{2}mv_i^2 + mgR + 0 = \frac{1}{2}mv_r^2 + mgR\cos\phi$$
$$gR + 2gR = v_r^2 + 2gR\cos\phi$$
$$v_r = \sqrt{3gR - 2gR\cos\phi}$$

The components of velocity at release are $v_x = v_r\cos\phi$ and $v_y = v_r\sin\phi$ so for the projectile motion we have

$$x = v_x t \qquad R\sin\phi = v_r\cos\phi t$$
$$y = v_y t - \frac{1}{2}gt^2 \qquad -R\cos\phi = v_r\sin\phi t - \frac{1}{2}gt^2$$

By substitution

$$-R\cos\phi = v_r\sin\phi\frac{R\sin\phi}{v_r\cos\phi} - \frac{g}{2}\frac{R^2\sin^2\phi}{v_r^2\cos^2\phi}$$

with $\sin^2\phi + \cos^2\phi = 1$,

$$gR\sin^2\phi = 2v_r^2\cos\phi = 2\cos\phi(3gR - 2gR\cos\phi)$$
$$\sin^2\phi = 6\cos\phi - 4\cos^2\phi = 1 - \cos^2\phi$$
$$3\cos^2\phi - 6\cos\phi + 1 = 0$$
$$\cos\phi = \frac{6 \pm \sqrt{36-12}}{6}$$

Only the – sign gives a value for $\cos\phi$ that is less than one:

$$\cos\phi = 0.1835 \qquad \phi = 79.43° \qquad \text{so } \theta = \boxed{100.6°}$$

P8.71 Applying Newton's second law at the bottom (b) and top (t) of the circle gives

$$T_b - mg = \frac{mv_b^2}{R} \text{ and } -T_t - mg = -\frac{mv_t^2}{R}$$

Adding these gives $$T_b = T_t + 2mg + \frac{m\left(v_b^2 - v_t^2\right)}{R}$$

Also, energy must be conserved and $\Delta U + \Delta K = 0$

So, $\frac{m\left(v_b^2 - v_t^2\right)}{2} + (0 - 2mgR) = 0$ and $\frac{m\left(v_b^2 - v_t^2\right)}{R} = 4mg$

Substituting into the above equation gives $\boxed{T_b = T_t + 6mg}$.

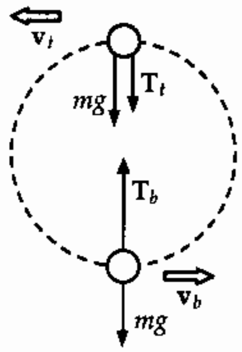


FIG. P8.71

P8.72 (a) Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leaves the peg, and the ball swings equally high on both sides.

(b) Relative to the point of suspension,

$$U_i = 0,\ U_f = -mg[d - (L - d)]$$

From this we find that

$$-mg(2d - L) + \frac{1}{2}mv^2 = 0$$

Also for centripetal motion,

$$mg = \frac{mv^2}{R} \text{ where } R = L - d.$$

Upon solving, we get $\boxed{d = \frac{3L}{5}}$.

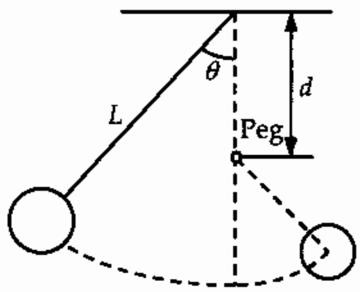


FIG. P8.72

***P8.73** (a) At the top of the loop the car and riders are in free fall:

$\sum F_y = ma_y$: $mg \text{ down} = \dfrac{mv^2}{R} \text{ down}$

$v = \sqrt{Rg}$

Energy of the car-riders-Earth system is conserved between release and top of loop:

$K_i + U_{gi} = K_f + U_{gf}$: $0 + mgh = \dfrac{1}{2}mv^2 + mg(2R)$

$gh = \dfrac{1}{2}Rg + g(2R)$

$\boxed{h = 2.50R}$

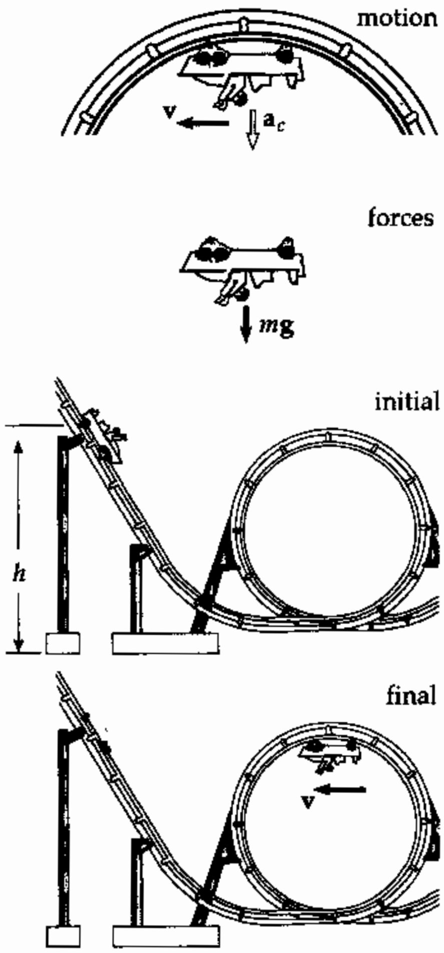


FIG. P8.73

(b) Let h now represent the height $\geq 2.5R$ of the release point. At the bottom of the loop we have

$mgh = \dfrac{1}{2}mv_b^2$ or $v_b^2 = 2gh$

$\sum F_y = ma_y$: $n_b - mg = \dfrac{mv_b^2}{R}(\text{up})$

$n_b = mg + \dfrac{m(2gh)}{R}$

At the top of the loop, $mgh = \dfrac{1}{2}mv_t^2 + mg(2R)$

$v_t^2 = 2gh - 4gR$

$\sum F_y = ma_y$: $-n_t - mg = -\dfrac{mv_t^2}{R}$

$n_t = -mg + \dfrac{m}{R}(2gh - 4gR)$

$n_t = \dfrac{m(2gh)}{R} - 5mg$

Then the normal force at the bottom is larger by

$$n_b - n_t = mg + \frac{m(2gh)}{R} - \frac{m(2gh)}{R} + 5mg = \boxed{6mg}.$$

***P8.74** (a) Conservation of energy for the sled-rider-Earth system, between A and C:

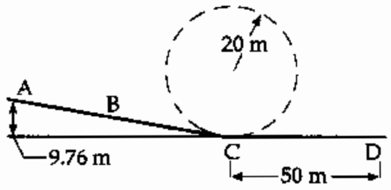


FIG. P8.74(a)

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}m(2.5\ \text{m/s})^2 + m(9.80\ \text{m/s}^2)(9.76\ \text{m}) = \frac{1}{2}mv_C^2 + 0$$

$$v_C = \sqrt{(2.5\ \text{m/s})^2 + 2(9.80\ \text{m/s}^2)(9.76\ \text{m})} = \boxed{14.1\ \text{m/s}}$$

(b) Incorporating the loss of mechanical energy during the portion of the motion in the water, we have, for the entire motion between A and D (the rider's stopping point),

$$K_i + U_{gi} - f_k\Delta x = K_f + U_{gf}: \quad \frac{1}{2}(80\ \text{kg})(2.5\ \text{m/s})^2 + (80\ \text{kg})(9.80\ \text{m/s}^2)(9.76\ \text{m}) - f_k\Delta x = 0 + 0$$

$$-f_k\Delta x = \boxed{-7.90\times10^3\ \text{J}}$$

(c) The water exerts a frictional force $f_k = \dfrac{7.90\times10^3\ \text{J}}{\Delta x} = \dfrac{7.90\times10^3\ \text{N}\cdot\text{m}}{50\ \text{m}} = 158\ \text{N}$

and also a normal force of $n = mg = (80\ \text{kg})(9.80\ \text{m/s}^2) = 784\ \text{N}$

The magnitude of the water force is $\sqrt{(158\ \text{N})^2 + (784\ \text{N})^2} = \boxed{800\ \text{N}}$

(d) The angle of the slide is

$$\theta = \sin^{-1}\frac{9.76\ \text{m}}{54.3\ \text{m}} = 10.4°$$

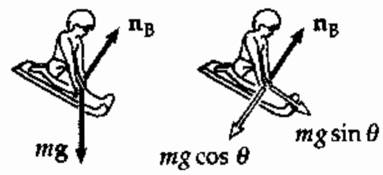


FIG. P8.74(d)

For forces perpendicular to the track at B,

$$\sum F_y = ma_y: \quad n_B - mg\cos\theta = 0$$

$$n_B = (80.0\ \text{kg})(9.80\ \text{m/s}^2)\cos 10.4° = \boxed{771\ \text{N}}$$

(e)

$$\sum F_y = ma_y: \quad +n_C - mg = \frac{mv_C^2}{r}$$

$$n_C = (80.0\ \text{kg})(9.80\ \text{m/s}^2) + \frac{(80.0\ \text{kg})(14.1\ \text{m/s})^2}{20\ \text{m}}$$

$$n_C = \boxed{1.57\times10^3\ \text{N up}}$$

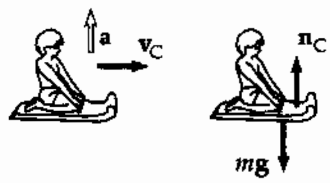


FIG. P8.74(e)

The rider pays for the thrills of a giddy height at A, and a high speed and tremendous splash at C. As a bonus, he gets the quick change in direction and magnitude among the forces we found in parts (d), (e), and (c).

ANSWERS TO EVEN PROBLEMS

P8.2 (a) 800 J; (b) 107 J; (c) 0

P8.4 (a) 1.11×10^9 J; (b) 0.2

P8.6 1.84 m

P8.8 (a) 10.2 kW; (b) 10.6 kW; (c) 5.82×10^6 J

P8.10 $d = \dfrac{kx^2}{2mg\sin\theta} - x$

P8.12 (a) see the solution; (b) 60.0°

P8.14 (a) $\sqrt{\dfrac{2(m_1 - m_2)gh}{(m_1 + m_2)}}$; (b) $\dfrac{2m_1 h}{m_1 + m_2}$

P8.16 160 L/min

P8.18 40.8°

P8.20 $\left(\dfrac{8gh}{15}\right)^{1/2}$

P8.22 (a) see the solution; (b) 35.0 J

P8.24 (a) $v_B = 5.94$ m/s; $v_C = 7.67$ m/s; (b) 147 J

P8.26 (a) $U_f = 22.0$ J; $E = 40.0$ J; (b) Yes. The total mechanical energy changes.

P8.28 194 m

P8.30 2.06 kN up

P8.32 168 J

P8.34 (a) 24.5 m/s; (b) yes; (c) 206 m; (d) Air drag depends strongly on speed.

P8.36 3.92 kJ

P8.38 44.1 kW

P8.40 (a) $\dfrac{Ax^2}{2} - \dfrac{Bx^3}{3}$;
(b) $\Delta U = \dfrac{5A}{2} - \dfrac{19B}{3}$; $\Delta K = \dfrac{19B}{3} - \dfrac{5A}{2}$

P8.42 $\left(7 - 9x^2 y\right)\hat{\mathrm{i}} - 3x^3\hat{\mathrm{j}}$

P8.44 see the solution

P8.46 (a) $r = 1.5$ mm and 3.2 mm, stable; 2.3 mm and unstable; $r \to \infty$ neutral;
(b) $-5.6\text{ J} \le E < 1\text{ J}$; (c) $0.6\text{ mm} \le r \le 3.6\text{ mm}$;
(d) 2.6 J; (e) 1.5 mm; (f) 4 J

P8.48 see the solution

P8.50 33.4 kW

P8.52 (a) 0.588 J; (b) 0.588 J; (c) 2.42 m/s;
(d) 0.196 J; 0.392 J

P8.54 0.115

P8.56 (a) 100 J; (b) 0.410 m; (c) 2.84 m/s;
(d) −9.80 mm; (e) 2.85 m/s

P8.58 (a) $\left(3x^2 - 4x - 3\right)\hat{\mathrm{i}}$; (b) 1.87; −0.535;
(c) see the solution

P8.60 (a) 0.378 m; (b) 2.30 m/s; (c) 1.08 m

P8.62 (a) see the solution; (b) 7.42 m/s

P8.64 (a) see the solution; (b) 1.35 m/s;
(c) 0.958 m/s; (d) see the solution

P8.66 0.923 m/s

P8.68 $2m$

P8.70 100.6°

P8.72 see the solution

P8.74 (a) 14.1 m/s; (b) −7.90 J; (c) 800 N;
(d) 771 N; (e) 1.57 kN up

9

Linear Momentum and Collisions

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q9.1 No. Impulse, $F\Delta t$, depends on the force and the time for which it is applied.

Q9.2 The momentum doubles since it is proportional to the speed. The kinetic energy quadruples, since it is proportional to the speed-squared.

Q9.3 The momenta of two particles will only be the same if the masses of the particles of the same.

Q9.4 (a) It does not carry force, for if it did, it could accelerate itself.

(b) It cannot deliver more kinetic energy than it possesses. This would violate the law of energy conservation.

(c) It can deliver more momentum in a collision than it possesses in its flight, by bouncing from the object it strikes.

Q9.5 Provided there is some form of potential energy in the system, the parts of an isolated system can move if the system is initially at rest. Consider two air-track gliders on a horizontal track. If you compress a spring between them and then tie them together with a string, it is possible for the system to start out at rest. If you then burn the string, the potential energy stored in the spring will be converted into kinetic energy of the gliders.

Q9.6 No. Only in a precise head-on collision with momenta with equal magnitudes and opposite directions can both objects wind up at rest. Yes. Assume that ball 2, originally at rest, is struck squarely by an equal-mass ball 1. Then ball 2 will take off with the velocity of ball 1, leaving ball 1 at rest.

Q9.7 Interestingly, mutual gravitation brings the ball and the Earth together. As the ball moves downward, the Earth moves upward, although with an acceleration 10^{25} times smaller than that of the ball. The two objects meet, rebound, and separate. Momentum of the ball-Earth system is conserved.

Q9.8 (a) Linear momentum is conserved since there are no external forces acting on the system.

(b) Kinetic energy is not conserved because the chemical potential energy initially in the explosive is converted into kinetic energy of the pieces of the bomb.

Q9.9 Momentum conservation is not violated if we make our system include the Earth along with the clay. When the clay receives an impulse backwards, the Earth receives the same size impulse forwards. The resulting acceleration of the Earth due to this impulse is significantly smaller than the acceleration of the clay, but the planet absorbs all of the momentum that the clay loses.

Q9.10 Momentum conservation is not violated if we choose as our system the planet along with you. When you receive an impulse forward, the Earth receives the same size impulse backwards. The resulting acceleration of the Earth due to this impulse is significantly smaller than your acceleration forward, but the planet's backward momentum is equal in magnitude to your forward momentum.

Q9.11 As a ball rolls down an incline, the Earth receives an impulse of the same size and in the opposite direction as that of the ball. If you consider the Earth-ball system, momentum conservation is not violated.

Q9.12 Suppose car and truck move along the same line. If one vehicle overtakes the other, the faster-moving one loses more energy than the slower one gains. In a head-on collision, if the speed of the truck is less than $\frac{m_T + 3m_c}{3m_T + m_c}$ times the speed of the car, the car will lose more energy.

Q9.13 The rifle has a much lower speed than the bullet and much less kinetic energy. The butt distributes the recoil force over an area much larger than that of the bullet.

Q9.14 His impact speed is determined by the acceleration of gravity and the distance of fall, in $v_f^2 = v_i^2 - 2g(0 - y_i)$. The force exerted by the pad depends also on the unknown stiffness of the pad.

Q9.15 The product of the mass flow rate and velocity of the water determines the force the firefighters must exert.

Q9.16 The sheet stretches and pulls the two students toward each other. These effects are larger for a faster-moving egg. The time over which the egg stops is extended so that the force stopping it is never too large.

Q9.17 (c) In this case, the impulse on the Frisbee is largest. According to Newton's third law, the impulse on the skater and thus the final speed of the skater will also be largest.

Q9.18 Usually but not necessarily. In a one-dimensional collision between two identical particles with the same initial speed, the kinetic energy of the particles will not change.

Q9.19 g downward.

Q9.20 As one finger slides towards the center, the normal force exerted by the sliding finger on the ruler increases. At some point, this normal force will increase enough so that static friction between the sliding finger and the ruler will stop their relative motion. At this moment the other finger starts sliding along the ruler towards the center. This process repeats until the fingers meet at the center of the ruler.

Q9.21 The planet is in motion around the sun, and thus has momentum and kinetic energy of its own. The spacecraft is directed to cross the planet's orbit behind it, so that the planet's gravity has a component pulling forward on the spacecraft. Since this is an elastic collision, and the velocity of the planet remains nearly unchanged, the probe must both increase speed and change direction for both momentum and kinetic energy to be conserved.

Q9.22 No—an external force of gravity acts on the moon. Yes, because its speed is constant.

Q9.23 The impulse given to the egg is the same regardless of how it stops. If you increase the impact time by dropping the egg onto foam, you will decrease the impact force.

Q9.24 Yes. A boomerang, a kitchen stool.

Q9.25 The center of mass of the balls is in free fall, moving up and then down with the acceleration due to gravity, during the 40% of the time when the juggler's hands are empty. During the 60% of the time when the juggler is engaged in catching and tossing, the center of mass must accelerate up with a somewhat smaller average acceleration. The center of mass moves around in a little circle, making three revolutions for every one revolution that one ball makes. Letting T represent the time for one cycle and F_g the weight of one ball, we have $F_J 0.60T = 3F_g T$ and $F_J = 5F_g$. The average force exerted by the juggler is five times the weight of one ball.

Q9.26 In empty space, the center of mass of a rocket-plus-fuel system does not accelerate during a burn, because no outside force acts on this system. According to the text's 'basic expression for rocket propulsion,' the change in speed of the rocket body will be larger than the speed of the exhaust relative to the rocket, if the final mass is less than 37% of the original mass.

Q9.27 The gun recoiled.

Q9.28 Inflate a balloon and release it. The air escaping from the balloon gives the balloon an impulse.

Q9.29 There was a time when the English favored position (a), the Germans position (b), and the French position (c). A Frenchman, Jean D'Alembert, is most responsible for showing that each theory is consistent with the others. All are equally correct. Each is useful for giving a mathematically simple solution for some problems.

Section 9.1 Linear Momentum and Its Conservation

P9.1 $m = 3.00\text{ kg}$, $\mathbf{v} = \left(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right)\text{ m/s}$

(a) $\mathbf{p} = m\mathbf{v} = \left(9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}}\right)\text{ kg}\cdot\text{m/s}$

Thus, $\boxed{p_x = 9.00\text{ kg}\cdot\text{m/s}}$

and $\boxed{p_y = -12.0\text{ kg}\cdot\text{m/s}}$

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00)^2 + (12.0)^2} = \boxed{15.0\text{ kg}\cdot\text{m/s}}$

$\theta = \tan^{-1}\left(\dfrac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$

P9.2 (a) At maximum height $\mathbf{v} = 0$, so $\mathbf{p} = \boxed{0}$.

(b) Its original kinetic energy is its constant total energy,

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(0.100)\text{kg}(15.0\text{ m/s})^2 = 11.2\text{ J}.$$

At the top all of this energy is gravitational. Halfway up, one-half of it is gravitational and the other half is kinetic:

$$K = 5.62\text{ J} = \frac{1}{2}(0.100\text{ kg})v^2$$

$$v = \sqrt{\frac{2 \times 5.62\text{ J}}{0.100\text{ kg}}} = 10.6\text{ m/s}$$

Then $\mathbf{p} = m\mathbf{v} = (0.100\text{ kg})(10.6\text{ m/s})\hat{\mathbf{j}}$

$$\mathbf{p} = \boxed{1.06\text{ kg}\cdot\text{m/s}\,\hat{\mathbf{j}}}.$$

P9.3 I have mass 85.0 kg and can jump to raise my center of gravity 25.0 cm. I leave the ground with speed given by

$$v_f^2 - v_i^2 = 2a(x_f - x_i): \qquad 0 - v_i^2 = 2(-9.80\text{ m/s}^2)(0.250\text{ m})$$

$$v_i = 2.20\text{ m/s}$$

Total momentum of the system of the Earth and me is conserved as I push the earth down and myself up:

$$0 = (5.98 \times 10^{24}\text{ kg})v_e + (85.0\text{ kg})(2.20\text{ m/s})$$

$$v_e \sim \boxed{10^{-23}\text{ m/s}}$$

P9.4 (a) For the system of two blocks $\Delta p = 0$,

or $\qquad p_i = p_f$

Therefore, $\qquad 0 = Mv_m + (3M)(2.00\text{ m/s})$

Solving gives $\qquad v_m = \boxed{-6.00\text{ m/s}}$ (motion toward the left).

(b) $\frac{1}{2}kx^2 = \frac{1}{2}Mv_M^2 + \frac{1}{2}(3M)v_{3M}^2 = \boxed{8.40\text{ J}}$

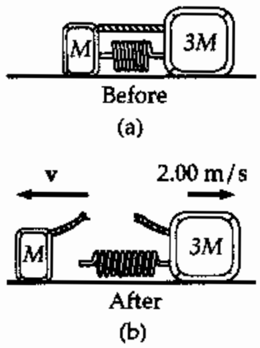


FIG. P9.4

P9.5 (a) The momentum is $p = mv$, so $v = \frac{p}{m}$ and the kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$.

(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{\frac{2K}{m}}$, so $p = mv = m\sqrt{\frac{2K}{m}} = \boxed{\sqrt{2mK}}$.

Section 9.2 Impulse and Momentum

***P9.6** From the impulse-momentum theorem, $\overline{F}(\Delta t) = \Delta p = mv_f - mv_i$, the average force required to hold onto the child is

$$\overline{F} = \frac{m(v_f - v_i)}{(\Delta t)} = \frac{(12\text{ kg})(0 - 60\text{ mi/h})}{0.050\text{ s} - 0}\left(\frac{1\text{ m/s}}{2.237\text{ mi/h}}\right) = -6.44 \times 10^3\text{ N}.$$

Therefore, the magnitude of the needed retarding force is $\boxed{6.44 \times 10^3\text{ N}}$, or 1 400 lb. A person cannot exert a force of this magnitude and a safety device should be used.

P9.7 (a) $I = \int F dt =$ area under curve

$$I = \frac{1}{2}(1.50 \times 10^{-3}\text{ s})(18\,000\text{ N}) = \boxed{13.5\text{ N}\cdot\text{s}}$$

(b) $F = \frac{13.5\text{ N}\cdot\text{s}}{1.50 \times 10^{-3}\text{ s}} = \boxed{9.00\text{ kN}}$

(c) From the graph, we see that $F_{max} = \boxed{18.0\text{ kN}}$

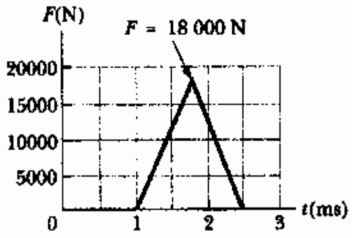


FIG. P9.7

***P9.8** The impact speed is given by $\frac{1}{2}mv_1^2 = mgy_1$. The rebound speed is given by $mgy_2 = \frac{1}{2}mv_2^2$. The impulse of the floor is the change in momentum,

$$\begin{aligned} mv_2\text{ up} - mv_1\text{ down} &= m(v_2 + v_1)\text{ up} \\ &= m\left(\sqrt{2gh_2} + \sqrt{2gh_1}\right)\text{ up} \\ &= 0.15\text{ kg}\sqrt{2(9.8\text{ m/s}^2)}\left(\sqrt{0.960\text{ m}} + \sqrt{1.25\text{ m}}\right)\text{ up} \\ &= \boxed{1.39\text{ kg}\cdot\text{m/s upward}} \end{aligned}$$

P9.9 $\Delta \mathbf{p} = \mathbf{F}\Delta t$

$$\Delta p_y = m\left(v_{fy} - v_{iy}\right) = m(v\cos 60.0°) - mv\cos 60.0° = 0$$

$$\Delta p_x = m(-v\sin 60.0° - v\sin 60.0°) = -2mv\sin 60.0°$$

$$= -2(3.00\text{ kg})(10.0\text{ m/s})(0.866)$$

$$= -52.0\text{ kg}\cdot\text{m/s}$$

$$F_{\text{ave}} = \frac{\Delta p_x}{\Delta t} = \frac{-52.0\text{ kg}\cdot\text{m/s}}{0.200\text{ s}} = \boxed{-260\text{ N}}$$

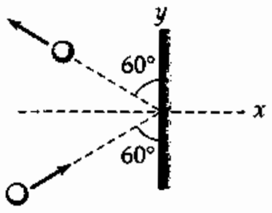


FIG. P9.9

P9.10 Assume the initial direction of the ball in the $-x$ direction.

(a) Impulse, $\mathbf{I} = \Delta\mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = (0.060\,0)(40.0)\hat{\mathbf{i}} - (0.060\,0)(50.0)\left(-\hat{\mathbf{i}}\right) = \boxed{5.40\hat{\mathbf{i}}\text{ N}\cdot\text{s}}$

(b) Work $= K_f - K_i = \frac{1}{2}(0.060\,0)\left[(40.0)^2 - (50.0)^2\right] = \boxed{-27.0\text{ J}}$

P9.11 Take x-axis toward the pitcher

(a) $p_{ix} + I_x = p_{fx}$: $(0.200\text{ kg})(15.0\text{ m/s})(-\cos 45.0°) + I_x = (0.200\text{ kg})(40.0\text{ m/s})\cos 30.0°$

$I_x = 9.05\text{ N}\cdot\text{s}$

$p_{iy} + I_y = p_{fy}$: $(0.200\text{ kg})(15.0\text{ m/s})(-\sin 45.0°) + I_y = (0.200\text{ kg})(40.0\text{ m/s})\sin 30.0°$

$$\mathbf{I} = \boxed{\left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right)\text{ N}\cdot\text{s}}$$

(b) $\mathbf{I} = \frac{1}{2}(0 + \mathbf{F}_m)(4.00\text{ ms}) + \mathbf{F}_m(20.0\text{ ms}) + \frac{1}{2}\mathbf{F}_m(4.00\text{ ms})$

$$\mathbf{F}_m \times 24.0\times 10^{-3}\text{ s} = \left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right)\text{ N}\cdot\text{s}$$

$$\mathbf{F}_m = \boxed{\left(377\hat{\mathbf{i}} + 255\hat{\mathbf{j}}\right)\text{ N}}$$

P9.12 If the diver starts from rest and drops vertically into the water, the velocity just before impact is found from

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv_{\text{impact}}^2 + 0 = 0 + mgh \Rightarrow v_{\text{impact}} = \sqrt{2gh}$$

With the diver at rest after an impact time of Δt, the average force during impact is given by

$$\overline{\mathbf{F}} = \frac{m\left(0 - v_{\text{impact}}\right)}{\Delta t} = \frac{-m\sqrt{2gh}}{\Delta t} \text{ or } \overline{F} = \frac{m\sqrt{2gh}}{\Delta t} \text{ (directed upward).}$$

Assuming a mass of 55 kg and an impact time of ≈ 1.0 s, the magnitude of this average force is

$$\left|\overline{F}\right| = \frac{(55\text{ kg})\sqrt{2\left(9.8\text{ m/s}^2\right)(10\text{ m})}}{1.0\text{ s}} = 770\text{ N, or } \boxed{\sim 10^3\text{ N}}.$$

P9.13 The force exerted on the water by the hose is

$$F = \frac{\Delta p_{\text{water}}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.600\text{ kg})(25.0\text{ m/s}) - 0}{1.00\text{ s}} = \boxed{15.0\text{ N}}.$$

According to Newton's third law, the water exerts a force of equal magnitude back on the hose. Thus, the gardener must apply a 15.0 N force (in the direction of the velocity of the exiting water stream) to hold the hose stationary.

***P9.14** (a) Energy is conserved for the spring-mass system:

$$K_i + U_{si} = K_f + U_{sf}: \quad 0 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2 + 0$$

$$v = x\sqrt{\frac{k}{m}}$$

(b) From the equation, a $\boxed{\text{smaller}}$ value of m makes $v = x\sqrt{\frac{k}{m}}$ larger.

(c) $I = \left|\mathbf{p}_f - \mathbf{p}_i\right| = mv_f = 0 = mx\sqrt{\frac{k}{m}} = x\sqrt{km}$

(d) From the equation, a $\boxed{\text{larger}}$ value of m makes $I = x\sqrt{km}$ larger.

(e) For the glider, $W = K_f - K_i = \frac{1}{2}mv^2 - 0 = \frac{1}{2}kx^2$

The mass makes $\boxed{\text{no difference}}$ to the work.

Section 9.3 **Collisions in One Dimension**

P9.15 $(200\text{ g})(55.0\text{ m/s}) = (46.0\text{ g})v + (200\text{ g})(40.0\text{ m/s})$

$v = \boxed{65.2\text{ m/s}}$

***P9.16** $(m_1v_1 + m_2v_2)_i = (m_1v_1 + m_2v_2)_f$

$22.5\text{ g}(35\text{ m/s}) + 300\text{ g}(-2.5\text{ m/s}) = 22.5\text{ g}v_{1f} + 0$

$$v_{1f} = \frac{37.5\text{ g}\cdot\text{m/s}}{22.5\text{ g}} = \boxed{1.67\text{ m/s}}$$

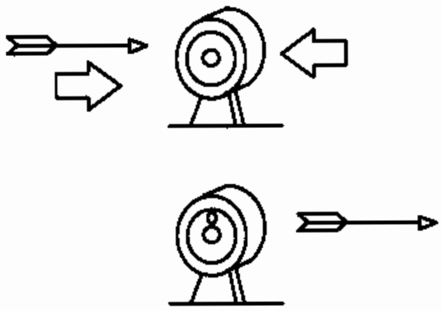


FIG. P9.16

P9.17 Momentum is conserved

$$(10.0\times 10^{-3}\text{ kg})v=(5.01\text{ kg})(0.600\text{ m/s})$$

$$v=\boxed{301\text{ m/s}}$$

P9.18 (a) $mv_{1i}+3mv_{2i}=4mv_f$ where $m=2.50\times 10^4$ kg

$$v_f=\frac{4.00+3(2.00)}{4}=\boxed{2.50\text{ m/s}}$$

(b) $$K_f-K_i=\frac{1}{2}(4m)v_f^2-\left[\frac{1}{2}mv_{1i}^2+\frac{1}{2}(3m)v_{2i}^2\right]=(2.50\times 10^4)(12.5-8.00-6.00)=\boxed{-3.75\times 10^4\text{ J}}$$

P9.19 (a) The internal forces exerted by the actor do not change the total momentum of the system of the four cars and the movie actor

$$(4m)v_i=(3m)(2.00\text{ m/s})+m(4.00\text{ m/s})$$

$$v_i=\frac{6.00\text{ m/s}+4.00\text{ m/s}}{4}=\boxed{2.50\text{ m/s}}$$

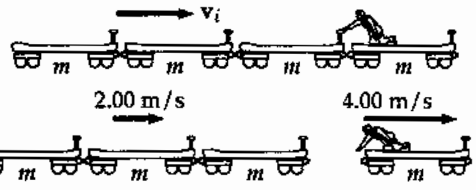


FIG. P9.19

(b) $$W_{\text{actor}}=K_f-K_i=\frac{1}{2}\left[(3m)(2.00\text{ m/s})^2+m(4.00\text{ m/s})^2\right]-\frac{1}{2}(4\text{ m})(2.50\text{ m/s})^2$$

$$W_{\text{actor}}=\frac{(2.50\times 10^4\text{ kg})}{2}(12.0+16.0-25.0)(\text{m/s})^2=\boxed{37.5\text{ kJ}}$$

(c) The event considered here is the time reversal of the perfectly inelastic collision in the previous problem. The same momentum conservation equation describes both processes.

P9.20 v_1, speed of m_1 at B before collision.

$$\frac{1}{2}m_1v_1^2=m_1gh$$

$$v_1=\sqrt{2(9.80)(5.00)}=9.90\text{ m/s}$$

v_{1f}, speed of m_1 at B just after collision.

$$v_{1f}=\frac{m_1-m_2}{m_1+m_2}v_1=-\frac{1}{3}(9.90)\text{ m/s}=-3.30\text{ m/s}$$

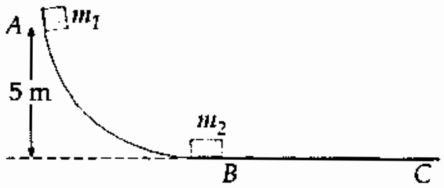


FIG. P9.20

At the highest point (after collision)

$$m_1gh_{\max}=\frac{1}{2}m_1(-3.30)^2 \qquad h_{\max}=\frac{(-3.30\text{ m/s})^2}{2(9.80\text{ m/s}^2)}=\boxed{0.556\text{ m}}$$

P9.21 (a), (b) Let v_g and v_p be the velocity of the girl and the plank relative to the ice surface. Then we may say that $v_g - v_p$ is the velocity of the girl relative to the plank, so that

$$v_g - v_p = 1.50 \qquad (1)$$

But also we must have $m_g v_g + m_p v_p = 0$, since total momentum of the girl-plank system is zero relative to the ice surface. Therefore

$$45.0v_g + 150v_p = 0, \text{ or } v_g = -3.33v_p$$

Putting this into the equation (1) above gives

$$-3.33v_p - v_p = 1.50 \text{ or } v_p = \boxed{-0.346 \text{ m/s}}$$

Then $v_g = -3.33(-0.346) = \boxed{1.15 \text{ m/s}}$

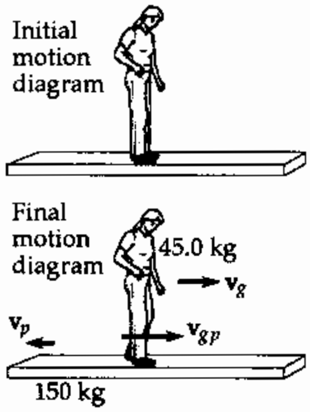


FIG. P9.21

***P9.22** For the car-truck-driver-driver system, momentum is conserved:

$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$: $\quad (4\,000 \text{ kg})(8 \text{ m/s})\hat{\mathbf{i}} + (800 \text{ kg})(8 \text{ m/s})(-\hat{\mathbf{i}}) = (4\,800 \text{ kg})v_f\hat{\mathbf{i}}$

$$v_f = \frac{25\,600 \text{ kg}\cdot\text{m/s}}{4\,800 \text{ kg}} = 5.33 \text{ m/s}$$

For the driver of the truck, the impulse-momentum theorem is

$\mathbf{F}\Delta t = \mathbf{p}_f - \mathbf{p}_i$: $\quad \mathbf{F}(0.120 \text{ s}) = (80 \text{ kg})(5.33 \text{ m/s})\hat{\mathbf{i}} - (80 \text{ kg})(8 \text{ m/s})\hat{\mathbf{i}}$

$$\mathbf{F} = \boxed{1.78\times10^3 \text{ N}(-\hat{\mathbf{i}}) \text{ on the truck driver}}$$

For the driver of the car, $\mathbf{F}(0.120 \text{ s}) = (80 \text{ kg})(5.33 \text{ m/s})\hat{\mathbf{i}} - (80 \text{ kg})(8 \text{ m/s})(-\hat{\mathbf{i}})$

$$\mathbf{F} = \boxed{8.89\times10^3 \text{ N}\hat{\mathbf{i}} \text{ on the car driver}}, \text{ 5 times larger.}$$

P9.23 (a) According to the Example in the chapter text, the fraction of total kinetic energy transferred to the moderator is

$$f_2 = \frac{4m_1m_2}{(m_1+m_2)^2}$$

where m_2 is the moderator nucleus and in this case, $m_2 = 12m_1$

$$f_2 = \frac{4m_1(12m_1)}{(13m_1)^2} = \frac{48}{169} = \boxed{0.284 \text{ or } 28.4\%}$$

of the neutron energy is transferred to the carbon nucleus.

(b) $K_C = (0.284)(1.6\times10^{-13} \text{ J}) = \boxed{4.54\times10^{-14} \text{ J}}$

$K_n = (0.716)(1.6\times10^{-13} \text{ J}) = \boxed{1.15\times10^{-13} \text{ J}}$

P9.24 Energy is conserved for the bob-Earth system between bottom and top of swing. At the top the stiff rod is in compression and the bob nearly at rest.

$$K_i + U_i = K_f + U_f: \qquad \frac{1}{2}Mv_b^2 + 0 = 0 + Mg2\ell$$
$$v_b^2 = g4\ell \text{ so } v_b = 2\sqrt{g\ell}$$

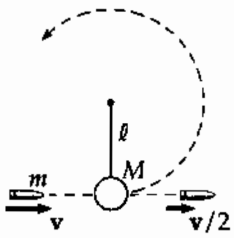


FIG. P9.24

Momentum of the bob-bullet system is conserved in the collision:

$$mv = m\frac{v}{2} + M\left(2\sqrt{g\ell}\right) \qquad \boxed{v = \frac{4M}{m}\sqrt{g\ell}}$$

P9.25 At impact, momentum of the clay-block system is conserved, so:

$$mv_1 = (m_1 + m_2)v_2$$

After impact, the change in kinetic energy of the clay-block-surface system is equal to the increase in internal energy:

$$\frac{1}{2}(m_1 + m_2)v_2^2 = f_f d = \mu(m_1 + m_2)gd$$
$$\frac{1}{2}(0.112 \text{ kg})v_2^2 = 0.650(0.112 \text{ kg})\left(9.80 \text{ m/s}^2\right)(7.50 \text{ m})$$
$$v_2^2 = 95.6 \text{ m}^2/\text{s}^2 \qquad v_2 = 9.77 \text{ m/s}$$
$$\left(12.0 \times 10^{-3} \text{ kg}\right)v_1 = (0.112 \text{ kg})(9.77 \text{ m/s}) \qquad v_1 = \boxed{91.2 \text{ m/s}}$$

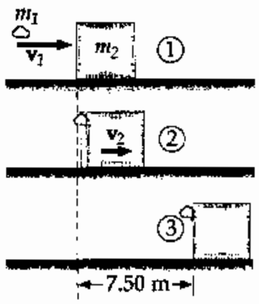


FIG. P9.25

P9.26 We assume equal firing speeds v and equal forces F required for the two bullets to push wood fibers apart. These equal forces act backward on the two bullets.

For the first, $\quad K_i + \Delta E_{\text{mech}} = K_f \quad \frac{1}{2}\left(7.00 \times 10^{-3} \text{ kg}\right)v^2 - F\left(8.00 \times 10^{-2} \text{ m}\right) = 0$

For the second, $\quad p_i = p_f \quad \left(7.00 \times 10^{-3} \text{ kg}\right)v = (1.014 \text{ kg})v_f$

$$v_f = \frac{\left(7.00 \times 10^{-3}\right)v}{1.014}$$

Again, $\quad K_i + \Delta E_{\text{mech}} = K_f: \quad \frac{1}{2}\left(7.00 \times 10^{-3} \text{ kg}\right)v^2 - Fd = \frac{1}{2}(1.014 \text{ kg})v_f^2$

Substituting for v_f, $\quad \frac{1}{2}\left(7.00 \times 10^{-3} \text{ kg}\right)v^2 - Fd = \frac{1}{2}(1.014 \text{ kg})\left(\frac{7.00 \times 10^{-3} v}{1.014}\right)^2$

$$Fd = \frac{1}{2}\left(7.00 \times 10^{-3}\right)v^2 - \frac{1}{2}\frac{\left(7.00 \times 10^{-3}\right)^2}{1.014}v^2$$

Substituting for v, $\quad Fd = F\left(8.00 \times 10^{-2} \text{ m}\right)\left(1 - \frac{7.00 \times 10^{-3}}{1.014}\right) \qquad d = \boxed{7.94 \text{ cm}}$

***P9.27** (a) Using conservation of momentum, $\left(\sum \mathbf{p}\right)_{\text{after}} = \left(\sum \mathbf{p}\right)_{\text{before}}$, gives

$$[(4.0+10+3.0)\text{ kg}]v = (4.0\text{ kg})(5.0\text{ m/s}) + (10\text{ kg})(3.0\text{ m/s}) + (3.0\text{ kg})(-4.0\text{ m/s}).$$

Therefore, $v = +2.24\text{ m/s}$, or $\boxed{2.24\text{ m/s toward the right}}$.

(b) $\boxed{\text{No}}$. For example, if the 10-kg and 3.0-kg mass were to stick together first, they would move with a speed given by solving

$$(13\text{ kg})v_1 = (10\text{ kg})(3.0\text{ m/s}) + (3.0\text{ kg})(-4.0\text{ m/s}),\text{ or } v_1 = +1.38\text{ m/s}.$$

Then when this 13 kg combined mass collides with the 4.0 kg mass, we have

$$(17\text{ kg})v = (13\text{ kg})(1.38\text{ m/s}) + (4.0\text{ kg})(5.0\text{ m/s}),\text{ and } v = +2.24\text{ m/s}$$

just as in part (a). Coupling order makes no difference.

Section 9.4 Two-Dimensional Collisions

P9.28 (a) First, we conserve momentum for the system of two football players in the x direction (the direction of travel of the fullback).

$$(90.0\text{ kg})(5.00\text{ m/s}) + 0 = (185\text{ kg})V\cos\theta$$

where θ is the angle between the direction of the final velocity V and the x axis. We find

$$V\cos\theta = 2.43\text{ m/s} \qquad (1)$$

Now consider conservation of momentum of the system in the y direction (the direction of travel of the opponent).

$$(95.0\text{ kg})(3.00\text{ m/s}) + 0 = (185\text{ kg})(V\sin\theta)$$

which gives, $V\sin\theta = 1.54\text{ m/s}$ (2)

Divide equation (2) by (1) $\tan\theta = \dfrac{1.54}{2.43} = 0.633$

From which $\boxed{\theta = 32.3°}$

Then, either (1) or (2) gives $V = \boxed{2.88\text{ m/s}}$

(b) $K_i = \frac{1}{2}(90.0\text{ kg})(5.00\text{ m/s})^2 + \frac{1}{2}(95.0\text{ kg})(3.00\text{ m/s})^2 = 1.55\times10^3\text{ J}$

$K_f = \frac{1}{2}(185\text{ kg})(2.88\text{ m/s})^2 = 7.67\times10^2\text{ J}$

Thus, the kinetic energy lost is $\boxed{783\text{ J into internal energy.}}$

P9.29 $p_{xf} = p_{xi}$

$$mv_O \cos 37.0° + mv_Y \cos 53.0° = m(5.00 \text{ m/s})$$

$$0.799v_O + 0.602v_Y = 5.00 \text{ m/s} \qquad (1)$$

$$p_{yf} = p_{yi}$$

$$mv_O \sin 37.0° - mv_Y \sin 53.0° = 0$$

$$0.602v_O = 0.799v_Y \qquad (2)$$

Solving (1) and (2) simultaneously,

$$\boxed{v_O = 3.99 \text{ m/s}} \text{ and } \boxed{v_Y = 3.01 \text{ m/s}}.$$

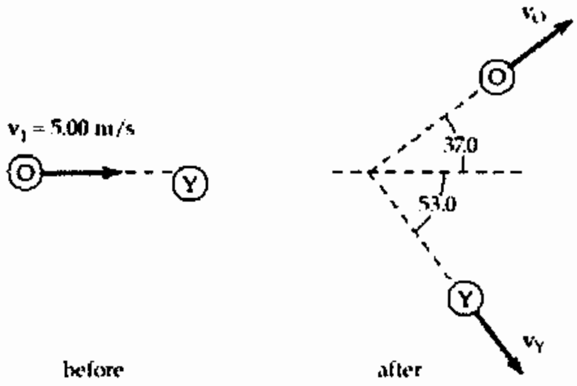


FIG. P9.29

P9.30 $p_{xf} = p_{xi}$: $\quad mv_O \cos\theta + mv_Y \cos(90.0° - \theta) = mv_i$

$$v_O \cos\theta + v_Y \sin\theta = v_i \qquad (1)$$

$p_{yf} = p_{yi}$: $\quad mv_O \sin\theta - mv_Y \sin(90.0° - \theta) = 0$

$$v_O \sin\theta = v_Y \cos\theta \qquad (2)$$

From equation (2),

$$v_O = v_Y\left(\frac{\cos\theta}{\sin\theta}\right) \qquad (3)$$

Substituting into equation (1),

$$v_Y\left(\frac{\cos^2\theta}{\sin\theta}\right) + v_Y \sin\theta = v_i$$

so $\quad v_Y\left(\cos^2\theta + \sin^2\theta\right) = v_i \sin\theta$, and $\boxed{v_Y = v_i \sin\theta}$.

Then, from equation (3), $\boxed{v_O = v_i \cos\theta}$.

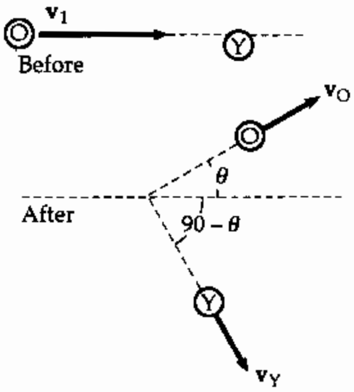


FIG. P9.30

We did not need to write down an equation expressing conservation of mechanical energy. In the problem situation, the requirement of perpendicular final velocities is equivalent to the condition of elasticity.

P9.31 The initial momentum of the system is 0. Thus,

$$(1.20m)v_{Bi} = m(10.0\ \text{m/s})$$

and $v_{Bi} = 8.33\ \text{m/s}$

$$K_i = \frac{1}{2}m(10.0\ \text{m/s})^2 + \frac{1}{2}(1.20m)(8.33\ \text{m/s})^2 = \frac{1}{2}m\left(183\ \text{m}^2/\text{s}^2\right)$$

$$K_f = \frac{1}{2}m(v_G)^2 + \frac{1}{2}(1.20m)(v_B)^2 = \frac{1}{2}\left(\frac{1}{2}m\left(183\ \text{m}^2/\text{s}^2\right)\right)$$

or $v_G^2 + 1.20v_B^2 = 91.7\ \text{m}^2/\text{s}^2$ (1)

From conservation of momentum,

$$mv_G = (1.20m)v_B$$

or $v_G = 1.20v_B$ (2)

Solving (1) and (2) simultaneously, we find

$\boxed{v_G = 7.07\ \text{m/s}}$ (speed of green puck after collision)

and $\boxed{v_B = 5.89\ \text{m/s}}$ (speed of blue puck after collision)

P9.32 We use conservation of momentum for the system of two vehicles for both northward and eastward components.

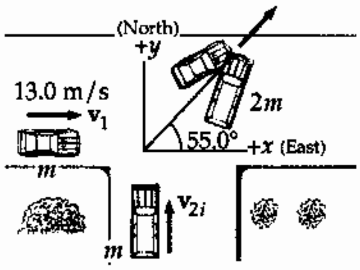


FIG. P9.32

For the eastward direction:

$$M(13.0\ \text{m/s}) = 2MV_f\cos 55.0°$$

For the northward direction:

$$Mv_{2i} = 2MV_f\sin 55.0°$$

Divide the northward equation by the eastward equation to find:

$$v_{2i} = (13.0\ \text{m/s})\tan 55.0° = 18.6\ \text{m/s} = \boxed{41.5\ \text{mi/h}}$$

Thus, the driver of the north bound car was untruthful.

P9.33 By conservation of momentum for the system of the two billiard balls (with all masses equal),

$$5.00 \text{ m/s} + 0 = (4.33 \text{ m/s})\cos 30.0° + v_{2fx}$$
$$v_{2fx} = 1.25 \text{ m/s}$$
$$0 = (4.33 \text{ m/s})\sin 30.0° + v_{2fy}$$
$$v_{2fy} = -2.16 \text{ m/s}$$
$$\mathbf{v}_{2f} = \boxed{2.50 \text{ m/s at } -60.0°}$$

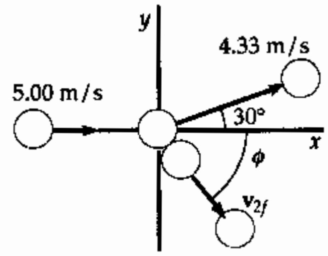


FIG. P9.33

Note that we did not need to use the fact that the collision is perfectly elastic.

P9.34 (a) $\mathbf{p}_i = \mathbf{p}_f$ so $p_{xi} = p_{xf}$

and $p_{yi} = p_{yf}$

$$mv_i = mv\cos\theta + mv\cos\phi \quad (1)$$
$$0 = mv\sin\theta + mv\sin\phi \quad (2)$$

From (2), $\sin\theta = -\sin\phi$

so $\theta = -\phi$

Furthermore, energy conservation for the system of two protons requires

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

so $\boxed{v = \frac{v_i}{\sqrt{2}}}$

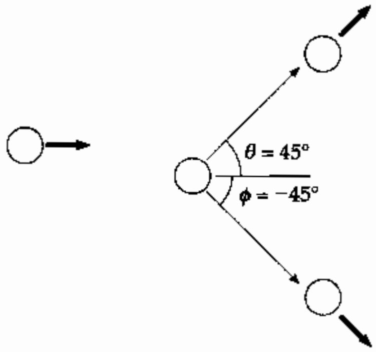


FIG. P9.34

(b) Hence, (1) gives $v_i = \frac{2v_i\cos\theta}{\sqrt{2}}$ $\theta = \boxed{45.0°}$ $\phi = \boxed{-45.0°}$

P9.35 $m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = (m_1 + m_2)\mathbf{v}_f$: $3.00(5.00)\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}} = 5.00\mathbf{v}$

$$\mathbf{v} = \boxed{(3.00\hat{\mathbf{i}} - 1.20\hat{\mathbf{j}}) \text{ m/s}}$$

P9.36 x-component of momentum for the system of the two objects:

$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$: $-mv_i + 3mv_i = 0 + 3mv_{2x}$

y-component of momentum of the system: $0 + 0 = -mv_{1y} + 3mv_{2y}$

by conservation of energy of the system: $+\frac{1}{2}mv_i^2 + \frac{1}{2}3mv_i^2 = \frac{1}{2}mv_{1y}^2 + \frac{1}{2}3m\left(v_{2x}^2 + v_{2y}^2\right)$

we have $v_{2x} = \frac{2v_i}{3}$

also $v_{1y} = 3v_{2y}$

So the energy equation becomes $4v_i^2 = 9v_{2y}^2 + \frac{4v_i^2}{3} + 3v_{2y}^2$

$$\frac{8v_i^2}{3} = 12v_{2y}^2$$

or $v_{2y} = \frac{\sqrt{2}v_i}{3}$

continued on next page

(a) The object of mass m has final speed $v_{1y} = 3v_{2y} = \boxed{\sqrt{2}v_i}$

and the object of mass 3 m moves at $\sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\frac{4v_i^2}{9} + \frac{2v_i^2}{9}}$

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \boxed{\sqrt{\frac{2}{3}}v_i}$$

(b) $\theta = \tan^{-1}\left(\frac{v_{2y}}{v_{2x}}\right)$ $\qquad \theta = \tan^{-1}\left(\frac{\sqrt{2}v_i}{3}\frac{3}{2v_i}\right) = \boxed{35.3°}$

P9.37 $m_0 = 17.0 \times 10^{-27}$ kg $\qquad \mathbf{v}_i = 0$ (the parent nucleus)

$m_1 = 5.00 \times 10^{-27}$ kg $\qquad \mathbf{v}_1 = 6.00 \times 10^6 \hat{\mathbf{j}}$ m/s

$m_2 = 8.40 \times 10^{-27}$ kg $\qquad \mathbf{v}_2 = 4.00 \times 10^6 \hat{\mathbf{i}}$ m/s

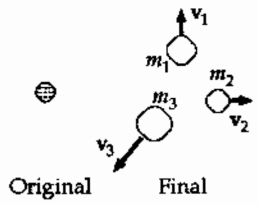


FIG. P9.37

(a) $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + m_3\mathbf{v}_3 = 0$

where $m_3 = m_0 - m_1 - m_2 = 3.60 \times 10^{-27}$ kg

$$\left(5.00 \times 10^{-27}\right)\left(6.00 \times 10^6 \hat{\mathbf{j}}\right) + \left(8.40 \times 10^{-27}\right)\left(4.00 \times 10^6 \hat{\mathbf{i}}\right) + \left(3.60 \times 10^{-27}\right)\mathbf{v}_3 = 0$$

$$\mathbf{v}_3 = \boxed{\left(-9.33 \times 10^6 \hat{\mathbf{i}} - 8.33 \times 10^6 \hat{\mathbf{j}}\right) \text{ m/s}}$$

(b) $E = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2$

$$E = \frac{1}{2}\left[\left(5.00 \times 10^{-27}\right)\left(6.00 \times 10^6\right)^2 + \left(8.40 \times 10^{-27}\right)\left(4.00 \times 10^6\right)^2 + \left(3.60 \times 10^{-27}\right)\left(12.5 \times 10^6\right)^2\right]$$

$$\boxed{E = 4.39 \times 10^{-13} \text{ J}}$$

Section 9.5 The Center of Mass

P9.38 The x-coordinate of the center of mass is

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{0 + 0 + 0 + 0}{(2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg})}$$

$$\boxed{x_{CM} = 0}$$

and the y-coordinate of the center of mass is

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(2.00 \text{ kg})(3.00 \text{ m}) + (3.00 \text{ kg})(2.50 \text{ m}) + (2.50 \text{ kg})(0) + (4.00 \text{ kg})(-0.500 \text{ m})}{2.00 \text{ kg} + 3.00 \text{ kg} + 2.50 \text{ kg} + 4.00 \text{ kg}}$$

$$\boxed{y_{CM} = 1.00 \text{ m}}$$

P9.39 Take x-axis starting from the oxygen nucleus and pointing toward the middle of the V.

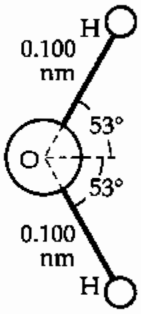


FIG. P9.39

Then $\boxed{y_{CM} = 0}$

and $x_{CM} = \dfrac{\sum m_i x_i}{\sum m_i} =$

$$x_{CM} = \frac{0 + 1.008\text{ u}(0.100\text{ nm})\cos 53.0° + 1.008\text{ u}(0.100\text{ nm})\cos 53.0°}{15.999\text{ u} + 1.008\text{ u} + 1.008\text{ u}}$$

$\boxed{x_{CM} = 0.006\,73\text{ nm from the oxygen nucleus}}$

***P9.40** Let the x axis start at the Earth's center and point toward the Moon.

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{5.98 \times 10^{24}\text{ kg }0 + 7.36 \times 10^{22}\text{ kg}\left(3.84 \times 10^{8}\text{ m}\right)}{6.05 \times 10^{24}\text{ kg}}$$

$$= \boxed{4.67 \times 10^{6}\text{ m from the Earth's center}}$$

The center of mass is within the Earth, which has radius 6.37×10^{6} m.

P9.41 Let A_1 represent the area of the bottom row of squares, A_2 the middle square, and A_3 the top pair.

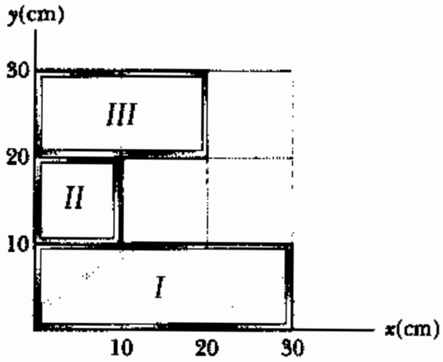


FIG. P9.41

$A = A_1 + A_2 + A_3$

$M = M_1 + M_2 + M_3$

$\dfrac{M_1}{A_1} = \dfrac{M}{A}$

$A_1 = 300\text{ cm}^2,\ A_2 = 100\text{ cm}^2,\ A_3 = 200\text{ cm}^2,\ A = 600\text{ cm}^2$

$$M_1 = M\left(\frac{A_1}{A}\right) = \frac{300\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{2}$$

$$M_2 = M\left(\frac{A_2}{A}\right) = \frac{100\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{6}$$

$$M_3 = M\left(\frac{A_3}{A}\right) = \frac{200\text{ cm}^2}{600\text{ cm}^2}M = \frac{M}{3}$$

$$x_{CM} = \frac{x_1 M_1 + x_2 M_2 + x_3 M_3}{M} = \frac{15.0\text{ cm}\left(\frac{1}{2}M\right) + 5.00\text{ cm}\left(\frac{1}{6}M\right) + 10.0\text{ cm}\left(\frac{1}{3}M\right)}{M}$$

$x_{CM} = \boxed{11.7\text{ cm}}$

$$y_{CM} = \frac{\frac{1}{2}M(5.00\text{ cm}) + \frac{1}{6}M(15.0\text{ cm}) + \left(\frac{1}{3}M\right)(25.0\text{ cm})}{M} = 13.3\text{ cm}$$

$y_{CM} = \boxed{13.3\text{ cm}}$

***P9.42** (a) Represent the height of a particle of mass dm within the object as y. Its contribution to the gravitational energy of the object-Earth system is $(dm)gy$. The total gravitational energy is $U_g = \int_{\text{all mass}} gy\,dm = g\int y\,dm$. For the center of mass we have $y_{CM} = \frac{1}{M}\int y\,dm$, so $U_g = gMy_{CM}$.

(b) The volume of the ramp is $\frac{1}{2}(3.6\text{ m})(15.7\text{ m})(64.8\text{ m}) = 1.83\times10^3\text{ m}^3$. Its mass is $\rho V = (3\,800\text{ kg/m}^3)(1.83\times10^3\text{ m}^3) = 6.96\times10^6\text{ kg}$. Its center of mass is above its base by one-third of its height, $y_{CM} = \frac{1}{3}15.7\text{ m} = 5.23\text{ m}$. Then

$$U_g = Mgy_{CM} = 6.96\times10^6\text{ kg}(9.8\text{ m/s}^2)5.23\text{ m} = \boxed{3.57\times10^8\text{ J}}.$$

P9.43 (a)

$$M = \int_0^{0.300\text{ m}} \lambda dx = \int_0^{0.300\text{ m}} \left[50.0\text{ g/m} + 20.0x\text{ g/m}^2\right]dx$$

$$M = \left[50.0x\text{ g/m} + 10.0x^2\text{ g/m}^2\right]_0^{0.300\text{ m}} = \boxed{15.9\text{ g}}$$

(b)

$$x_{CM} = \frac{\int_{\text{all mass}} x\,dm}{M} = \frac{1}{M}\int_0^{0.300\text{ m}} \lambda x\,dx = \frac{1}{M}\int_0^{0.300\text{ m}} \left[50.0x\text{ g/m} + 20.0x^2\text{ g/m}^2\right]dx$$

$$x_{CM} = \frac{1}{15.9\text{ g}}\left[25.0x^2\text{ g/m} + \frac{20x^3\text{ g/m}^2}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153\text{ m}}$$

***P9.44** Take the origin at the center of curvature. We have $L = \frac{1}{4}2\pi r$, $r = \frac{2L}{\pi}$. An incremental bit of the rod at angle θ from the x axis has mass given by $\frac{dm}{rd\theta} = \frac{M}{L}$, $dm = \frac{Mr}{L}d\theta$ where we have used the definition of radian measure. Now

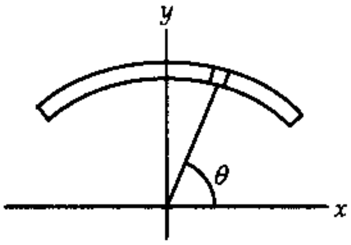


FIG. P9.44

$$y_{CM} = \frac{1}{M}\int_{\text{all mass}} y\,dm = \frac{1}{M}\int_{\theta=45°}^{135°} r\sin\theta\frac{Mr}{L}d\theta = \frac{r^2}{L}\int_{45°}^{135°}\sin\theta\,d\theta$$

$$= \left(\frac{2L}{\pi}\right)^2\frac{1}{L}(-\cos\theta)\Big|_{45°}^{135°} = \frac{4L}{\pi^2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{4\sqrt{2}L}{\pi^2}$$

The top of the bar is above the origin by $r = \frac{2L}{\pi}$, so the center of mass is below the middle of the bar by $\frac{2L}{\pi} - \frac{4\sqrt{2}L}{\pi^2} = \frac{2}{\pi}\left(1 - \frac{2\sqrt{2}}{\pi}\right)L = \boxed{0.063\,5L}$.

Section 9.6 **Motion of a System of Particles**

P9.45 (a) $\mathbf{v}_{CM} = \dfrac{\sum m_i \mathbf{v}_i}{M} = \dfrac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{M}$

$$= \frac{(2.00\text{ kg})\left(2.00\hat{\mathbf{i}}\text{ m/s} - 3.00\hat{\mathbf{j}}\text{ m/s}\right) + (3.00\text{ kg})\left(1.00\hat{\mathbf{i}}\text{ m/s} + 6.00\hat{\mathbf{j}}\text{ m/s}\right)}{5.00\text{ kg}}$$

$$\mathbf{v}_{CM} = \boxed{\left(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) $\mathbf{p} = M\mathbf{v}_{CM} = (5.00\text{ kg})\left(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\text{ m/s} = \boxed{\left(7.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}}\right)\text{ kg}\cdot\text{m/s}}$

P9.46 (a) See figure to the right.

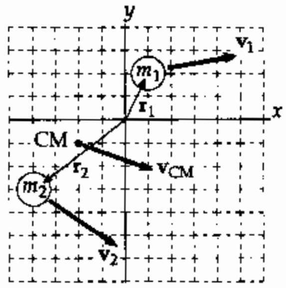


FIG. P9.46

(b) Using the definition of the position vector at the center of mass,

$$\mathbf{r}_{CM} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}$$

$$\mathbf{r}_{CM} = \frac{(2.00\text{ kg})(1.00\text{ m}, 2.00\text{ m}) + (3.00\text{ kg})(-4.00\text{ m}, -3.00\text{ m})}{2.00\text{ kg} + 3.00\text{ kg}}$$

$$\mathbf{r}_{CM} = \boxed{\left(-2.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)\text{ m}}$$

(c) The velocity of the center of mass is

$$\mathbf{v}_{CM} = \frac{\mathbf{P}}{M} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2} = \frac{(2.00\text{ kg})(3.00\text{ m/s}, 0.50\text{ m/s}) + (3.00\text{ kg})(3.00\text{ m/s}, -2.00\text{ m/s})}{(2.00\text{ kg} + 3.00\text{ kg})}$$

$$\mathbf{v}_{CM} = \boxed{\left(3.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)\text{ m/s}}$$

(d) The total linear momentum of the system can be calculated as $\mathbf{P} = M\mathbf{v}_{CM}$

or as $\mathbf{P} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$

Either gives $\mathbf{P} = \boxed{\left(15.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)\text{ kg}\cdot\text{m/s}}$

P9.47 Let x = distance from shore to center of boat
ℓ = length of boat
x' = distance boat moves as Juliet moves toward Romeo
The center of mass stays fixed.

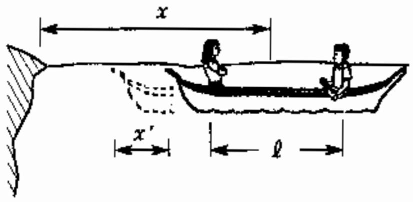


FIG. P9.47

Before: $x_{CM} = \dfrac{\left[M_b x + M_J\left(x - \frac{\ell}{2}\right) + M_R\left(x + \frac{\ell}{2}\right)\right]}{\left(M_B + M_J + M_R\right)}$

After: $x_{CM} = \dfrac{\left[M_B(x - x') + M_J\left(x + \frac{\ell}{2} - x'\right) + M_R\left(x + \frac{\ell}{2} - x'\right)\right]}{\left(M_B + M_J + M_R\right)}$

$$\ell\left(-\frac{55.0}{2} + \frac{77.0}{2}\right) = x'(-80.0 - 55.0 - 77.0) + \frac{\ell}{2}(55.0 + 77.0)$$

$$x' = \frac{55.0\ell}{212} = \frac{55.0(2.70)}{212} = \boxed{0.700\text{ m}}$$

P9.48 (a) Conservation of momentum for the two-ball system gives us:

$$0.200\text{ kg}(1.50\text{ m/s}) + 0.300\text{ kg}(-0.400\text{ m/s}) = 0.200\text{ kg}\,v_{1f} + 0.300\text{ kg}\,v_{2f}$$

Relative velocity equation:

$$v_{2f} - v_{1f} = 1.90\text{ m/s}$$

Then $0.300 - 0.120 = 0.200 v_{1f} + 0.300(1.90 + v_{1f})$

$v_{1f} = -0.780\text{ m/s}$ $\qquad$ $v_{2f} = 1.12\text{ m/s}$

$\boxed{\mathbf{v}_{1f} = -0.780\hat{\mathbf{i}}\text{ m/s}}$ $\qquad$ $\boxed{\mathbf{v}_{2f} = 1.12\hat{\mathbf{i}}\text{ m/s}}$

(b) Before, $\mathbf{v}_{CM} = \dfrac{(0.200\text{ kg})(1.50\text{ m/s})\hat{\mathbf{i}} + (0.300\text{ kg})(-0.400\text{ m/s})\hat{\mathbf{i}}}{0.500\text{ kg}}$

$\boxed{\mathbf{v}_{CM} = (0.360\text{ m/s})\hat{\mathbf{i}}}$

Afterwards, the center of mass must move at the same velocity, as momentum of the system is conserved.

Section 9.7 Rocket Propulsion

P9.49 (a) $\text{Thrust} = \left|v_e \dfrac{dM}{dt}\right|$ $\qquad$ $\text{Thrust} = (2.60\times10^3\text{ m/s})(1.50\times10^4\text{ kg/s}) = \boxed{3.90\times10^7\text{ N}}$

(b) $\sum F_y = \text{Thrust} - Mg = Ma$: $\qquad$ $3.90\times10^7 - (3.00\times10^6)(9.80) = (3.00\times10^6)a$

$a = \boxed{3.20\text{ m/s}^2}$

***P9.50** (a) The fuel burns at a rate $\qquad$ $\dfrac{dM}{dt} = \dfrac{12.7\text{ g}}{1.90\text{ s}} = 6.68\times10^{-3}\text{ kg/s}$

$\text{Thrust} = v_e \dfrac{dM}{dt}$: $\qquad$ $5.26\text{ N} = v_e(6.68\times10^{-3}\text{ kg/s})$

$v_e = \boxed{787\text{ m/s}}$

(b) $v_f - v_i = v_e \ln\left(\dfrac{M_i}{M_f}\right)$: $\qquad$ $v_f - 0 = (797\text{ m/s})\ln\left(\dfrac{53.5\text{ g} + 25.5\text{ g}}{53.5\text{ g} + 25.5\text{ g} - 12.7\text{ g}}\right)$

$v_f = \boxed{138\text{ m/s}}$

P9.51 $v = v_e \ln\dfrac{M_i}{M_f}$

(a) $M_i = e^{v/v_e} M_f$ $\qquad$ $M_i = e^5(3.00\times10^3\text{ kg}) = 4.45\times10^5\text{ kg}$

The mass of fuel and oxidizer is $\quad$ $\Delta M = M_i - M_f = (445 - 3.00)\times10^3\text{ kg} = \boxed{442\text{ metric tons}}$

(b) $\Delta M = e^2(3.00\text{ metric tons}) - 3.00\text{ metric tons} = \boxed{19.2\text{ metric tons}}$

Because of the exponential, a relatively small increase in fuel and/or engine efficiency causes a large change in the amount of fuel and oxidizer required.

P9.52 (a) From Equation 9.41, $v - 0 = v_e \ln\left(\frac{M_i}{M_f}\right) = -v_e \ln\left(\frac{M_f}{M_i}\right)$

Now, $M_f = M_i - kt$, so $v = -v_e \ln\left(\frac{M_i - kt}{M_i}\right) = -v_e \ln\left(1 - \frac{k}{M_i} t\right)$

With the definition, $T_p \equiv \frac{M_i}{k}$, this becomes

$$v(t) = \boxed{-v_e \ln\left(1 - \frac{t}{T_p}\right)}$$

(b) With $v_e = 1\,500$ m/s, and $T_p = 144$ s, $v = -(1\,500 \text{ m/s}) \ln\left(1 - \frac{t}{144 \text{ s}}\right)$

$t(s)$	$v(\text{m/s})$
0	0
20	224
40	488
60	808
80	1220
100	1780
120	2690
132	3730

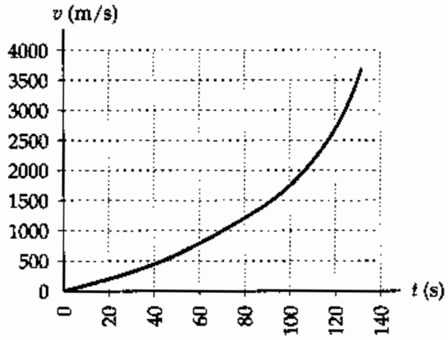


FIG. P9.52(b)

(c) $a(t) = \frac{dv}{dt} = \frac{d\left[-v_e \ln\left(1 - \frac{t}{T_p}\right)\right]}{dt} = -v_e \left(\frac{1}{1 - \frac{t}{T_p}}\right)\left(-\frac{1}{T_p}\right) = \left(\frac{v_e}{T_p}\right)\left(\frac{1}{1 - \frac{t}{T_p}}\right)$, or

$$a(t) = \boxed{\frac{v_e}{T_p - t}}$$

(d) With $v_e = 1\,500$ m/s, and $T_p = 144$ s, $a = \frac{1\,500 \text{ m/s}}{144 \text{ s} - t}$

$t(s)$	$a(\text{m/s}^2)$
0	10.4
20	12.1
40	14.4
60	17.9
80	23.4
100	34.1
120	62.5
132	125

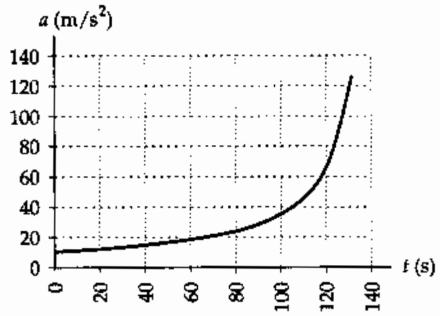


FIG. P9.52(d)

continued on next page

(e) $x(t)=0+\int_0^t v\,dt=\int_0^t\left[-v_e\ln\left(1-\frac{t}{T_p}\right)\right]dt=v_eT_p\int_0^t\ln\left[1-\frac{t}{T_p}\right]\left(-\frac{dt}{T_p}\right)$

$$x(t)=v_eT_p\left[\left(1-\frac{t}{T_p}\right)\ln\left(1-\frac{t}{T_p}\right)-\left(1-\frac{t}{T_p}\right)\right]_0^t$$

$$x(t)=\boxed{v_e\left(T_p-t\right)\ln\left(1-\frac{t}{T_p}\right)+v_et}$$

(f) With $v_e=1\,500\ \text{m/s}=1.50\ \text{km/s}$, and $T_p=144\ \text{s}$,

$$x=1.50(144-t)\ln\left(1-\frac{t}{144}\right)+1.50t$$

t(s)	x(km)
0	0
20	2.19
40	9.23
60	22.1
80	42.2
100	71.7
120	115
132	153

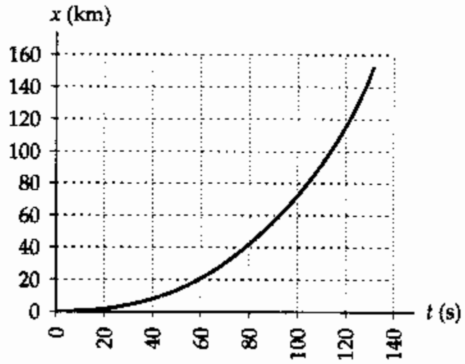


FIG. P9.52(f)

***P9.53** The thrust acting on the spacecraft is

$\sum F=ma$: $\sum F=(3\,500\ \text{kg})\left(2.50\times10^{-6}\right)\left(9.80\ \text{m/s}^2\right)=8.58\times10^{-2}\ \text{N}$

thrust $=\left(\frac{dM}{dt}\right)v_e$: $8.58\times10^{-2}\ \text{N}=\left(\frac{\Delta M}{3\,600\ \text{s}}\right)(70\ \text{m/s})$

$$\Delta M=\boxed{4.41\ \text{kg}}$$

Additional Problems

P9.54 (a) When the spring is fully compressed, each cart moves with same velocity **v**. Apply conservation of momentum for the system of two gliders

$$\mathbf{p}_i = \mathbf{p}_f: \qquad m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v} \qquad \boxed{\mathbf{v} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}}$$

(b) Only conservative forces act, therefore $\Delta E = 0$. $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}kx_m^2$

Substitute for v from (a) and solve for x_m.

$$x_m^2 = \frac{(m_1 + m_2)m_1v_1^2 + (m_1 + m_2)m_2v_2^2 - (m_1v_1)^2 - (m_2v_2)^2 - 2m_1m_2v_1v_2}{k(m_1 + m_2)}$$

$$x_m = \sqrt{\frac{m_1m_2\left(v_1^2 + v_2^2 - 2v_1v_2\right)}{k(m_1 + m_2)}} = \boxed{(v_1 - v_2)\sqrt{\frac{m_1m_2}{k(m_1 + m_2)}}}$$

(c) $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$

Conservation of momentum: $$m_1\left(\mathbf{v}_1 - \mathbf{v}_{1f}\right) = m_2\left(\mathbf{v}_{2f} - \mathbf{v}_2\right) \qquad (1)$$

Conservation of energy: $$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

which simplifies to: $$m_1\left(v_1^2 - v_{1f}^2\right) = m_2\left(v_{2f}^2 - v_2^2\right)$$

Factoring gives $$m_1\left(\mathbf{v}_1 - \mathbf{v}_{1f}\right)\cdot\left(\mathbf{v}_1 + \mathbf{v}_{1f}\right) = m_2\left(\mathbf{v}_{2f} - \mathbf{v}_2\right)\cdot\left(\mathbf{v}_{2f} + \mathbf{v}_2\right)$$

and with the use of the momentum equation (equation (1)),

this reduces to $$\left(\mathbf{v}_1 + \mathbf{v}_{1f}\right) = \left(\mathbf{v}_{2f} + \mathbf{v}_2\right)$$

or $$\mathbf{v}_{1f} = \mathbf{v}_{2f} + \mathbf{v}_2 - \mathbf{v}_1 \qquad (2)$$

Substituting equation (2) into equation (1) and simplifying yields:

$$\mathbf{v}_{2f} = \boxed{\left(\frac{2m_1}{m_1 + m_2}\right)\mathbf{v}_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\mathbf{v}_2}$$

Upon substitution of this expression for $\mathbf{v}_{2f}$ into equation 2, one finds

$$\mathbf{v}_{1f} = \boxed{\left(\frac{m_1 - m_2}{m_1 + m_2}\right)\mathbf{v}_1 + \left(\frac{2m_2}{m_1 + m_2}\right)\mathbf{v}_2}$$

Observe that these results are the same as Equations 9.20 and 9.21, which should have been expected since this is a perfectly elastic collision in one dimension.

P9.55 (a) $(60.0\text{ kg})4.00\text{ m/s} = (120 + 60.0)\text{ kg}v_f$

$\mathbf{v}_f = \boxed{1.33\text{ m/s}\,\hat{\mathbf{i}}}$

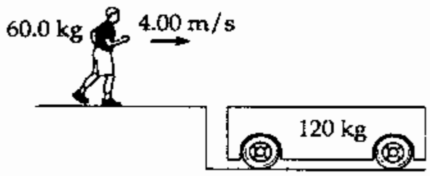


FIG. P9.55

(b) $\sum F_y = 0$: $n - (60.0\text{ kg})9.80\text{ m/s}^2 = 0$

$f_k = \mu_k n = 0.400(588\text{ N}) = 235\text{ N}$

$\mathbf{f}_k = \boxed{-235\text{ N}\hat{\mathbf{i}}}$

(c) For the person, $p_i + I = p_f$

$mv_i + Ft = mv_f$

$(60.0\text{ kg})4.00\text{ m/s} - (235\text{ N})t = (60.0\text{ kg})1.33\text{ m/s}$

$t = \boxed{0.680\text{ s}}$

(d) person: $m\mathbf{v}_f - m\mathbf{v}_i = 60.0\text{ kg}(1.33 - 4.00)\text{ m/s} = \boxed{-160\text{ N}\cdot\text{s}\hat{\mathbf{i}}}$

cart: $120\text{ kg}(1.33\text{ m/s}) - 0 = \boxed{+160\text{ N}\cdot\text{s}\hat{\mathbf{i}}}$

(e) $x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}[(4.00 + 1.33)\text{ m/s}]0.680\text{ s} = \boxed{1.81\text{ m}}$

(f) $x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(0 + 1.33\text{ m/s})0.680\text{ s} = \boxed{0.454\text{ m}}$

(g) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}60.0\text{ kg}(1.33\text{ m/s})^2 - \frac{1}{2}60.0\text{ kg}(4.00\text{ m/s})^2 = \boxed{-427\text{ J}}$

(h) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}120.0\text{ kg}(1.33\text{ m/s})^2 - 0 = \boxed{107\text{ J}}$

(i) The force exerted by the person on the cart must equal in magnitude and opposite in direction to the force exerted by the cart on the person. The changes in momentum of the two objects must be equal in magnitude and must add to zero. Their changes in kinetic energy are different in magnitude and do not add to zero. The following represent two ways of thinking about 'why.' The distance the cart moves is different from the distance moved by the point of application of the friction force to the cart. The total change in mechanical energy for both objects together, -320 J, becomes $+320$ J of additional internal energy in this perfectly inelastic collision.

P9.56 The equation for the horizontal range of a projectile is $R = \frac{v_i^2 \sin 2\theta}{g}$. Thus, with $\theta = 45.0°$, the initial velocity is

$$v_i = \sqrt{Rg} = \sqrt{(200\text{ m})(9.80\text{ m/s}^2)} = 44.3\text{ m/s}$$

$$I = \overline{F}(\Delta t) = \Delta p = mv_i - 0$$

Therefore, the magnitude of the average force acting on the ball during the impact is:

$$\overline{F} = \frac{mv_i}{\Delta t} = \frac{(46.0 \times 10^{-3}\text{ kg})(44.3\text{ m/s})}{7.00 \times 10^{-3}\text{ s}} = \boxed{291\text{ N}}.$$

P9.57 We hope the momentum of the wrench provides enough recoil so that the astronaut can reach the ship before he loses life support! We might expect the elapsed time to be on the order of several minutes based on the description of the situation.

No external force acts on the system (astronaut plus wrench), so the total momentum is constant. Since the final momentum (wrench plus astronaut) must be zero, we have final momentum = initial momentum = 0.

$$m_{\text{wrench}}v_{\text{wrench}} + m_{\text{astronaut}}v_{\text{astronaut}} = 0$$

Thus $v_{\text{astronaut}} = -\frac{m_{\text{wrench}}v_{\text{wrench}}}{m_{\text{astronaut}}} = -\frac{(0.500\text{ kg})(20.0\text{ m/s})}{80.0\text{ kg}} = -0.125\text{ m/s}$

At this speed, the time to travel to the ship is

$$t = \frac{30.0\text{ m}}{0.125\text{ m/s}} = \boxed{240\text{ s}} = 4.00\text{ minutes}$$

The astronaut is fortunate that the wrench gave him sufficient momentum to return to the ship in a reasonable amount of time! In this problem, we were told that the astronaut was not drifting away from the ship when he threw the wrench. However, this is not quite possible since he did not encounter an external force that would reduce his velocity away from the ship (there is no air friction beyond earth's atmosphere). If this were a real-life situation, the astronaut would have to throw the wrench hard enough to overcome his momentum caused by his original push away from the ship.

P9.58 Using conservation of momentum from just before to just after the impact of the bullet with the block:

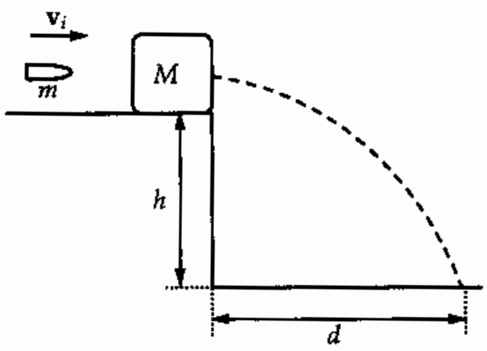


FIG. P9.58

$$mv_i = (M+m)v_f$$

or $$v_i = \left(\frac{M+m}{m}\right)v_f \qquad (1)$$

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \text{ and } h = \frac{1}{2}gt^2$$

Thus, $t = \sqrt{\frac{2h}{g}}$ and $v_f = \frac{d}{t} = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$

Substituting into (1) from above gives $v_i = \boxed{\left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}}}$.

***P9.59** (a) Conservation of momentum:

$$0.5\text{ kg}\left(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+1\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg}\left(-1\hat{\mathbf{i}}+2\hat{\mathbf{j}}-3\hat{\mathbf{k}}\right)\text{ m/s}$$

$$=0.5\text{ kg}\left(-1\hat{\mathbf{i}}+3\hat{\mathbf{j}}-8\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg }\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f}=\frac{\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}+\left(0.5\hat{\mathbf{i}}-1.5\hat{\mathbf{j}}+4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}=\boxed{0}$$

The original kinetic energy is

$$\frac{1}{2}0.5\text{ kg}\left(2^2+3^2+1^2\right)\text{ m}^2/\text{s}^2+\frac{1}{2}1.5\text{ kg}\left(1^2+2^2+3^2\right)\text{ m}^2/\text{s}^2=14.0\text{ J}$$

The final kinetic energy is $\frac{1}{2}0.5\text{ kg}\left(1^2+3^2+8^2\right)\text{ m}^2/\text{s}^2+0=18.5\text{ J}$ different from the original energy so the collision is $\boxed{\text{inelastic}}$.

(b) We follow the same steps as in part (a):

$$\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}=0.5\text{ kg}\left(-0.25\hat{\mathbf{i}}+0.75\hat{\mathbf{j}}-2\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg }\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f}=\frac{\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}+\left(0.125\hat{\mathbf{i}}-0.375\hat{\mathbf{j}}+1\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}$$

$$=\boxed{\left(-0.250\hat{\mathbf{i}}+0.750\hat{\mathbf{j}}-2.00\hat{\mathbf{k}}\right)\text{ m/s}}$$

We see $\mathbf{v}_{2f}=\mathbf{v}_{1f}$, so the collision is $\boxed{\text{perfectly inelastic}}$.

(c) Conservation of momentum:

$$\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}=0.5\text{ kg}\left(-1\hat{\mathbf{i}}+3\hat{\mathbf{j}}+a\hat{\mathbf{k}}\right)\text{ m/s}+1.5\text{ kg }\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f}=\frac{\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}+\left(0.5\hat{\mathbf{i}}-1.5\hat{\mathbf{j}}-0.5a\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}}{1.5\text{ kg}}$$

$$=\boxed{(-2.67-0.333a)\hat{\mathbf{k}}\text{ m/s}}$$

Conservation of energy:

$$14.0\text{ J}=\frac{1}{2}0.5\text{ kg}\left(1^2+3^2+a^2\right)\text{ m}^2/\text{s}^2+\frac{1}{2}1.5\text{ kg}(2.67+0.333a)^2\text{ m}^2/\text{s}^2$$

$$=2.5\text{ J}+0.25a^2+5.33\text{ J}+1.33a+0.083\,3a^2$$

$$0=0.333a^2+1.33a-6.167$$

$$a=\frac{-1.33\pm\sqrt{1.33^2-4(0.333)(-6.167)}}{0.667}$$

$a=2.74$ or -6.74. Either value is possible.

$$\therefore\ \boxed{a=2.74},\quad \mathbf{v}_{2f}=\left(-2.67-0.333(2.74)\right)\hat{\mathbf{k}}\text{ m/s}=\boxed{-3.58\hat{\mathbf{k}}\text{ m/s}}$$

$$\therefore\ \boxed{a=-6.74},\quad \mathbf{v}_{2f}=\left(-2.67-0.333(-6.74)\right)\hat{\mathbf{k}}\text{ m/s}=\boxed{-0.419\hat{\mathbf{k}}\text{ m/s}}$$

P9.60 (a) The initial momentum of the system is zero, which remains constant throughout the motion. Therefore, when m_1 leaves the wedge, we must have

$$m_2 v_{\text{wedge}} + m_1 v_{\text{block}} = 0$$

or $(3.00 \text{ kg})v_{\text{wedge}} + (0.500 \text{ kg})(+4.00 \text{ m/s}) = 0$

so $v_{\text{wedge}} = \boxed{-0.667 \text{ m/s}}$

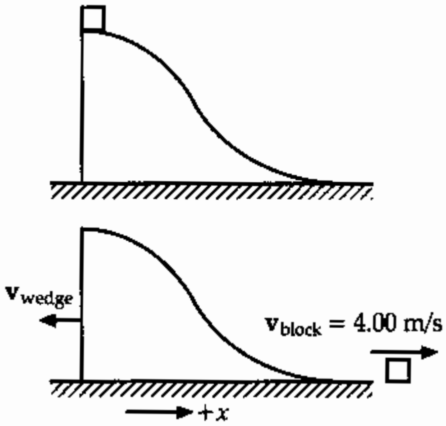


FIG. P9.60

(b) Using conservation of energy for the block-wedge-Earth system as the block slides down the smooth (frictionless) wedge, we have

$$\left[K_{\text{block}} + U_{\text{system}}\right]_i + \left[K_{\text{wedge}}\right]_i = \left[K_{\text{block}} + U_{\text{system}}\right]_f + \left[K_{\text{wedge}}\right]_f$$

or $\left[0 + m_1 gh\right] + 0 = \left[\frac{1}{2}m_1(4.00)^2 + 0\right] + \frac{1}{2}m_2(-0.667)^2$ which gives $\boxed{h = 0.952 \text{ m}}$.

***P9.61** (a) Conservation of the x component of momentum for the cart-bucket-water system:

$$mv_i + 0 = (m + \rho V)v \qquad \boxed{v_i = \frac{m + \rho V}{m}v}$$

(b) Raindrops with zero x-component of momentum stop in the bucket and slow its horizontal motion. When they drip out, they carry with them horizontal momentum. Thus the cart slows with constant acceleration.

P9.62 Consider the motion of the firefighter during the three intervals:

(1) before, (2) during, and (3) after collision with the platform.

(a) While falling a height of 4.00 m, his speed changes from $v_i = 0$ to v_1 as found from

$$\Delta E = \left(K_f + U_f\right) - \left(K_i - U_i\right), \text{ or}$$
$$K_f = \Delta E - U_f + K_i + U_i$$

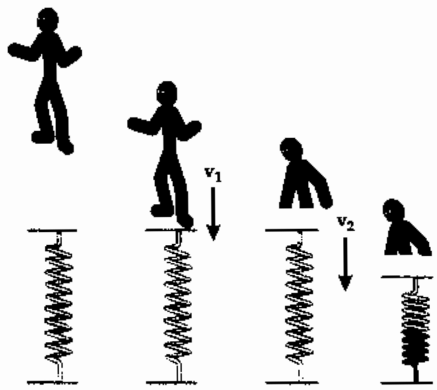


FIG. P9.62

When the initial position of the platform is taken as the zero level of gravitational potential, we have

$$\frac{1}{2}mv_1^2 = fh\cos(180°) - 0 + 0 + mgh$$

Solving for v_1 gives

$$v_1 = \sqrt{\frac{2(-fh + mgh)}{m}} = \sqrt{\frac{2(-300(4.00) + 75.0(9.80)4.00)}{75.0}} = \boxed{6.81 \text{ m/s}}$$

(b) During the inelastic collision, momentum is conserved; and if v_2 is the speed of the firefighter and platform just after collision, we have $mv_1 = (m+M)v_2$ or

$$v_2 = \frac{m_1 v_1}{m + M} = \frac{75.0(6.81)}{75.0 + 20.0} = 5.38 \text{ m/s}$$

Following the collision and again solving for the work done by non-conservative forces, using the distances as labeled in the figure, we have (with the zero level of gravitational potential at the initial position of the platform):

$$\Delta E = K_f + U_{fg} + U_{fs} - K_i - U_{ig} - U_{is}, \text{ or}$$
$$-fs = 0 + (m+M)g(-s) + \frac{1}{2}ks^2 - \frac{1}{2}(m+M)v^2 - 0 - 0$$

This results in a quadratic equation in s:

$$2\,000s^2 - (931)s + 300s - 1\,375 = 0 \text{ or } \boxed{s = 1.00 \text{ m}}$$

***P9.63** (a) Each object swings down according to

$$mgR = \frac{1}{2}mv_1^2 \qquad MgR = \frac{1}{2}Mv_1^2 \qquad v_1 = \sqrt{2gR}$$

The collision: $-mv_1 + Mv_1 = +(m+M)v_2$

$$v_2 = \frac{M-m}{M+m}v_1$$

Swinging up: $\frac{1}{2}(M+m)v_2^2 = (M+m)gR(1-\cos 35°)$

$$v_2 = \sqrt{2gR(1-\cos 35°)}$$

$$\sqrt{2gR(1-\cos 35°)}(M+m) = (M-m)\sqrt{2gR}$$

$$0.425M + 0.425m = M - m$$

$$1.425m = 0.575M$$

$$\boxed{\frac{m}{M} = 0.403}$$

(b) No change is required if the force is different. The nature of the forces within the system of colliding objects does not affect the total momentum of the system. With strong magnetic attraction, the heavier object will be moving somewhat faster and the lighter object faster still. Their extra kinetic energy will all be immediately converted into extra internal energy when the objects latch together. Momentum conservation guarantees that none of the extra kinetic energy remains after the objects join to make them swing higher.

P9.64 (a) Use conservation of the horizontal component of momentum for the system of the shell, the cannon, and the carriage, from just before to just after the cannon firing.

$p_{xf} = p_{xi}$: $\quad m_{\text{shell}}v_{\text{shell}}\cos 45.0° + m_{\text{cannon}}v_{\text{recoil}} = 0$

$$(200)(125)\cos 45.0° + (5\,000)v_{\text{recoil}} = 0$$

or $\quad v_{\text{recoil}} = \boxed{-3.54 \text{ m/s}}$

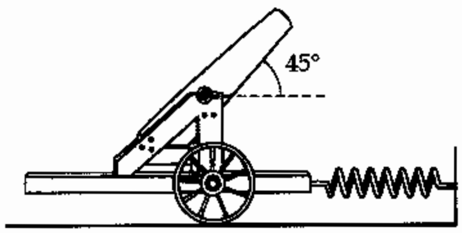


FIG. P9.64

(b) Use conservation of energy for the system of the cannon, the carriage, and the spring from right after the cannon is fired to the instant when the cannon comes to rest.

$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}$: $\quad 0 + 0 + \frac{1}{2}kx_{\max}^2 = \frac{1}{2}mv_{\text{recoil}}^2 + 0 + 0$

$$x_{\max} = \sqrt{\frac{mv_{\text{recoil}}^2}{k}} = \sqrt{\frac{(5\,000)(-3.54)^2}{2.00\times 10^4}} \text{ m} = \boxed{1.77 \text{ m}}$$

(c) $|F_{s,\max}| = kx_{\max}$ $\qquad |F_{s,\max}| = (2.00\times 10^4 \text{ N/m})(1.77 \text{ m}) = \boxed{3.54\times 10^4 \text{ N}}$

(d) No. The rail exerts a vertical external force (the normal force) on the cannon and prevents it from recoiling vertically. Momentum is not conserved in the vertical direction. The spring does not have time to stretch during the cannon firing. Thus, no external horizontal force is exerted on the system (cannon, carriage, and shell) from just before to just after firing. Momentum of this system is conserved in the horizontal direction during this interval.

P9.65 (a) Utilizing conservation of momentum,

$$m_1 v_{1A} = (m_1 + m_2) v_B$$

$$v_{1A} = \frac{m_1 + m_2}{m_1}\sqrt{2gh}$$

$$v_{1A} \cong \boxed{6.29 \text{ m/s}}$$

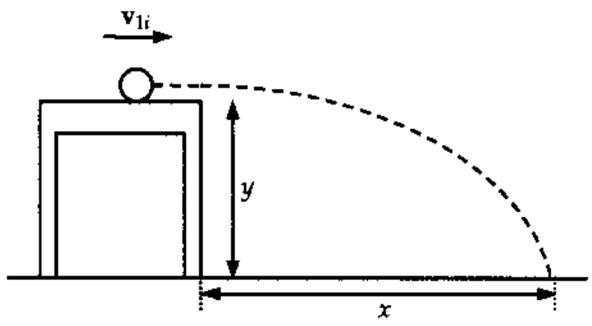


FIG. P9.65

(b) Utilizing the two equations,

$$\frac{1}{2}gt^2 = y \text{ and } x = v_{1A}t$$

we combine them to find

$$v_{1A} = \frac{x}{\sqrt{\frac{2y}{g}}}$$

From the data, $v_{1A} = \boxed{6.16 \text{ m/s}}$

Most of the 2% difference between the values for speed is accounted for by the uncertainty in the data, estimated as $\frac{0.01}{8.68} + \frac{0.1}{68.8} + \frac{1}{263} + \frac{1}{257} + \frac{0.1}{85.3} = 1.1\%$.

***P9.66** The ice cubes leave the track with speed determined by $mgy_i = \frac{1}{2}mv^2$;

$$v = \sqrt{2\left(9.8 \text{ m/s}^2\right)1.5 \text{ m}} = 5.42 \text{ m/s}.$$

Its speed at the apex of its trajectory is $5.42 \text{ m/s} \cos 40° = 4.15 \text{ m/s}$. For its collision with the wall we have

$$mv_i + F\Delta t = mv_f$$

$$0.005 \text{ kg } 4.15 \text{ m/s} + F\Delta t = 0.005 \text{ kg}\left(-\frac{1}{2}4.15 \text{ m/s}\right)$$

$$F\Delta t = -3.12 \times 10^{-2} \text{ kg} \cdot \text{m/s}$$

The impulse exerted by the cube on the wall is to the right, $+3.12 \times 10^{-2} \text{ kg} \cdot \text{m/s}$. Here F could refer to a large force over a short contact time. It can also refer to the average force if we interpret Δt as $\frac{1}{10}$ s, the time between one cube's tap and the next's.

$$F_{\text{av}} = \frac{3.12 \times 10^{-2} \text{ kg} \cdot \text{m/s}}{0.1 \text{ s}} = \boxed{0.312 \text{ N to the right}}$$

P9.67 (a) Find the speed when the bullet emerges from the block by using momentum conservation:

$$mv_i = MV_i + mv$$

The block moves a distance of 5.00 cm. Assume for an approximation that the block quickly reaches its maximum velocity, V_i, and the bullet kept going with a constant velocity, v. The block then compresses the spring and stops.

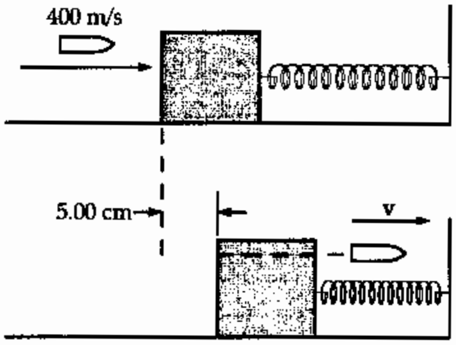


FIG. P9.67

$$\frac{1}{2}MV_i^2 = \frac{1}{2}kx^2$$

$$V_i = \sqrt{\frac{(900\ \text{N/m})(5.00\times10^{-2}\ \text{m})^2}{1.00\ \text{kg}}} = 1.50\ \text{m/s}$$

$$v = \frac{mv_i - MV_i}{m} = \frac{(5.00\times10^{-3}\ \text{kg})(400\ \text{m/s}) - (1.00\ \text{kg})(1.50\ \text{m/s})}{5.00\times10^{-3}\ \text{kg}}$$

$$v = \boxed{100\ \text{m/s}}$$

(b) $$\Delta E = \Delta K + \Delta U = \frac{1}{2}(5.00\times10^{-3}\ \text{kg})(100\ \text{m/s})^2 - \frac{1}{2}(5.00\times10^{-3}\ \text{kg})(400\ \text{m/s})^2 + \frac{1}{2}(900\ \text{N/m})(5.00\times10^{-2}\ \text{m})^2$$

$\Delta E = -374$ J, or there is an energy loss of $\boxed{374\ \text{J}}$.

***P9.68** The orbital speed of the Earth is

$$v_E = \frac{2\pi r}{T} = \frac{2\pi 1.496\times10^{11}\ \text{m}}{3.156\times10^{7}\ \text{s}} = 2.98\times10^{4}\ \text{m/s}$$

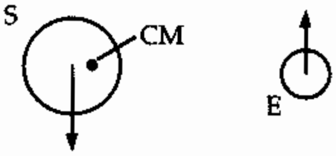


FIG. P9.68

In six months the Earth reverses its direction, to undergo momentum change

$$m_E|\Delta \mathbf{v}_E| = 2m_E v_E = 2(5.98\times10^{24}\ \text{kg})(2.98\times10^{4}\ \text{m/s}) = 3.56\times10^{25}\ \text{kg}\cdot\text{m/s}.$$

Relative to the center of mass, the sun always has momentum of the same magnitude in the opposite direction. Its 6-month momentum change is the same size, $m_S|\Delta \mathbf{v}_S| = 3.56\times10^{25}\ \text{kg}\cdot\text{m/s}$.

Then $|\Delta \mathbf{v}_S| = \dfrac{3.56\times10^{25}\ \text{kg}\cdot\text{m/s}}{1.991\times10^{30}\ \text{kg}} = \boxed{0.179\ \text{m/s}}$.

P9.69 (a) $\mathbf{p}_i + \mathbf{F}t = \mathbf{p}_f$: $(3.00\text{ kg})(7.00\text{ m/s})\hat{\mathbf{j}} + (12.0\text{ N}\hat{\mathbf{i}})(5.00\text{ s}) = (3.00\text{ kg})\mathbf{v}_f$

$$\mathbf{v}_f = \boxed{(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})\text{ m/s}}$$

(b) $\mathbf{a} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$: $\mathbf{a} = \frac{(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}} - 7.00\hat{\mathbf{j}})\text{ m/s}}{5.00\text{ s}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$

(c) $\mathbf{a} = \frac{\sum \mathbf{F}}{m}$: $\mathbf{a} = \frac{12.0\text{ N}\hat{\mathbf{i}}}{3.00\text{ kg}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$

(d) $\Delta\mathbf{r} = \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$: $\Delta\mathbf{r} = (7.00\text{ m/s}\,\hat{\mathbf{j}})(5.00\text{ s}) + \frac{1}{2}(4.00\text{ m/s}^2\,\hat{\mathbf{i}})(5.00\text{ s})^2$

$$\Delta\mathbf{r} = \boxed{(50.0\hat{\mathbf{i}} + 35.0\hat{\mathbf{j}})\text{ m}}$$

(e) $W = \mathbf{F}\cdot\Delta\mathbf{r}$: $W = (12.0\text{ N}\hat{\mathbf{i}})\cdot(50.0\text{ m}\hat{\mathbf{i}} + 35.0\text{ m}\hat{\mathbf{j}}) = \boxed{600\text{ J}}$

(f) $\frac{1}{2}mv_f^2 = \frac{1}{2}(3.00\text{ kg})(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})\cdot(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})\text{ m}^2/\text{s}^2$

$$\frac{1}{2}mv_f^2 = (1.50\text{ kg})(449\text{ m}^2/\text{s}^2) = \boxed{674\text{ J}}$$

(g) $\frac{1}{2}mv_i^2 + W = \frac{1}{2}(3.00\text{ kg})(7.00\text{ m/s})^2 + 600\text{ J} = \boxed{674\text{ J}}$

P9.70 We find the mass from $M = 360\text{ kg} - (2.50\text{ kg/s})t$.

We find the acceleration from $a = \frac{\text{Thrust}}{M} = \frac{v_e|dM/dt|}{M} = \frac{(1\,500\text{ m/s})(2.50\text{ kg/s})}{M} = \frac{3\,750\text{ N}}{M}$

We find the velocity and position according to Euler,

from $v_{\text{new}} = v_{\text{old}} + a(\Delta t)$

and $x_{\text{new}} = x_{\text{old}} + v(\Delta t)$

If we take $\Delta t = 0.132$ s, a portion of the output looks like this:

Time t(s)	Total mass (kg)	Acceleration $a(\text{m/s}^2)$	Speed, v (m/s)	Position x(m)
0.000	360.00	10.4167	0.0000	0.0000
0.132	359.67	10.4262	1.3750	0.1815
0.264	359.34	10.4358	2.7513	0.54467
...				
65.868	195.330	19.1983	916.54	27191
66.000	195.000	19.2308	919.08	27312
66.132	194.670	19.2634	921.61	27433
...				
131.736	30.660	122.3092	3687.3	152382
131.868	30.330	123.6400	3703.5	152871
132.000	30.000	125.0000	3719.8	153362

(a) The final speed is $v_f = \boxed{3.7\text{ km/s}}$

(b) The rocket travels $\boxed{153\text{ km}}$

P9.71 The force exerted by the table is equal to the change in momentum of each of the links in the chain.

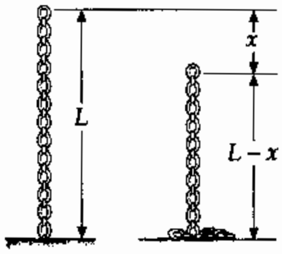


FIG. P9.71

By the calculus chain rule of derivatives,

$$F_1 = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}.$$

We choose to account for the change in momentum of each link by having it pass from our area of interest just before it hits the table, so that

$$v\frac{dm}{dt} \neq 0 \text{ and } m\frac{dv}{dt} = 0.$$

Since the mass per unit length is uniform, we can express each link of length dx as having a mass dm:

$$dm = \frac{M}{L}dx.$$

The magnitude of the force on the falling chain is the force that will be necessary to stop each of the elements dm.

$$F_1 = v\frac{dm}{dt} = v\left(\frac{M}{L}\right)\frac{dx}{dt} = \left(\frac{M}{L}\right)v^2$$

After falling a distance x, the square of the velocity of each link $v^2 = 2gx$ (from kinematics), hence

$$F_1 = \frac{2Mgx}{L}.$$

The links already on the table have a total length x, and their weight is supported by a force F_2:

$$F_2 = \frac{Mgx}{L}.$$

Hence, the *total* force on the chain is

$$F_{\text{total}} = F_1 + F_2 = \boxed{\frac{3Mgx}{L}}.$$

That is, *the total force is three times the weight of the chain on the table at that instant.*

P9.72 A picture one second later differs by showing five extra kilograms of sand moving on the belt.

(a) $\frac{\Delta p_x}{\Delta t} = \frac{(5.00 \text{ kg})(0.750 \text{ m/s})}{1.00 \text{ s}} = \boxed{3.75 \text{ N}}$

(b) The only horizontal force on the sand is belt friction,

so from $p_{xi} + f\Delta t = p_{xf}$ this is $f = \frac{\Delta p_x}{\Delta t} = \boxed{3.75 \text{ N}}$

(c) The belt is in equilibrium:

$\sum F_x = ma_x$: $+F_{\text{ext}} - f = 0$ and $F_{\text{ext}} = \boxed{3.75 \text{ N}}$

(d) $W = F\Delta r \cos\theta = 3.75 \text{ N}(0.750 \text{ m})\cos 0° = \boxed{2.81 \text{ J}}$

(e) $\frac{1}{2}(\Delta m)v^2 = \frac{1}{2}5.00 \text{ kg}(0.750 \text{ m/s})^2 = \boxed{1.41 \text{ J}}$

(f) Friction between sand and belt converts half of the input work into extra internal energy.

***P9.73** $x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_1\left(R + \frac{\ell}{2}\right) + m_2(0)}{m_1 + m_2} = \boxed{\frac{m_1\left(R + \frac{\ell}{2}\right)}{m_1 + m_2}}$

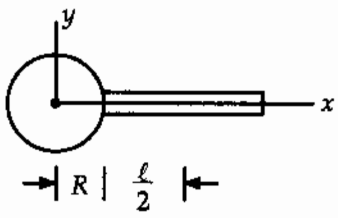


FIG. P9.73

ANSWERS TO EVEN PROBLEMS

P9.2 (a) 0; (b) $1.06 \text{ kg}\cdot\text{m/s}$; upward

P9.4 (a) 6.00 m/s to the left; (b) 8.40 J

P9.6 The force is 6.44 kN

P9.8 $1.39 \text{ kg}\cdot\text{m/s}$ upward

P9.10 (a) $5.40 \text{ N}\cdot\text{s}$ toward the net; (b) −27.0 J

P9.12 $\sim 10^3$ N upward

P9.14 (a) and (c) see the solution; (b) small; (d) large; (e) no difference

P9.16 1.67 m/s

P9.18 (a) 2.50 m/s; (b) 3.75×10^4 J

P9.20 0.556 m

P9.22 1.78 kN on the truck driver; 8.89 kN in the opposite direction on the car driver

P9.24 $v = \frac{4M}{m}\sqrt{g\ell}$

P9.26 7.94 cm

P9.28 (a) 2.88 m/s at 32.3°; (b) 783 J becomes internal energy

P9.30 $v_Y = v_i \sin\theta$; $v_O = v_i \cos\theta$

P9.32 No; his speed was 41.5 mi/h

P9.34 (a) $v = \frac{v_i}{\sqrt{2}}$; (b) 45.0° and −45.0°

P9.36 (a) $\sqrt{2}v_i$; $\sqrt{\frac{2}{3}}v_i$; (b) 35.3°

P9.38 (0, 1.00 m)

P9.40 4.67×10^{6} m from the Earth's center

P9.42 (a) see the solution; (b) 3.57×10^{8} J

P9.44 0.063 5L

P9.46 (a) see the solution;
(b) (−2.00 m, −1.00 m);
(c) $\left(3.00\hat{\mathbf{i}}-1.00\hat{\mathbf{j}}\right)$ m/s;
(d) $\left(15.0\hat{\mathbf{i}}-5.00\hat{\mathbf{j}}\right)$ kg·m/s

P9.48 (a) $-0.780\hat{\mathbf{i}}$ m/s; $1.12\hat{\mathbf{i}}$ m/s; (b) $0.360\hat{\mathbf{i}}$ m/s

P9.50 (a) 787 m/s; (b) 138 m/s

P9.52 see the solution

P9.54 (a) $\frac{m_1\mathbf{v}_1+m_2\mathbf{v}_2}{m_1+m_2}$;
(b) $(v_1-v_2)\sqrt{\frac{m_1m_2}{k(m_1+m_2)}}$;
(c) $\mathbf{v}_{1f}=\left(\frac{m_1-m_2}{m_1+m_2}\right)\mathbf{v}_1+\left(\frac{2m_2}{m_1+m_2}\right)\mathbf{v}_2$;
$\mathbf{v}_{2f}=\left(\frac{2m_1}{m_1+m_2}\right)\mathbf{v}_1+\left(\frac{m_2-m_1}{m_1+m_2}\right)\mathbf{v}_2$

P9.56 291 N

P9.58 $\left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}}$

P9.60 (a) −0.667 m/s; (b) 0.952 m

P9.62 (a) 6.81 m/s; (b) 1.00 m

P9.64 (a) −3.54 m/s; (b) 1.77 m; (c) 35.4 kN; (d) No. The rails exert a vertical force to change the momentum

P9.66 0.312 N to the right

P9.68 0.179 m/s

P9.70 (a) 3.7 km/s; (b) 153 km

P9.72 (a) 3.75 N to the right; (b) 3.75 N to the right; (c) 3.75 N; (d) 2.81 J; (e) 1.41 J; (f) Friction between sand and belt converts half of the input work into extra internal energy.

10

Rotation of a Rigid Object About a Fixed Axis

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q10.1 1 rev/min, or $\frac{\pi}{30}$ rad/s. Into the wall (clockwise rotation). $\alpha = 0$.

FIG. Q10.1

Q10.2 $+\hat{\mathbf{k}}$, $-\hat{\mathbf{k}}$

Q10.3 Yes, they are valid provided that ω is measured in degrees per second and α is measured in degrees per second-squared.

Q10.4 The speedometer will be inaccurate. The speedometer measures the number of revolutions per second of the tires. A larger tire will travel more distance in one full revolution as $2\pi r$.

Q10.5 Smallest I is about x axis and largest I is about y axis.

Q10.6 The moment of inertia would no longer be $\frac{ML^2}{12}$ if the mass was nonuniformly distributed, nor could it be calculated if the mass distribution was not known.

Q10.7 The object will start to rotate if the two forces act along different lines. Then the torques of the forces will not be equal in magnitude and opposite in direction.

Q10.8 No horizontal force acts on the pencil, so its center of mass moves straight down.

Q10.9 You could measure the time that it takes the hanging object, m, to fall a measured distance after being released from rest. Using this information, the linear acceleration of the mass can be calculated, and then the torque on the rotating object and its angular acceleration.

Q10.10 You could use $\omega = \alpha t$ and $v = at$. The equation $v = R\omega$ is valid in this situation since $a = R\alpha$.

Q10.11 The angular speed ω would decrease. The center of mass is farther from the pivot, but the moment of inertia increases also.

Q10.12 The moment of inertia depends on the distribution of mass with respect to a given axis. If the axis is changed, then each bit of mass that makes up the object is a different distance from the axis. In example 10.6 in the text, the moment of inertia of a uniform rigid rod about an axis perpendicular to the rod and passing through the center of mass is derived. If you spin a pencil back and forth about this axis, you will get a feeling for its stubbornness against changing rotation. Now change the axis about which you rotate it by spinning it back and forth about the axis that goes down the middle of the graphite. Easier, isn't it? The moment of inertia about the graphite is much smaller, as the mass of the pencil is concentrated near this axis.

Q10.13 Compared to an axis through the center of mass, any other parallel axis will have larger average squared distance from the axis to the particles of which the object is composed.

Q10.14 A quick flip will set the hard–boiled egg spinning faster and more smoothly. The raw egg loses mechanical energy to internal fluid friction.

Q10.15 $I_{CM} = MR^2$, $I_{CM} = MR^2$, $I_{CM} = \frac{1}{3}MR^2$, $I_{CM} = \frac{1}{2}MR^2$

Q10.16 Yes. If you drop an object, it will gain translational kinetic energy from decreasing gravitational potential energy.

Q10.17 No, just as an object need not be moving to have mass.

Q10.18 No, only if its angular momentum changes.

Q10.19 Yes. Consider a pendulum at its greatest excursion from equilibrium. It is momentarily at rest, but must have an angular acceleration or it would not oscillate.

Q10.20 Since the source reel stops almost instantly when the tape stops playing, the friction on the source reel axle must be fairly large. Since the source reel appears to us to rotate at almost constant angular velocity, the angular acceleration must be very small. Therefore, the torque on the source reel due to the tension in the tape must almost exactly balance the frictional torque. In turn, the frictional torque is nearly constant because kinetic friction forces don't depend on velocity, and the radius of the axle where the friction is applied is constant. Thus we conclude that the torque exerted by the tape on the source reel is essentially constant in time as the tape plays.

As the source reel radius R shrinks, the reel's angular speed $\omega = \frac{v}{R}$ must increase to keep the tape speed v constant. But the biggest change is to the reel's moment of inertia. We model the reel as a roll of tape, ignoring any spool or platter carrying the tape. If we think of the roll of tape as a uniform disk, then its moment of inertia is $I = \frac{1}{2}MR^2$. But the roll's mass is proportional to its base area πR^2. Thus, on the whole the moment of inertia is proportional to R^4. The moment of inertia decreases very rapidly as the reel shrinks!

The tension in the tape coming into the read-and-write heads is normally dominated by balancing frictional torque on the source reel, according to $TR \approx \tau_{\text{friction}}$. Therefore, as the tape plays the tension is largest when the reel is smallest. However, in the case of a sudden jerk on the tape, the rotational dynamics of the source reel becomes important. If the source reel is full, then the moment of inertia, proportional to R^4, will be so large that higher tension in the tape will be required to give the source reel its angular acceleration. If the reel is nearly empty, then the same tape acceleration will require a smaller tension. Thus, the tape will be more likely to break when the source reel is nearly full. One sees the same effect in the case of paper towels; it is easier to snap a towel free when the roll is new than when it is nearly empty.

Q10.21 The moment of inertia would decrease. This would result in a higher angular speed of the earth, shorter days, and more days in the year!

Q10.22 There is very little resistance to motion that can reduce the kinetic energy of the rolling ball. Even though there is static friction between the ball and the floor (if there were none, then no rotation would occur and the ball would slide), there is no relative motion of the two surfaces—by the definition of "rolling"—and so no force of kinetic friction acts to reduce K. Air resistance and friction associated with deformation of the ball eventually stop the ball.

Q10.23 In the frame of reference of the ground, no. Every point moves perpendicular to the line joining it to the instantaneous contact point. The contact point is not moving at all. The leading and trailing edges of the cylinder have velocities at 45° to the vertical as shown.

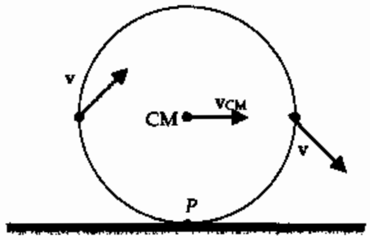


FIG. Q10.23

Q10.24 The sphere would reach the bottom first; the hoop would reach the bottom last. If each object has the same mass and the same radius, they all have the same torque due to gravity acting on them. The one with the smallest moment of inertia will thus have the largest angular acceleration and reach the bottom of the plane first.

Q10.25 To win the race, you want to decrease the moment of inertia of the wheels as much as possible. Small, light, solid disk-like wheels would be best!

SOLUTIONS TO PROBLEMS

Section 10.1 Angular Position, Velocity, and Acceleration

P10.1 (a)

$$\theta\big|_{t=0} = \boxed{5.00 \text{ rad}}$$

$$\omega\big|_{t=0} = \frac{d\theta}{dt}\bigg|_{t=0} = 10.0 + 4.00t\big|_{t=0} = \boxed{10.0 \text{ rad/s}}$$

$$\alpha_{t=0} = \frac{d\omega}{dt}\bigg|_{t=0} = \boxed{4.00 \text{ rad/s}^2}$$

(b)

$$\theta\big|_{t=3.00\text{ s}} = 5.00 + 30.0 + 18.0 = \boxed{53.0 \text{ rad}}$$

$$\omega\big|_{t=3.00\text{ s}} = \frac{d\theta}{dt}\bigg|_{t=3.00\text{ s}} = 10.0 + 4.00t\big|_{t=3.00\text{ s}} = \boxed{22.0 \text{ rad/s}}$$

$$\alpha\big|_{t=3.00\text{ s}} = \frac{d\omega}{dt}\bigg|_{t=3.00\text{ s}} = \boxed{4.00 \text{ rad/s}^2}$$

Section 10.2 Rotational Kinematics: Rotational Motion with Constant Angular Acceleration

***P10.2** $\omega_f = 2.51\times10^4 \text{ rev/min} = 2.63\times10^3 \text{ rad/s}$

(a) $\alpha = \dfrac{\omega_f - \omega_i}{t} = \dfrac{2.63\times10^3 \text{ rad/s} - 0}{3.2 \text{ s}} = \boxed{8.22\times10^2 \text{ rad/s}^2}$

(b) $\theta_f = \omega_i t + \dfrac{1}{2}\alpha t^2 = 0 + \dfrac{1}{2}\left(8.22\times10^2 \text{ rad/s}^2\right)(3.2 \text{ s})^2 = \boxed{4.21\times10^3 \text{ rad}}$

P10.3 (a) $\alpha = \dfrac{\omega - \omega_i}{t} = \dfrac{12.0 \text{ rad/s}}{3.00 \text{ s}} = \boxed{4.00 \text{ rad/s}^2}$

(b) $\theta = \omega_i t + \dfrac{1}{2}\alpha t^2 = \dfrac{1}{2}\left(4.00 \text{ rad/s}^2\right)(3.00 \text{ s})^2 = \boxed{18.0 \text{ rad}}$

P10.4 $\omega_i = 2\,000 \text{ rad/s}$, $\alpha = -80.0 \text{ rad/s}^2$

(a) $\omega_f = \omega_i + \alpha t = 2\,000 - (80.0)(10.0) = \boxed{1\,200 \text{ rad/s}}$

(b) $0 = \omega_i + \alpha t$

$t = \dfrac{\omega_i}{-\alpha} = \dfrac{2\,000}{80.0} = \boxed{25.0 \text{ s}}$

P10.5 $\omega_i = \dfrac{100 \text{ rev}}{1.00 \text{ min}}\left(\dfrac{1 \text{ min}}{60.0 \text{ s}}\right)\left(\dfrac{2\pi \text{ rad}}{1.00 \text{ rev}}\right) = \dfrac{10\pi}{3} \text{ rad/s}$, $\omega_f = 0$

(a) $t = \dfrac{\omega_f - \omega_i}{\alpha} = \dfrac{0 - \frac{10\pi}{3}}{-2.00} s = \boxed{5.24 \text{ s}}$

(b) $\theta_f = \bar{\omega} t = \left(\dfrac{\omega_f + \omega_i}{2}\right)t = \left(\dfrac{10\pi}{6} \text{ rad/s}\right)\left(\dfrac{10\pi}{6} \text{ s}\right) = \boxed{27.4 \text{ rad}}$

P10.6 $\omega_i = 3\,600 \text{ rev/min} = 3.77\times10^2 \text{ rad/s}$

$\theta = 50.0 \text{ rev} = 3.14\times10^2 \text{ rad}$ and $\omega_f = 0$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$0 = \left(3.77\times10^2 \text{ rad/s}\right)^2 + 2\alpha\left(3.14\times10^2 \text{ rad}\right)$

$\alpha = \boxed{-2.26\times10^2 \text{ rad/s}^2}$

P10.7 $\omega = 5.00 \text{ rev/s} = 10.0\pi \text{ rad/s}$. We will break the motion into two stages: (1) a period during which the tub speeds up and (2) a period during which it slows down.

While speeding up, $\theta_1 = \bar{\omega} t = \dfrac{0 + 10.0\pi \text{ rad/s}}{2}(8.00 \text{ s}) = 40.0\pi \text{ rad}$

While slowing down, $\theta_2 = \bar{\omega} t = \dfrac{10.0\pi \text{ rad/s} + 0}{2}(12.0 \text{ s}) = 60.0\pi \text{ rad}$

So, $\theta_{\text{total}} = \theta_1 + \theta_2 = 100\pi \text{ rad} = \boxed{50.0 \text{ rev}}$

P10.8 $\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$ and $\omega_f = \omega_i + \alpha t$ are two equations in two unknowns ω_i and α

$\omega_i = \omega_f - \alpha t$: $\theta_f - \theta_i = (\omega_f - \alpha t)t + \frac{1}{2}\alpha t^2 = \omega_f t - \frac{1}{2}\alpha t^2$

$$37.0 \text{ rev}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 98.0 \text{ rad/s}(3.00 \text{ s}) - \frac{1}{2}\alpha(3.00 \text{ s})^2$$

$232 \text{ rad} = 294 \text{ rad} - (4.50 \text{ s}^2)\alpha$: $\alpha = \frac{61.5 \text{ rad}}{4.50 \text{ s}^2} = \boxed{13.7 \text{ rad/s}^2}$

P10.9 (a) $\omega = \frac{\Delta\theta}{\Delta t} = \frac{1 \text{ rev}}{1 \text{ day}} = \frac{2\pi \text{ rad}}{86\,400 \text{ s}} = \boxed{7.27 \times 10^{-5} \text{ rad/s}}$

(b) $\Delta t = \frac{\Delta\theta}{\omega} = \frac{107°}{7.27 \times 10^{-5} \text{ rad/s}}\left(\frac{2\pi \text{ rad}}{360°}\right) = \boxed{2.57 \times 10^4 \text{ s}}$ or 428 min

***P10.10** The location of the dog is described by $\theta_d = (0.750 \text{ rad/s})t$. For the bone,

$$\theta_b = \frac{1}{3}2\pi \text{ rad} + \frac{1}{2}0.015 \text{ rad/s}^2 t^2.$$

We look for a solution to

$$0.75t = \frac{2\pi}{3} + 0.007\,5t^2$$
$$0 = 0.007\,5t^2 - 0.75t + 2.09 = 0$$
$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)2.09}}{0.015} = 2.88 \text{ s or } 97.1 \text{ s}$$

The dog and bone will also pass if $0.75t = \frac{2\pi}{3} - 2\pi + 0.007\,5t^2$ or if $0.75t = \frac{2\pi}{3} + 2\pi + 0.007\,5t^2$ that is, if either the dog or the turntable gains a lap on the other. The first equation has

$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)(-4.19)}}{0.015} = 105 \text{ s or } -5.30 \text{ s}$$

only one positive root representing a physical answer. The second equation has

$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)8.38}}{0.015} = 12.8 \text{ s or } 87.2 \text{ s}.$$

In order, the dog passes the bone at $\boxed{2.88 \text{ s}}$ after the merry-go-round starts to turn, and again at $\boxed{12.8 \text{ s}}$ and 26.6 s, after gaining laps on the bone. The bone passes the dog at 73.4 s, 87.2 s, 97.1 s, 105 s, and so on, after the start.

Section 10.3 **Angular and Linear Quantities**

P10.11 Estimate the tire's radius at 0.250 m and miles driven as 10 000 per year.

$$\theta = \frac{s}{r} = \frac{1.00\times10^4 \text{ mi}}{0.250 \text{ m}}\left(\frac{1\,609 \text{ m}}{1 \text{ mi}}\right) = 6.44\times10^7 \text{ rad/yr}$$

$$\theta = 6.44\times10^7 \text{ rad/yr}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = 1.02\times10^7 \text{ rev/yr or } \boxed{\sim10^7 \text{ rev/yr}}$$

P10.12 (a) $v = r\omega;\ \omega = \frac{v}{r} = \frac{45.0 \text{ m/s}}{250 \text{ m}} = \boxed{0.180 \text{ rad/s}}$

(b) $a_r = \frac{v^2}{r} = \frac{(45.0 \text{ m/s})^2}{250 \text{ m}} = \boxed{8.10 \text{ m/s}^2 \text{ toward the center of track}}$

P10.13 Given $r = 1.00$ m, $\alpha = 4.00 \text{ rad/s}^2$, $\omega_i = 0$ and $\theta_i = 57.3° = 1.00$ rad

(a) $\omega_f = \omega_i + \alpha t = 0 + \alpha t$

At $t = 2.00$ s, $\omega_f = 4.00 \text{ rad/s}^2 (2.00 \text{ s}) = \boxed{8.00 \text{ rad/s}}$

(b) $v = r\omega = 1.00 \text{ m}(8.00 \text{ rad/s}) = \boxed{8.00 \text{ m/s}}$

$|a_r| = a_c = r\omega^2 = 1.00 \text{ m}(8.00 \text{ rad/s})^2 = 64.0 \text{ m/s}^2$

$a_t = r\alpha = 1.00 \text{ m}\left(4.00 \text{ rad/s}^2\right) = 4.00 \text{ m/s}^2$

The magnitude of the total acceleration is:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{\left(64.0 \text{ m/s}^2\right)^2 + \left(4.00 \text{ m/s}^2\right)^2} = \boxed{64.1 \text{ m/s}^2}$$

The direction of the total acceleration vector makes an angle ϕ with respect to the radius to point *P*:

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{4.00}{64.0}\right) = \boxed{3.58°}$$

(c) $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = (1.00 \text{ rad}) + \frac{1}{2}\left(4.00 \text{ rad/s}^2\right)(2.00 \text{ s})^2 = \boxed{9.00 \text{ rad}}$

***P10.14** (a) Consider a tooth on the front sprocket. It gives this speed, relative to the frame, to the link of the chain it engages:

$$v = r\omega = \left(\frac{0.152\text{ m}}{2}\right)76\text{ rev/min}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{0.605\text{ m/s}}$$

(b) Consider the chain link engaging a tooth on the rear sprocket:

$$\omega = \frac{v}{r} = \frac{0.605\text{ m/s}}{\left(\frac{0.07\text{ m}}{2}\right)} = \boxed{17.3\text{ rad/s}}$$

(c) Consider the wheel tread and the road. A thread could be unwinding from the tire with this speed relative to the frame:

$$v = r\omega = \left(\frac{0.673\text{ m}}{2}\right)17.3\text{ rad/s} = \boxed{5.82\text{ m/s}}$$

(d) We did not need to know the length of the pedal cranks, but we could use that information to find the linear speed of the pedals:

$$v = r\omega = 0.175\text{ m }7.96\text{ rad/s}\left(\frac{1}{1\text{ rad}}\right) = 1.39\text{ m/s}$$

P10.15 (a) $\omega = \frac{v}{r} = \frac{25.0\text{ m/s}}{1.00\text{ m}} = \boxed{25.0\text{ rad/s}}$

(b) $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2(\Delta\theta)} = \frac{(25.0\text{ rad/s})^2 - 0}{2\left[(1.25\text{ rev})(2\pi\text{ rad/rev})\right]} = \boxed{39.8\text{ rad/s}^2}$$

(c) $\Delta t = \frac{\Delta\omega}{\alpha} = \frac{25.0\text{ rad/s}}{39.8\text{ rad/s}^2} = \boxed{0.628\text{ s}}$

P10.16 (a) $s = \bar{v}t = (11.0\text{ m/s})(9.00\text{ s}) = 99.0\text{ m}$

$$\theta = \frac{s}{r} = \frac{99.0\text{ m}}{0.290\text{ m}} = 341\text{ rad} = \boxed{54.3\text{ rev}}$$

(b) $\omega_f = \frac{v_f}{r} = \frac{22.0\text{ m/s}}{0.290\text{ m}} = 75.9\text{ rad/s} = \boxed{12.1\text{ rev/s}}$

P10.17 (a) $\omega = 2\pi f = \frac{2\pi\text{ rad}}{1\text{ rev}}\left(\frac{1\,200\text{ rev}}{60.0\text{ s}}\right) = \boxed{126\text{ rad/s}}$

(b) $v = \omega r = (126\text{ rad/s})(3.00\times 10^{-2}\text{ m}) = \boxed{3.77\text{ m/s}}$

(c) $a_c = \omega^2 r = (126)^2(8.00\times 10^{-2}) = 1\,260\text{ m/s}^2$ so $\mathbf{a}_r = \boxed{1.26\text{ km/s}^2\text{ toward the center}}$

(d) $s = r\theta = \omega r t = (126\text{ rad/s})(8.00\times 10^{-2}\text{ m})(2.00\text{ s}) = \boxed{20.1\text{ m}}$

P10.18 The force of static friction must act forward and then more and more inward on the tires, to produce both tangential and centripetal acceleration. Its tangential component is $m(1.70\text{ m/s}^2)$. Its radially inward component is $\frac{mv^2}{r}$. This takes the maximum value

$$m\omega_f^2 r = mr(\omega_i^2 + 2\alpha\Delta\theta) = mr\left(0 + 2\alpha\frac{\pi}{2}\right) = m\pi r\alpha = m\pi a_t = m\pi(1.70\text{ m/s}^2).$$

With skidding impending we have $\sum F_y = ma_y$, $+n - mg = 0$, $n = mg$

$$f_s = \mu_s n = \mu_s mg = \sqrt{m^2(1.70\text{ m/s}^2)^2 + m^2\pi^2(1.70\text{ m/s}^2)^2}$$

$$\mu_s = \frac{1.70\text{ m/s}^2}{g}\sqrt{1+\pi^2} = \boxed{0.572}$$

***P10.19** (a) Let R_E represent the radius of the Earth. The base of the building moves east at $v_1 = \omega R_E$ where ω is one revolution per day. The top of the building moves east at $v_2 = \omega(R_E + h)$. Its eastward speed relative to the ground is $v_2 - v_1 = \omega h$. The object's time of fall is given by $\Delta y = 0 + \frac{1}{2}gt^2$, $t = \sqrt{\frac{2h}{g}}$. During its fall the object's eastward motion is unimpeded so its deflection distance is $\Delta x = (v_2 - v_1)t = \omega h\sqrt{\frac{2h}{g}} = \boxed{\omega h^{3/2}\left(\frac{2}{g}\right)^{1/2}}$.

(b) $\frac{2\pi\text{ rad}}{86\,400\text{ s}}(50\text{ m})^{3/2}\left(\frac{2\text{ s}^2}{9.8\text{ m}}\right)^{1/2} = \boxed{1.16\text{ cm}}$

(c) The deflection is only 0.02% of the original height, so it is negligible in many practical cases.

Section 10.4 Rotational Energy

P10.20 $m_1 = 4.00\text{ kg}$, $r_1 = |y_1| = 3.00\text{ m}$;

$m_2 = 2.00\text{ kg}$, $r_2 = |y_2| = 2.00\text{ m}$;

$m_3 = 3.00\text{ kg}$, $r_3 = |y_3| = 4.00\text{ m}$;

$\omega = 2.00\text{ rad/s}$ about the x-axis

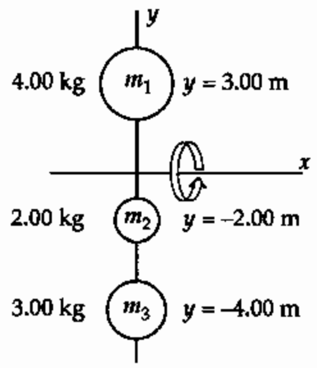


FIG. P10.20

(a) $$I_x = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

$$I_x = 4.00(3.00)^2 + 2.00(2.00)^2 + 3.00(4.00)^2 = \boxed{92.0\text{ kg}\cdot\text{m}^2}$$

$$K_R = \frac{1}{2}I_x\omega^2 = \frac{1}{2}(92.0)(2.00)^2 = \boxed{184\text{ J}}$$

(b) $$v_1 = r_1\omega = 3.00(2.00) = \boxed{6.00\text{ m/s}} \qquad K_1 = \frac{1}{2}m_1 v_1^2 = \frac{1}{2}(4.00)(6.00)^2 = 72.0\text{ J}$$

$$v_2 = r_2\omega = 2.00(2.00) = \boxed{4.00\text{ m/s}} \qquad K_2 = \frac{1}{2}m_2 v_2^2 = \frac{1}{2}(2.00)(4.00)^2 = 16.0\text{ J}$$

$$v_3 = r_3\omega = 4.00(2.00) = \boxed{8.00\text{ m/s}} \qquad K_3 = \frac{1}{2}m_3 v_3^2 = \frac{1}{2}(3.00)(8.00)^2 = 96.0\text{ J}$$

$$K = K_1 + K_2 + K_3 = 72.0 + 16.0 + 96.0 = \boxed{184\text{ J}} = \frac{1}{2}I_x\omega^2$$

P10.21 (a) $$I = \sum_j m_j r_j^2$$

In this case,

$$r_1 = r_2 = r_3 = r_4$$

$$r = \sqrt{(3.00\text{ m})^2 + (2.00\text{ m})^2} = \sqrt{13.0}\text{ m}$$

$$I = \left[\sqrt{13.0}\text{ m}\right]^2[3.00 + 2.00 + 2.00 + 4.00]\text{ kg}$$

$$= \boxed{143\text{ kg}\cdot\text{m}^2}$$

(b) $$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(143\text{ kg}\cdot\text{m}^2\right)(6.00\text{ rad/s})^2$$

$$= \boxed{2.57\times 10^3\text{ J}}$$

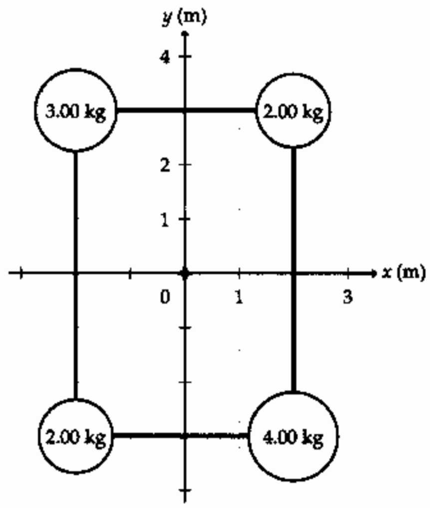


FIG. P10.21

P10.22 $I = Mx^2 + m(L-x)^2$

$$\frac{dI}{dx} = 2Mx - 2m(L-x) = 0 \text{ (for an extremum)}$$

$$\therefore x = \frac{mL}{M+m}$$

$\dfrac{d^2I}{dx^2} = 2m + 2M$; therefore I is minimum when the axis of rotation passes through $x = \dfrac{mL}{M+m}$ which is also the center of mass of the system. The moment of inertia about an axis passing through x is

$$I_{CM} = M\left[\frac{mL}{M+m}\right]^2 + m\left[1 - \frac{m}{M+m}\right]^2 L^2 = \frac{Mm}{M+m}L^2 = \mu L^2$$

where $\mu = \dfrac{Mm}{M+m}$.

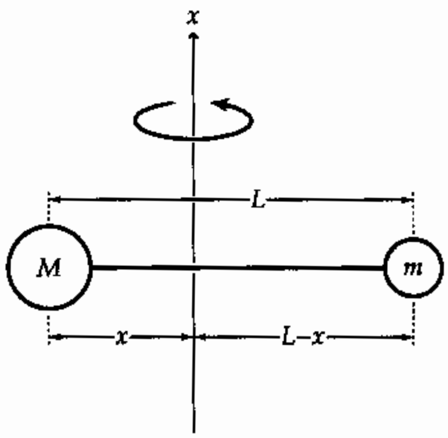


FIG. P10.22

Section 10.5 **Calculation of Moments of Inertia**

P10.23 We assume the rods are thin, with radius much less than L. Call the junction of the rods the origin of coordinates, and the axis of rotation the z-axis.

For the rod along the y-axis, $I = \frac{1}{3}mL^2$ from the table.

For the rod parallel to the z-axis, the parallel-axis theorem gives

$$I = \frac{1}{2}mr^2 + m\left(\frac{L}{2}\right)^2 \cong \frac{1}{4}mL^2$$

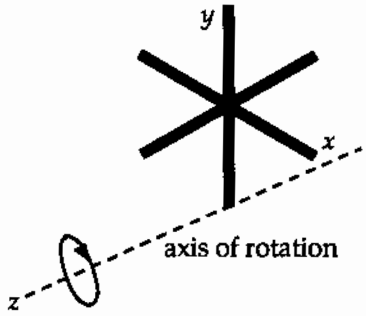


FIG. P10.23

In the rod along the x-axis, the bit of material between x and $x + dx$ has mass $\left(\frac{m}{L}\right)dx$ and is at distance $r = \sqrt{x^2 + \left(\frac{L}{2}\right)^2}$ from the axis of rotation. The total rotational inertia is:

$$I_{\text{total}} = \frac{1}{3}mL^2 + \frac{1}{4}mL^2 + \int_{-L/2}^{L/2}\left(x^2 + \frac{L^2}{4}\right)\left(\frac{m}{L}\right)dx$$

$$= \frac{7}{12}mL^2 + \left(\frac{m}{L}\right)\frac{x^3}{3}\Bigg|_{-L/2}^{L/2} + \frac{mL}{4}x\Bigg|_{-L/2}^{L/2}$$

$$= \frac{7}{12}mL^2 + \frac{mL^2}{12} + \frac{mL^2}{4} = \boxed{\frac{11mL^2}{12}}$$

Note: The moment of inertia of the rod along the x axis can also be calculated from the parallel-axis theorem as $\frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2$.

P10.24 Treat the tire as consisting of three parts. The two sidewalls are each treated as a hollow cylinder of inner radius 16.5 cm, outer radius 30.5 cm, and height 0.635 cm. The tread region is treated as a hollow cylinder of inner radius 30.5 cm, outer radius 33.0 cm, and height 20.0 cm.

Use $I=\frac{1}{2}m\left(R_1^2+R_2^2\right)$ for the moment of inertia of a hollow cylinder.

Sidewall:

$$m=\pi\left[(0.305\text{ m})^2-(0.165\text{ m})^2\right]\left(6.35\times10^{-3}\text{ m}\right)\left(1.10\times10^3\text{ kg/m}^3\right)=1.44\text{ kg}$$

$$I_{\text{side}}=\frac{1}{2}(1.44\text{ kg})\left[(0.165\text{ m})^2+(0.305\text{ m})^2\right]=8.68\times10^{-2}\text{ kg}\cdot\text{m}^2$$

Tread:

$$m=\pi\left[(0.330\text{ m})^2-(0.305\text{ m})^2\right](0.200\text{ m})\left(1.10\times10^3\text{ kg/m}^3\right)=11.0\text{ kg}$$

$$I_{\text{tread}}=\frac{1}{2}(11.0\text{ kg})\left[(0.330\text{ m})^2+(0.305\text{ m})^2\right]=1.11\text{ kg}\cdot\text{m}^2$$

Entire Tire:

$$I_{\text{total}}=2I_{\text{side}}+I_{\text{tread}}=2\left(8.68\times10^{-2}\text{ kg}\cdot\text{m}^2\right)+1.11\text{ kg}\cdot\text{m}^2=\boxed{1.28\text{ kg}\cdot\text{m}^2}$$

P10.25 Every particle in the door could be slid straight down into a high-density rod across its bottom, without changing the particle's distance from the rotation axis of the door. Thus, a rod 0.870 m long with mass 23.0 kg, pivoted about one end, has the same rotational inertia as the door:

$$I=\frac{1}{3}ML^2=\frac{1}{3}(23.0\text{ kg})(0.870\text{ m})^2=\boxed{5.80\text{ kg}\cdot\text{m}^2}.$$

The $\boxed{\text{height of the door is unnecessary}}$ data.

P10.26 Model your body as a cylinder of mass 60.0 kg and circumference 75.0 cm. Then its radius is

$$\frac{0.750\text{ m}}{2\pi}=0.120\text{ m}$$

and its moment of inertia is

$$\frac{1}{2}MR^2=\frac{1}{2}(60.0\text{ kg})(0.120\text{ m})^2=0.432\text{ kg}\cdot\text{m}^2\sim\boxed{10^0\text{ kg}\cdot\text{m}^2=1\text{ kg}\cdot\text{m}^2}.$$

P10.27 For a spherical shell $dI = \frac{2}{3}dmr^2 = \frac{2}{3}\left[\left(4\pi r^2 dr\right)\rho\right]r^2$

$$I = \int dI = \int \frac{2}{3}\left(4\pi r^2\right)r^2 \rho(r)dr$$

$$I = \int_0^R \frac{2}{3}\left(4\pi r^4\right)\left(14.2 - 11.6\frac{r}{R}\right)\left(10^3 \text{ kg/m}^3\right)dr$$

$$= \left(\frac{2}{3}\right)4\pi\left(14.2\times 10^3\right)\frac{R^5}{5} - \left(\frac{2}{3}\right)4\pi\left(11.6\times 10^3\right)\frac{R^5}{6}$$

$$I = \frac{8\pi}{3}\left(10^3\right)R^5\left(\frac{14.2}{5} - \frac{11.6}{6}\right)$$

$$M = \int dm = \int_0^R 4\pi r^2\left(14.2 - 11.6\frac{r}{R}\right)10^3\,dr$$

$$= 4\pi\times 10^3\left(\frac{14.2}{3} - \frac{11.6}{4}\right)R^3$$

$$\frac{I}{MR^2} = \frac{(8\pi/3)\left(10^3\right)R^5(14.2/5 - 11.6/6)}{4\pi\times 10^3 R^3 R^2(14.2/3 - 11.6/4)} = \frac{2}{3}\left(\frac{.907}{1.83}\right) = 0.330$$

$$\therefore I = \boxed{0.330MR^2}$$

***P10.28** (a) By similar triangles, $\frac{y}{x} = \frac{h}{L}$, $y = \frac{hx}{L}$. The area of the front face is $\frac{1}{2}hL$. The volume of the plate is $\frac{1}{2}hLw$. Its density is $\rho = \frac{M}{V} = \frac{M}{\frac{1}{2}hLw} = \frac{2M}{hLw}$. The mass of the ribbon is

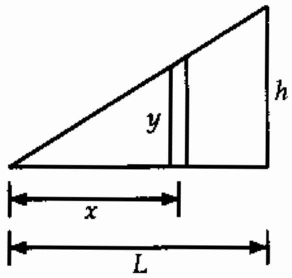


FIG. P10.28

$$dm = \rho dV = \rho y w dx = \frac{2Mywdx}{hLw} = \frac{2Mhx}{hLL}dx = \frac{2Mxdx}{L^2}.$$

The moment of inertia is

$$I = \int_{\text{all mass}} r^2 dm = \int_{x=0}^{L} x^2 \frac{2Mxdx}{L^2} = \frac{2M}{L^2}\int_0^L x^3 dx = \frac{2M}{L^2}\frac{L^4}{4} = \boxed{\frac{ML^2}{2}}.$$

(b) From the parallel axis theorem $I = I_{CM} + M\left(\frac{2L}{3}\right)^2 = I_{CM} + \frac{4ML^2}{9}$ and $I_h = I_{CM} + M\left(\frac{L}{3}\right)^2 = I_{CM} + \frac{ML^2}{9}$. The two triangles constitute a rectangle with moment of inertia $I_{CM} + \frac{4ML^2}{9} + I_{CM} + \frac{ML^2}{9} = \frac{1}{3}(2M)L^2$. Then $2I_{CM} = \frac{1}{9}ML^2$

$$I = I_{CM} + \frac{4ML^2}{9} = \frac{1}{18}ML^2 + \frac{8}{18}ML^2 = \boxed{\frac{1}{2}ML^2}.$$

***P10.29** We consider the cam as the superposition of the original solid disk and a disk of negative mass cut from it. With half the radius, the cut-away part has one-quarter the face area and one-quarter the volume and one-quarter the mass M_0 of the original solid cylinder:

$$M_0 - \frac{1}{4}M_0 = M \qquad M_0 = \frac{4}{3}M.$$

By the parallel-axis theorem, the original cylinder had moment of inertia

$$I_{CM} + M_0\left(\frac{R}{2}\right)^2 = \frac{1}{2}M_0R^2 + M_0\frac{R^2}{4} = \frac{3}{4}M_0R^2.$$

The negative-mass portion has $I = \frac{1}{2}\left(-\frac{1}{4}M_0\right)\left(\frac{R}{2}\right)^2 = -\frac{M_0R^2}{32}$. The whole cam has

$$I = \frac{3}{4}M_0R^2 - \frac{M_0R^2}{32} = \frac{23}{32}M_0R^2 = \frac{23}{32}\frac{4}{3}MR^2 = \frac{23}{24}MR^2 \text{ and } K = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{23}{24}MR^2\omega^2 = \boxed{\frac{23}{48}MR^2\omega^2}.$$

Section 10.6 Torque

P10.30 Resolve the 100 N force into components perpendicular to and parallel to the rod, as

$$F_{par} = (100 \text{ N})\cos 57.0° = 54.5 \text{ N}$$

and $F_{perp} = (100 \text{ N})\sin 57.0° = 83.9 \text{ N}$

The torque of F_{par} is zero since its line of action passes through the pivot point.

The torque of F_{perp} is $\tau = 83.9 \text{ N}(2.00 \text{ m}) = \boxed{168 \text{ N}\cdot\text{m}}$ (clockwise)

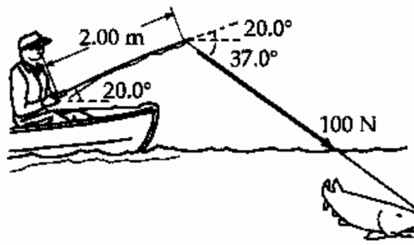


FIG. P10.30

P10.31 $\sum\tau = 0.100 \text{ m}(12.0 \text{ N}) - 0.250 \text{ m}(9.00 \text{ N}) - 0.250 \text{ m}(10.0 \text{ N}) = \boxed{-3.55 \text{ N}\cdot\text{m}}$

The thirty-degree angle is unnecessary information.

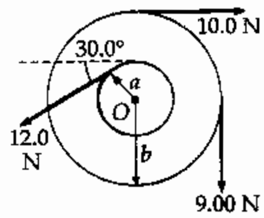


FIG. P10.31

P10.32 The normal force exerted by the ground on each wheel is

$$n = \frac{mg}{4} = \frac{(1\,500 \text{ kg})(9.80 \text{ m/s}^2)}{4} = 3\,680 \text{ N}$$

The torque of friction can be as large as

$$\tau_{max} = f_{max}r = (\mu_s n)r = (0.800)(3\,680 \text{ N})(0.300 \text{ m}) = \boxed{882 \text{ N}\cdot\text{m}}$$

The torque of the axle on the wheel can be equally as large as the light wheel starts to turn without slipping.

P10.33 In the previous problem we calculated the maximum torque that can be applied without skidding to be 882 N · m. This same torque is to be applied by the frictional force, f, between the brake pad and the rotor for this wheel. Since the wheel is slipping against the brake pad, we use the coefficient of kinetic friction to calculate the normal force.

$$\tau = fr = (\mu_k n)r, \text{ so } n = \frac{\tau}{\mu_k r} = \frac{882 \text{ N}\cdot\text{m}}{(0.500)(0.220 \text{ m})} = 8.02\times10^3 \text{ N} = \boxed{8.02 \text{ kN}}$$

Section 10.7 **Relationship Between Torque and Angular Acceleration**

P10.34 (a)

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2.00 \text{ kg})(7.00\times10^{-2} \text{ m})^2 = 4.90\times10^{-3} \text{ kg}\cdot\text{m}^2$$

$$\alpha = \frac{\tau}{I} = \frac{0.600}{4.90\times10^{-3}} = 122 \text{ rad/s}^2$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{1\,200\left(\frac{2\pi}{60}\right)}{122} = \boxed{1.03 \text{ s}}$$

(b)

$$\Delta\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(122 \text{ rad/s})(1.03 \text{ s})^2 = 64.7 \text{ rad} = \boxed{10.3 \text{ rev}}$$

P10.35 $m = 0.750$ kg, $F = 0.800$ N

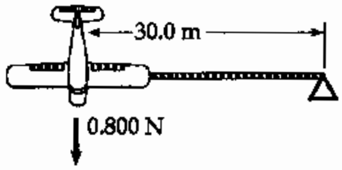


FIG. P10.35

(a) $\tau = rF = 30.0 \text{ m}(0.800 \text{ N}) = \boxed{24.0 \text{ N}\cdot\text{m}}$

(b) $\alpha = \frac{\tau}{I} = \frac{rF}{mr^2} = \frac{24.0}{0.750(30.0)^2} = \boxed{0.035\,6 \text{ rad/s}^2}$

(c) $a_t = \alpha r = 0.035\,6(30.0) = \boxed{1.07 \text{ m/s}^2}$

P10.36 $\omega_f = \omega_i + \alpha t$:

$$10.0 \text{ rad/s} = 0 + \alpha(6.00 \text{ s})$$

$$\alpha = \frac{10.00}{6.00} \text{ rad/s}^2 = 1.67 \text{ rad/s}^2$$

(a) $\sum\tau = 36.0 \text{ N}\cdot\text{m} = I\alpha$: $I = \frac{\sum\tau}{\alpha} = \frac{36.0 \text{ N}\cdot\text{m}}{1.67 \text{ rad/s}^2} = \boxed{21.6 \text{ kg}\cdot\text{m}^2}$

(b) $\omega_f = \omega_i + \alpha t$:

$$0 = 10.0 + \alpha(60.0)$$

$$\alpha = -0.167 \text{ rad/s}^2$$

$$|\tau| = |I\alpha| = (21.6 \text{ kg}\cdot\text{m}^2)(0.167 \text{ rad/s}^2) = \boxed{3.60 \text{ N}\cdot\text{m}}$$

(c) Number of revolutions $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$

During first 6.00 s $\quad \theta_f = \frac{1}{2}(1.67)(6.00)^2 = 30.1 \text{ rad}$

During next 60.0 s $\quad \theta_f = 10.0(60.0) - \frac{1}{2}(0.167)(60.0)^2 = 299 \text{ rad}$

$$\theta_{\text{total}} = 329 \text{ rad}\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{52.4 \text{ rev}}$$

P10.37 For m_1,

$\sum F_y = ma_y$: $+n - m_1 g = 0$

$n_1 = m_1 g = 19.6\text{ N}$

$f_{k1} = \mu_k n_1 = 7.06\text{ N}$

$\sum F_x = ma_x$: $-7.06\text{ N} + T_1 = (2.00\text{ kg})a$ (1)

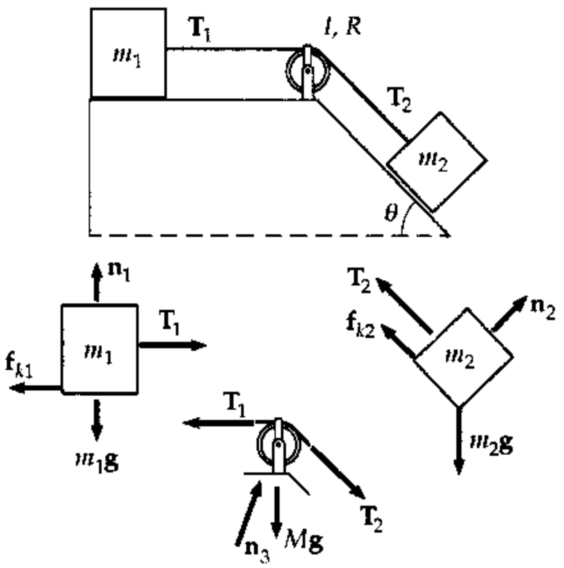


FIG. P10.37

For the pulley,

$\sum \tau = I\alpha$: $-T_1 R + T_2 R = \frac{1}{2} MR^2\left(\frac{a}{R}\right)$

$-T_1 + T_2 = \frac{1}{2}(10.0\text{ kg})a$

$-T_1 + T_2 = (5.00\text{ kg})a$ (2)

For m_2, $+n_2 - m_2 g\cos\theta = 0$

$n_2 = 6.00\text{ kg}(9.80\text{ m/s}^2)(\cos 30.0°)$

$= 50.9\text{ N}$

$f_{k2} = \mu_k n_2$

$= 18.3\text{ N}$: $-18.3\text{ N} - T_2 + m_2\sin\theta = m_2 a$

$-18.3\text{ N} - T_2 + 29.4\text{ N} = (6.00\text{ kg})a$ (3)

(a) Add equations (1), (2), and (3):

$$-7.06\text{ N} - 18.3\text{ N} + 29.4\text{ N} = (13.0\text{ kg})a$$

$$a = \frac{4.01\text{ N}}{13.0\text{ kg}} = \boxed{0.309\text{ m/s}^2}$$

(b) $T_1 = 2.00\text{ kg}(0.309\text{ m/s}^2) + 7.06\text{ N} = \boxed{7.67\text{ N}}$

$T_2 = 7.67\text{ N} + 5.00\text{ kg}(0.309\text{ m/s}^2) = \boxed{9.22\text{ N}}$

P10.38 $I = \frac{1}{2}mR^2 = \frac{1}{2}(100\text{ kg})(0.500\text{ m})^2 = 12.5\text{ kg}\cdot\text{m}^2$

$\omega_i = 50.0\text{ rev/min} = 5.24\text{ rad/s}$

$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{0 - 5.24\text{ rad/s}}{6.00\text{ s}} = -0.873\text{ rad/s}^2$

$\tau = I\alpha = 12.5\text{ kg}\cdot\text{m}^2(-0.873\text{ rad/s}^2) = -10.9\text{ N}\cdot\text{m}$

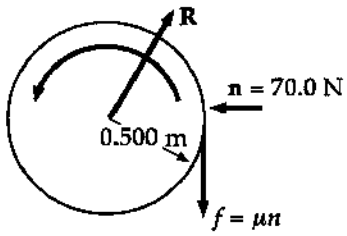


FIG. P10.38

The magnitude of the torque is given by $fR = 10.9\text{ N}\cdot\text{m}$, where f is the force of friction.

Therefore, $f = \frac{10.9\text{ N}\cdot\text{m}}{0.500\text{ m}}$ and $f = \mu_k n$

yields $\mu_k = \frac{f}{n} = \frac{21.8\text{ N}}{70.0\text{ N}} = \boxed{0.312}$

***P10.39** $\sum \tau = I\alpha = \frac{1}{2}MR^2\alpha$

$$-135\text{ N}(0.230\text{ m}) + T(0.230\text{ m}) = \frac{1}{2}(80\text{ kg})\left(\frac{1.25}{2}\text{ m}\right)^2\left(-1.67\text{ rad/s}^2\right)$$

$T = \boxed{21.5\text{ N}}$

Section 10.8 **Work, Power, and Energy in Rotational Motion**

P10.40 The moment of inertia of a thin rod about an axis through one end is $I = \frac{1}{3}ML^2$. The total rotational kinetic energy is given as

$$K_R = \frac{1}{2}I_h\omega_h^2 + \frac{1}{2}I_m\omega_m^2$$

with $I_h = \frac{m_hL_h^2}{3} = \frac{60.0\text{ kg}(2.70\text{ m})^2}{3} = 146\text{ kg}\cdot\text{m}^2$

and $I_m = \frac{m_mL_m^2}{3} = \frac{100\text{ kg}(4.50\text{ m})^2}{3} = 675\text{ kg}\cdot\text{m}^2$

In addition, $\omega_h = \frac{2\pi\text{ rad}}{12\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.45\times10^{-4}\text{ rad/s}$

while $\omega_m = \frac{2\pi\text{ rad}}{1\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.75\times10^{-3}\text{ rad/s}$

Therefore, $K_R = \frac{1}{2}(146)\left(1.45\times10^{-4}\right)^2 + \frac{1}{2}(675)\left(1.75\times10^{-3}\right)^2 = \boxed{1.04\times10^{-3}\text{ J}}$

***P10.41** The power output of the bus is $\wp = \frac{E}{\Delta t}$ where $E = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{1}{2}MR^2\omega^2$ is the stored energy and $\Delta t = \frac{\Delta x}{v}$ is the time it can roll. Then $\frac{1}{4}MR^2\omega^2 = \wp\Delta t = \frac{\wp\Delta x}{v}$ and

$$\Delta x = \frac{MR^2\omega^2 v}{4\wp} = \frac{1\,600\text{ kg}(0.65\text{ m})^2\left(4\,000\cdot\frac{2\pi}{60\text{ s}}\right)^2 11.1\text{ m/s}}{4(18\cdot746\text{ W})} = \boxed{24.5\text{ km}}.$$

P10.42 Work done $= F\Delta r = (5.57\text{ N})(0.800\text{ m}) = 4.46\text{ J}$

and Work $= \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$

(The last term is zero because the top starts from rest.)

Thus, $4.46\text{ J} = \frac{1}{2}\left(4.00\times10^{-4}\text{ kg}\cdot\text{m}^2\right)\omega_f^2$

and from this, $\omega_f = \boxed{149\text{ rad/s}}$.

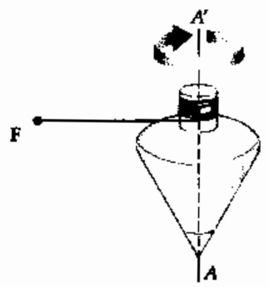


FIG. P10.42

***P10.43** (a) $I=\frac{1}{2}M\left(R_1^2+R_2^2\right)=\frac{1}{2}(0.35\text{ kg})\left[(0.02\text{ m})^2+(0.03\text{ m})^2\right]=2.28\times10^{-4}\text{ kg}\cdot\text{m}^2$

$$\left(K_1+K_2+K_{\text{rot}}+U_{g2}\right)_i-f_k\Delta x=\left(K_1+K_2+K_{\text{rot}}\right)_f$$

$$\frac{1}{2}(0.850\text{ kg})(0.82\text{ m/s})^2+\frac{1}{2}(0.42\text{ kg})(0.82\text{ m/s})^2+\frac{1}{2}\left(2.28\times10^{-4}\text{ kg}\cdot\text{m}^2\right)\left(\frac{0.82\text{ m/s}}{0.03\text{ m}}\right)^2$$
$$+0.42\text{ kg}\left(9.8\text{ m/s}^2\right)(0.7\text{ m})-0.25(0.85\text{ kg})\left(9.8\text{ m/s}^2\right)(0.7\text{ m})$$
$$=\frac{1}{2}(0.85\text{ kg})v_f^2+\frac{1}{2}(0.42\text{ kg})v_f^2+\frac{1}{2}\left(2.28\times10^{-4}\text{ kg}\cdot\text{m}^2\right)\left(\frac{v_f}{0.03\text{ m}}\right)^2$$

$$0.512\text{ J}+2.88\text{ J}-1.46\text{ J}=(0.761\text{ kg})v_f^2$$

$$v_f=\sqrt{\frac{1.94\text{ J}}{0.761\text{ kg}}}=\boxed{1.59\text{ m/s}}$$

(b) $\omega=\frac{v}{r}=\frac{1.59\text{ m/s}}{0.03\text{ m}}=\boxed{53.1\text{ rad/s}}$

P10.44 We assume the rod is thin. For the compound object

$$I=\frac{1}{3}M_{\text{rod}}L^2+\left[\frac{2}{5}m_{\text{ball}}R^2+M_{\text{ball}}D^2\right]$$
$$I=\frac{1}{3}1.20\text{ kg}(0.240\text{ m})^2+\frac{2}{5}2.00\text{ kg}\left(4.00\times10^{-2}\text{ m}\right)^2+2.00\text{ kg}(0.280\text{ m})^2$$
$$I=0.181\text{ kg}\cdot\text{m}^2$$

(a) $K_f+U_f=K_i+U_i+\Delta E$

$$\frac{1}{2}I\omega^2+0=0+M_{\text{rod}}g\left(\frac{L}{2}\right)+M_{\text{ball}}g(L+R)+0$$
$$\frac{1}{2}\left(0.181\text{ kg}\cdot\text{m}^2\right)\omega^2=1.20\text{ kg}\left(9.80\text{ m/s}^2\right)(0.120\text{ m})+2.00\text{ kg}\left(9.80\text{ m/s}^2\right)(0.280\text{ m})$$
$$\frac{1}{2}\left(0.181\text{ kg}\cdot\text{m}^2\right)\omega^2=\boxed{6.90\text{ J}}$$

(b) $\omega=\boxed{8.73\text{ rad/s}}$

(c) $v=r\omega=(0.280\text{ m})8.73\text{ rad/s}=\boxed{2.44\text{ m/s}}$

(d) $v_f^2=v_i^2+2a\left(y_f-y_i\right)$

$$v_f=\sqrt{0+2\left(9.80\text{ m/s}^2\right)(0.280\text{ m})}=2.34\text{ m/s}$$

The speed it attains in swinging is greater by $\frac{2.44}{2.34}=\boxed{1.043\,2\text{ times}}$

P10.45 (a) For the counterweight,

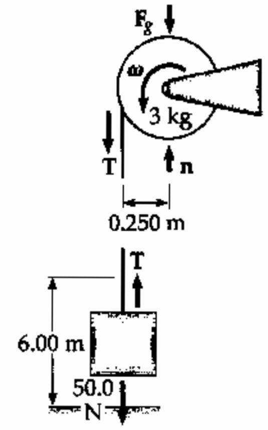


FIG. P10.45

$\sum F_y = ma_y$ becomes: $$50.0 - T = \left(\frac{50.0}{9.80}\right)a$$

For the reel $\sum \tau = I\alpha$ reads $TR = I\alpha = I\frac{a}{R}$

where $$I = \frac{1}{2}MR^2 = 0.093\,8\ \text{kg}\cdot\text{m}^2$$

We substitute to eliminate the acceleration:

$$50.0 - T = 5.10\left(\frac{TR^2}{I}\right)$$

$$T = \boxed{11.4\ \text{N}} \quad \text{and}$$

$$a = \frac{50.0 - 11.4}{5.10} = \boxed{7.57\ \text{m/s}^2}$$

$v_f^2 = v_i^2 + 2a(x_f - x_i)$: $$v_f = \sqrt{2(7.57)6.00} = \boxed{9.53\ \text{m/s}}$$

(b) Use conservation of energy for the system of the object, the reel, and the Earth:

$(K+U)_i = (K+U)_f$: $$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$2mgh = mv^2 + I\left(\frac{v^2}{R^2}\right) = v^2\left(m + \frac{I}{R^2}\right)$$

$$v = \sqrt{\frac{2mgh}{m + \frac{I}{R^2}}} = \sqrt{\frac{2(50.0\ \text{N})(6.00\ \text{m})}{5.10\ \text{kg} + \frac{0.093\,8}{(0.250)^2}}} = \boxed{9.53\ \text{m/s}}$$

P10.46 Choose the zero gravitational potential energy at the level where the masses pass.

$$K_f + U_{gf} = K_i + U_{gi} + \Delta E$$

$$\frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2 = 0 + m_1gh_{1i} + m_2gh_{2i} + 0$$

$$\frac{1}{2}(15.0 + 10.0)v^2 + \frac{1}{2}\left[\frac{1}{2}(3.00)R^2\right]\left(\frac{v}{R}\right)^2 = 15.0(9.80)(1.50) + 10.0(9.80)(-1.50)$$

$$\frac{1}{2}(26.5\ \text{kg})v^2 = 73.5\ \text{J} \Rightarrow v = \boxed{2.36\ \text{m/s}}$$

P10.47 From conservation of energy for the object-turntable-cylinder-Earth system,

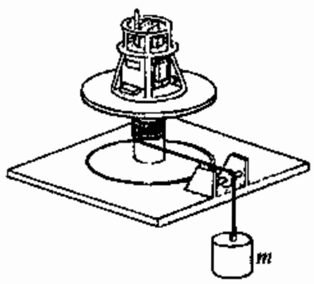


FIG. P10.47

$$\frac{1}{2}I\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2 = mgh$$

$$I\frac{v^2}{r^2} = 2mgh - mv^2$$

$$I = \boxed{mr^2\left(\frac{2gh}{v^2} - 1\right)}$$

P10.48 The moment of inertia of the cylinder is

$$I=\frac{1}{2}mr^2=\frac{1}{2}(81.6\text{ kg})(1.50\text{ m})^2=91.8\text{ kg}\cdot\text{m}^2$$

and the angular acceleration of the merry-go-round is found as

$$\alpha=\frac{\tau}{I}=\frac{(Fr)}{I}=\frac{(50.0\text{ N})(1.50\text{ m})}{\left(91.8\text{ kg}\cdot\text{m}^2\right)}=0.817\text{ rad/s}^2.$$

At $t=3.00$ s, we find the angular velocity

$$\omega=\omega_i+\alpha t$$

$$\omega=0+\left(0.817\text{ rad/s}^2\right)(3.00\text{ s})=2.45\text{ rad/s}$$

and $K=\frac{1}{2}I\omega^2=\frac{1}{2}\left(91.8\text{ kg}\cdot\text{m}^2\right)(2.45\text{ rad/s})^2=\boxed{276\text{ J}}$.

P10.49 (a) Find the velocity of the CM

$$(K+U)_i=(K+U)_f$$

$$0+mgR=\frac{1}{2}I\omega^2$$

$$\omega=\sqrt{\frac{2mgR}{I}}=\sqrt{\frac{2mgR}{\frac{3}{2}mR^2}}$$

$$v_{CM}=R\sqrt{\frac{4g}{3R}}=\boxed{2\sqrt{\frac{Rg}{3}}}$$

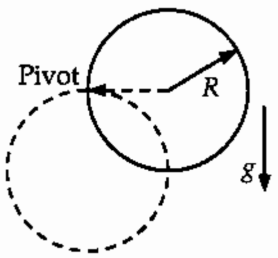


FIG. P10.49

(b) $v_L=2v_{CM}=\boxed{4\sqrt{\frac{Rg}{3}}}$

(c) $v_{CM}=\sqrt{\frac{2mgR}{2m}}=\boxed{\sqrt{Rg}}$

***P10.50** (a) The moment of inertia of the cord on the spool is

$$\frac{1}{2}M\left(R_1^2+R_2^2\right)=\frac{1}{2}0.1\text{ kg}\left((0.015\text{ m})^2+(0.09\text{ m})^2\right)=4.16\times10^{-4}\text{ kg}\cdot\text{m}^2.$$

The protruding strand has mass $\left(10^{-2}\text{ kg/m}\right)0.16\text{ m}=1.6\times10^{-3}\text{ kg}$ and

$$I=I_{CM}+Md^2=\frac{1}{12}ML^2+Md^2=1.6\times10^{-3}\text{ kg}\left(\frac{1}{12}(0.16\text{ m})^2+(0.09\text{ m}+0.08\text{ m})^2\right)$$

$$=4.97\times10^{-5}\text{ kg}\cdot\text{m}^2$$

For the whole cord, $I=4.66\times10^{-4}\text{ kg}\cdot\text{m}^2$. In speeding up, the average power is

$$\mathcal{P}=\frac{E}{\Delta t}=\frac{\frac{1}{2}I\omega^2}{\Delta t}=\frac{4.66\times10^{-4}\text{ kg}\cdot\text{m}^2}{2(0.215\text{ s})}\left(\frac{2\,500\cdot2\pi}{60\text{ s}}\right)^2=\boxed{74.3\text{ W}}$$

(b) $\mathcal{P}=\tau\omega=(7.65\text{ N})(0.16\text{ m}+0.09\text{ m})\left(\frac{2\,000\cdot2\pi}{60\text{ s}}\right)=\boxed{401\text{ W}}$

Section 10.9 Rolling Motion of a Rigid Object

P10.51 (a) $K_{\text{trans}} = \frac{1}{2}mv^2 = \frac{1}{2}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{500\text{ J}}$

(b) $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v^2}{r^2}\right) = \frac{1}{4}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{250\text{ J}}$

(c) $K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \boxed{750\text{ J}}$

P10.52 $W = K_f - K_i = (K_{\text{trans}} + K_{\text{rot}})_f - (K_{\text{trans}} + K_{\text{rot}})_i$

$$W = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 - 0 - 0 = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$

or $W = \boxed{\left(\frac{7}{10}\right)Mv^2}$

P10.53 (a) $\tau = I\alpha$

$$mgR\sin\theta = \left(I_{CM} + mR^2\right)\alpha$$

$$a = \frac{mgR^2\sin\theta}{I_{CM} + mR^2}$$

$$a_{\text{hoop}} = \frac{mgR^2\sin\theta}{2mR^2} = \boxed{\frac{1}{2}g\sin\theta}$$

$$a_{\text{disk}} = \frac{mgR^2\sin\theta}{\frac{3}{2}mR^2} = \boxed{\frac{2}{3}g\sin\theta}$$

The disk moves with $\frac{4}{3}$ the acceleration of the hoop.

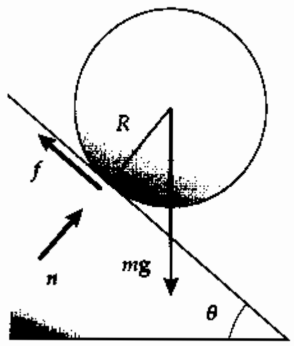


FIG. P10.53

(b) $Rf = I\alpha$

$$f = \mu n = \mu mg\cos\theta$$

$$\mu = \frac{f}{mg\cos\theta} = \frac{\frac{I\alpha}{R}}{mg\cos\theta} = \frac{\left(\frac{2}{3}g\sin\theta\right)\left(\frac{1}{2}mR^2\right)}{R^2 mg\cos\theta} = \boxed{\frac{1}{3}\tan\theta}$$

P10.54 $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2$ where $\omega = \frac{v}{R}$ since no slipping.

Also, $U_i = mgh$, $U_f = 0$, and $v_i = 0$

Therefore, $\frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2 = mgh$

Thus, $v^2 = \frac{2gh}{\left[1 + \left(\frac{I}{mR^2}\right)\right]}$

For a disk, $I = \frac{1}{2}mR^2$

So $v^2 = \frac{2gh}{1 + \frac{1}{2}}$ or $\boxed{v_{\text{disk}} = \sqrt{\frac{4gh}{3}}}$

For a ring, $I = mR^2$ so $v^2 = \frac{2gh}{2}$ or $\boxed{v_{\text{ring}} = \sqrt{gh}}$

Since $v_{\text{disk}} > v_{\text{ring}}$, $\boxed{\text{the disk}}$ reaches the bottom first.

P10.55 $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{3.00\text{ m}}{1.50\text{ s}} = 2.00\text{ m/s} = \frac{1}{2}(0 + v_f)$

$$v_f = 4.00\text{ m/s and } \omega_f = \frac{v_f}{r} = \frac{4.00\text{ m/s}}{(6.38\times10^{-2}\text{ m})/2} = \frac{8.00}{6.38\times10^{-2}}\text{ rad/s}$$

We ignore internal friction and suppose the can rolls without slipping.

$$(K_{\text{trans}} + K_{\text{rot}} + U_g)_i + \Delta E_{\text{mech}} = (K_{\text{trans}} + K_{\text{rot}} + U_g)_f$$

$$(0 + 0 + mgy_i) + 0 = \left(\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + 0\right)$$

$$0.215\text{ kg}(9.80\text{ m/s}^2)[(3.00\text{ m})\sin 25.0°] = \frac{1}{2}(0.215\text{ kg})(4.00\text{ m/s})^2 + \frac{1}{2}I\left(\frac{8.00}{6.38\times10^{-2}}\text{ rad/s}\right)^2$$

$$2.67\text{ J} = 1.72\text{ J} + (7\,860\text{ s}^{-2})I$$

$$I = \frac{0.951\text{ kg}\cdot\text{m}^2/\text{s}^2}{7\,860\text{ s}^{-2}} = \boxed{1.21\times10^{-4}\text{ kg}\cdot\text{m}^2}$$

The $\boxed{\text{height of the can}}$ is unnecessary data.

P10.56 (a) Energy conservation for the system of the ball and the Earth between the horizontal section and top of loop:

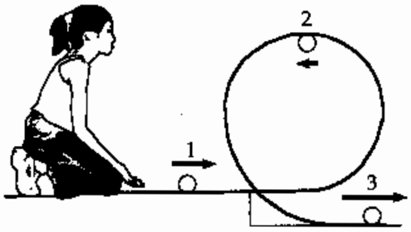


FIG. P10.56

$$\frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2$$

$$\frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_2}{r}\right)^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$\frac{5}{6}v_2^2 + gy_2 = \frac{5}{6}v_1^2$$

$$v_2 = \sqrt{v_1^2 - \frac{6}{5}gy_2} = \sqrt{(4.03\text{ m/s})^2 - \frac{6}{5}(9.80\text{ m/s}^2)(0.900\text{ m})} = \boxed{2.38\text{ m/s}}$$

The centripetal acceleration is $\frac{v_2^2}{r} = \frac{(2.38\text{ m/s})^2}{0.450\text{ m}} = 12.6\text{ m/s}^2 > g$

Thus, the ball must be in contact with the track, with the track pushing downward on it.

(b) $$\frac{1}{2}mv_3^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_3}{r}\right)^2 + mgy_3 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$v_3 = \sqrt{v_1^2 - \frac{6}{5}gy_3} = \sqrt{(4.03\text{ m/s})^2 - \frac{6}{5}(9.80\text{ m/s}^2)(-0.200\text{ m})} = \boxed{4.31\text{ m/s}}$$

(c) $$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2$$

$$v_2 = \sqrt{v_1^2 - 2gy_2} = \sqrt{(4.03\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(0.900\text{ m})} = \sqrt{-1.40\text{ m}^2/\text{s}^2}$$

This result is imaginary. In the case where the ball does not roll, the ball starts with less energy than in part (a) and $\boxed{\text{never makes it to the top}}$ of the loop.

Additional Problems

P10.57 $mg\dfrac{\ell}{2}\sin\theta = \dfrac{1}{3}m\ell^2\alpha$

$\alpha = \dfrac{3}{2}\dfrac{g}{\ell}\sin\theta$

$a_t = \left(\dfrac{3}{2}\dfrac{g}{\ell}\sin\theta\right)r$

Then $\left(\dfrac{3}{2}\dfrac{g}{\ell}\right)r > g\sin\theta$

for $r > \dfrac{2}{3}\ell$

$\therefore$ About $\boxed{\dfrac{1}{3}\text{ the length of the chimney}}$ will have a tangential acceleration greater than $g\sin\theta$.

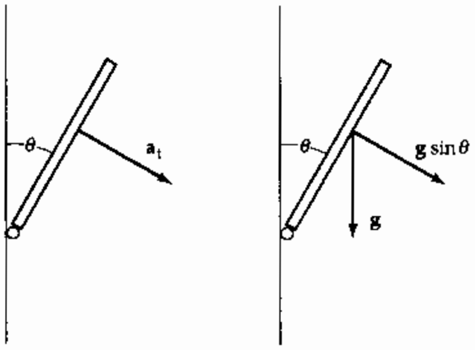


FIG. P10.57

P10.58 The resistive force on each ball is $R = D\rho Av^2$. Here $v = r\omega$, where r is the radius of each ball's path. The resistive torque on each ball is $\tau = rR$, so the total resistive torque on the three ball system is $\tau_{\text{total}} = 3rR$.

The power required to maintain a constant rotation rate is $\mathcal{P} = \tau_{\text{total}}\omega = 3rR\omega$. This required power may be written as

$$\mathcal{P} = \tau_{\text{total}}\omega = 3r\left[D\rho A(r\omega)^2\right]\omega = \left(3r^3DA\omega^3\right)\rho$$

With $$\omega = \frac{2\pi\text{ rad}}{1\text{ rev}}\left(\frac{10^3\text{ rev}}{1\text{ min}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = \frac{1\,000\pi}{30.0}\text{ rad/s}$$

$$\mathcal{P} = 3(0.100\text{ m})^3(0.600)\left(4.00\times10^{-4}\text{ m}^2\right)\left(\frac{1\,000\pi}{30.0\text{ s}}\right)^3\rho$$

or $\mathcal{P} = \left(0.827\ \text{m}^5/\text{s}^3\right)\rho$, where ρ is the density of the resisting medium.

(a) In air, $\rho = 1.20\ \text{kg/m}^3$,
and $\mathcal{P} = 0.827\ \text{m}^5/\text{s}^3\left(1.20\ \text{kg/m}^3\right) = 0.992\ \text{N}\cdot\text{m/s} = \boxed{0.992\text{ W}}$

(b) In water, $\rho = 1\,000\ \text{kg/m}^3$ and $\mathcal{P} = \boxed{827\text{ W}}$.

P10.59 (a) $W = \Delta K = \dfrac{1}{2}I\omega_f^2 - \dfrac{1}{2}I\omega_i^2 = \dfrac{1}{2}I\left(\omega_f^2 - \omega_i^2\right)$ where $I = \dfrac{1}{2}mR^2$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1.00\text{ kg})(0.500\text{ m})^2\left[(8.00\text{ rad/s})^2 - 0\right] = \boxed{4.00\text{ J}}$$

(b) $$t = \frac{\omega_f - 0}{\alpha} = \frac{\omega r}{a} = \frac{(8.00\text{ rad/s})(0.500\text{ m})}{2.50\text{ m/s}^2} = \boxed{1.60\text{ s}}$$

(c) $\theta_f = \theta_i + \omega_i t + \dfrac{1}{2}\alpha t^2$; $\theta_i = 0$; $\omega_i = 0$

$$\theta_f = \frac{1}{2}\alpha t^2 = \frac{1}{2}\left(\frac{2.50\text{ m/s}^2}{0.500\text{ m}}\right)(1.60\text{ s})^2 = 6.40\text{ rad}$$

$s = r\theta = (0.500\text{ m})(6.40\text{ rad}) = \boxed{3.20\text{ m} < 4.00\text{ m Yes}}$

*P10.60 The quantity of tape is constant. Then the area of the rings you see it fill is constant. This is expressed by

$\pi r_t^2 - \pi r_s^2 = \pi r^2 - \pi r_s^2 + \pi r_2^2 - \pi r_s^2$ or $r_2 = \sqrt{r_t^2 + r_s^2 - r^2}$ is the outer radius of spool 2.

(a) Where the tape comes off spool 1, $\omega_1 = \frac{v}{r}$. Where the tape joins spool 2, $\omega_2 = \frac{v}{r_2} = v\left(r_s^2 + r_t^2 - r^2\right)^{-1/2}$.

(b) At the start, $r = r_t$ and $r_2 = r_s$ so $\omega_1 = \frac{v}{r_t}$ and $\omega_2 = \frac{v}{r_s}$. The takeup reel must spin at maximum speed. At the end, $r = r_s$ and $r_2 = r_t$ so $\omega_2 = \frac{v}{r_t}$ and $\omega_1 = \frac{v}{r_s}$. The angular speeds are just reversed.

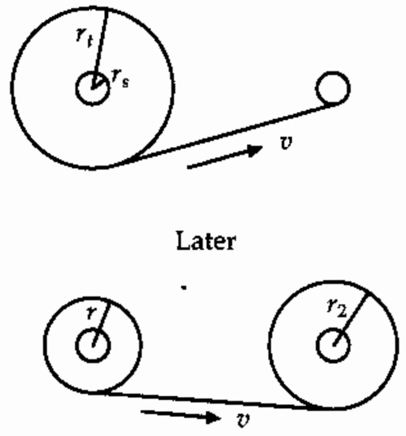


FIG. P10.60

P10.61 (a) Since only conservative forces act within the system of the rod and the Earth,

$$\Delta E = 0 \quad \text{so} \quad K_f + U_f = K_i + U_i$$

$$\frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{L}{2}\right)$$

where $I = \frac{1}{3}ML^2$

Therefore, $\boxed{\omega = \sqrt{\frac{3g}{L}}}$

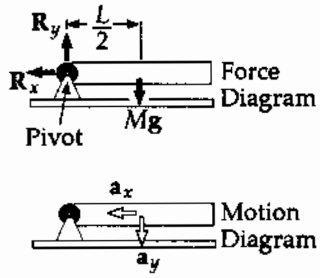


FIG. P10.61

(b) $\sum \tau = I\alpha$, so that in the horizontal orientation,

$$Mg\left(\frac{L}{2}\right) = \frac{ML^2}{3}\alpha$$

$$\boxed{\alpha = \frac{3g}{2L}}$$

(c) $a_x = a_r = -r\omega^2 = -\left(\frac{L}{2}\right)\omega^2 = \boxed{-\frac{3g}{2}}$ $\quad a_y = -a_t = -r\alpha = -\alpha\left(\frac{L}{2}\right) = \boxed{-\frac{3g}{4}}$

(d) Using Newton's second law, we have

$$R_x = Ma_x = \boxed{-\frac{3Mg}{2}}$$

$$R_y - Mg = Ma_y = -\frac{3Mg}{4} \qquad R_y = \boxed{\frac{Mg}{4}}$$

P10.62 $\alpha = -10.0\ \text{rad/s}^2 - \left(5.00\ \text{rad/s}^3\right)t = \frac{d\omega}{dt}$

$$\int_{65.0}^{\omega} d\omega = \int_0^t [-10.0 - 5.00t]dt = -10.0t - 2.50t^2 = \omega - 65.0\ \text{rad/s}$$

$$\omega = \frac{d\theta}{dt} = 65.0\ \text{rad/s} - \left(10.0\ \text{rad/s}^2\right)t - \left(2.50\ \text{rad/s}^3\right)t^2$$

(a) At $t = 3.00$ s,

$$\omega = 65.0\ \text{rad/s} - \left(10.0\ \text{rad/s}^2\right)(3.00\ \text{s}) - \left(2.50\ \text{rad/s}^3\right)\left(9.00\ \text{s}^2\right) = \boxed{12.5\ \text{rad/s}}$$

(b) $\int_0^{\theta} d\theta = \int_0^t \omega\, dt = \int_0^t \left[65.0\ \text{rad/s} - \left(10.0\ \text{rad/s}^2\right)t - \left(2.50\ \text{rad/s}^3\right)t^2\right]dt$

$\theta = (65.0\ \text{rad/s})t - \left(5.00\ \text{rad/s}^2\right)t^2 - \left(0.833\ \text{rad/s}^3\right)t^3$

At $t = 3.00$ s,

$$\theta = (65.0\ \text{rad/s})(3.00\ \text{s}) - \left(5.00\ \text{rad/s}^2\right)9.00\ \text{s}^2 - \left(0.833\ \text{rad/s}^3\right)27.0\ \text{s}^3$$

$$\theta = \boxed{128\ \text{rad}}$$

P10.63 The first drop has a velocity leaving the wheel given by $\frac{1}{2}mv_i^2 = mgh_1$, so

$$v_1 = \sqrt{2gh_1} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(0.540\ \text{m})} = 3.25\ \text{m/s}$$

The second drop has a velocity given by

$$v_2 = \sqrt{2gh_2} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(0.510\ \text{m})} = 3.16\ \text{m/s}$$

From $\omega = \frac{v}{r}$, we find

$$\omega_1 = \frac{v_1}{r} = \frac{3.25\ \text{m/s}}{0.381\ \text{m}} = 8.53\ \text{rad/s and } \omega_2 = \frac{v_2}{r} = \frac{3.16\ \text{m/s}}{0.381\ \text{m}} = 8.29\ \text{rad/s}$$

or

$$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{(8.29\ \text{rad/s})^2 - (8.53\ \text{rad/s})^2}{4\pi} = \boxed{-0.322\ \text{rad/s}^2}$$

P10.64 At the instant it comes off the wheel, the first drop has a velocity v_1, directed upward. The magnitude of this velocity is found from

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh_1 \text{ or } v_1 = \sqrt{2gh_1}$$

and the angular velocity of the wheel at the instant the first drop leaves is

$$\omega_1 = \frac{v_1}{R} = \sqrt{\frac{2gh_1}{R^2}}.$$

Similarly for the second drop: $v_2 = \sqrt{2gh_2}$ and $\omega_2 = \frac{v_2}{R} = \sqrt{\frac{2gh_2}{R^2}}$.

The angular acceleration of the wheel is then

$$a = \frac{\omega_2^2 - \omega_1^2}{2\theta} = \frac{\frac{2gh_2}{R^2} - \frac{2gh_1}{R^2}}{2(2\pi)} = \boxed{\frac{g(h_2 - h_1)}{2\pi R^2}}.$$

P10.65 $K_f = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2 : U_f = Mgh_f = 0;\ K_i = \frac{1}{2}Mv_i^2 + \frac{1}{2}I\omega_i^2 = 0$

$U_i = (Mgh)_i : f = \mu N = \mu Mg\cos\theta;\ \omega = \frac{v}{r};\ h = d\sin\theta$ and $I = \frac{1}{2}mr^2$

(a)

$$\Delta E = E_f - E_i \text{ or } -fd = K_f + U_f - K_i - U_i$$

$$-fd = \frac{1}{2}Mv_f^2 + \frac{1}{2}I\omega_f^2 - Mgh$$

$$-(\mu Mg\cos\theta)d = \frac{1}{2}Mv^2 + \left(\frac{mr^2}{2}\right)\frac{\frac{v^2}{r^2}}{2} - Mgd\sin\theta$$

$$\frac{1}{2}\left[M + \frac{m}{2}\right]v^2 = Mgd\sin\theta - (\mu Mg\cos\theta)d \text{ or}$$

$$v^2 = 2Mgd\frac{(\sin\theta - \mu\cos\theta)}{\frac{m}{2} + M}$$

$$v_d = \left[4gd\frac{M}{(m + 2M)}(\sin\theta - \mu\cos\theta)\right]^{1/2}$$

(b)

$$v_f^2 = v_i^2 + 2a\Delta x,\ v_d^2 = 2ad$$

$$a = \frac{v_d^2}{2d} = \boxed{2g\left(\frac{M}{m + 2M}\right)(\sin\theta - \mu\cos\theta)}$$

P10.66 (a) $E=\frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\omega^2\right)$

$$E=\frac{1}{2}\cdot\frac{2}{5}\left(5.98\times10^{24}\right)\left(6.37\times10^{6}\right)^2\left(\frac{2\pi}{86\,400}\right)^2=\boxed{2.57\times10^{29}\ \text{J}}$$

(b)
$$\frac{dE}{dt}=\frac{d}{dt}\left[\frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{2\pi}{T}\right)^2\right]$$
$$=\frac{1}{5}MR^2(2\pi)^2\left(-2T^{-3}\right)\frac{dT}{dt}$$
$$=\frac{1}{5}MR^2\left(\frac{2\pi}{T}\right)^2\left(\frac{-2}{T}\right)\frac{dT}{dt}$$
$$=\left(2.57\times10^{29}\ \text{J}\right)\left(\frac{-2}{86\,400\ \text{s}}\right)\left(\frac{10\times10^{-6}\ \text{s}}{3.16\times10^{7}\ \text{s}}\right)(86\,400\ \text{s/day})$$
$$\frac{dE}{dt}=\boxed{-1.63\times10^{17}\ \text{J/day}}$$

***P10.67** (a) $\omega_f=\omega_i+\alpha t$

$$\alpha=\frac{\omega_f-\omega_i}{t}=\frac{\frac{2\pi}{T_f}-\frac{2\pi}{T_i}}{t}=\frac{2\pi\left(T_i-T_f\right)}{T_iT_ft}$$
$$\sim\frac{2\pi\left(-10^{-3}\ \text{s}\right)}{1\ \text{d}\ 1\ \text{d}\ 100\ \text{yr}}\left(\frac{1\ \text{d}}{86\,400\ \text{s}}\right)^2\left(\frac{1\ \text{yr}}{3.156\times10^{7}\ \text{s}}\right)=\boxed{-10^{-22}\ \text{s}^{-2}}$$

(b) The Earth, assumed uniform, has moment of inertia

$$I=\frac{2}{5}MR^2=\frac{2}{5}\left(5.98\times10^{24}\ \text{kg}\right)\left(6.37\times10^{6}\ \text{m}\right)^2=9.71\times10^{37}\ \text{kg}\cdot\text{m}^2$$
$$\sum\tau=I\alpha\sim9.71\times10^{37}\ \text{kg}\cdot\text{m}^2\left(-2.67\times10^{-22}\ \text{s}^{-2}\right)=\boxed{-10^{16}\ \text{N}\cdot\text{m}}$$

The negative sign indicates clockwise, to slow the planet's counterclockwise rotation.

(c) $|\tau|=Fd$. Suppose the person can exert a 900-N force.

$$d=\frac{|\tau|}{F}=\frac{2.59\times10^{16}\ \text{N}\cdot\text{m}}{900\ \text{N}}\sim\boxed{10^{13}\ \text{m}}$$

This is the order of magnitude of the size of the planetary system.

P10.68 $\Delta\theta = \omega t$

$$t = \frac{\Delta\theta}{\omega} = \frac{\left(\frac{31.0^\circ}{360^\circ}\right)\text{rev}}{\frac{900\text{ rev}}{60\text{ s}}} = 0.005\,74\text{ s}$$

$$v = \frac{0.800\text{ m}}{0.005\,74\text{ s}} = \boxed{139\text{ m/s}}$$

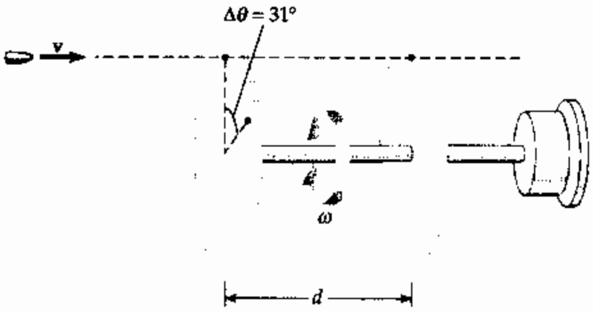


FIG. P10.68

P10.69 τ_f will oppose the torque due to the hanging object:

$$\sum \tau = I\alpha = TR - \tau_f: \qquad \tau_f = TR - I\alpha \qquad (1)$$

Now find T, I and α in given or known terms and substitute into equation (1).

$$\sum F_y = T - mg = -ma: \qquad T = m(g-a) \qquad (2)$$

$$\text{also } \Delta y = v_i t + \frac{at^2}{2} \qquad a = \frac{2y}{t^2} \qquad (3)$$

$$\text{and} \qquad \alpha = \frac{a}{R} = \frac{2y}{Rt^2}: \qquad (4)$$

$$I = \frac{1}{2}M\left[R^2 + \left(\frac{R}{2}\right)^2\right] = \frac{5}{8}MR^2 \qquad (5)$$

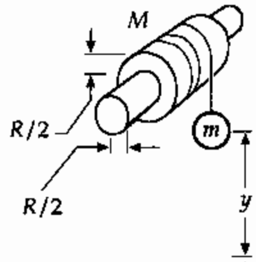


FIG. P10.69

Substituting (2), (3), (4), and (5) into (1),

we find
$$\tau_f = m\left(g - \frac{2y}{t^2}\right)R - \frac{5}{8}\frac{MR^2(2y)}{Rt^2} = \boxed{R\left[m\left(g - \frac{2y}{t^2}\right) - \frac{5}{4}\frac{My}{t^2}\right]}$$

P10.70 (a) $W = \Delta K + \Delta U$

$$W = K_f - K_i + U_f - U_i$$

$$0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - mgd\sin\theta - \frac{1}{2}kd^2$$

$$\frac{1}{2}\omega^2\left(I + mR^2\right) = mgd\sin\theta + \frac{1}{2}kd^2$$

$$\boxed{\omega = \sqrt{\frac{2mgd\sin\theta + kd^2}{I + mR^2}}}$$

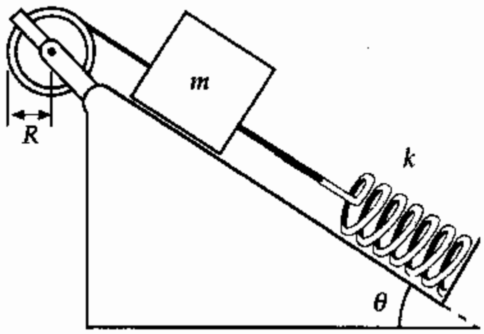


FIG. P10.70

(b)
$$\omega = \sqrt{\frac{2(0.500\text{ kg})\left(9.80\text{ m/s}^2\right)(0.200\text{ m})(\sin 37.0^\circ) + 50.0\text{ N/m}(0.200\text{ m})^2}{1.00\text{ kg}\cdot\text{m}^2 + 0.500\text{ kg}(0.300\text{ m})^2}}$$

$$\omega = \sqrt{\frac{1.18 + 2.00}{1.05}} = \sqrt{3.04} = \boxed{1.74\text{ rad/s}}$$

P10.71 (a) $m_2g - T_2 = m_2a$

$$T_2 = m_2(g-a) = 20.0\text{ kg}\left(9.80\text{ m/s}^2 - 2.00\text{ m/s}^2\right) = \boxed{156\text{ N}}$$

$$T_1 - m_1 g\sin 37.0° = m_1 a$$

$$T_1 = (15.0\text{ kg})(9.80\sin 37.0° + 2.00)\text{ m/s}^2 = \boxed{118\text{ N}}$$

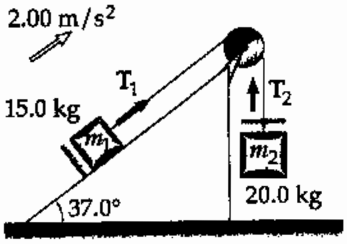


FIG. P10.71

(b) $(T_2 - T_1)R = I\alpha = I\left(\frac{a}{R}\right)$

$$I = \frac{(T_2-T_1)R^2}{a} = \frac{(156\text{ N} - 118\text{ N})(0.250\text{ m})^2}{2.00\text{ m/s}^2} = \boxed{1.17\text{ kg}\cdot\text{m}^2}$$

P10.72 For the board just starting to move,

$$\sum \tau = I\alpha: \qquad mg\left(\frac{\ell}{2}\right)\cos\theta = \left(\frac{1}{3}m\ell^2\right)\alpha$$

$$\alpha = \frac{3}{2}\left(\frac{g}{\ell}\right)\cos\theta$$

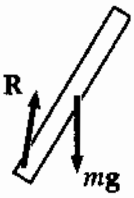


FIG. P10.72

The tangential acceleration of the end is $a_t = \ell\alpha = \frac{3}{2}g\cos\theta$

The vertical component is $a_y = a_t\cos\theta = \frac{3}{2}g\cos^2\theta$

If this is greater than g, the board will pull ahead of the ball falling:

(a) $\frac{3}{2}g\cos^2\theta \ge g$ gives $\cos^2\theta \ge \frac{2}{3}$ so $\cos\theta \ge \sqrt{\frac{2}{3}}$ and $\boxed{\theta \le 35.3°}$

(b) When $\theta = 35.3°$, the cup will land underneath the release-point of the ball if $r_c = \ell\cos\theta$

When $\ell = 1.00$ m, and $\theta = 35.3°$ $\qquad r_c = 1.00\text{ m}\sqrt{\frac{2}{3}} = 0.816\text{ m}$

so the cup should be $(1.00\text{ m} - 0.816\text{ m}) =$ $\boxed{0.184\text{ m from the moving end}}$

P10.73 At $t = 0$, $\omega = 3.50\text{ rad/s} = \omega_0 e^0$. Thus, $\omega_0 = 3.50$ rad/s

At $t = 9.30$ s, $\omega = 2.00\text{ rad/s} = \omega_0 e^{-\sigma(9.30\text{ s})}$, yielding $\sigma = 6.02\times10^{-2}\text{ s}^{-1}$

(a) $$\alpha = \frac{d\omega}{dt} = \frac{d\left(\omega_0 e^{-\sigma t}\right)}{dt} = \omega_0(-\sigma)e^{-\sigma t}$$

At $t = 3.00$ s,

$$\alpha = (3.50\text{ rad/s})\left(-6.02\times10^{-2}\text{ s}^{-1}\right)e^{-3.00\left(6.02\times10^{-2}\right)} = \boxed{-0.176\text{ rad/s}^2}$$

(b) $$\theta = \int_0^t \omega_0 e^{-\sigma t}dt = \frac{\omega_0}{-\sigma}\left[e^{-\sigma t} - 1\right] = \frac{\omega_0}{\sigma}\left[1 - e^{-\sigma t}\right]$$

At $t = 2.50$ s,

$$\theta = \frac{3.50\text{ rad/s}}{\left(6.02\times10^{-2}\right)1/\text{s}}\left[1 - e^{-\left(6.02\times10^{-2}\right)(2.50)}\right] = 8.12\text{ rad} = \boxed{1.29\text{ rev}}$$

(c) As $t\to\infty$, $\theta \to \frac{\omega_0}{\sigma}\left(1 - e^{-\infty}\right) = \frac{3.50\text{ rad/s}}{6.02\times10^{-2}\text{ s}^{-1}} = 58.2\text{ rad} = \boxed{9.26\text{ rev}}$

P10.74 Consider the total weight of each hand to act at the center of gravity (mid-point) of that hand. Then the total torque (taking CCW as positive) of these hands about the center of the clock is given by

$$\tau = -m_h g\left(\frac{L_h}{2}\right)\sin\theta_h - m_m g\left(\frac{L_m}{2}\right)\sin\theta_m = -\frac{g}{2}\left(m_h L_h \sin\theta_h + m_m L_m \sin\theta_m\right)$$

If we take $t = 0$ at 12 o'clock, then the angular positions of the hands at time t are

$$\theta_h = \omega_h t,$$

where $\omega_h = \frac{\pi}{6}$ rad/h

and $\theta_m = \omega_m t,$

where $\omega_m = 2\pi$ rad/h

Therefore, $\tau = -4.90 \text{ m/s}^2\left[60.0 \text{ kg}(2.70 \text{ m})\sin\left(\frac{\pi t}{6}\right) + 100 \text{ kg}(4.50 \text{ m})\sin 2\pi t\right]$

or $\tau = -794 \text{ N}\cdot\text{m}\left[\sin\left(\frac{\pi t}{6}\right) + 2.78\sin 2\pi t\right]$, where t is in hours.

(a) (i) At 3:00, $t = 3.00$ h,

so $\tau = -794 \text{ N}\cdot\text{m}\left[\sin\left(\frac{\pi}{2}\right) + 2.78\sin 6\pi\right] = \boxed{-794 \text{ N}\cdot\text{m}}$

(ii) At 5:15, $t = 5\text{ h} + \frac{15}{60}\text{ h} = 5.25\text{ h}$, and substitution gives:

$\tau = \boxed{-2\,510 \text{ N}\cdot\text{m}}$

(iii) At 6:00, $\tau = \boxed{0 \text{ N}\cdot\text{m}}$

(iv) At 8:20, $\tau = \boxed{-1\,160 \text{ N}\cdot\text{m}}$

(v) At 9:45, $\tau = \boxed{-2\,940 \text{ N}\cdot\text{m}}$

(b) The total torque is zero at those times when

$$\sin\left(\frac{\pi t}{6}\right) + 2.78\sin 2\pi t = 0$$

We proceed numerically, to find 0, 0.515 295 5, ..., corresponding to the times

12:00:00	12:30:55	12:58:19	1:32:31	1:57:01
2:33:25	2:56:29	3:33:22	3:56:55	4:32:24
4:58:14	5:30:52	6:00:00	6:29:08	7:01:46
7:27:36	8:03:05	8:26:38	9:03:31	9:26:35
10:02:59	10:27:29	11:01:41	11:29:05	

***P10.75** (a) As the bicycle frame moves forward at speed v, the center of each wheel moves forward at the same speed and the wheels turn at angular speed $\omega = \frac{v}{R}$. The total kinetic energy of the bicycle is

$$K = K_{\text{trans}} + K_{\text{rot}}$$

or

$$K = \frac{1}{2}(m_{\text{frame}} + 2m_{\text{wheel}})v^2 + 2\left(\frac{1}{2}I_{\text{wheel}}\omega^2\right) = \frac{1}{2}(m_{\text{frame}} + 2m_{\text{wheel}})v^2 + \left(\frac{1}{2}m_{\text{wheel}}R^2\right)\left(\frac{v^2}{R^2}\right).$$

This yields

$$K = \frac{1}{2}(m_{\text{frame}} + 3m_{\text{wheel}})v^2 = \frac{1}{2}[8.44\text{ kg} + 3(0.820\text{ kg})](3.35\text{ m/s})^2 = \boxed{61.2\text{ J}}.$$

(b) As the block moves forward with speed v, the top of each trunk moves forward at the same speed and the center of each trunk moves forward at speed $\frac{v}{2}$. The angular speed of each roller is $\omega = \frac{v}{2R}$. As in part (a), we have one object undergoing pure translation and two identical objects rolling without slipping. The total kinetic energy of the system of the stone and the trees is

$$K = K_{\text{trans}} + K_{\text{rot}}$$

or

$$K = \frac{1}{2}m_{\text{stone}}v^2 + 2\frac{1}{2}m_{\text{tree}}\left(\frac{v}{2}\right)^2 + 2\left(\frac{1}{2}I_{\text{tree}}\omega^2\right) = \frac{1}{2}\left(m_{\text{stone}} + \frac{1}{2}m_{\text{tree}}\right)v^2 + \left(\frac{1}{2}m_{\text{tree}}R^2\right)\left(\frac{v^2}{4R^2}\right).$$

This gives

$$K = \frac{1}{2}\left(m_{\text{stone}} + \frac{3}{4}m_{\text{tree}}\right)v^2 = \frac{1}{2}[844\text{ kg} + 0.75(82.0\text{ kg})](0.335\text{ m/s})^2 = \boxed{50.8\text{ J}}.$$

P10.76 Energy is conserved so $\Delta U + \Delta K_{\text{rot}} + \Delta K_{\text{trans}} = 0$

$$mg(R-r)(\cos\theta - 1) + \left[\frac{1}{2}mv^2 - 0\right] + \frac{1}{2}\left[\frac{2}{5}mr^2\right]\omega^2 = 0$$

Since $r\omega = v$, this gives

$$\omega = \sqrt{\frac{10}{7}\frac{(R-r)(1-\cos\theta)g}{r^2}}$$

or $\omega = \boxed{\sqrt{\frac{10Rg(1-\cos\theta)}{7r^2}}}$ since $R >> r$.

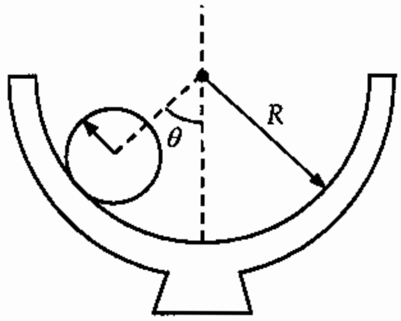


FIG. P10.76

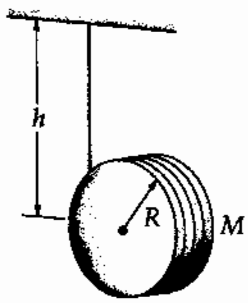


FIG. P10.77

P10.77 $\sum F = T - Mg = -Ma$: $\sum \tau = TR = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right)$

(a) Combining the above two equations we find

$$T = M(g - a)$$

and

$$a = \frac{2T}{M}$$

thus $\boxed{T = \frac{Mg}{3}}$

(b) $a = \frac{2T}{M} = \frac{2}{M}\left(\frac{Mg}{3}\right) = \boxed{\frac{2}{3}g}$

(c) $v_f^2 = v_i^2 + 2a(x_f - x_i)$ $\quad v_f^2 = 0 + 2\left(\frac{2}{3}g\right)(h - 0)$

$$v_f = \boxed{\sqrt{\frac{4gh}{3}}}$$

For comparison, from conservation of energy for the system of the disk and the Earth we have

$U_{gi} + K_{\text{rot}\,i} + K_{\text{trans}\,i} = U_{gf} + K_{\text{rot}\,f} + K_{\text{trans}\,f}$: $\quad Mgh + 0 + 0 = 0 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_f}{R}\right)^2 + \frac{1}{2}Mv_f^{\,2}$

$$v_f = \sqrt{\frac{4gh}{3}}$$

P10.78 (a) $\sum F_x = F - f = Ma$: $\sum \tau = fR = I\alpha$

Using $I = \frac{1}{2}MR^2$ and $\alpha = \frac{a}{R}$, we find $a = \boxed{\frac{2F}{3M}}$

(b) When there is no slipping, $f = \mu Mg$.

Substituting this into the torque equation of part (a), we have

$$\mu MgR = \frac{1}{2}MRa \text{ and } \mu = \boxed{\frac{F}{3Mg}}.$$

P10.79 (a) $\Delta K_{rot} + \Delta K_{trans} + \Delta U = 0$

Note that initially the center of mass of the sphere is a distance $h+r$ above the bottom of the loop; and as the mass reaches the top of the loop, this distance above the reference level is $2R-r$. The conservation of energy requirement gives

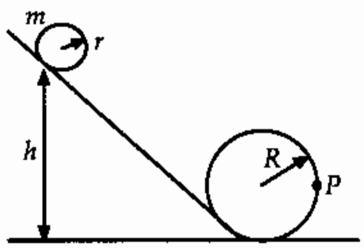


FIG. P10.79

$$mg(h+r) = mg(2R-r) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For the sphere $I = \frac{2}{5}mr^2$ and $v = r\omega$ so that the expression becomes

$$gh + 2gr = 2gR + \frac{7}{10}v^2 \qquad (1)$$

Note that $h = h_{min}$ when the speed of the sphere at the top of the loop satisfies the condition

$$\sum F = mg = \frac{mv^2}{(R-r)} \text{ or } v^2 = g(R-r)$$

Substituting this into Equation (1) gives

$$h_{min} = 2(R-r) + 0.700(R-r) \text{ or } \boxed{h_{min} = 2.70(R-r) = 2.70R}$$

(b) When the sphere is initially at $h = 3R$ and finally at point P, the conservation of energy equation gives

$$mg(3R+r) = mgR + \frac{1}{2}mv^2 + \frac{1}{5}mv^2, \text{ or}$$

$$v^2 = \frac{10}{7}(2R+r)g$$

Turning clockwise as it rolls without slipping past point P, the sphere is slowing down with counterclockwise angular acceleration caused by the torque of an upward force f of static friction. We have $\sum F_y = ma_y$ and $\sum \tau = I\alpha$ becoming $f - mg = -m\alpha r$ and $fr = \left(\frac{2}{5}\right)mr^2\alpha$.

Eliminating f by substitution yields $\alpha = \frac{5g}{7r}$ so that $\sum F_y = \boxed{-\frac{5}{7}mg}$

$$\sum F_x = -n = -\frac{mv^2}{R-r} = -\frac{\left(\frac{10}{7}\right)(2R+r)}{R-r}mg = \boxed{\frac{-20mg}{7}} \text{ (since } R >> r\text{)}$$

P10.80 Consider the free-body diagram shown. The sum of torques about the chosen pivot is

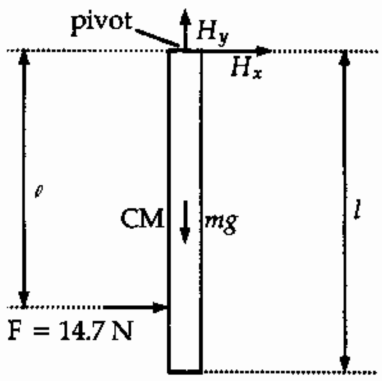


FIG. P10.80

$$\sum \tau = I\alpha \Rightarrow F\ell = \left(\frac{1}{3}ml^2\right)\left(\frac{a_{\text{CM}}}{\frac{l}{2}}\right) = \left(\frac{2}{3}ml\right)a_{\text{CM}} \qquad (1)$$

(a) $\ell = l = 1.24$ m: In this case, Equation (1) becomes

$$a_{\text{CM}} = \frac{3F}{2m} = \frac{3(14.7\text{ N})}{2(0.630\text{ kg})} = 35.0\text{ m/s}^2$$

$$\sum F_x = ma_{\text{CM}} \Rightarrow F + H_x = ma_{\text{CM}} \text{ or } H_x = ma_{\text{CM}} - F$$

Thus, $H_x = (0.630\text{ kg})(35.0\text{ m/s}^2) - 14.7\text{ N} = +7.35\text{ N}$ or

$\mathbf{H}_x = \boxed{7.35\hat{\mathbf{i}}\text{ N}}$.

(b) $\ell = \frac{1}{2} = 0.620$ m: For this situation, Equation (1) yields

$$a_{\text{CM}} = \frac{3F}{4m} = \frac{3(14.7\text{ N})}{4(0.630\text{ kg})} = 17.5\text{ m/s}^2.$$

Again, $\sum F_x = ma_{\text{CM}} \Rightarrow H_x = ma_{\text{CM}} - F$, so

$$H_x = (0.630\text{ kg})(17.5\text{ m/s}^2) - 14.7\text{ N} = -3.68\text{ N or } \mathbf{H}_x = \boxed{-3.68\hat{\mathbf{i}}\text{ N}}.$$

(c) If $H_x = 0$, then $\sum F_x = ma_{\text{CM}} \Rightarrow F = ma_{\text{CM}}$, or $a_{\text{CM}} = \frac{F}{m}$.

Thus, Equation (1) becomes

$$F\ell = \left(\frac{2}{3}ml\right)\left(\frac{F}{m}\right) \text{ so } \ell = \frac{2}{3}l = \frac{2}{3}(1.24\text{ m}) = \boxed{0.827\text{ m (from the top)}}.$$

P10.81 Let the ball have mass m and radius r. Then $I = \frac{2}{5}mr^2$. If the ball takes four seconds to go down twenty-meter alley, then $\bar{v} = 5$ m/s. The translational speed of the ball will decrease somewhat as the ball loses energy to sliding friction and some translational kinetic energy is converted to rotational kinetic energy; but its speed will always be on the order of 5.00 m/s, including at the starting point.

As the ball slides, the kinetic friction force exerts a torque on the ball to increase the angular speed. When $\omega = \frac{v}{r}$, the ball has achieved pure rolling motion, and kinetic friction ceases. To determine the elapsed time before pure rolling motion is achieved, consider:

$$\sum \tau = I\alpha \Rightarrow (\mu_k mg)r = \left(\frac{2}{5}mr^2\right)\left[\frac{(5.00\text{ m/s})/r}{t}\right] \text{ which gives}$$

$$t = \frac{2(5.00\text{ m/s})}{5\mu_k g} = \frac{2.00\text{ m/s}}{\mu_k g}$$

Note that the mass and radius of the ball have canceled. If $\mu_k = 0.100$ for the polished alley, the sliding distance will be given by

$$\Delta x = \bar{v}t = (5.00\text{ m/s})\left[\frac{2.00\text{ m/s}}{(0.100)(9.80\text{ m/s}^2)}\right] = 10.2\text{ m or } \Delta x \sim \boxed{10^1\text{ m}}.$$

P10.82 Conservation of energy between apex and the point where the grape leaves the surface:

$$mg\Delta y = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

$$mgR(1-\cos\theta) = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v_f}{R}\right)^2$$

which gives $g(1-\cos\theta) = \frac{7}{10}\left(\frac{v_f^2}{R}\right)$ (1)

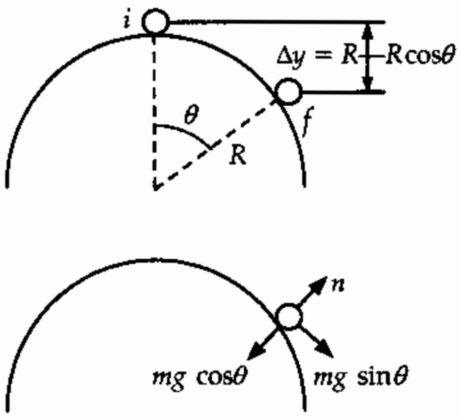


FIG. P10.82

Consider the radial forces acting on the grape:

$$mg\cos\theta - n = \frac{mv_f^2}{R}.$$

At the point where the grape leaves the surface, $n \to 0$.

Thus, $mg\cos\theta = \frac{mv_f^2}{R}$ or $\frac{v_f^2}{R} = g\cos\theta$.

Substituting this into Equation (1) gives

$g - g\cos\theta = \frac{7}{10}g\cos\theta$ or $\cos\theta = \frac{10}{17}$ and $\theta = \boxed{54.0°}$.

P10.83 (a) There are not any horizontal forces acting on the rod, so the center of mass will not move horizontally. Rather, the center of mass drops straight downward (distance $h/2$) with the rod rotating about the center of mass as it falls. From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}Mv_{CM}^2 + \frac{1}{2}\left(\frac{1}{12}Mh^2\right)\left(\frac{v_{CM}}{\frac{h}{2}}\right)^2 = Mg\left(\frac{h}{2}\right) \text{ which reduces to}$$

$$v_{CM} = \boxed{\sqrt{\frac{3gh}{4}}}$$

(b) In this case, the motion is a pure rotation about a fixed pivot point (the lower end of the rod) with the center of mass moving in a circular path of radius $h/2$. From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}\left(\frac{1}{3}Mh^2\right)\left(\frac{v_{CM}}{\frac{h}{2}}\right)^2 = Mg\left(\frac{h}{2}\right) \text{ which reduces to}$$

$$v_{CM} = \boxed{\sqrt{\frac{3gh}{4}}}$$

P10.84 (a) The mass of the roll decreases as it unrolls. We have $m=\frac{Mr^2}{R^2}$ where M is the initial mass of the roll. Since $\Delta E=0$, we then have $\Delta U_g+\Delta K_{\text{trans}}+\Delta K_{\text{rot}}=0$. Thus, when $I=\frac{mr^2}{2}$,

$$(mgr-MgR)+\frac{mv^2}{2}+\left[\frac{mr^2}{2}\frac{\omega^2}{2}\right]=0$$

Since $\omega r=v$, this becomes $v=\boxed{\sqrt{\frac{4g\left(R^3-r^3\right)}{3r^2}}}$

(b) Using the given data, we find $v=\boxed{5.31\times10^4\text{ m/s}}$

(c) We have assumed that $\Delta E=0$. When the roll gets to the end, we will have an inelastic collision with the surface. $\boxed{\text{The energy goes into internal energy}}$. With the assumption we made, there are problems with this question. It would take an infinite time to unwrap the tissue since $dr\to0$. Also, as r approaches zero, the velocity of the center of mass approaches infinity, which is physically impossible.

P10.85 (a) $\sum F_x=F+f=Ma_{\text{CM}}$

$\sum\tau=FR-fR=I\alpha$

$FR-(Ma_{\text{CM}}-F)R=\frac{Ia_{\text{CM}}}{R}$ $\qquad \boxed{a_{\text{CM}}=\frac{4F}{3M}}$

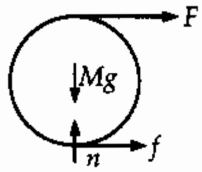


FIG. P10.85

(b) $f=Ma_{\text{CM}}-F=M\left(\frac{4F}{3M}\right)-F=\boxed{\frac{1}{3}F}$

(c) $v_f^2=v_i^2+2a\left(x_f-x_i\right)$

$v_f=\boxed{\sqrt{\frac{8Fd}{3M}}}$

P10.86 Call f_t the frictional force exerted by each roller backward on the plank. Name as f_b the rolling resistance exerted backward by the ground on each roller. Suppose the rollers are equally far from the ends of the plank.

FIG. P10.86

For the plank,

$$\sum F_x = ma_x \qquad 6.00\text{ N} - 2f_t = (6.00\text{ kg})a_p$$

The center of each roller moves forward only half as far as the plank. Each roller has acceleration $\frac{a_p}{2}$ and angular acceleration

$$\frac{a_p/2}{(5.00\text{ cm})} = \frac{a_p}{(0.100\text{ m})}$$

Then for each,

$$\sum F_x = ma_x \qquad +f_t - f_b = (2.00\text{ kg})\frac{a_p}{2}$$

$$\sum \tau = I\alpha \qquad f_t(5.00\text{ cm}) + f_b(5.00\text{ cm}) = \frac{1}{2}(2.00\text{ kg})(5.00\text{ cm})^2\frac{a_p}{10.0\text{ cm}}$$

So $\quad f_t + f_b = \left(\frac{1}{2}\text{ kg}\right)a_p$

Add to eliminate f_b:

$$2f_t = (1.50\text{ kg})a_p$$

(a) And $6.00\text{ N} - (1.50\text{ kg})a_p = (6.00\text{ kg})a_p$

$$a_p = \frac{(6.00\text{ N})}{(7.50\text{ kg})} = \boxed{0.800\text{ m/s}^2}$$

For each roller, $a = \frac{a_p}{2} = \boxed{0.400\text{ m/s}^2}$

(b) Substituting back, $2f_t = (1.50\text{ kg})0.800\text{ m/s}^2$

$$f_t = \boxed{0.600\text{ N}}$$

$$0.600\text{ N} + f_b = \frac{1}{2}\text{ kg}\left(0.800\text{ m/s}^2\right)$$

$$f_b = -0.200\text{ N}$$

The negative sign means that the horizontal force of ground on each roller is $\boxed{0.200\text{ N forward}}$ rather than backward as we assumed.

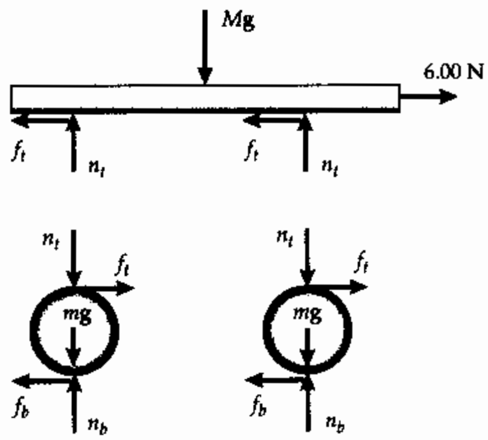


FIG. P10.86(b)

P10.87 Rolling is instantaneous rotation about the contact point P. The weight and normal force produce no torque about this point.

Now F_1 produces [a clockwise torque] about P and makes the spool roll forward.

[Counterclockwise torques] result from F_3 and F_4, making the spool roll to the left.

The force F_2 produces [zero torque] about point P and does not cause the spool to roll. If F_2 were strong enough, it would cause the spool to slide to the right, but not roll.

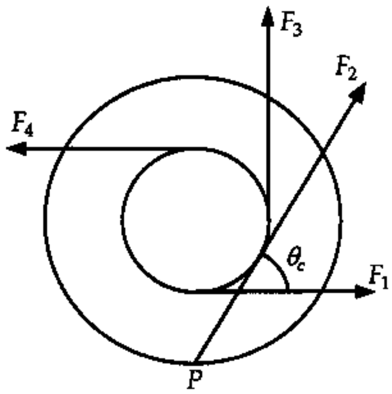


FIG. P10.87

P10.88 The force applied at the critical angle exerts zero torque about the spool's contact point with the ground and so will not make the spool roll.

From the right triangle shown in the sketch, observe that $\theta_c = 90°-\phi = 90°-(90°-\gamma) = \gamma$.

Thus, $\boxed{\cos\theta_c = \cos\gamma = \frac{r}{R}}$.

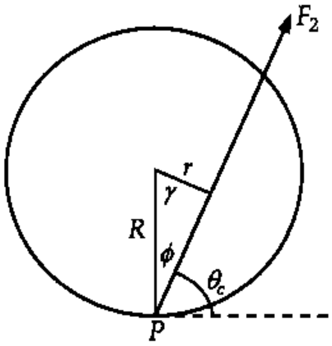


FIG. P10.88

P10.89 (a) Consider motion starting from rest over distance x along the incline:

$$(K_{\text{trans}} + K_{\text{rot}} + U)_i + \Delta E = (K_{\text{trans}} + K_{\text{rot}} + U)_f$$

$$0 + 0 + Mgx\sin\theta + 0 = \frac{1}{2}Mv^2 + 2\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2 + 0$$

$$2Mgx\sin\theta = (M+2m)v^2$$

Since acceleration is constant,

$$v^2 = v_i^2 + 2ax = 0 + 2ax, \text{ so}$$

$$2Mgx\sin\theta = (M+2m)2ax$$

$$a = \boxed{\frac{Mg\sin\theta}{(M+2m)}}$$

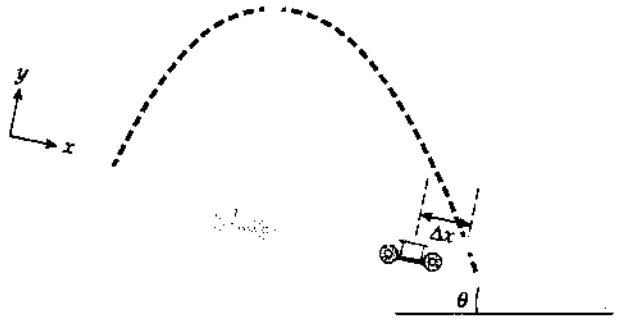


FIG. P10.88

continued on next page

(c) Suppose the ball is fired from a cart at rest. It moves with acceleration $g\sin\theta = a_x$ down the incline and $a_y = -g\cos\theta$ perpendicular to the incline. For its range along the ramp, we have

$$y - y_i = v_{yi}t - \frac{1}{2}g\cos\theta t^2 = 0 - 0$$

$$t = \frac{2v_{yi}}{g\cos\theta}$$

$$x - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

$$d = 0 + \frac{1}{2}g\sin\theta\left(\frac{4v_{yi}^2}{g^2\cos^2\theta}\right)$$

$$d = \boxed{\frac{2v_{yi}^2\sin\theta}{g\cos^2\theta}}$$

(b) In the same time the cart moves

$$x - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

$$d_c = 0 + \frac{1}{2}\left(\frac{g\sin\theta M}{(M+2m)}\right)\left(\frac{4v_{yi}^2}{g^2\cos^2\theta}\right)$$

$$d_c = \frac{2v_{yi}^2\sin\theta M}{g(M+2m)\cos^2\theta}$$

So the ball overshoots the cart by

$$\Delta x = d - d_c = \frac{2v_{yi}^2\sin\theta}{g\cos^2\theta} - \frac{2v_{yi}^2\sin\theta M}{g\cos^2\theta(M+2m)}$$

$$\Delta x = \frac{2v_{yi}^2\sin\theta M + 4v_{yi}^2\sin\theta m - 2v_{yi}^2\sin\theta M}{g\cos^2\theta(M+2m)}$$

$$\Delta x = \boxed{\frac{4mv_{yi}^2\sin\theta}{(M+2m)g\cos^2\theta}}$$

P10.90 $\sum F_x = ma_x$ reads $-f + T = ma$. If we take torques around the center of mass, we can use $\sum \tau = I\alpha$, which reads $+fR_2 - TR_1 = I\alpha$. For rolling without slipping, $\alpha = \frac{a}{R_2}$. By substitution,

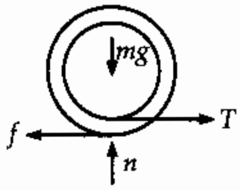


FIG. P10.90

$$fR_2 - TR_1 = \frac{Ia}{R_2} = \frac{I}{R_2 m}(T - f)$$
$$fR_2^2 m - TR_1 R_2 m = IT - If$$
$$f\left(I + mR_2^2\right) = T\left(I + mR_1 R_2\right)$$
$$f = \left(\frac{I + mR_1 R_2}{I + mR_2^2}\right)T$$

Since the answer is positive, the friction force is confirmed to be **to the left.**

ANSWERS TO EVEN PROBLEMS

P10.2 (a) $822\ \text{rad/s}^2$; (b) 4.21×10^3 rad

P10.4 (a) $1.20 \times 10^2\ \text{rad/s}$; (b) 25.0 s

P10.6 $-226\ \text{rad/s}^2$

P10.8 $13.7\ \text{rad/s}^2$

P10.10 (a) 2.88 s; (b) 12.8 s

P10.12 (a) 0.180 rad/s;
(b) $8.10\ \text{m/s}^2$ toward the center of the track

P10.14 (a) 0.605 m/s; (b) 17.3 rad/s; (c) 5.82 m/s;
(d) The crank length is unnecessary

P10.16 (a) 54.3 rev; (b) 12.1 rev/s

P10.18 0.572

P10.20 (a) $92.0\ \text{kg}\cdot\text{m}^2$; 184 J;
(b) 6.00 m/s; 4.00 m/s; 8.00 m/s; 184 J

P10.22 see the solution

P10.24 $1.28\ \text{kg}\cdot\text{m}^2$

P10.26 $\sim 10^0\ \text{kg}\cdot\text{m}^2$

P10.28 $\frac{1}{2}ML^2$

P10.30 $168\ \text{N}\cdot\text{m}$ clockwise

P10.32 $882\ \text{N}\cdot\text{m}$

P10.34 (a) 1.03 s; (b) 10.3 rev

P10.36 (a) $21.6\ \text{kg}\cdot\text{m}^2$; (b) $3.60\ \text{N}\cdot\text{m}$; (c) 52.4 rev

P10.38 0.312

P10.40 1.04×10^{-3} J

P10.42 149 rad/s

P10.44 (a) 6.90 J; (b) 8.73 rad/s; (c) 2.44 m/s;
(d) 1.043 2 times larger

P10.46 2.36 m/s

P10.48 276 J

P10.50 (a) 74.3 W; (b) 401 W

P10.52 $\frac{7Mv^2}{10}$

P10.54 The disk; $\sqrt{\frac{4gh}{3}}$ versus $\sqrt{gh}$

P10.56 (a) 2.38 m/s; (b) 4.31 m/s; (c) It will not reach the top of the loop.

P10.58 (a) 0.992 W; (b) 827 W

P10.60 see the solution

P10.62 (a) 12.5 rad/s; (b) 128 rad

P10.64 $\dfrac{g(h_2 - h_1)}{2\pi R^2}$

P10.66 (a) 2.57×10^{29} J; (b) -1.63×10^{17} J/day

P10.68 139 m/s

P10.70 (a) $\sqrt{\dfrac{2mgd\sin\theta + kd^2}{I + mR^2}}$; (b) 1.74 rad/s

P10.72 see the solution

P10.74 (a) −794 N·m; −2 510 N·m; 0; −1 160 N·m; −2 940 N·m; (b) see the solution

P10.76 $\sqrt{\dfrac{10Rg(1-\cos\theta)}{7r^2}}$

P10.78 see the solution

P10.80 (a) 35.0 m/s^2; $7.35\hat{\mathbf{i}}$ N; (b) 17.5 m/s^2; $-3.68\hat{\mathbf{i}}$ N; (c) At 0.827 m from the top.

P10.82 54.0°

P10.84 (a) $\sqrt{\dfrac{4g\left(R^3 - r^3\right)}{3r^2}}$; (b) 5.31×10^4 m/s; (c) It becomes internal energy.

P10.86 (a) 0.800 m/s^2; 0.400 m/s^2; (b) 0.600 N between each cylinder and the plank; 0.200 N forward on each cylinder by the ground

P10.88 see the solution

P10.90 see the solution; to the left

11

Angular Momentum

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q11.1 No to both questions. An axis of rotation must be defined to calculate the torque acting on an object. The moment arm of each force is measured from the axis.

Q11.2 $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$ is a scalar quantity, since $(\mathbf{B}\times\mathbf{C})$ is a vector. Since $\mathbf{A}\cdot\mathbf{B}$ is a scalar, and the cross product between a scalar and a vector is not defined, $(\mathbf{A}\cdot\mathbf{B})\times\mathbf{C}$ is undefined.

Q11.3 (a) Down–cross–left is away from you: $-\hat{\mathbf{j}}\times\left(-\hat{\mathbf{i}}\right)=-\hat{\mathbf{k}}$

(b) Left–cross–down is toward you: $-\hat{\mathbf{i}}\times\left(-\hat{\mathbf{j}}\right)=\hat{\mathbf{k}}$

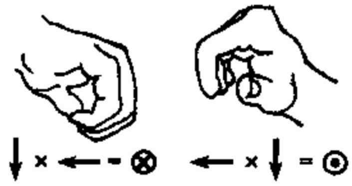


FIG. Q11.3

Q11.4 The torque about the point of application of the force is zero.

Q11.5 You cannot conclude anything about the magnitude of the angular momentum vector without first defining your axis of rotation. Its direction will be perpendicular to its velocity, but you cannot tell its direction in three-dimensional space until an axis is specified.

Q11.6 Yes. If the particles are moving in a straight line, then the angular momentum of the particles about any point on the path is zero.

Q11.7 Its angular momentum about that axis is constant in time. You cannot conclude anything about the magnitude of the angular momentum.

Q11.8 No. The angular momentum about any axis that does not lie along the instantaneous line of motion of the ball is nonzero.

Q11.9 There must be two rotors to balance the torques on the body of the helicopter. If it had only one rotor, the engine would cause the body of the helicopter to swing around rapidly with angular momentum opposite to the rotor.

Q11.10 The angular momentum of the particle about the center of rotation is constant. The angular momentum about any point that does not lie along the axis through the center of rotation and perpendicular to the plane of motion of the particle is not constant in time.

Q11.11 The long pole has a large moment of inertia about an axis along the rope. An unbalanced torque will then produce only a small angular acceleration of the performer-pole system, to extend the time available for getting back in balance. To keep the center of mass above the rope, the performer can shift the pole left or right, instead of having to bend his body around. The pole sags down at the ends to lower the system center of gravity.

Q11.12 The diver leaves the platform with some angular momentum about a horizontal axis through her center of mass. When she draws up her legs, her moment of inertia decreases and her angular speed increases for conservation of angular momentum. Straightening out again slows her rotation.

Q11.13 Suppose we look at the motorcycle moving to the right. Its drive wheel is turning clockwise. The wheel speeds up when it leaves the ground. No outside torque about its center of mass acts on the airborne cycle, so its angular momentum is conserved. As the drive wheel's clockwise angular momentum increases, the frame of the cycle acquires counterclockwise angular momentum. The cycle's front end moves up and its back end moves down.

Q11.14 The angular speed must increase. Since gravity does not exert a torque on the system, its angular momentum remains constant as the gas contracts.

Q11.15 Mass moves away from axis of rotation, so moment of inertia increases, angular speed decreases, and period increases.

Q11.16 The turntable will rotate counterclockwise. Since the angular momentum of the mouse-turntable system is initially zero, as both are at rest, the turntable must rotate in the direction opposite to the motion of the mouse, for the angular momentum of the system to remain zero.

Q11.17 Since the cat cannot apply an external torque to itself while falling, its angular momentum cannot change. Twisting in this manner changes the orientation of the cat to feet-down without changing the total angular momentum of the cat. Unfortunately, humans aren't flexible enough to accomplish this feat.

Q11.18 The angular speed of the ball must increase. Since the angular momentum of the ball is constant, as the radius decreases, the angular speed must increase.

Q11.19 Rotating the book about the axis that runs across the middle pages perpendicular to the binding—most likely where you put the rubber band—is the one that has the intermediate moment of inertia and gives unstable rotation.

Q11.20 The suitcase might contain a spinning gyroscope. If the gyroscope is spinning about an axis that is oriented horizontally passing through the bellhop, the force he applies to turn the corner results in a torque that could make the suitcase swing away. If the bellhop turns quickly enough, anything at all could be in the suitcase and need not be rotating. Since the suitcase is massive, it will want to follow an inertial path. This could be perceived as the suitcase swinging away by the bellhop.

SOLUTIONS TO PROBLEMS

Section 11.1 The Vector Product and Torque

P11.1 $\mathbf{M}\times\mathbf{N}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 6 & 2 & -1\\ 2 & -1 & -3\end{vmatrix}=\boxed{-7.00\hat{\mathbf{i}}+16.0\hat{\mathbf{j}}-10.0\hat{\mathbf{k}}}$

P11.2 (a) $\text{area}=|\mathbf{A}\times\mathbf{B}|=AB\sin\theta=(42.0\text{ cm})(23.0\text{ cm})\sin(65.0°-15.0°)=\boxed{740\text{ cm}^2}$

(b) $\mathbf{A}+\mathbf{B}=\left[(42.0\text{ cm})\cos 15.0°+(23.0\text{ cm})\cos 65.0°\right]\hat{\mathbf{i}}+\left[(42.0\text{ cm})\sin 15.0°+(23.0\text{ cm})\sin 65.0°\right]\hat{\mathbf{j}}$

$\mathbf{A}+\mathbf{B}=(50.3\text{ cm})\hat{\mathbf{i}}+(31.7\text{ cm})\hat{\mathbf{j}}$

$\text{length}=|\mathbf{A}+\mathbf{B}|=\sqrt{(50.3\text{ cm})^2+(31.7\text{ cm})^2}=\boxed{59.5\text{ cm}}$

P11.3 (a) $\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ -3 & 4 & 0\\ 2 & 3 & 0\end{vmatrix}=\boxed{-17.0\hat{\mathbf{k}}}$

(b) $|\mathbf{A}\times\mathbf{B}|=|\mathbf{A}||\mathbf{B}|\sin\theta$

$17=5\sqrt{13}\sin\theta$

$\theta=\arcsin\left(\frac{17}{5\sqrt{13}}\right)=\boxed{70.6°}$

P11.4 $\mathbf{A}\cdot\mathbf{B}=-3.00(6.00)+7.00(-10.0)+(-4.00)(9.00)=-124$

$AB=\sqrt{(-3.00)^2+(7.00)^2+(-4.00)^2}\cdot\sqrt{(6.00)^2+(-10.0)^2+(9.00)^2}=127$

(a) $\cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right)=\cos^{-1}(-0.979)=\boxed{168°}$

(b) $\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ -3.00 & 7.00 & -4.00\\ 6.00 & -10.0 & 9.00\end{vmatrix}=23.0\hat{\mathbf{i}}+3.00\hat{\mathbf{j}}-12.0\hat{\mathbf{k}}$

$|\mathbf{A}\times\mathbf{B}|=\sqrt{(23.0)^2+(3.00)^2+(-12.0)^2}=26.1$

$\sin^{-1}\left(\frac{|\mathbf{A}\times\mathbf{B}|}{AB}\right)=\sin^{-1}(0.206)=\boxed{11.9°}$ or 168°

(c) Only $\boxed{\text{the first method}}$ gives the angle between the vectors unambiguously.

***P11.5** $\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F} = 0.450\text{ m}(0.785\text{ N})\sin(90°-14°)\text{ up}\times\text{east}$

$= \boxed{0.343\text{ N}\cdot\text{m north}}$

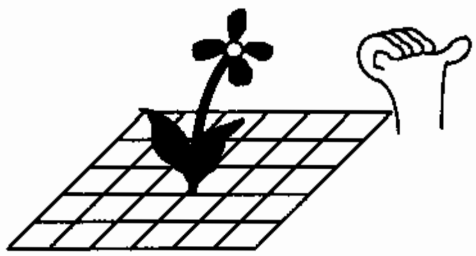


FIG. P11.5

P11.6 The cross-product vector must be perpendicular to both of the factors, so its dot product with either factor must be zero:

Does $\left(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}}\right)\cdot\left(4\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}}\right)=0$?

$$8-9-4=-5\neq 0$$

$\boxed{\text{No}}$. The cross product could not work out that way.

P11.7 $|\mathbf{A}\times\mathbf{B}| = \mathbf{A}\cdot\mathbf{B} \Rightarrow AB\sin\theta = AB\cos\theta \Rightarrow \tan\theta = 1$ or

$\theta = \boxed{45.0°}$

P11.8 (a) $$\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F} = \begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & 3 & 0\\ 3 & 2 & 0\end{vmatrix} = \hat{\mathbf{i}}(0-0)-\hat{\mathbf{j}}(0-0)+\hat{\mathbf{k}}(2-9) = \boxed{(-7.00\text{ N}\cdot\text{m})\hat{\mathbf{k}}}$$

(b) The particle's position vector relative to the new axis is $1\hat{\mathbf{i}}+3\hat{\mathbf{j}}-6\hat{\mathbf{j}}=1\hat{\mathbf{i}}-3\hat{\mathbf{j}}$.

$$\boldsymbol{\tau} = \begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 1 & -3 & 0\\ 3 & 2 & 0\end{vmatrix} = \boxed{(11.0\text{ N}\cdot\text{m})\hat{\mathbf{k}}}$$

P11.9 $\boxed{|\mathbf{F}_3| = |\mathbf{F}_1| + |\mathbf{F}_2|}$

The torque produced by $\mathbf{F}_3$ depends on the perpendicular distance **OD**, therefore translating the point of application of $\mathbf{F}_3$ to any other point along **BC** $\boxed{\text{will not change the net torque}}$.

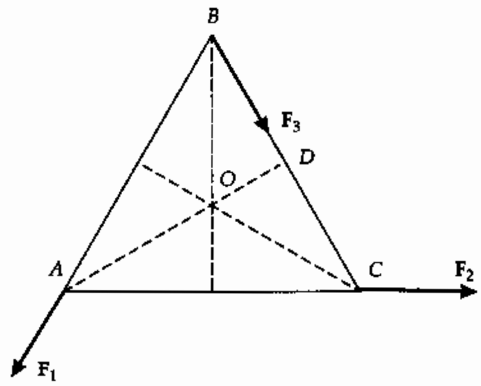


FIG. P11.9

***P11.10** $\left|\hat{\mathbf{i}}\times\hat{\mathbf{i}}\right| = 1\cdot 1\cdot \sin 0° = 0$

$\hat{\mathbf{j}}\times\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}\times\hat{\mathbf{k}}$ are zero similarly since the vectors being multiplied are parallel.

$\left|\hat{\mathbf{i}}\times\hat{\mathbf{j}}\right| = 1\cdot 1\cdot \sin 90° = 1$

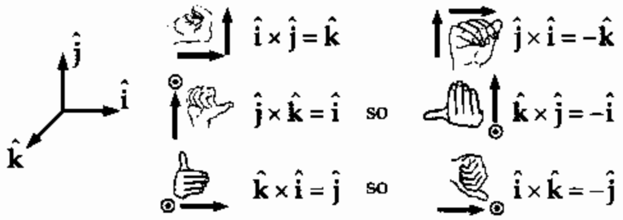


FIG. P11.10

Section 11.2 Angular Momentum

P11.11 $L = \sum m_i v_i r_i$

$= (4.00\text{ kg})(5.00\text{ m/s})(0.500\text{ m}) + (3.00\text{ kg})(5.00\text{ m/s})(0.500\text{ m})$

$L = 17.5\text{ kg}\cdot\text{m}^2/\text{s}$, and

$\boxed{\mathbf{L} = \left(17.5\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$

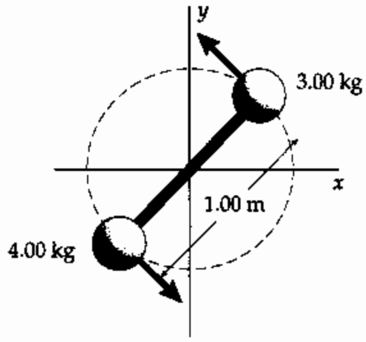


FIG. P11.11

P11.12 $\mathbf{L} = \mathbf{r}\times\mathbf{p}$

$\mathbf{L} = \left(1.50\hat{\mathbf{i}} + 2.20\hat{\mathbf{j}}\right)\text{ m}\times(1.50\text{ kg})\left(4.20\hat{\mathbf{i}} - 3.60\hat{\mathbf{j}}\right)\text{ m/s}$

$\mathbf{L} = \left(-8.10\hat{\mathbf{k}} - 13.9\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m}^2/\text{s} = \boxed{\left(-22.0\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$

P11.13 $\mathbf{r} = \left(6.00\hat{\mathbf{i}} + 5.00t\hat{\mathbf{j}}\text{ m}\right)$ $\qquad \mathbf{v} = \frac{d\mathbf{r}}{dt} = 5.00\hat{\mathbf{j}}\text{ m/s}$

so $\quad \mathbf{p} = m\mathbf{v} = 2.00\text{ kg}\left(5.00\hat{\mathbf{j}}\text{ m/s}\right) = 10.0\hat{\mathbf{j}}\text{ kg}\cdot\text{m/s}$

and $\quad \mathbf{L} = \mathbf{r}\times\mathbf{p} = \begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 6.00 & 5.00t & 0\\ 0 & 10.0 & 0\end{vmatrix} = \boxed{\left(60.0\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$

P11.14 $\sum F_x = ma_x \qquad T\sin\theta = \frac{mv^2}{r}$

$\sum F_y = ma_y \qquad T\cos\theta = mg$

So $\frac{\sin\theta}{\cos\theta} = \frac{v^2}{rg} \qquad v = \sqrt{rg\frac{\sin\theta}{\cos\theta}}$

$L = rmv\sin 90.0°$

$L = rm\sqrt{rg\frac{\sin\theta}{\cos\theta}}$

$L = \sqrt{m^2 g r^3 \frac{\sin\theta}{\cos\theta}}$

$r = \ell\sin\theta$, so

$L = \boxed{\sqrt{m^2 g \ell^3 \frac{\sin^4\theta}{\cos\theta}}}$

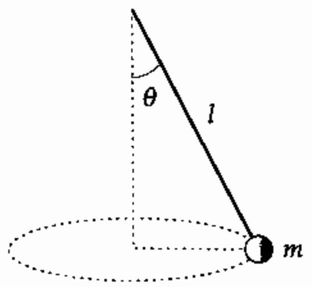


FIG. P11.14

P11.15 The angular displacement of the particle around the circle is $\theta = \omega t = \frac{vt}{R}$.

The vector from the center of the circle to the mass is then $R\cos\theta\,\hat{\mathbf{i}} + R\sin\theta\,\hat{\mathbf{j}}$.

The vector from point P to the mass is

$$\mathbf{r} = R\hat{\mathbf{i}} + R\cos\theta\,\hat{\mathbf{i}} + R\sin\theta\,\hat{\mathbf{j}}$$

$$\mathbf{r} = R\left[\left(1 + \cos\left(\frac{vt}{R}\right)\right)\hat{\mathbf{i}} + \sin\left(\frac{vt}{R}\right)\hat{\mathbf{j}}\right]$$

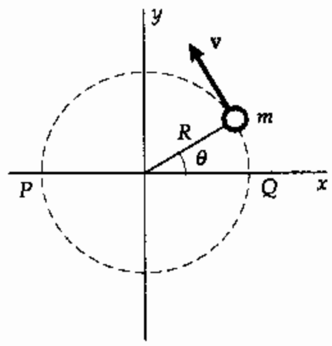


FIG. P11.15

The velocity is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = -v\sin\left(\frac{vt}{R}\right)\hat{\mathbf{i}} + v\cos\left(\frac{vt}{R}\right)\hat{\mathbf{j}}$$

So $\quad \mathbf{L} = \mathbf{r} \times m\mathbf{v}$

$$\mathbf{L} = mvR\left[(1 + \cos\omega t)\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}}\right] \times \left[-\sin\omega t\hat{\mathbf{i}} + \cos\omega t\hat{\mathbf{j}}\right]$$

$$\mathbf{L} = \boxed{mvR\hat{\mathbf{k}}\left[\cos\left(\frac{vt}{R}\right) + 1\right]}$$

P11.16 (a) The net torque on the counterweight-cord-spool system is:

$$|\tau| = |\mathbf{r} \times \mathbf{F}| = 8.00 \times 10^{-2}\ \text{m}(4.00\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{3.14\ \text{N}\cdot\text{m}}.$$

(b) $|\mathbf{L}| = |\mathbf{r} \times m\mathbf{v}| + I\omega \qquad |\mathbf{L}| = Rmv + \frac{1}{2}MR^2\left(\frac{v}{R}\right) = R\left(m + \frac{M}{2}\right)v = \boxed{(0.400\ \text{kg}\cdot\text{m})v}$

(c) $\tau = \frac{dL}{dt} = (0.400\ \text{kg}\cdot\text{m})a \qquad a = \frac{3.14\ \text{N}\cdot\text{m}}{0.400\ \text{kg}\cdot\text{m}} = \boxed{7.85\ \text{m/s}^2}$

P11.17 (a) $\boxed{\text{zero}}$

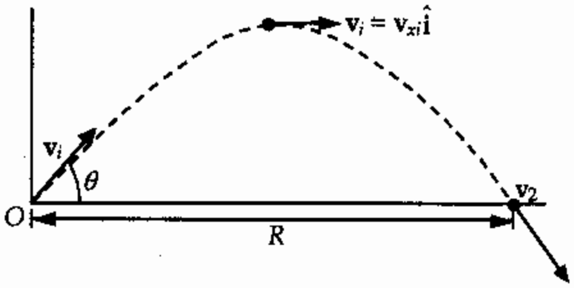


FIG. P11.17

(b) At the highest point of the trajectory,

$$x = \frac{1}{2}R = \frac{v_i^2 \sin 2\theta}{2g} \text{ and}$$

$$y = h_{\max} = \frac{(v_i \sin\theta)^2}{2g}$$

$$\mathbf{L}_1 = \mathbf{r}_1 \times m\mathbf{v}_1$$

$$= \left[\frac{v_i^2 \sin 2\theta}{2g}\hat{\mathbf{i}} + \frac{(v_i \sin\theta)^2}{2g}\hat{\mathbf{j}}\right] \times m v_{xi}\hat{\mathbf{i}}$$

$$= \boxed{\frac{-m(v_i \sin\theta)^2 v_i \cos\theta}{2g}\hat{\mathbf{k}}}$$

(c) $\mathbf{L}_2 = R\hat{\mathbf{i}} \times m\mathbf{v}_2$, where $R = \dfrac{v_i^2 \sin 2\theta}{g}$

$$= mR\hat{\mathbf{i}} \times \left(v_i \cos\theta\,\hat{\mathbf{i}} - v_i \sin\theta\,\hat{\mathbf{j}}\right)$$

$$= -mRv_i \sin\theta\,\hat{\mathbf{k}} = \boxed{\frac{-mv_i^3 \sin 2\theta \sin\theta}{g}\hat{\mathbf{k}}}$$

(d) The downward force of gravity exerts a torque in the $-z$ direction.

P11.18 Whether we think of the Earth's surface as curved or flat, we interpret the problem to mean that the plane's line of flight extended is precisely tangent to the mountain at its peak, and nearly parallel to the wheat field. Let the positive x direction be eastward, positive y be northward, and positive z be vertically upward.

(a)
$$\mathbf{r} = (4.30\text{ km})\hat{\mathbf{k}} = \left(4.30 \times 10^3\text{ m}\right)\hat{\mathbf{k}}$$

$$\mathbf{p} = m\mathbf{v} = 12\,000\text{ kg}\left(-175\hat{\mathbf{i}}\text{ m/s}\right) = -2.10 \times 10^6\,\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s}$$

$$\mathbf{L} = \mathbf{r} \times \mathbf{p} = \left(4.30 \times 10^3\,\hat{\mathbf{k}}\text{ m}\right) \times \left(-2.10 \times 10^6\,\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s}\right) = \boxed{\left(-9.03 \times 10^9\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{j}}}$$

(b) $\boxed{\text{No}}$. $L = |\mathbf{r}||\mathbf{p}|\sin\theta = mv(r\sin\theta)$, and $r\sin\theta$ is the altitude of the plane. Therefore, $L =$ *constant* as the plane moves in level flight with constant velocity.

(c) $\boxed{\text{Zero}}$. The position vector from Pike's Peak to the plane is anti-parallel to the velocity of the plane. That is, it is directed along the same line and opposite in direction.
Thus, $L = mvr\sin 180° = 0$.

P11.19 The vector from P to the falling ball is

$$\mathbf{r} = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{r} = \left(\ell\cos\theta\,\hat{\mathbf{i}} + \ell\sin\theta\,\hat{\mathbf{j}}\right) + 0 - \left(\frac{1}{2}gt^2\right)\hat{\mathbf{j}}$$

The velocity of the ball is

$$\mathbf{v} = \mathbf{v}_i + \mathbf{a}t = 0 - gt\hat{\mathbf{j}}$$

So $\mathbf{L} = \mathbf{r} \times m\mathbf{v}$

$$\mathbf{L} = m\left[\left(\ell\cos\theta\,\hat{\mathbf{i}} + \ell\sin\theta\,\hat{\mathbf{j}}\right) + 0 - \left(\frac{1}{2}gt^2\right)\hat{\mathbf{j}}\right] \times \left(-gt\hat{\mathbf{j}}\right)$$

$$\mathbf{L} = \boxed{-m\ell gt\cos\theta\,\hat{\mathbf{k}}}$$

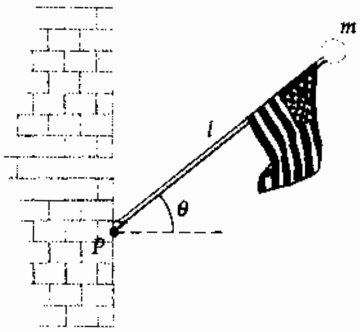


FIG. P11.19

P11.20 In the vertical section of the hose, the water has zero angular momentum about our origin (point O between the fireman's feet). As it leaves the nozzle, a parcel of mass m has angular momentum:

$$L = |\mathbf{r} \times m\mathbf{v}| = mrv\sin 90.0° = m(1.30\text{ m})(12.5\text{ m/s})$$

$$L = \left(16.3\text{ m}^2/\text{s}\right)m$$

The torque on the hose is the rate of change in angular momentum. Thus,

$$\tau = \frac{dL}{dt} = \left(16.3\text{ m}^2/\text{s}\right)\frac{dm}{dt} = \left(16.3\text{ m}^2/\text{s}\right)(6.31\text{ kg/s}) = \boxed{103\text{ N}\cdot\text{m}}$$

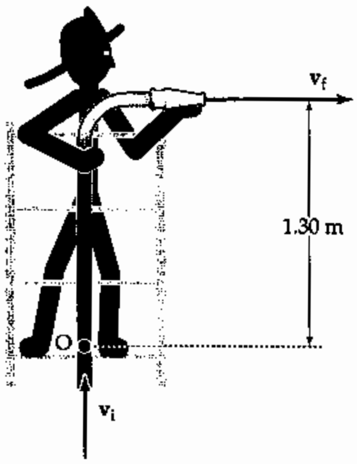


FIG. P11.20

Section 11.3 Angular Momentum of a Rotating Rigid Object

***P11.21** $K = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{I^2\omega^2}{I} = \frac{L^2}{2I}$

P11.22 The moment of inertia of the sphere about an axis through its center is

$$I = \frac{2}{5}MR^2 = \frac{2}{5}(15.0\text{ kg})(0.500\text{ m})^2 = 1.50\text{ kg}\cdot\text{m}^2$$

Therefore, the magnitude of the angular momentum is

$$L = I\omega = \left(1.50\text{ kg}\cdot\text{m}^2\right)(3.00\text{ rad/s}) = 4.50\text{ kg}\cdot\text{m}^2/\text{s}$$

Since the sphere rotates counterclockwise about the vertical axis, the angular momentum vector is directed upward in the $+z$ direction.

Thus, $\boxed{\mathbf{L} = \left(4.50\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$.

P11.23 (a) $L = I\omega = \left(\frac{1}{2}MR^2\right)\omega = \frac{1}{2}(3.00\text{ kg})(0.200\text{ m})^2(6.00\text{ rad/s}) = \boxed{0.360\text{ kg}\cdot\text{m}^2/\text{s}}$

(b) $L = I\omega = \left[\frac{1}{2}MR^2 + M\left(\frac{R}{2}\right)^2\right]\omega$

$= \frac{3}{4}(3.00\text{ kg})(0.200\text{ m})^2(6.00\text{ rad/s}) = \boxed{0.540\text{ kg}\cdot\text{m}^2/\text{s}}$

P11.24 The total angular momentum about the center point is given by $L = I_h\omega_h + I_m\omega_m$

with $I_h = \frac{m_h L_h^2}{3} = \frac{60.0\text{ kg}(2.70\text{ m})^2}{3} = 146\text{ kg}\cdot\text{m}^2$

and $I_{m3} = \frac{m_m L_m^2}{3} = \frac{100\text{ kg}(4.50\text{ m})^2}{3} = 675\text{ kg}\cdot\text{m}^2$

In addition, $\omega_h = \frac{2\pi\text{ rad}}{12\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.45\times10^{-4}\text{ rad/s}$

while $\omega_m = \frac{2\pi\text{ rad}}{1\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.75\times10^{-3}\text{ rad/s}$

Thus, $L = 146\text{ kg}\cdot\text{m}^2(1.45\times10^{-4}\text{ rad/s}) + 675\text{ kg}\cdot\text{m}^2(1.75\times10^{-3}\text{ rad/s})$

or $\boxed{L = 1.20\text{ kg}\cdot\text{m}^2/\text{s}}$

P11.25 (a) $I = \frac{1}{12}m_1L^2 + m_2(0.500)^2 = \frac{1}{12}(0.100)(1.00)^2 + 0.400(0.500)^2 = 0.108\,3\text{ kg}\cdot\text{m}^2$

$L = I\omega = 0.108\,3(4.00) = \boxed{0.433\text{ kg}\cdot\text{m}^2/\text{s}}$

(b) $I = \frac{1}{3}m_1L^2 + m_2R^2 = \frac{1}{3}(0.100)(1.00)^2 + 0.400(1.00)^2 = 0.433$

$L = I\omega = 0.433(4.00) = \boxed{1.73\text{ kg}\cdot\text{m}^2/\text{s}}$

***P11.26** $\sum F_x = ma_x: \quad +f_s = ma_x$

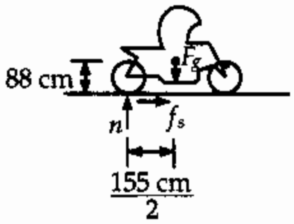


FIG. P11.26

We must use the center of mass as the axis in

$\sum\tau = I\alpha: \quad F_g(0) - n(77.5\text{ cm}) + f_s(88\text{ cm}) = 0$

$\sum F_y = ma_y: \quad +n - F_g = 0$

We combine the equations by substitution:

$-mg(77.5\text{ cm}) + ma_x(88\text{ cm}) = 0$

$a_x = \frac{(9.80\text{ m/s}^2)77.5\text{ cm}}{88\text{ cm}} = \boxed{8.63\text{ m/s}^2}$

***P11.27** We require $a_c = g = \frac{v^2}{r} = \omega^2 r$

$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{(9.80\ \text{m/s}^2)}{100\ \text{m}}} = 0.313\ \text{rad/s}$$

$$I = Mr^2 = 5\times10^4\ \text{kg}(100\ \text{m})^2 = 5\times10^8\ \text{kg}\cdot\text{m}^2$$

(a) $L = I\omega = 5\times10^8\ \text{kg}\cdot\text{m}^2\, 0.313/\text{s} = \boxed{1.57\times10^8\ \text{kg}\cdot\text{m}^2/\text{s}}$

(c) $\sum\tau = I\alpha = \frac{I(\omega_f - \omega_i)}{\Delta t}$

$\sum\tau\Delta t = I\omega_f - I\omega_i = L_f - L_i$

This is the angular impulse-angular momentum theorem.

(b) $\Delta t = \frac{L_f - 0}{\sum\tau} = \frac{1.57\times10^8\ \text{kg}\cdot\text{m}^2/\text{s}}{2(125\ \text{N})(100\ \text{m})} = \boxed{6.26\times10^3\ \text{s}} = 1.74\ \text{h}$

Section 11.4 Conservation of Angular Momentum

P11.28 (a) From conservation of angular momentum for the system of two cylinders:

$(I_1 + I_2)\omega_f = I_1\omega_i$ or $\omega_f = \boxed{\frac{I_1}{I_1 + I_2}\omega_i}$

(b) $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$ and $K_i = \frac{1}{2}I_1\omega_i^2$

so $\frac{K_f}{K_i} = \frac{\frac{1}{2}(I_1 + I_2)}{\frac{1}{2}I_1\omega_i^2}\left(\frac{I_1}{I_1 + I_2}\omega_i\right)^2 = \boxed{\frac{I_1}{I_1 + I_2}\ \text{which is less than 1}}$.

P11.29 $I_i\omega_i = I_f\omega_f$: $(250\ \text{kg}\cdot\text{m}^2)(10.0\ \text{rev/min}) = \left[250\ \text{kg}\cdot\text{m}^2 + 25.0\ \text{kg}(2.00\ \text{m})^2\right]\omega_2$

$\omega_2 = \boxed{7.14\ \text{rev/min}}$

P11.30 (a) The total angular momentum of the system of the student, the stool, and the weights about the axis of rotation is given by

$$I_{\text{total}} = I_{\text{weights}} + I_{\text{student}} = 2\left(mr^2\right) + 3.00\ \text{kg}\cdot\text{m}^2$$

Before: $r = 1.00\ \text{m}$.

Thus, $I_i = 2(3.00\ \text{kg})(1.00\ \text{m})^2 + 3.00\ \text{kg}\cdot\text{m}^2 = 9.00\ \text{kg}\cdot\text{m}^2$

After: $r = 0.300\ \text{m}$

Thus, $I_f = 2(3.00\ \text{kg})(0.300\ \text{m})^2 + 3.00\ \text{kg}\cdot\text{m}^2 = 3.54\ \text{kg}\cdot\text{m}^2$

We now use conservation of angular momentum.

$$I_f\omega_f = I_i\omega_i$$

or $$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{9.00}{3.54}\right)(0.750\ \text{rad/s}) = \boxed{1.91\ \text{rad/s}}$$

(b) $$K_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}\left(9.00\ \text{kg}\cdot\text{m}^2\right)(0.750\ \text{rad/s})^2 = \boxed{2.53\ \text{J}}$$

$$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}\left(3.54\ \text{kg}\cdot\text{m}^2\right)(1.91\ \text{rad/s})^2 = \boxed{6.44\ \text{J}}$$

P11.31 (a) Let $M =$ mass of rod and $m =$ mass of each bead. From $I_i\omega_i = I_f\omega_f$, we have

$$\left[\frac{1}{12}M\ell^2 + 2mr_1^2\right]\omega_i = \left[\frac{1}{12}M\ell^2 + 2mr_2^2\right]\omega_f$$

When $\ell = 0.500\ \text{m}$, $r_1 = 0.100\ \text{m}$, $r_2 = 0.250\ \text{m}$, and with other values as stated in the problem, we find

$$\boxed{\omega_f = 9.20\ \text{rad/s}}.$$

(b) Since there is no external torque on the rod,

$L =$ constant and $\boxed{\omega \text{ is unchanged}}$.

***P11.32** Let M represent the mass of all the ribs together and L the length of each. The original moment of inertia is $\frac{1}{3}ML^2$. The final effective length of each rib is $L\sin 22.5°$ and the final moment of inertia is $\frac{1}{3}M(L\sin 22.5°)^2$ angular momentum of the umbrella is conserved:

$$\frac{1}{3}ML^2\omega_i = \frac{1}{3}ML^2\sin^2 22.5°\,\omega_f$$

$$\omega_f = \frac{1.25\ \text{rad/s}}{\sin^2 22.5°} = \boxed{8.54\ \text{rad/s}}$$

P11.33 (a) The table turns opposite to the way the woman walks, so its angular momentum cancels that of the woman. From conservation of angular momentum for the system of the woman and the turntable, we have $L_f = L_i = 0$

so, $$L_f = I_{\text{woman}}\omega_{\text{woman}} + I_{\text{table}}\omega_{\text{table}} = 0$$

and $$\omega_{\text{table}} = \left(-\frac{I_{\text{woman}}}{I_{\text{table}}}\right)\omega_{\text{woman}} = \left(-\frac{m_{\text{woman}}r^2}{I_{\text{table}}}\right)\left(\frac{v_{\text{woman}}}{r}\right) = -\frac{m_{\text{woman}}rv_{\text{woman}}}{I_{\text{table}}}$$

$$\omega_{\text{table}} = -\frac{60.0\text{ kg}(2.00\text{ m})(1.50\text{ m/s})}{500\text{ kg}\cdot\text{m}^2} = -0.360\text{ rad/s}$$

or $$\omega_{\text{table}} = \boxed{0.360\text{ rad/s (counterclockwise)}}$$

(b) $$\text{work done} = \Delta K = K_f - 0 = \frac{1}{2}m_{\text{woman}}v_{\text{woman}}^2 + \frac{1}{2}I\omega_{\text{table}}^2$$

$$W = \frac{1}{2}(60\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}\left(500\text{ kg}\cdot\text{m}^2\right)(0.360\text{ rad/s})^2 = \boxed{99.9\text{ J}}$$

P11.34 When they touch, the center of mass is distant from the center of the larger puck by

$$y_{\text{CM}} = \frac{0 + 80.0\text{ g}(4.00\text{ cm} + 6.00\text{ cm})}{120\text{ g} + 80.0\text{ g}} = 4.00\text{ cm}$$

(a) $$L = r_1m_1v_1 + r_2m_2v_2 = 0 + \left(6.00\times10^{-2}\text{ m}\right)\left(80.0\times10^{-3}\text{ kg}\right)(1.50\text{ m/s}) = \boxed{7.20\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}}$$

(b) The moment of inertia about the CM is

$$I = \left(\frac{1}{2}m_1r_1^2 + m_1d_1^2\right) + \left(\frac{1}{2}m_2r_2^2 + m_2d_2^2\right)$$

$$I = \frac{1}{2}(0.120\text{ kg})\left(6.00\times10^{-2}\text{ m}\right)^2 + (0.120\text{ kg})\left(4.00\times10^{-2}\right)^2$$

$$+\frac{1}{2}\left(80.0\times10^{-3}\text{ kg}\right)\left(4.00\times10^{-2}\text{ m}\right)^2 + \left(80.0\times10^{-3}\text{ kg}\right)\left(6.00\times10^{-2}\text{ m}\right)^2$$

$$I = 7.60\times10^{-4}\text{ kg}\cdot\text{m}^2$$

Angular momentum of the two-puck system is conserved: $L = I\omega$

$$\omega = \frac{L}{I} = \frac{7.20\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}}{7.60\times10^{-4}\text{ kg}\cdot\text{m}^2} = \boxed{9.47\text{ rad/s}}$$

P11.35 (a) $L_i = mv\ell \quad \sum \tau_{\text{ext}} = 0$, so $L_f = L_i = \boxed{mv\ell}$

$L_f = (m+M)v_f\ell$

$v_f = \left(\frac{m}{m+M}\right)v$

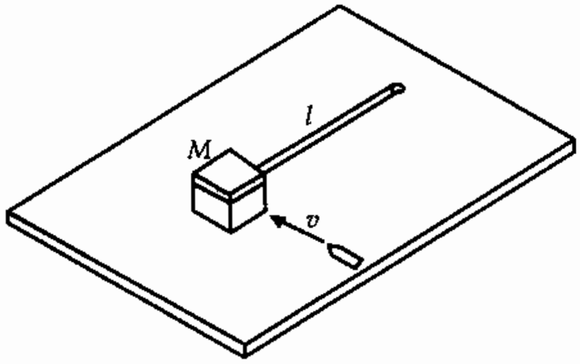


FIG. P11.35

(b) $K_i = \frac{1}{2}mv^2$

$K_f = \frac{1}{2}(M+m)v_f^2$

$v_f = \left(\frac{m}{M+m}\right)v \Rightarrow$ velocity of the bullet and block

Fraction of K lost $= \dfrac{\frac{1}{2}mv^2 - \frac{1}{2}\frac{m^2}{M+m}v^2}{\frac{1}{2}mv^2} = \boxed{\dfrac{M}{M+m}}$

P11.36 For one of the crew,

$\sum F_r = ma_r: \qquad n = \dfrac{mv^2}{r} = m\omega_i^2 r$

We require $\quad n = mg$, so $\omega_i = \sqrt{\dfrac{g}{r}}$

Now, $\quad I_i\omega_i = I_f\omega_f$

$$\left[5.00\times10^8\ \text{kg}\cdot\text{m}^2 + 150\times65.0\ \text{kg}\times(100\ \text{m})^2\right]\sqrt{\frac{g}{r}} = \left[5.00\times10^8\ \text{kg}\cdot\text{m}^2 + 50\times65.0\ \text{kg}(100\ \text{m})^2\right]\omega_f$$

$$\left(\frac{5.98\times10^8}{5.32\times10^8}\right)\sqrt{\frac{g}{r}} = \omega_f = 1.12\sqrt{\frac{g}{r}}$$

Now, $\quad |a_r| = \omega_f^2 r = 1.26g = \boxed{12.3\ \text{m/s}^2}$

P11.37 (a) Consider the system to consist of the wad of clay and the cylinder. No external forces acting on this system have a torque about the center of the cylinder. Thus, angular momentum of the system is conserved about the axis of the cylinder.

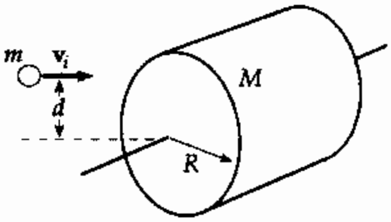


FIG. P11.37

$L_f = L_i: \qquad I\omega = mv_i d$

or $\qquad \left[\frac{1}{2}MR^2 + mR^2\right]\omega = mv_i d$

Thus, $\qquad \omega = \boxed{\dfrac{2mv_i d}{(M+2m)R^2}}$.

(b) $\boxed{\text{No}}$. Some mechanical energy changes to internal energy in this perfectly inelastic collision.

***P11.38** (a) Let ω be the angular speed of the signboard when it is vertical.

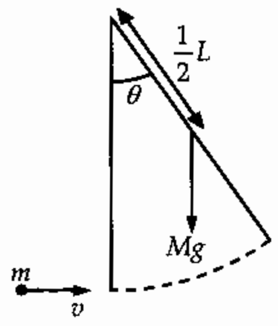


FIG. P11.38

$$\frac{1}{2}I\omega^2 = Mgh$$

$$\therefore \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 = Mg\frac{1}{2}L(1-\cos\theta)$$

$$\therefore \omega = \sqrt{\frac{3g(1-\cos\theta)}{L}}$$

$$= \sqrt{\frac{3(9.80\ \text{m/s}^2)(1-\cos 25.0°)}{0.50\ \text{m}}}$$

$$= \boxed{2.35\ \text{rad/s}}$$

(b) $I_f\omega_f = I_i\omega_i - mvL$ represents angular momentum conservation

$$\therefore \left(\frac{1}{3}ML^2 + mL^2\right)\omega_f = \frac{1}{3}ML^2\omega_i - mvL$$

$$\therefore \omega_f = \frac{\frac{1}{3}ML\omega_i - mv}{\left(\frac{1}{3}M + m\right)L}$$

$$= \frac{\frac{1}{3}(2.40\ \text{kg})(0.5\ \text{m})(2.347\ \text{rad/s}) - (0.4\ \text{kg})(1.6\ \text{m/s})}{\left[\frac{1}{3}(2.40\ \text{kg}) + 0.4\ \text{kg}\right](0.5\ \text{m})} = \boxed{0.498\ \text{rad/s}}$$

(c) Let h_{CM} = distance of center of mass from the axis of rotation.

$$h_{CM} = \frac{(2.40\ \text{kg})(0.25\ \text{m}) + (0.4\ \text{kg})(0.50\ \text{m})}{2.40\ \text{kg} + 0.4\ \text{kg}} = 0.2857\ \text{m}.$$

Apply conservation of mechanical energy:

$$(M+m)gh_{CM}(1-\cos\theta) = \frac{1}{2}\left(\frac{1}{3}ML^2 + mL^2\right)\omega^2$$

$$\therefore \theta = \cos^{-1}\left[1 - \frac{\left(\frac{1}{3}M + m\right)L^2\omega^2}{2(M+m)gh_{CM}}\right]$$

$$= \cos^{-1}\left\{1 - \frac{\left[\frac{1}{3}(2.40\ \text{kg}) + 0.4\ \text{kg}\right](0.50\ \text{m})^2(0.498\ \text{rad/s})^2}{2(2.40\ \text{kg} + 0.4\ \text{kg})(9.80\ \text{m/s}^2)(0.2857\ \text{m})}\right\}$$

$$= \boxed{5.58°}$$

P11.39 The meteor will slow the rotation of the Earth by the largest amount if its line of motion passes farthest from the Earth's axis. The meteor should be headed west and strike a point on the equator tangentially.

Let the z axis coincide with the axis of the Earth with $+z$ pointing northward. Then, conserving angular momentum about this axis,

$$\sum \mathbf{L}_f = \sum \mathbf{L}_i \Rightarrow I\omega_f = I\omega_i + m\mathbf{v}\times\mathbf{r}$$

or $$\frac{2}{5}MR^2\omega_f\hat{\mathbf{k}} = \frac{2}{5}MR^2\omega_i\hat{\mathbf{k}} - mvR\hat{\mathbf{k}}$$

Thus, $$\omega_i - \omega_f = \frac{mvR}{\frac{2}{5}MR^2} = \frac{5mv}{2MR} \text{ or}$$

$$\omega_i - \omega_f = \frac{5(3.00\times10^{13}\text{ kg})(30.0\times10^3\text{ m/s})}{2(5.98\times10^{24}\text{ kg})(6.37\times10^6\text{ m})} = 5.91\times10^{-14}\text{ rad/s}$$

$$\boxed{|\Delta\omega_{\max}| \sim 10^{-13}\text{ rad/s}}$$

Section 11.5 The Motion of Gyroscopes and Tops

***P11.40** Angular momentum of the system of the spacecraft and the gyroscope is conserved. The gyroscope and spacecraft turn in opposite directions.

$$0 = I_1\omega_1 + I_2\omega_2: \quad -I_1\omega_1 = I_2\frac{\theta}{t}$$

$$-20\text{ kg}\cdot\text{m}^2(-100\text{ rad/s}) = 5\times10^5\text{ kg}\cdot\text{m}^2\left(\frac{30°}{t}\right)\left(\frac{\pi\text{ rad}}{180°}\right)$$

$$t = \frac{2.62\times10^5\text{ s}}{2\,000} = \boxed{131\text{ s}}$$

***P11.41** $$I = \frac{2}{5}MR^2 = \frac{2}{5}(5.98\times10^{24}\text{ kg})(6.37\times10^6\text{ m})^2 = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2$$

$$L = I\omega = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2\left(\frac{2\pi\text{ rad}}{86\,400\text{ s}}\right) = 7.06\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s}^2$$

$$\tau = L\omega_p = (7.06\times10^{33}\text{ kg}\cdot\text{m}^2/\text{s})\left(\frac{2\pi\text{ rad}}{2.58\times10^4\text{ yr}}\right)\left(\frac{1\text{ yr}}{365.25\text{ d}}\right)\left(\frac{1\text{ d}}{86\,400\text{ s}}\right) = \boxed{5.45\times10^{22}\text{ N}\cdot\text{m}}$$

Section 11.6 Angular Momentum as a Fundamental Quantity

P11.42 (a) $$L = \frac{h}{2\pi} = mvr \text{ so } v = \frac{h}{2\pi mr} \qquad v = \frac{6.626\,1\times10^{-34}\text{ J}\cdot\text{s}}{2\pi(9.11\times10^{-31}\text{ kg})(0.529\times10^{-10}\text{ m})} = \boxed{2.19\times10^6\text{ m/s}}$$

(b) $$K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11\times10^{-31}\text{ kg})(2.19\times10^6\text{ m/s})^2 = \boxed{2.18\times10^{-18}\text{ J}}$$

(c) $$\omega = \frac{L}{I} = \frac{\hbar}{mr^2} = \frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{(9.11\times10^{-31}\text{ kg})(0.529\times10^{-10}\text{ m})^2} = \boxed{4.13\times10^{16}\text{ rad/s}}$$

Additional Problems

***P11.43** First, we define the following symbols:

I_P = moment of inertia due to mass of people on the equator
I_E = moment of inertia of the Earth alone (without people)
ω = angular velocity of the Earth (due to rotation on its axis)
$T = \frac{2\pi}{\omega}$ = rotational period of the Earth (length of the day)
R = radius of the Earth

The initial angular momentum of the system (before people start running) is

$$L_i = I_P\omega_i + I_E\omega_i = (I_P + I_E)\omega_i$$

When the Earth has angular speed ω, the tangential speed of a point on the equator is $v_t = R\omega$. Thus, when the people run eastward along the equator at speed v relative to the surface of the Earth, their tangential speed is $v_p = v_t + v = R\omega + v$ and their angular speed is $\omega_P = \frac{v_P}{R} = \omega + \frac{v}{R}$.

The angular momentum of the system after the people begin to run is

$$L_f = I_P\omega_p + I_E\omega = I_P\left(\omega + \frac{v}{R}\right) + I_E\omega = (I_P + I_E)\omega + \frac{I_P v}{R}.$$

Since no external torques have acted on the system, angular momentum is conserved $\left(L_f = L_i\right)$, giving $(I_P + I_E)\omega + \frac{I_P v}{R} = (I_P + I_E)\omega_i$. Thus, the final angular velocity of the Earth is

$\omega = \omega_i - \frac{I_P v}{(I_P + I_E)R} = \omega_i(1 - x) =$, where $x \equiv \frac{I_P v}{(I_P + I_E)R\omega_i}$.

The new length of the day is $T = \frac{2\pi}{\omega} = \frac{2\pi}{\omega_i(1-x)} = \frac{T_i}{1-x} \approx T_i(1+x)$, so the increase in the length of the day is $\Delta T = T - T_i \approx T_i x = T_i\left[\frac{I_P v}{(I_P + I_E)R\omega_i}\right]$. Since $\omega_i = \frac{2\pi}{T_i}$, this may be written as $\Delta T \approx \frac{T_i^2 I_P v}{2\pi(I_P + I_E)R}$.

To obtain a numeric answer, we compute

$$I_P = m_p R^2 = \left[\left(5.5\times10^9\right)(70\text{ kg})\right]\left(6.37\times10^6\text{ m}\right)^2 = 1.56\times10^{25}\text{ kg}\cdot\text{m}^2$$

and

$$I_E = \frac{2}{5}m_E R^2 = \frac{2}{5}\left(5.98\times10^{24}\text{ kg}\right)\left(6.37\times10^6\text{ m}\right)^2 = 9.71\times10^{37}\text{ kg}\cdot\text{m}^2.$$

Thus, $\Delta T \approx \frac{\left(8.64\times10^4\text{ s}\right)^2\left(1.56\times10^{25}\text{ kg}\cdot\text{m}^2\right)(2.5\text{ m/s})}{2\pi\left[\left(1.56\times10^{25} + 9.71\times10^{37}\right)\text{kg}\cdot\text{m}^2\right]\left(6.37\times10^6\text{ m}\right)} = \boxed{7.50\times10^{-11}\text{ s}}$.

***P11.44** (a) $(K+U_s)_A = (K+U_s)_B$

$$0+mgy_A = \frac{1}{2}mv_B^2 + 0$$

$$v_B = \sqrt{2gy_A} = \sqrt{2(9.8\ \text{m/s}^2)6.30\ \text{m}} = \boxed{11.1\ \text{m/s}}$$

(b) $L = mvr = 76\ \text{kg}\ 11.1\ \text{m/s}\ 6.3\ \text{m} = \boxed{5.32\times10^3\ \text{kg}\cdot\text{m}^2/\text{s}}$ toward you along the axis of the channel.

(c) The wheels on his skateboard prevent any tangential force from acting on him. Then no torque about the axis of the channel acts on him and his angular momentum is constant. His legs convert chemical into mechanical energy. They do work to increase his kinetic energy. The normal force acts forward on his body on its rising trajectory, to increase his linear momentum.

(d) $L = mvr \qquad v = \dfrac{5.32\times10^3\ \text{kg}\cdot\text{m}^2/\text{s}}{76\ \text{kg}\ 5.85\ \text{m}} = \boxed{12.0\ \text{m/s}}$

(e) $(K+U_g)_B + W = (K+U_g)_C$

$$\frac{1}{2}76\ \text{kg}(11.1\ \text{m/s})^2 + 0 + W = \frac{1}{2}76\ \text{kg}(12.0\ \text{m/s})^2 + 76\ \text{kg}\ 9.8\ \text{m/s}^2\ 0.45\ \text{m}$$

$$W = 5.44\ \text{kJ} - 4.69\ \text{kJ} + 335\ \text{J} = \boxed{1.08\ \text{kJ}}$$

(f) $(K+U_g)_C = (K+U_g)_D$

$$\frac{1}{2}76\ \text{kg}(12.0\ \text{m/s})^2 + 0 = \frac{1}{2}76\ \text{kg}v_D^2 + 76\ \text{kg}\ 9.8\ \text{m/s}^2\ 5.85\ \text{m}$$

$$v_D = \boxed{5.34\ \text{m/s}}$$

(g) Let point E be the apex of his flight:

$$(K+U_g)_D = (K+U_g)_E$$

$$\frac{1}{2}76\ \text{kg}(5.34\ \text{m/s})^2 + 0 = 0 + 76\ \text{kg}(9.8\ \text{m/s}^2)(y_E - y_D)$$

$$(y_E - y_D) = \boxed{1.46\ \text{m}}$$

(h) For the motion between takeoff and touchdown

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_yt^2$$

$$-2.34\ \text{m} = 0 + 5.34\ \text{m/s}\,t - 4.9\ \text{m/s}^2\,t^2$$

$$t = \frac{-5.34 \pm \sqrt{5.34^2 + 4(4.9)(2.34)}}{-9.8} = \boxed{1.43\ \text{s}}$$

(i) This solution is more accurate. In chapter 8 we modeled the normal force as constant while the skateboarder stands up. Really it increases as the process goes on.

P11.45 (a)
$$I=\sum m_i r_i^2$$
$$=m\left(\frac{4d}{3}\right)^2+m\left(\frac{d}{3}\right)^2+m\left(\frac{2d}{3}\right)^2$$
$$=\boxed{7m\frac{d^2}{3}}$$

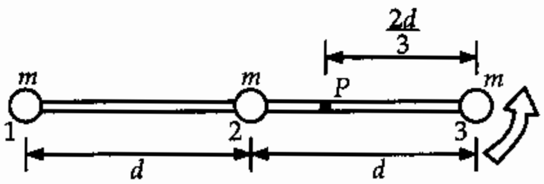


FIG. P11.45

(b) Think of the whole weight, $3mg$, acting at the center of gravity.

$$\boldsymbol{\tau}=\mathbf{r}\times\mathbf{F}=\left(\frac{d}{3}\right)(-\hat{\mathbf{i}})\times 3mg(-\hat{\mathbf{j}})=\boxed{(mgd)\hat{\mathbf{k}}}$$

(c) $\alpha=\dfrac{\tau}{I}=\dfrac{3mgd}{7md^2}=\boxed{\dfrac{3g}{7d}\text{ counterclockwise}}$

(d) $a=\alpha r=\left(\dfrac{3g}{7d}\right)\left(\dfrac{2d}{3}\right)=\boxed{\dfrac{2g}{7}\text{ up}}$

The angular acceleration is not constant, but energy is.

$$(K+U)_i+\Delta E=(K+U)_f$$
$$0+(3m)g\left(\frac{d}{3}\right)+0=\frac{1}{2}I\omega_f^2+0$$

(e) maximum kinetic energy $=\boxed{mgd}$

(f) $\omega_f=\boxed{\sqrt{\dfrac{6g}{7d}}}$

(g) $L_f=I\omega_f=\dfrac{7md^2}{3}\sqrt{\dfrac{6g}{7d}}=\boxed{\left(\dfrac{14g}{3}\right)^{1/2}md^{3/2}}$

(h) $v_f=\omega_f r=\sqrt{\dfrac{6g}{7d}}\dfrac{d}{3}=\boxed{\sqrt{\dfrac{2gd}{21}}}$

P11.46 (a) The radial coordinate of the sliding mass is $r(t) = (0.012\,5\ \text{m/s})t$. Its angular momentum is

$$L = mr^2\omega = (1.20\ \text{kg})(2.50\ \text{rev/s})(2\pi\ \text{rad/rev})(0.012\,5\ \text{m/s})^2 t^2$$

or $$L = \left(2.95 \times 10^{-3}\ \text{kg} \cdot \text{m}^2/\text{s}^3\right)t^2$$

The drive motor must supply torque equal to the rate of change of this angular momentum:

$$\tau = \frac{dL}{dt} = \left(2.95 \times 10^{-3}\ \text{kg} \cdot \text{m}^2/\text{s}^3\right)(2t) = \boxed{(0.005\,89\ \text{W})t}$$

(b) $$\tau_f = (0.005\,89\ \text{W})(440\ \text{s}) = \boxed{2.59\ \text{N} \cdot \text{m}}$$

(c) $$\mathcal{P} = \tau\omega = (0.005\,89\ \text{W})t(5\pi\ \text{rad/s}) = \boxed{(0.092\,5\ \text{W/s})t}$$

(d) $$\mathcal{P}_f = (0.092\,5\ \text{W/s})(440\ \text{s}) = \boxed{40.7\ \text{W}}$$

(e) $$T = m\frac{v^2}{r} = mr\omega^2 = (1.20\ \text{kg})(0.012\,5\ \text{m/s})t(5\pi\ \text{rad/s})^2 = \boxed{(3.70\ \text{N/s})t}$$

(f) $$W = \int_0^{440\ \text{s}} \mathcal{P}dt = \int_0^{440\ \text{s}} (0.092\,5\ \text{W/s})t\,dt = \frac{1}{2}\left(0.092\,5\ \text{J/s}^2\right)(440\ \text{s})^2 = \boxed{8.96\ \text{kJ}}$$

(g) The power the brake injects into the sliding block through the string is

$$\mathcal{P}_b = \mathbf{F} \cdot \mathbf{v} = Tv\cos 180° = -(3.70\ \text{N/s})t(0.012\,5\ \text{m/s}) = -(0.046\,3\ \text{W/s})t = \frac{dW_b}{dt}$$

$$W_b = \int_0^{440\ \text{s}} \mathcal{P}_b dt = -\int_0^{440\ \text{s}} (0.046\,3\ \text{W/s})t\,dt$$

$$= -\frac{1}{2}(0.046\,3\ \text{W/s})(440\ \text{s})^2 = \boxed{-4.48\ \text{kJ}}$$

(h) $$\sum W = W + W_b = 8.96\ \text{kJ} - 4.48\ \text{kJ} = \boxed{4.48\ \text{kJ}}$$

Just half of the work required to increase the angular momentum goes into rotational kinetic energy. The other half becomes internal energy in the brake.

P11.47 Using conservation of angular momentum, we have

$$L_{\text{aphelion}} = L_{\text{perihelion}} \text{ or } \left(mr_a^2\right)\omega_a = \left(mr_p^2\right)\omega_p.$$

Thus, $\left(mr_a^2\right)\frac{v_a}{r_a} = \left(mr_p^2\right)\frac{v_p}{r_p}$ giving

$$r_a v_a = r_p v_p \text{ or } v_a = \frac{r_p}{r_a}v_p = \frac{0.590\ \text{AU}}{35.0\ \text{AU}}(54.0\ \text{km/s}) = \boxed{0.910\ \text{km/s}}.$$

P11.48 (a) $\sum \tau = MgR - MgR = \boxed{0}$

(b) $\sum \tau = \frac{dL}{dt}$, and since $\sum \tau = 0$, $L =$ constant.

Since the total angular momentum of the system is zero, the $\boxed{\text{monkey and bananas move upward with the same speed}}$ at any instant, and $\boxed{\text{he will not reach the bananas}}$ (until they get tangled in the pulley). Also, since the tension in the rope is the same on both sides, Newton's second law applied to the monkey and bananas give the same acceleration upwards.

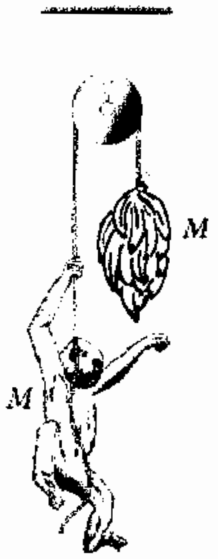


FIG. P11.48

P11.49 (a) $\tau = |\mathbf{r} \times \mathbf{F}| = |\mathbf{r}||\mathbf{F}|\sin 180° = 0$

Angular momentum is conserved.

$$L_f = L_i$$
$$mrv = mr_i v_i$$
$$v = \boxed{\frac{r_i v_i}{r}}$$

(b) $T = \frac{mv^2}{r} = \boxed{\frac{m(r_i v_i)^2}{r^3}}$

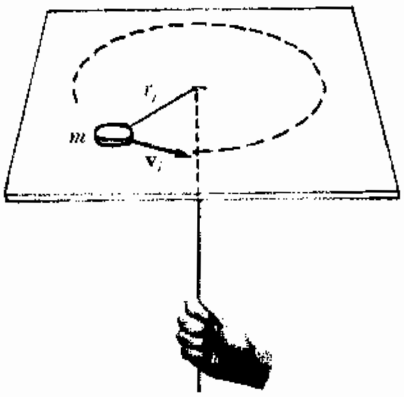


FIG. P11.49

(c) The work is done by the centripetal force in the *negative* direction.
Method 1:

$$W = \int F \cdot d\ell = -\int T dr' = -\int_{r_i}^{r} \frac{m(r_i v_i)^2}{(r')^3} dr' = \left.\frac{m(r_i v_i)^2}{2(r')^2}\right|_{r_i}^{r}$$

$$= \frac{m(r_i v_i)^2}{2}\left(\frac{1}{r^2} - \frac{1}{r_i^2}\right) = \boxed{\frac{1}{2}mv_i^2\left(\frac{r_i^2}{r^2} - 1\right)}$$

Method 2:

$$W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2 = \boxed{\frac{1}{2}mv_i^2\left(\frac{r_i^2}{r^2} - 1\right)}$$

(d) Using the data given, we find

$v = \boxed{4.50 \text{ m/s}}$ $\quad T = \boxed{10.1 \text{ N}}$ $\quad W = \boxed{0.450 \text{ J}}$

P11.50 (a) Angular momentum is conserved:

$$\frac{mv_i d}{2}=\left(\frac{1}{12}Md^2+m\left(\frac{d}{2}\right)^2\right)\omega$$

$$\omega=\boxed{\frac{6mv_i}{Md+3md}}$$

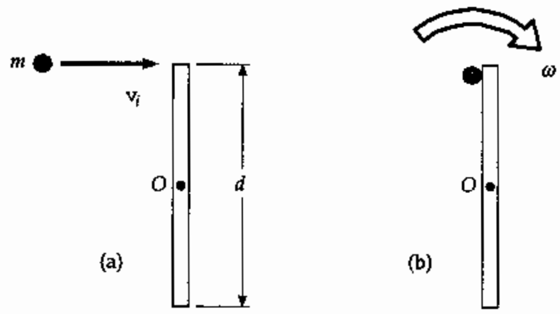


FIG. P11.50

(b) The original energy is $\frac{1}{2}mv_i^2$.

The final energy is

$$\frac{1}{2}I\omega^2=\frac{1}{2}\left(\frac{1}{12}Md^2+\frac{md^2}{4}\right)\frac{36m^2v_i^2}{(Md+3md)^2}=\frac{3m^2v_i^2d}{2(Md+3md)}.$$

The loss of energy is

$$\frac{1}{2}mv_i^2-\frac{3m^2v_i^2d}{2(Md+3md)}=\frac{mMv_i^2d}{2(Md+3md)}$$

and the fractional loss of energy is

$$\frac{mMv_i^2d2}{2(Md+3md)mv_i^2}=\boxed{\frac{M}{M+3m}}.$$

P11.51 (a) $L_i=m_1v_{1i}r_{1i}+m_2v_{2i}r_{2i}=2mv\left(\frac{d}{2}\right)$

$L_i=2(75.0\text{ kg})(5.00\text{ m/s})(5.00\text{ m})$

$L_i=\boxed{3\,750\text{ kg}\cdot\text{m}^2/\text{s}}$

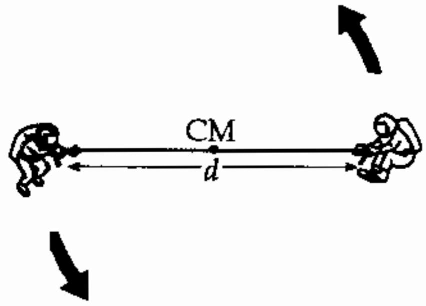


FIG. P11.51

(b) $K_i=\frac{1}{2}m_1v_{1i}^2+\frac{1}{2}m_2v_{2i}^2$

$K_i=2\left(\frac{1}{2}\right)(75.0\text{ kg})(5.00\text{ m/s})^2=\boxed{1.88\text{ kJ}}$

(c) Angular momentum is conserved: $L_f=L_i=\boxed{3\,750\text{ kg}\cdot\text{m}^2/\text{s}}$

(d) $v_f=\frac{L_f}{2(mr_f)}=\frac{3\,750\text{ kg}\cdot\text{m}^2/\text{s}}{2(75.0\text{ kg})(2.50\text{ m})}=\boxed{10.0\text{ m/s}}$

(e) $K_f=2\left(\frac{1}{2}\right)(75.0\text{ kg})(10.0\text{ m/s})^2=\boxed{7.50\text{ kJ}}$

(f) $W=K_f-K_i=\boxed{5.62\text{ kJ}}$

P11.52 (a) $L_i = 2\left[Mv\left(\frac{d}{2}\right)\right] = \boxed{Mvd}$

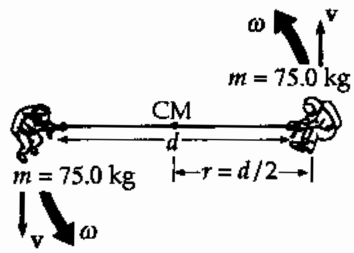


FIG. P11.52

(b) $K = 2\left(\frac{1}{2}Mv^2\right) = \boxed{Mv^2}$

(c) $L_f = L_i = \boxed{Mvd}$

(d) $v_f = \frac{L_f}{2Mr_f} = \frac{Mvd}{2M\left(\frac{d}{4}\right)} = \boxed{2v}$

(e) $K_f = 2\left(\frac{1}{2}Mv_f^2\right) = M(2v)^2 = \boxed{4Mv^2}$

(f) $W = K_f - K_i = \boxed{3Mv^2}$

***P11.53** The moment of inertia of the rest of the Earth is

$$I = \frac{2}{5}MR^2 = \frac{2}{5}5.98\times10^{24}\ \text{kg}\left(6.37\times10^6\ \text{m}\right)^2 = 9.71\times10^{37}\ \text{kg}\cdot\text{m}^2.$$

For the original ice disks,

$$I = \frac{1}{2}Mr^2 = \frac{1}{2}2.30\times10^{19}\ \text{kg}\left(6\times10^5\ \text{m}\right)^2 = 4.14\times10^{30}\ \text{kg}\cdot\text{m}^2.$$

For the final thin shell of water,

$$I = \frac{2}{3}Mr^2 = \frac{2}{3}2.30\times10^{19}\ \text{kg}\left(6.37\times10^6\ \text{m}\right)^2 = 6.22\times10^{32}\ \text{kg}\cdot\text{m}^2.$$

Conservation of angular momentum for the spinning planet is expressed by $I_i\omega_i = I_f\omega_f$

$$\left(4.14\times10^{30} + 9.71\times10^{37}\right)\frac{2\pi}{86\,400\ \text{s}} = \left(6.22\times10^{32} + 9.71\times10^{37}\right)\frac{2\pi}{(86\,400\ \text{s} + \delta)}$$

$$\left(1 + \frac{\delta}{86\,400\ \text{s}}\right)\left(1 + \frac{4.14\times10^{30}}{9.71\times10^{37}}\right) = \left(1 + \frac{6.22\times10^{32}}{9.71\times10^{37}}\right)$$

$$\frac{\delta}{86\,400\ \text{s}} = \frac{6.22\times10^{32}}{9.71\times10^{37}} - \frac{4.14\times10^{30}}{9.71\times10^{37}}$$

$$\boxed{\delta = 0.550\ \text{s}}$$

P11.54 For the cube to tip over, the center of mass (CM) must rise so that it is over the axis of rotation AB. To do this, the CM must be raised a distance of $a\left(\sqrt{2}-1\right)$.

$$\therefore Mga\left(\sqrt{2}-1\right)=\frac{1}{2}I_{\text{cube}}\omega^2$$

From conservation of angular momentum,

$$\frac{4a}{3}mv=\left(\frac{8Ma^2}{3}\right)\omega$$

$$\omega=\frac{mv}{2Ma}$$

$$\frac{1}{2}\left(\frac{8Ma^2}{3}\right)\frac{m^2v^2}{4M^2a^2}=Mga\left(\sqrt{2}-1\right)$$

$$v=\boxed{\frac{M}{m}\sqrt{3ga\left(\sqrt{2}-1\right)}}$$

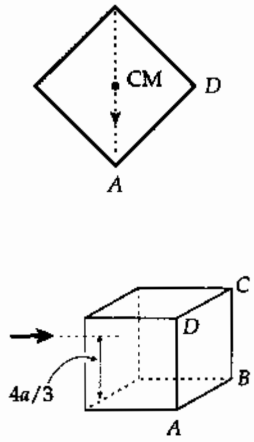


FIG. P11.54

P11.55 Angular momentum is conserved during the inelastic collision.

$$Mva=I\omega$$

$$\omega=\frac{Mva}{I}=\frac{3v}{8a}$$

The condition, that the box falls off the table, is that the center of mass must reach its maximum height as the box rotates, $h_{\max}=a\sqrt{2}$. Using conservation of energy:

$$\frac{1}{2}I\omega^2=Mg\left(a\sqrt{2}-a\right)$$

$$\frac{1}{2}\left(\frac{8Ma^2}{3}\right)\left(\frac{3v}{8a}\right)^2=Mg\left(a\sqrt{2}-a\right)$$

$$v^2=\frac{16}{3}ga\left(\sqrt{2}-1\right)$$

$$v=\boxed{4\left[\frac{ga}{3}\left(\sqrt{2}-1\right)\right]^{1/2}}$$

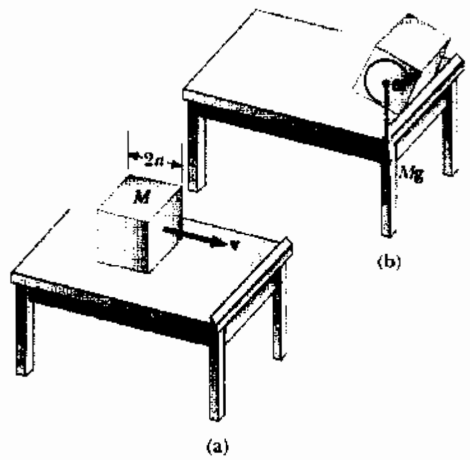


FIG. P11.55

P11.56 (a) The net torque is zero at the point of contact, so the angular momentum before and after the collision must be equal.

$$\left(\frac{1}{2}MR^2\right)\omega_i=\left(\frac{1}{2}MR^2\right)\omega+\left(MR^2\right)\omega \qquad \omega=\boxed{\frac{\omega_i}{3}}$$

(b) $$\frac{\Delta E}{E}=\frac{\frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{\omega_i}{3}\right)^2+\frac{1}{2}M\left(\frac{R\omega_i}{3}\right)^2-\frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_i^2}{\frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_i^2}=\boxed{-\frac{2}{3}}$$

P11.57 (a) $\Delta t = \dfrac{\Delta p}{f} = \dfrac{Mv}{\mu Mg} = \dfrac{MR\omega}{\mu Mg} = \boxed{\dfrac{R\omega_i}{3\mu g}}$

(b) $W = \Delta K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{18}MR^2\omega_i^2$ (See Problem 11.56)

$\mu Mgx = \dfrac{1}{18}MR^2\omega_i^2$ $\boxed{x = \dfrac{R^2\omega_i^2}{18\mu g}}$

ANSWERS TO EVEN PROBLEMS

P11.2 (a) 740 cm^2; (b) 59.5 cm

P11.4 (a) 168°; (b) 11.9° principal value; (c) Only the first is unambiguous.

P11.6 No; see the solution

P11.8 (a) $(-7.00\ \text{N}\cdot\text{m})\hat{\mathbf{k}}$; (b) $(11.0\ \text{N}\cdot\text{m})\hat{\mathbf{k}}$

P11.10 see the solution

P11.12 $\left(-22.0\ \text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}$

P11.14 see the solution

P11.16 (a) 3.14 N·m; (b) $(0.400\ \text{kg}\cdot\text{m})v$; (c) 7.85 m/s^2

P11.18 (a) $\left(+9.03\times10^9\ \text{kg}\cdot\text{m}^2/\text{s}\right)$ south; (b) No; (c) 0

P11.20 103 N·m

P11.22 $\left(4.50\ \text{kg}\cdot\text{m}^2/\text{s}\right)$ up

P11.24 1.20 $\text{kg}\cdot\text{m}^2/\text{s}$ perpendicularly into the clock face

P11.26 8.63 m/s^2

P11.28 (a) $\dfrac{I_1\omega_i}{I_1+I_2}$; (b) $\dfrac{K_f}{K_i} = \dfrac{I_1}{I_1+I_2}$

P11.30 (a) 1.91 rad/s; (b) 2.53 J; 6.44 J

P11.32 8.54 rad/s

P11.34 (a) $7.20\times10^{-3}\ \text{kg}\cdot\text{m}^2/\text{s}$; (b) 9.47 rad/s

P11.36 12.3 m/s^2

P11.38 (a) 2.35 rad/s; (b) 0.498 rad/s; (c) 5.58°

P11.40 131 s

P11.42 (a) 2.19×10^6 m/s; (b) 2.18×10^{-18} J; (c) 4.13×10^{16} rad/s

P11.44 (a) 11.1 m/s; (b) $5.32\times10^3\ \text{kg}\cdot\text{m}^2/\text{s}$; (c) see the solution; (d) 12.0 m/s; (e) 1.08 kJ; (f) 5.34 m/s; (g) 1.46 m; (h) 1.43 s; (i) see the solution

P11.46 (a) $(0.005\,89\ \text{W})t$; (b) 2.59 N·m; (c) $(0.092\,5\ \text{W/s})t$; (d) 40.7 W; (e) $(3.70\ \text{N/s})t$; (f) 8.96 kJ; (g) −4.48 kJ (h) +4.48 kJ

P11.48 (a) 0; (b) 0; no

P11.50 (a) $\dfrac{6mv_i}{Md+3md}$; (b) $\dfrac{M}{M+3m}$

P11.52 (a) Mvd; (b) Mv^2; (c) Mvd; (d) $2v$; (e) $4Mv^2$; (f) $3Mv^2$

P11.54 $\dfrac{M}{m}\sqrt{3ga\left(\sqrt{2}-1\right)}$

P11.56 (a) $\dfrac{\omega_i}{3}$; (b) $\dfrac{\Delta E}{E} = -\dfrac{2}{3}$

12

Static Equilibrium and Elasticity

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q12.1 When you bend over, your center of gravity shifts forward. Once your CG is no longer over your feet, gravity contributes to a nonzero net torque on your body and you begin to rotate.

Q12.2 Yes, it can. Consider an object on a spring oscillating back and forth. In the center of the motion both the sum of the torques and the sum of the forces acting on the object are (separately) zero. Again, a meteoroid flying freely through interstellar space feels essentially no forces and keeps moving with constant velocity.

Q12.3 No—one condition for equilibrium is that $\sum F = 0$. For this to be true with only a single force acting on an object, that force would have to be of zero magnitude; so really no forces act on that object.

Q12.4 (a) Consider pushing up with one hand on one side of a steering wheel and pulling down equally hard with the other hand on the other side. A pair of equal-magnitude oppositely-directed forces applied at different points is called a couple.

(b) An object in free fall has a non-zero net force acting on it, but a net torque of zero about its center of mass.

Q12.5 No. If the torques are all in the same direction, then the net torque cannot be zero.

Q12.6 (a) Yes, provided that its angular momentum is constant.

(b) Yes, provided that its linear momentum is constant.

Q12.7 A V-shaped boomerang, a barstool, an empty coffee cup, a satellite dish, and a curving plastic slide at the edge of a swimming pool each have a center of mass that is not within the bulk of the object.

Q12.8 Suspend the plywood from the nail, and hang the plumb bob from the nail. Trace on the plywood along the string of the plumb bob. Now suspend the plywood with the nail through a different point on the plywood, not along the first line you drew. Again hang the plumb bob from the nail and trace along the string. The center of gravity is located halfway through the thickness of the plywood under the intersection of the two lines you drew.

Q12.9 The center of gravity must be directly over the point where the chair leg contacts the floor. That way, no torque is applied to the chair by gravity. The equilibrium is unstable.

Q12.10 She can be correct. If the dog stands on a relatively thick scale, the dog's legs on the ground might support more of its weight than its legs on the scale. She can check for and if necessary correct for this error by having the dog stand like a bridge with two legs on the scale and two on a book of equal thickness—a physics textbook is a good choice.

Q12.11 If their base areas are equal, the tall crate will topple first. Its center of gravity is higher off the incline than that of the shorter crate. The taller crate can be rotated only through a smaller angle before its center of gravity is no longer over its base.

Q12.12 The free body diagram demonstrates that it is necessary to have friction on the ground to counterbalance the normal force of the wall and to keep the base of the ladder from sliding. Interestingly enough, if there is friction on the floor *and* on the wall, it is not possible to determine whether the ladder will slip from the equilibrium conditions alone.

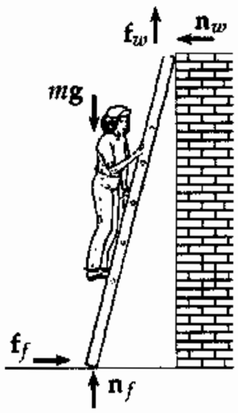


FIG. Q12.12

Q12.13 When you lift a load with your back, your back muscles must supply the torque not only to rotate your upper body to a vertical position, but also to lift the load. Since the distance from the pivot—your hips—to the load—essentially your shoulders—is great, the force required to supply the lifting torque is very large. When lifting from your knees, your back muscles need only keep your back straight. The force required to do that is much smaller than when lifting with your back, as the torque required is small, because the moment arm of the load is small—the line of action of the load passes close to your hips. When you lift from your knees, your much stronger leg and hip muscles do the work.

Q12.14 Shear deformation.

Q12.15 The vertical columns experience simple compression due to gravity acting upon their mass. The horizontal slabs, however, suffer significant shear stress due to gravity. The bottom surface of a sagging lintel is under tension. Stone is much stronger under compression than under tension, so horizontal slabs are more likely to fail.

SOLUTIONS TO PROBLEMS

Section 12.1 **The Conditions for Equilibrium**

P12.1 To hold the bat in equilibrium, the player must exert both a force and a torque on the bat to make

$$\sum F_x = \sum F_y = 0 \text{ and } \sum \tau = 0$$

$\sum F_y = 0 \Rightarrow F - 10.0 \text{ N} = 0$, or the player must exert a net upward force of $F = \boxed{10.0 \text{ N}}$

To satisfy the second condition of equilibrium, the player must exert an applied torque τ_a to make $\sum \tau = \tau_a - (0.600 \text{ m})(10.0 \text{ N}) = 0$. Thus, the required torque is

$$\tau_a = +6.00 \text{ N}\cdot\text{m or } \boxed{6.00 \text{ N}\cdot\text{m counterclockwise}}$$

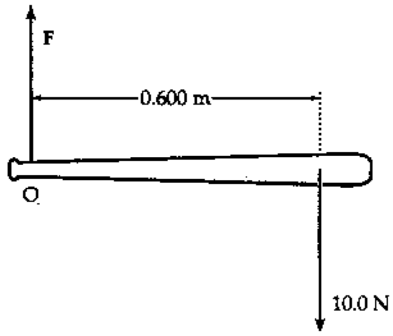


FIG. P12.1

P12.2 Use distances, angles, and forces as shown. The conditions of equilibrium are:

$$\sum F_y = 0 \Rightarrow \boxed{F_y + R_y - F_g = 0}$$
$$\sum F_x = 0 \Rightarrow \boxed{F_x - R_x = 0}$$
$$\sum \tau = 0 \Rightarrow \boxed{F_y \ell \cos\theta - F_g\left(\frac{\ell}{2}\right)\cos\theta - F_x \ell \sin\theta = 0}$$

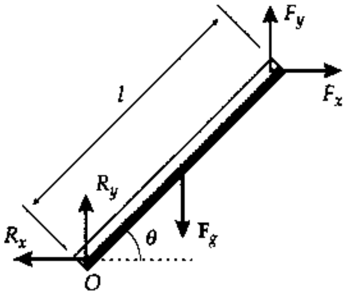


FIG. P12.2

P12.3 Take torques about *P*.

$$\sum \tau_P = -n_0\left[\frac{\ell}{2} + d\right] + m_1 g\left[\frac{\ell}{2} + d\right] + m_b g d - m_2 g x = 0$$

We want to find x for which $n_0 = 0$.

$$x = \frac{(m_1 g + m_b g)d + m_1 g \frac{\ell}{2}}{m_2 g} = \boxed{\frac{(m_1 + m_b)d + m_1 \frac{\ell}{2}}{m_2}}$$

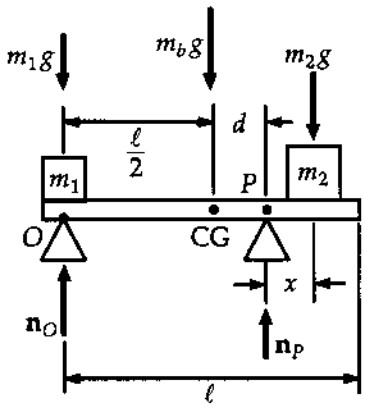


FIG. P12.3

Section 12.2 **More on the Center of Gravity**

P12.4 The hole we can count as negative mass

$$x_{CG} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2}$$

Call σ the mass of each unit of pizza area.

$$x_{CG} = \frac{\sigma \pi R^2 0 - \sigma \pi \left(\frac{R}{2}\right)^2 \left(-\frac{R}{2}\right)}{\sigma \pi R^2 - \sigma \pi \left(\frac{R}{2}\right)^2}$$

$$x_{CG} = \frac{\frac{R}{8}}{\frac{3}{4}} = \boxed{\frac{R}{6}}$$

P12.5 The coordinates of the center of gravity of piece 1 are

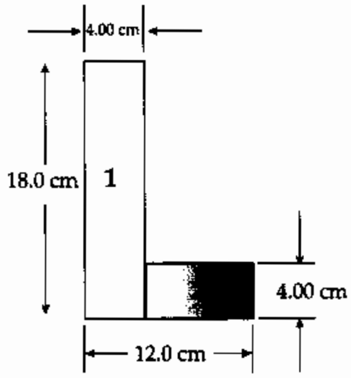


FIG. P12.5

$$x_1 = 2.00 \text{ cm} \text{ and } y_1 = 9.00 \text{ cm}.$$

The coordinates for piece 2 are

$$x_2 = 8.00 \text{ cm} \text{ and } y_2 = 2.00 \text{ cm}.$$

The area of each piece is

$$A_1 = 72.0 \text{ cm}^2 \text{ and } A_2 = 32.0 \text{ cm}^2.$$

And the mass of each piece is proportional to the area. Thus,

$$x_{CG} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(72.0 \text{ cm}^2)(2.00 \text{ cm}) + (32.0 \text{ cm}^2)(8.00 \text{ cm})}{72.0 \text{ cm}^2 + 32.0 \text{ cm}^2} = \boxed{3.85 \text{ cm}}$$

and

$$y_{CG} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(72.0 \text{ cm}^2)(9.00 \text{ cm}) + (32.0 \text{ cm}^2)(2.00 \text{ cm})}{104 \text{ cm}^2} = \boxed{6.85 \text{ cm}}.$$

P12.6 Let σ represent the mass-per-face area. A vertical strip at position x, with width dx and height $\frac{(x-3.00)^2}{9}$ has mass

$$dm = \frac{\sigma(x-3.00)^2 dx}{9}.$$

The total mass is

$$M = \int dm = \int_{x=0}^{3.00} \frac{\sigma(x-3)^2 dx}{9}$$

$$M = \left(\frac{\sigma}{9}\right)\int_0^{3.00}\left(x^2 - 6x + 9\right)dx$$

$$M = \left(\frac{\sigma}{9}\right)\left[\frac{x^3}{3} - \frac{6x^2}{2} + 9x\right]_0^{3.00} = \sigma$$

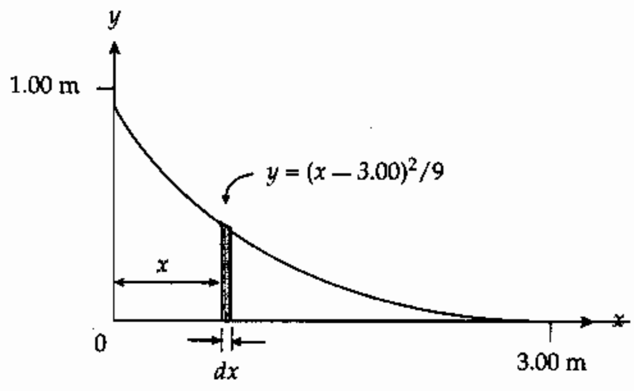


FIG. P12.6

The x-coordinate of the center of gravity is

$$x_{CG} = \frac{\int x\,dm}{M} = \frac{1}{9\sigma}\int_0^{3.00} \sigma x(x-3)^2 dx = \frac{\sigma}{9\sigma}\int_0^{3.00}\left(x^3 - 6x^2 + 9x\right)dx = \frac{1}{9}\left[\frac{x^4}{4} - \frac{6x^3}{3} + \frac{9x^2}{2}\right]_0^{3.00} = \frac{6.75 \text{ m}}{9.00} = \boxed{0.750 \text{ m}}$$

P12.7 Let the fourth mass (8.00 kg) be placed at (x, y), then

$$x_{CG} = 0 = \frac{(3.00)(4.00) + m_4(x)}{12.0 + m_4}$$

$$x = -\frac{12.0}{8.00} = \boxed{-1.50 \text{ m}}$$

Similarly, $$y_{CG} = 0 = \frac{(3.00)(4.00) + 8.00(y)}{12.0 + 8.00}$$

$$y = \boxed{-1.50 \text{ m}}$$

P12.8 In a uniform gravitational field, the center of mass and center of gravity of an object coincide. Thus, the center of gravity of the triangle is located at $x = 6.67$ m, $y = 2.33$ m (see the Example on the center of mass of a triangle in Chapter 9).

The coordinates of the center of gravity of the three-object system are then:

$$x_{CG} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(6.00 \text{ kg})(5.50 \text{ m}) + (3.00 \text{ kg})(6.67 \text{ m}) + (5.00 \text{ kg})(-3.50 \text{ m})}{(6.00 + 3.00 + 5.00) \text{ kg}}$$

$$x_{CG} = \frac{35.5 \text{ kg}\cdot\text{m}}{14.0 \text{ kg}} = \boxed{2.54 \text{ m}} \text{ and}$$

$$y_{CG} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(6.00 \text{ kg})(7.00 \text{ m}) + (3.00 \text{ kg})(2.33 \text{ m}) + (5.00 \text{ kg})(+3.50 \text{ m})}{14.0 \text{ kg}}$$

$$y_{CG} = \frac{66.5 \text{ kg}\cdot\text{m}}{14.0 \text{ kg}} = \boxed{4.75 \text{ m}}$$

Section 12.3 **Examples of Rigid Objects in Static Equilibrium**

P12.9 $\sum \tau = 0 = mg(3r) - Tr$

$2T - Mg\sin 45.0° = 0$

$$T = \frac{Mg\sin 45.0°}{2} = \frac{1\,500\text{ kg}(g)\sin 45.0°}{2}$$

$= (530)(9.80)$ N

$$m = \frac{T}{3g} = \frac{530g}{3g} = \boxed{177\text{ kg}}$$

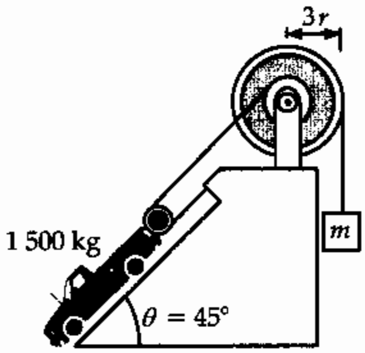


FIG. P12.9

***P12.10** (a) For rotational equilibrium of the lowest rod about its point of support, $\sum \tau = 0$.

$+12.0\text{ g } g\, 3\text{ cm} - m_1 g 4\text{ cm}$ $\boxed{m_1 = 9.00\text{ g}}$

(b) For the middle rod,

$+m_2 2\text{ cm} - (12.0\text{ g} + 9.0\text{ g})5\text{ cm} = 0$ $\boxed{m_2 = 52.5\text{ g}}$

(c) For the top rod,

$(52.5\text{ g} + 12.0\text{ g} + 9.0\text{ g})4\text{ cm} - m_3 6\text{ cm} = 0$ $\boxed{m_3 = 49.0\text{ g}}$

P12.11 $F_g \rightarrow$ standard weight

$F_g' \rightarrow$ weight of goods sold

$F_g(0.240) = F_g'(0.260)$

$$F_g = F_g'\left(\frac{13}{12}\right)$$

$$\left(\frac{F_g - F_g'}{F_g'}\right)100 = \left(\frac{13}{12} - 1\right) \times 100 = \boxed{8.33\%}$$

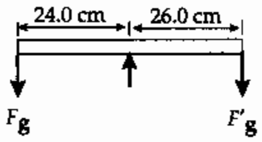


FIG. P12.11

***P12.12** (a) Consider the torques about an axis perpendicular to the page and through the left end of the horizontal beam.

$$\sum \tau = +(T\sin 30.0°)d - (196\text{ N})d = 0,$$

giving $T = \boxed{392\text{ N}}$.

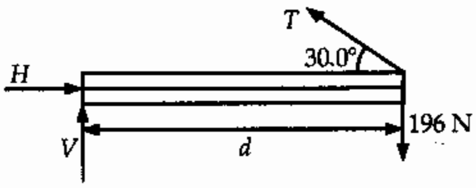


FIG. P12.12

(b) From $\sum F_x = 0$, $H - T\cos 30.0° = 0$, or $H = (392\text{ N})\cos 30.0° = \boxed{339\text{ N to the right}}$.

From $\sum F_y = 0$, $V + T\sin 30.0° - 200\text{ N} = 0$, or $V = 196\text{ N} - (392\text{ N})\sin 30.0° = \boxed{0}$.

P12.13 (a) $\sum F_x = f - n_w = 0$

$\sum F_y = n_g - 800 \text{ N} - 500 \text{ N} = 0$

Taking torques about an axis at the foot of the ladder,

$$(800 \text{ N})(4.00 \text{ m})\sin 30.0° + (500 \text{ N})(7.50 \text{ m})\sin 30.0° - n_w(15.0 \text{ cm})\cos 30.0° = 0$$

Solving the torque equation,

$$n_w = \frac{[(4.00 \text{ m})(800 \text{ N}) + (7.50 \text{ m})(500 \text{ N})]\tan 30.0°}{15.0 \text{ m}} = 268 \text{ N}.$$

Next substitute this value into the F_x equation to find

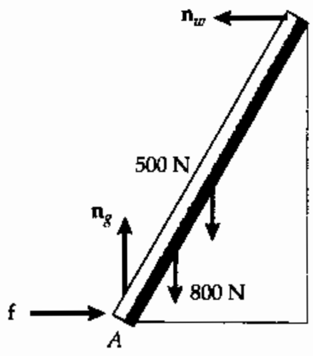


FIG. P12.13

$$f = n_w = \boxed{268 \text{ N}} \text{ in the positive } x \text{ direction.}$$

Solving the equation $\sum F_y = 0$,

$$n_g = \boxed{1\,300 \text{ N}} \text{ in the positive } y \text{ direction.}$$

(b) In this case, the torque equation $\sum \tau_A = 0$ gives:

$$(9.00 \text{ m})(800 \text{ N})\sin 30.0° + (7.50 \text{ m})(500 \text{ N})\sin 30.0° - (15.0 \text{ m})(n_w)\sin 60.0° = 0$$

or $n_w = 421 \text{ N}$.

Since $f = n_w = 421 \text{ N}$ and $f = f_{\max} = \mu n_g$, we find

$$\mu = \frac{f_{\max}}{n_g} = \frac{421 \text{ N}}{1\,300 \text{ N}} = \boxed{0.324}.$$

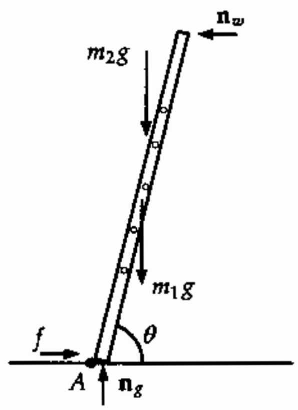
P12.14 (a) $\sum F_x = f - n_w = 0$ (1)

$\sum F_y = n_g - m_1 g - m_2 g = 0$ (2)

$$\sum \tau_A = -m_1 g\left(\frac{L}{2}\right)\cos\theta - m_2 g x \cos\theta + n_w L \sin\theta = 0$$

From the torque equation,

$$n_w = \left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta$$



Then, from equation (1): $f = n_w = \boxed{\left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta}$

and from equation (2): $n_g = \boxed{(m_1 + m_2)g}$

FIG. P12.14

(b) If the ladder is on the verge of slipping when $x = d$,

then $$\mu = \frac{f|_{x=d}}{n_g} = \boxed{\frac{\left(\frac{m_1}{2} + \frac{m_2 d}{L}\right)\cot\theta}{m_1 + m_2}}.$$

P12.15 (a) Taking moments about P,

$$(R\sin 30.0°)0 + (R\cos 30.0°)(5.00\text{ cm}) - (150\text{ N})(30.0\text{ cm}) = 0$$
$$R = 1\,039.2\text{ N} = 1.04\text{ kN}$$

The force exerted by the hammer on the nail is equal in magnitude and opposite in direction:

1.04 kN at 60° upward and to the right.

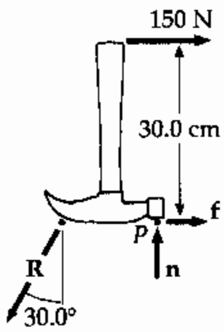


FIG. P12.15

(b) $f = R\sin 30.0° - 150\text{ N} = 370\text{ N}$

$n = R\cos 30.0° = 900\text{ N}$

$$\boxed{\mathbf{F}_{\text{surface}} = (370\text{ N})\hat{\mathbf{i}} + (900\text{ N})\hat{\mathbf{j}}}$$

P12.16 See the free-body diagram at the right.
When the plank is on the verge of tipping about point P, the normal force n_1 goes to zero. Then, summing torques about point P gives

$$\sum \tau_p = -mgd + Mgx = 0 \qquad \text{or} \qquad x = \left(\frac{m}{M}\right)d.$$

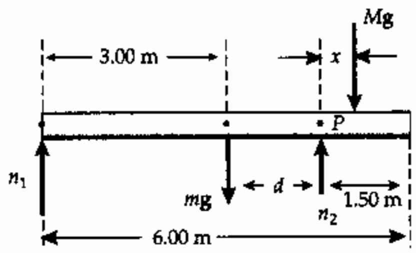


FIG. P12.16

From the dimensions given on the free-body diagram, observe that $d = 1.50$ m Thus, when the plank is about to tip,

$$x = \left(\frac{30.0\text{ kg}}{70.0\text{ kg}}\right)(1.50\text{ m}) = \boxed{0.643\text{ m}}.$$

P12.17 Torque about the front wheel is zero.

$$0 = (1.20\text{ m})(mg) - (3.00\text{ m})(2F_r)$$

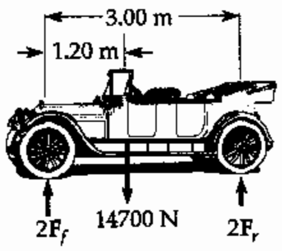


FIG. P12.17

Thus, the force at each rear wheel is

$$F_r = 0.200mg = \boxed{2.94\text{ kN}}.$$

The force at each front wheel is then

$$F_f = \frac{mg - 2F_r}{2} = \boxed{4.41\text{ kN}}.$$

P12.18 $\sum F_x = F_b - F_t + 5.50 \text{ N} = 0$ (1)

$\sum F_y = n - mg = 0$

Summing torques about point O,

$$\sum \tau_O = F_t(1.50 \text{ m}) - (5.50 \text{ m})(10.0 \text{ m}) = 0$$

which yields $F_t = \boxed{36.7 \text{ N to the left}}$

Then, from Equation (1),

$$F_b = 36.7 \text{ N} - 5.50 \text{ N} = \boxed{31.2 \text{ N to the right}}$$

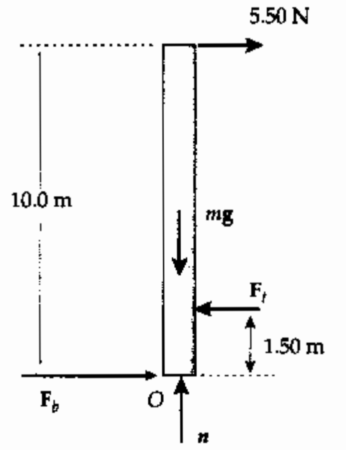


FIG. P12.18

P12.19 (a) $T_e \sin 42.0° = 20.0 \text{ N}$ $\boxed{T_e = 29.9 \text{ N}}$

(b) $T_e \cos 42.0° = T_m$ $\boxed{T_m = 22.2 \text{ N}}$

P12.20 Relative to the hinge end of the bridge, the cable is attached horizontally out a distance $x = (5.00 \text{ m})\cos 20.0° = 4.70 \text{ m}$ and vertically down a distance $y = (5.00 \text{ m})\sin 20.0° = 1.71 \text{ m}$. The cable then makes the following angle with the horizontal:

$$\theta = \tan^{-1}\left[\frac{(12.0 + 1.71) \text{ m}}{4.70 \text{ m}}\right] = 71.1° .$$

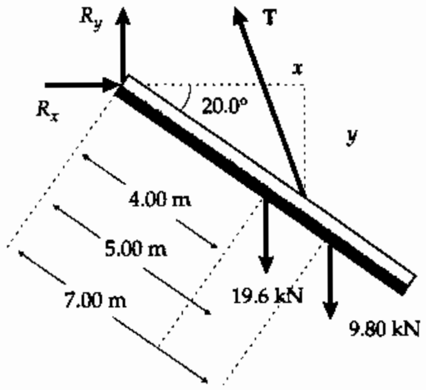


FIG. P12.20

(a) Take torques about the hinge end of the bridge:

$$\begin{aligned} &R_x(0) + R_y(0) - 19.6 \text{ kN}(4.00 \text{ m})\cos 20.0° \\ &\quad - T\cos 71.1°(1.71 \text{ m}) + T\sin 71.1°(4.70 \text{ m}) \\ &\quad - 9.80 \text{ kN}(7.00 \text{ m})\cos 20.0° = 0 \end{aligned}$$

which yields $T = \boxed{35.5 \text{ kN}}$

(b) $\sum F_x = 0 \Rightarrow R_x - T\cos 71.1° = 0$

or $R_x = (35.5 \text{ kN})\cos 71.1° = \boxed{11.5 \text{ kN (right)}}$

(c) $\sum F_y = 0 \Rightarrow R_y - 19.6 \text{ kN} + T\sin 71.1° - 9.80 \text{ kN} = 0$

Thus,

$$R_y = 29.4 \text{ kN} - (35.5 \text{ kN})\sin 71.1° = -4.19 \text{ kN}$$

$$= \boxed{4.19 \text{ kN down}}$$

***P12.21** (a) We model the horse as a particle. The drawbridge will fall out from under the horse.

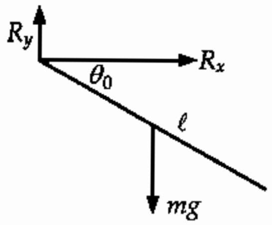


FIG. P12.21(a)

$$\alpha = mg\frac{\frac{1}{2}\ell\cos\theta_0}{\frac{1}{3}m\ell^2} = \frac{3g}{2\ell}\cos\theta_0$$

$$= \frac{3\left(9.80\ \text{m/s}^2\right)\cos 20.0°}{2(8.00\ \text{m})} = \boxed{1.73\ \text{rad/s}^2}$$

(b)

$$\frac{1}{2}I\omega^2 = mgh$$

$$\therefore \frac{1}{2}\cdot\frac{1}{3}m\ell^2\omega^2 = mg\cdot\frac{1}{2}\ell(1-\sin\theta_0)$$

$$\therefore \omega = \sqrt{\frac{3g}{\ell}(1-\sin\theta_0)} = \sqrt{\frac{3\left(9.80\ \text{m/s}^2\right)}{8.00\ \text{m}}(1-\sin 20°)} = \boxed{1.56\ \text{rad/s}}$$

(c) The linear acceleration of the bridge is:

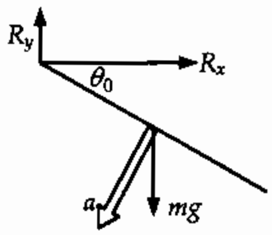


FIG. P12.21(c)

$$a = \frac{1}{2}\ell\alpha = \frac{1}{2}(8.0\ \text{m})\left(1.73\ \text{rad/s}^2\right) = 6.907\ \text{m/s}^2$$

The force at the hinge + the force of gravity produce the acceleration $a = 6.907\ \text{m/s}^2$ at right angles to the bridge.

$$R_x = ma_x = (2\,000\ \text{kg})\left(6.907\ \text{m/s}^2\right)\cos 250° = -4.72\ \text{kN}$$

$$R_y - mg = ma_y$$

$$\therefore R_y = m\left(g + a_y\right) = (2\,000\ \text{kg})\left[9.80\ \text{m/s}^2 + \left(6.907\ \text{m/s}^2\right)\sin 250°\right] = 6.62\ \text{kN}$$

Thus: $\boxed{\mathbf{R} = \left(-4.72\hat{\mathbf{i}} + 6.62\hat{\mathbf{j}}\right)\ \text{kN}}$.

(d)

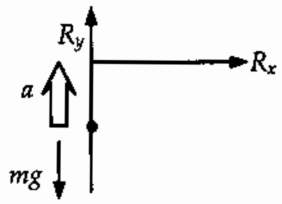


FIG. P12.21(d)

$$R_x = 0$$

$$a = \omega^2\left(\frac{1}{2}\ell\right) = (1.56\ \text{rad/s})^2(4.0\ \text{m}) = 9.67\ \text{m/s}^2$$

$$R_y - mg = ma$$

$$\therefore R_y = (2\,000\ \text{kg})\left(9.8\ \text{m/s}^2 + 9.67\ \text{m/s}^2\right) = 38.9\ \text{kN}$$

Thus: $\boxed{R_y = 38.9\hat{\mathbf{j}}\ \text{kN}}$

P12.22 Call the required force F, with components $F_x = F\cos 15.0°$ and $F_y = -F\sin 15.0°$, transmitted to the center of the wheel by the handles.

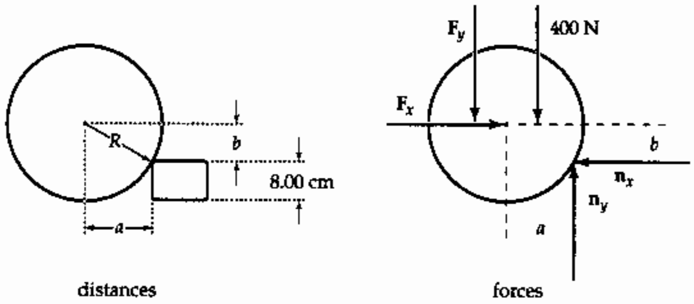


FIG. P12.22

Just as the wheel leaves the ground, the ground exerts no force on it.

$\sum F_x = 0: \; F\cos 15.0° - n_x$ (1)

$\sum F_y = 0: \; -F\sin 15.0° - 400 \text{ N} + n_y = 0$ (2)

Take torques about its contact point with the brick. The needed distances are seen to be:

$$b = R - 8.00 \text{ cm} = (20.0 - 8.00) \text{ cm} = 12.0 \text{ cm}$$
$$a = \sqrt{R^2 - b^2} = 16.0 \text{ cm}$$

(a) $\sum \tau = 0: \; -F_x b + F_y a + (400 \text{ N})a = 0$, or

$F[-(12.0 \text{ cm})\cos 15.0° + (16.0 \text{ cm})\sin 15.0°] + (400 \text{ N})(16.0 \text{ cm}) = 0$

so $F = \dfrac{6\,400 \text{ N}\cdot\text{cm}}{7.45 \text{ cm}} = \boxed{859 \text{ N}}$

(b) Then, using Equations (1) and (2),

$n_x = (859 \text{ N})\cos 15.0° = 830 \text{ N}$ and

$n_y = 400 \text{ N} + (859 \text{ N})\sin 15.0° = 622 \text{ N}$

$n = \sqrt{n_x^2 + n_y^2} = \boxed{1.04 \text{ kN}}$

$\theta = \tan^{-1}\left(\dfrac{n_y}{n_x}\right) = \tan^{-1}(0.749) = \boxed{36.9° \text{ to the left and upward}}$

***P12.23** When $x = x_{\min}$, the rod is on the verge of slipping, so

$$f = (f_s)_{\max} = \mu_s n = 0.50n.$$

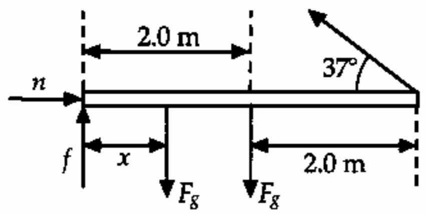


FIG. P12.23

From $\sum F_x = 0$, $n - T\cos 37° = 0$, or $n = 0.799T$.

Thus, $f = 0.50(0.799T) = 0.399T$

From $\sum F_y = 0$, $f + T\sin 37° - 2F_g = 0$, or $0.399T - 0.602T - 2F_g = 0$, giving $T = 2.00F_g$.

Using $\sum \tau = 0$ for an axis perpendicular to the page and through the left end of the beam gives $-F_g \cdot x_{\min} - F_g(2.0 \text{ m}) + [(2F_g)\sin 37°](4.0 \text{ m}) = 0$, which reduces to $x_{\min} = \boxed{2.82 \text{ m}}$.

P12.24 $x = \boxed{\dfrac{3L}{4}}$

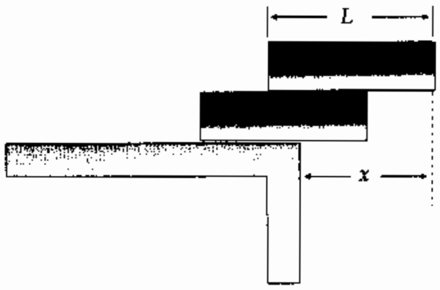


FIG. P12.24

If the CM of the two bricks does not lie over the edge, then the bricks balance.

If the lower brick is placed $\frac{L}{4}$ over the edge, then the second brick may be placed so that its end protrudes $\frac{3L}{4}$ over the edge.

P12.25 To find U, measure distances and forces from point A. Then, balancing torques,

$$(0.750)U = 29.4(2.25) \qquad \boxed{U = 88.2\text{ N}}$$

To find D, measure distances and forces from point B. Then, balancing torques,

$$(0.750)D = (1.50)(29.4) \qquad \boxed{D = 58.8\text{ N}}$$

Also, notice that $U = D + F_g$, so $\sum F_y = 0$.

***P12.26** Consider forces and torques on the beam.

$$\sum F_x = 0: \qquad R\cos\theta - T\cos 53° = 0$$
$$\sum F_y = 0: \qquad R\sin\theta + T\sin 53° - 800\text{ N} = 0$$
$$\sum \tau = 0: \qquad (T\sin 53°)8\text{ m} - (600\text{ N})x - (200\text{ N})4\text{ m} = 0$$

(a) Then $T = \dfrac{600\text{ N}x + 800\text{ N}\cdot\text{m}}{8\text{ m}\sin 53°} = (93.9\text{ N/m})x + 125\text{ N}$. As x increases from 2 m, this expression grows larger.

(b) From substituting back,

$$R\cos\theta = [93.9x + 125]\cos 53°$$
$$R\sin\theta = 800\text{ N} - [93.9x + 125]\sin 53°$$

Dividing, $\tan\theta = \dfrac{R\sin\theta}{R\cos\theta} = -\tan 53° + \dfrac{800\text{ N}}{(93.9x + 125)\cos 53°}$

$$\tan\theta = \tan 53°\left(\frac{32}{3x+4} - 1\right)$$

As x increases the fraction decreases and $\boxed{\theta\text{ decreases}}$.

continued on next page

(c) To find R we can work out $R^2\cos^2\theta + R^2\sin^2\theta = R^2$. From the expressions above for $R\cos\theta$ and $R\sin\theta$,

$$R^2 = T^2\cos^2 53° + T^2\sin^2 53° - 1\,600\text{ N}T\sin 53° + (800\text{ N})^2$$

$$R^2 = T^2 - 1\,600T\sin 53° + 640\,000$$

$$R^2 = (93.9x + 125)^2 - 1\,278(93.9x + 125) + 640\,000$$

$$R = \left(8\,819x^2 - 96\,482x + 495\,678\right)^{1/2}$$

At $x = 0$ this gives $R = 704\text{ N}$. At $x = 2\text{ m}$, $R = 581\text{ N}$. At $x = 8\text{ m}$, $R = 537\text{ N}$. Over the range of possible values for x, the negative term $-96\,482x$ dominates the positive term $8\,819x^2$, and $\boxed{R\text{ decreases}}$ as x increases.

Section 12.4 Elastic Properties of Solids

P12.27 $\frac{F}{A} = Y\frac{\Delta L}{L_i}$

$$\Delta L = \frac{FL_i}{AY} = \frac{(200)(9.80)(4.00)}{\left(0.200\times 10^{-4}\right)\left(8.00\times 10^{10}\right)} = \boxed{4.90\text{ mm}}$$

P12.28 (a) $\text{stress} = \frac{F}{A} = \frac{F}{\pi r^2}$

$$F = (\text{stress})\pi\left(\frac{d}{2}\right)^2$$

$$F = \left(1.50\times 10^8\text{ N/m}^2\right)\pi\left(\frac{2.50\times 10^{-2}\text{ m}}{2}\right)^2$$

$$F = \boxed{73.6\text{ kN}}$$

(b) $\text{stress} = Y(\text{strain}) = \frac{Y\Delta L}{L_i}$

$$\Delta L = \frac{(\text{stress})L_i}{Y} = \frac{\left(1.50\times 10^8\text{ N/m}^2\right)(0.250\text{ m})}{1.50\times 10^{10}\text{ N/m}^2} = \boxed{2.50\text{ mm}}$$

***P12.29** The definition of $Y = \frac{\text{stress}}{\text{strain}}$ means that Y is the slope of the graph:

$$Y = \frac{300\times 10^6\text{ N/m}^2}{0.003} = \boxed{1.0\times 10^{11}\text{ N/m}^2}.$$

P12.30 Count the wires. If they are wrapped together so that all support nearly equal stress, the number should be

$$\frac{20.0\text{ kN}}{0.200\text{ kN}}=100.$$

Since cross-sectional area is proportional to diameter squared, the diameter of the cable will be

$$(1\text{ mm})\sqrt{100}\ \boxed{\sim 1\text{ cm}}.$$

P12.31 From the defining equation for the shear modulus, we find Δx as

$$\Delta x=\frac{hf}{SA}=\frac{\left(5.00\times10^{-3}\text{ m}\right)(20.0\text{ N})}{\left(3.0\times10^{6}\text{ N/m}^2\right)\left(14.0\times10^{-4}\text{ m}^2\right)}=2.38\times10^{-5}\text{ m}$$

or $\Delta x=\boxed{2.38\times10^{-2}\text{ mm}}$.

P12.32 The force acting on the hammer changes its momentum according to

$$mv_i+\overline{F}(\Delta t)=mv_f\ \text{ so }\left|\overline{F}\right|=\frac{m\left|v_f-v_i\right|}{\Delta t}.$$

Hence, $\left|\overline{F}\right|=\dfrac{30.0\text{ kg}\left|-10.0\text{ m/s}-20.0\text{ m/s}\right|}{0.110\text{ s}}=8.18\times10^{3}\text{ N}.$

By Newton's third law, this is also the magnitude of the average force exerted on the spike by the hammer during the blow. Thus, the stress in the spike is:

$$\text{stress}=\frac{F}{A}=\frac{8.18\times10^{3}\text{ N}}{\pi\frac{(0.0230\text{ m})^2}{4}}=1.97\times10^{7}\text{ N/m}^2$$

and the strain is: strain $=\dfrac{\text{stress}}{Y}=\dfrac{1.97\times10^{7}\text{ N/m}^2}{20.0\times10^{10}\text{ N/m}^2}=\boxed{9.85\times10^{-5}}$.

P12.33 (a) $F=(A)(\text{stress})$

$$=\pi\left(5.00\times10^{-3}\text{ m}\right)^2\left(4.00\times10^{8}\text{ N/m}^2\right)$$

$$=\boxed{3.14\times10^{4}\text{ N}}$$

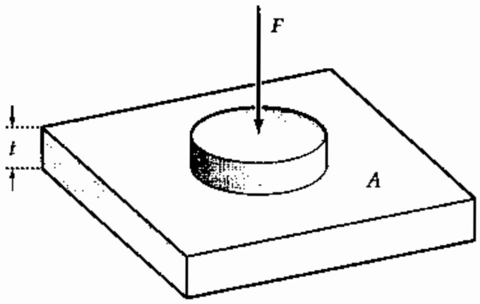


FIG. P12.33

(b) The area over which the shear occurs is equal to the circumference of the hole times its thickness. Thus,

$$A=(2\pi r)t=2\pi\left(5.00\times10^{-3}\text{ m}\right)\left(5.00\times10^{-3}\text{ m}\right)$$

$$=1.57\times10^{-4}\text{ m}^2$$

So, $F=(A)\text{Stress}=\left(1.57\times10^{-4}\text{ m}^2\right)\left(4.00\times10^{8}\text{ N/m}^2\right)=\boxed{6.28\times10^{4}\text{ N}}$.

P12.34 Let the 3.00 kg mass be mass #1, with the 5.00 kg mass, mass # 2. Applying Newton's second law to each mass gives:

$$m_1 a = T - m_1 g \quad (1) \qquad \text{and} \qquad m_2 a = m_2 g - T \quad (2)$$

where T is the tension in the wire.

Solving equation (1) for the acceleration gives: $a = \frac{T}{m_1} - g$,

and substituting this into equation (2) yields: $\frac{m_2}{m_1}T - m_2 g = m_2 g - T$.

Solving for the tension T gives

$$T = \frac{2m_1 m_2 g}{m_2 + m_1} = \frac{2(3.00\text{ kg})(5.00\text{ kg})(9.80\text{ m/s}^2)}{8.00\text{ kg}} = 36.8\text{ N}.$$

From the definition of Young's modulus, $Y = \frac{FL_i}{A(\Delta L)}$, the elongation of the wire is:

$$\Delta L = \frac{TL_i}{YA} = \frac{(36.8\text{ N})(2.00\text{ m})}{(2.00\times10^{11}\text{ N/m}^2)\pi(2.00\times10^{-3}\text{ m})^2} = \boxed{0.029\,3\text{ mm}}.$$

P12.35 Consider recompressing the ice, which has a volume $1.09V_0$.

$$\Delta P = -B\left(\frac{\Delta V}{V_i}\right) = \frac{-(2.00\times10^9\text{ N/m}^2)(-0.090)}{1.09} = \boxed{1.65\times10^8\text{ N/m}^2}$$

***P12.36** $B = -\frac{\Delta P}{\frac{\Delta V}{V_i}} = -\frac{\Delta P V_i}{\Delta V}$

(a) $$\Delta V = -\frac{\Delta P V_i}{B} = -\frac{(1.13\times10^8\text{ N/m}^2)1\text{ m}^3}{0.21\times10^{10}\text{ N/m}^2} = \boxed{-0.053\,8\text{ m}^3}$$

(b) The quantity of water with mass 1.03×10^3 kg occupies volume at the bottom $1\text{ m}^3 - 0.053\,8\text{ m}^3 = 0.946\text{ m}^3$. So its density is $\frac{1.03\times10^3\text{ kg}}{0.946\text{ m}^3} = \boxed{1.09\times10^3\text{ kg/m}^3}$.

(c) With only a 5% volume change in this extreme case, liquid water is indeed nearly incompressible.

***P12.37** Part of the load force extends the cable and part compresses the column by the same distance $\Delta\ell$:

$$F = \frac{Y_A A_A \Delta\ell}{\ell_A} + \frac{Y_s A_s \Delta\ell}{\ell_s}$$

$$\Delta\ell = \frac{F}{\frac{Y_A A_A}{\ell_A} + \frac{Y_s A_s}{\ell_s}} = \frac{8\,500\text{ N}}{\frac{7\times10^{10}\pi(0.162\,4^2 - 0.161\,4^2)}{4(3.25)} + \frac{20\times10^{10}\pi(0.012\,7)^2}{4(5.75)}}$$

$$= \boxed{8.60\times10^{-4}\text{ m}}$$

Additional Problems

***P12.38** (a) The beam is perpendicular to the wall, since $3^2+4^2=5^2$. Then $\sin\theta=\frac{4\text{ m}}{5\text{ m}}$; $\boxed{\theta=53.1°}$.

(b) $\sum\tau_{\text{hinge}}=0$: $+T\sin\theta(3\text{ m})-250\text{ N}(10\text{ m})=0$

$$T=\frac{2\,500\text{ Nm}}{3\text{ m}\sin 53.1°}=\boxed{1.04\times10^3\text{ N}}$$

(c) $x=\frac{T}{k}=\frac{1.04\times10^3\text{ N}}{8.25\times10^3\text{ N/m}}=\boxed{0.126\text{ m}}$

The cable is 5.126 m long. From the law of cosines,

$$4^2=5.126^2+3^2-2(3)(5.126)\cos\theta$$

$$\theta=\cos^{-1}\frac{3^2+5.126^2-4^2}{2(3)(5.126)}=\boxed{51.2°}$$

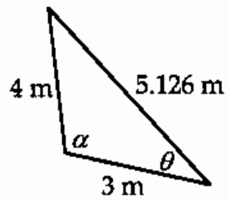


FIG. P12.38

(d) From the law of sines, the angle the hinge makes with the wall satisfies $\frac{\sin\alpha}{5.126\text{ m}}=\frac{\sin 51.2°}{4\text{ m}}$

$$\sin\alpha=0.998\,58$$

$$\sum\tau_{\text{hinge}}=0$$

$$+T(3\text{ m})\sin 51.2°-250\text{ N}(10\text{ m})(0.998\,58)=0$$

$$T=\boxed{1.07\times10^3\text{ N}}$$

(e) $x=\frac{1.07\times10^3\text{ N}}{8.25\times10^3\text{ N/m}}=\boxed{0.129\text{ m}}$

$$\theta=\cos^{-1}\frac{3^2+5.129^2-4^2}{2(3)(5.129)}=\boxed{51.1°}$$

(f) Now the answers are self-consistent:

$$\sin\alpha=5.129\text{ m}\frac{\sin 51.1°}{4\text{ m}}=0.998\,51$$

$$T(3\text{ m})\sin 51.1°-250\text{ N}(10\text{ m})(0.998\,51)=0$$

$$T=1.07\times10^3\text{ N}$$

$$x=0.129\,5\text{ m}$$

$$\boxed{\theta=51.1°}$$

P12.39 Let n_A and n_B be the normal forces at the points of support.

Choosing the origin at point A with $\sum F_y=0$ and $\sum\tau=0$, we find:

$$n_A+n_B-\left(8.00\times10^4\right)g-\left(3.00\times10^4\right)g=0\text{ and}$$

$$-\left(3.00\times10^4\right)(g)15.0-\left(8.00\times10^4\right)(g)25.0+n_B(50.0)=0$$

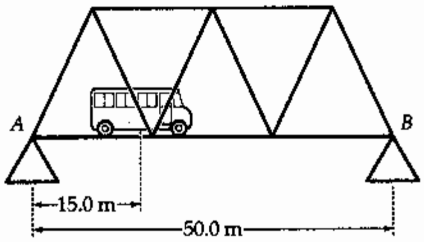


FIG. P12.39

The equations combine to give $n_A=\boxed{5.98\times10^5\text{ N}}$ and $b_B=\boxed{4.80\times10^5\text{ N}}$.

P12.40 When the concrete has cured and the pre-stressing tension has been released, the rod presses in on the concrete and with equal force, T_2, the concrete produces tension in the rod.

(a) In the concrete: stress $= 8.00\times10^6\ \text{N/m}^2 = Y\cdot(\text{strain}) = Y\left(\frac{\Delta L}{L_i}\right)$

$$\text{Thus, } \Delta L = \frac{(\text{stress})L_i}{Y} = \frac{(8.00\times10^6\ \text{N/m}^2)(1.50\ \text{m})}{30.0\times10^9\ \text{N/m}^2}$$

or $\Delta L = 4.00\times10^{-4}\ \text{m} = \boxed{0.400\ \text{mm}}$.

(b) In the concrete: stress $= \frac{T_2}{A_c} = 8.00\times10^6\ \text{N/m}^2$, so

$$T_2 = (8.00\times10^6\ \text{N/m}^2)(50.0\times10^{-4}\ \text{m}^2) = \boxed{40.0\ \text{kN}}$$

(c) For the rod: $\frac{T_2}{A_R} = \left(\frac{\Delta L}{L_i}\right)Y_{\text{steel}}$ so $\Delta L = \frac{T_2 L_i}{A_R Y_{\text{steel}}}$

$$\Delta L = \frac{(4.00\times10^4\ \text{N})(1.50\ \text{m})}{(1.50\times10^{-4}\ \text{m}^2)(20.0\times10^{10}\ \text{N/m}^2)} = 2.00\times10^{-3}\ \text{m} = \boxed{2.00\ \text{mm}}$$

(d) The rod in the finished concrete is 2.00 mm longer than its unstretched length. To remove stress from the concrete, one must stretch the rod 0.400 mm farther, by a total of $\boxed{2.40\ \text{mm}}$.

(e) For the stretched rod around which the concrete is poured:

$$\frac{T_1}{A_R} = \left(\frac{\Delta L_{\text{total}}}{L_i}\right)Y_{\text{steel}} \quad \text{or} \quad T_1 = \left(\frac{\Delta L_{\text{total}}}{L_i}\right)A_R Y_{\text{steel}}$$

$$T_1 = \left(\frac{2.40\times10^{-3}\ \text{m}}{1.50\ \text{m}}\right)(1.50\times10^{-4}\ \text{m}^2)(20.0\times10^{10}\ \text{N/m}^2) = \boxed{48.0\ \text{kN}}$$

***P12.41** With ℓ as large as possible, n_1 and n_2 will both be large. The equality sign in $f_2 \le \mu_s n_2$ will be true, but the less-than sign in $f_1 < \mu_s n_1$. Take torques about the lower end of the pole.

$$n_2\ell\cos\theta + F_g\left(\frac{1}{2}\ell\right)\cos\theta - f_2\ell\sin\theta = 0$$

Setting $f_2 = 0.576n_2$, the torque equation becomes

$$n_2(1-0.576\tan\theta) + \frac{1}{2}F_g = 0$$

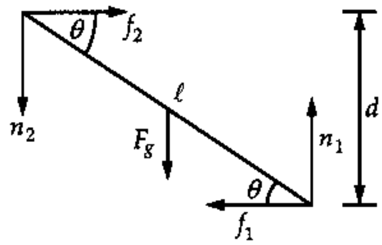


FIG. P12.41

Since $n_2 > 0$, it is necessary that

$$1 - 0.576\tan\theta < 0$$

$$\therefore \tan\theta > \frac{1}{0.576} = 1.736$$

$$\therefore \theta > 60.1°$$

$$\therefore \ell = \frac{d}{\sin\theta} < \frac{7.80\ \text{ft}}{\sin 60.1°} = \boxed{9.00\ \text{ft}}$$

P12.42 Call the normal forces A and B. They make angles α and β with the vertical.

$$\sum F_x = 0: \quad A\sin\alpha - B\sin\beta = 0$$
$$\sum F_y = 0: \quad A\cos\alpha - Mg + B\cos\beta = 0$$

Substitute $B = \dfrac{A\sin\alpha}{\sin\beta}$

$$A\cos\alpha + A\cos\beta\frac{\sin\alpha}{\sin\beta} = Mg$$
$$A(\cos\alpha\sin\beta + \sin\alpha\cos\beta) = Mg\sin\beta$$
$$A = \boxed{Mg\frac{\sin\beta}{\sin(\alpha+\beta)}}$$
$$B = \boxed{Mg\frac{\sin\alpha}{\sin(\alpha+\beta)}}$$

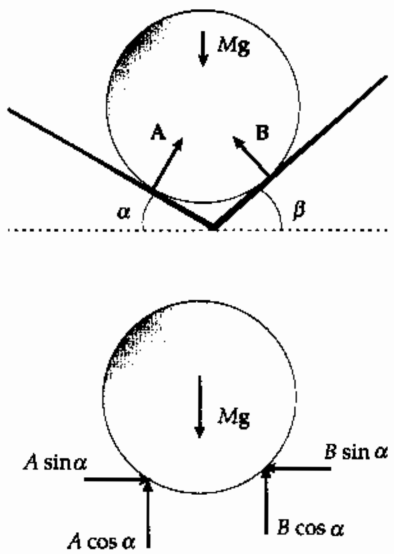


FIG. P12.42

P12.43 (a) See the diagram.

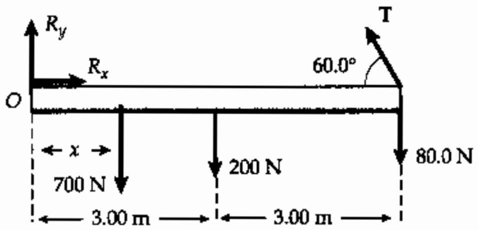


FIG. P12.43

(b) If $x = 1.00$ m, then

$$\sum \tau_O = (-700\text{ N})(1.00\text{ m}) - (200\text{ N})(3.00\text{ m}) -(80.0\text{ N})(6.00\text{ m}) +(T\sin 60.0°)(6.00\text{ m}) = 0$$

Solving for the tension gives: $T = \boxed{343\text{ N}}$.

From $\sum F_x = 0$, $R_x = T\cos 60.0° = \boxed{171\text{ N}}$.

From $\sum F_y = 0$, $R_y = 980\text{ N} - T\sin 60.0° = \boxed{683\text{ N}}$.

(c) If $T = 900$ N:

$$\sum \tau_O = (-700\text{ N})x - (200\text{ N})(3.00\text{ m}) - (80.0\text{ N})(6.00\text{ m}) + \left[(900\text{ N})\sin 60.0°\right](6.00\text{ m}) = 0.$$

Solving for x gives: $x = \boxed{5.13\text{ m}}$.

P12.44 (a) Sum the torques about top hinge:

$\sum \tau = 0$:

$$C(0)+D(0)+200\text{ N}\cos 30.0°(0)$$
$$+200\text{ N}\sin 30.0°(3.00\text{ m})$$
$$-392\text{ N}(1.50\text{ m})+A(1.80\text{ m})$$
$$+B(0)=0$$

Giving $A = \boxed{160\text{ N (right)}}$.

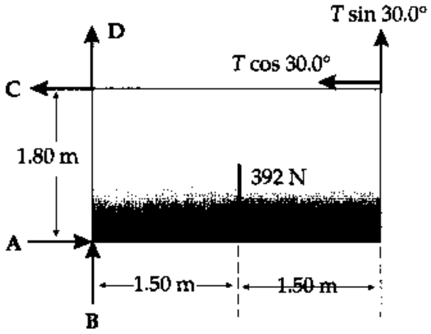


FIG. P12.44

(b) $\sum F_x = 0$:

$$-C-200\text{ N}\cos 30.0°+A=0$$
$$C=160\text{ N}-173\text{ N}=-13.2\text{ N}$$

In our diagram, this means $\boxed{13.2\text{ N to the right}}$.

(c) $\sum F_y = 0$: $+B+D-392\text{ N}+200\text{ N}\sin 30.0°=0$

$B+D=392\text{ N}-100\text{ N}=\boxed{292\text{ N(up)}}$

(d) Given $C = 0$: Take torques about bottom hinge to obtain

$$A(0)+B(0)+0(1.80\text{ m})+D(0)-392\text{ N}(1.50\text{ m})+T\sin 30.0°(3.00\text{ m})+T\cos 30.0°(1.80\text{ m})=0$$

so $T=\dfrac{588\text{ N}\cdot\text{m}}{(1.50\text{ m}+1.56\text{ m})}=\boxed{192\text{ N}}$.

P12.45 Using $\sum F_x = \sum F_y = \sum \tau = 0$, choosing the origin at the left end of the beam, we have (neglecting the weight of the beam)

$$\sum F_x = R_x - T\cos\theta = 0,$$
$$\sum F_y = R_y + T\sin\theta - F_g = 0,$$

and $\sum \tau = -F_g(L+d)+T\sin\theta(2L+d)=0$.

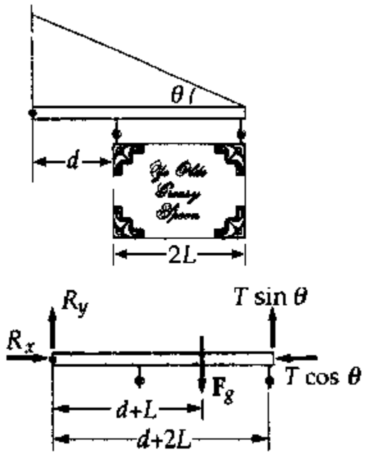


FIG. P12.45

Solving these equations, we find:

(a) $T=\boxed{\dfrac{F_g(L+d)}{\sin\theta(2L+d)}}$

(b) $R_x=\boxed{\dfrac{F_g(L+d)\cot\theta}{2L+d}}$ $\qquad R_y=\boxed{\dfrac{F_gL}{2L+d}}$

P12.46 $\sum \tau_{\text{point } 0} = 0$ gives

$$(T\cos 25.0°)\left(\frac{3\ell}{4}\sin 65.0°\right) + (T\sin 25.0°)\left(\frac{3\ell}{4}\cos 65.0°\right)$$
$$= (2\,000\text{ N})(\ell\cos 65.0°) + (1\,200\text{ N})\left(\frac{\ell}{2}\cos 65.0°\right)$$

From which, $T = 1\,465\text{ N} = \boxed{1.46\text{ kN}}$

From $\sum F_x = 0$,

$$H = T\cos 25.0° = 1\,328\text{ N(toward right)} = \boxed{1.33\text{ kN}}$$

From $\sum F_y = 0$,

$$V = 3\,200\text{ N} - T\sin 25.0° = 2\,581\text{ N(upward)} = \boxed{2.58\text{ kN}}$$

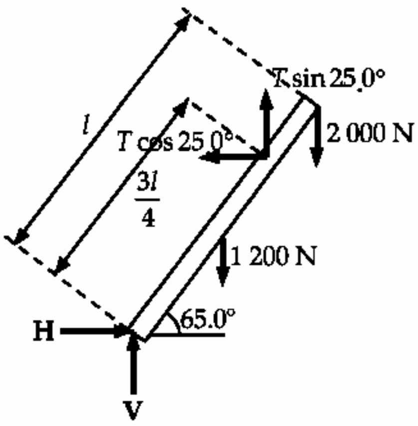


FIG. P12.46

P12.47 We interpret the problem to mean that the support at point B is frictionless. Then the support exerts a force in the x direction and

$$\boxed{F_{By} = 0}$$
$$\sum F_x = F_{Bx} - F_{Ax} = 0$$
$$F_{Ay} - (3\,000 + 10\,000)g = 0$$

and $\sum \tau = -(3\,000g)(2.00) - (10\,000g)(6.00) + F_{Bx}(1.00) = 0$.

These equations combine to give

$$F_{Ax} = F_{Bx} = \boxed{6.47 \times 10^5\text{ N}}$$
$$F_{Ay} = \boxed{1.27 \times 10^5\text{ N}}$$

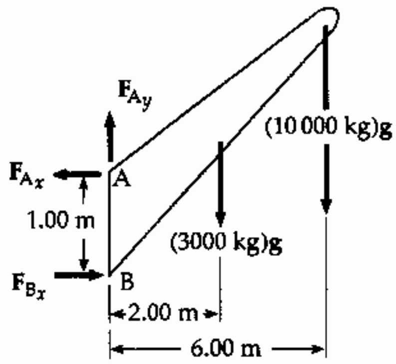


FIG. P12.47

P12.48 $n = (M+m)g \quad H = f$

$$H_{\max} = f_{\max} = \mu_s(m+M)g$$
$$\sum \tau_A = 0 = \frac{mgL}{2}\cos 60.0° + Mgx\cos 60.0° - HL\sin 60.0°$$
$$\frac{x}{L} = \frac{H\tan 60.0°}{Mg} - \frac{m}{2M} = \frac{\mu_s(m+M)\tan 60.0°}{M} - \frac{m}{2M}$$
$$= \frac{3}{2}\mu_s\tan 60.0° - \frac{1}{4} = \boxed{0.789}$$

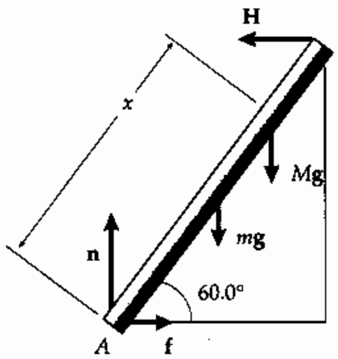


FIG. P12.48

P12.49 From the free-body diagram, the angle **T** makes with the rod is

$$\theta = 60.0° + 20.0° = 80.0°$$

and the perpendicular component of **T** is $T\sin 80.0°$.
Summing torques around the base of the rod,

$$\sum \tau = 0: \quad -(4.00\text{ m})(10\,000\text{ N})\cos 60° + T(4.00\text{ m})\sin 80° = 0$$

$$T = \frac{(10\,000\text{ N})\cos 60.0°}{\sin 80.0°} = \boxed{5.08 \times 10^3\text{ N}}$$

$$\sum F_x = 0: \quad F_H - T\cos 20.0° = 0$$

$$F_H = T\cos 20.0° = \boxed{4.77 \times 10^3\text{ N}}$$

$$\sum F_y = 0: \quad F_V + T\sin 20.0° - 10\,000\text{ N} = 0$$

and $F_V = (10\,000\text{ N}) - T\sin 20.0° = \boxed{8.26 \times 10^3\text{ N}}$

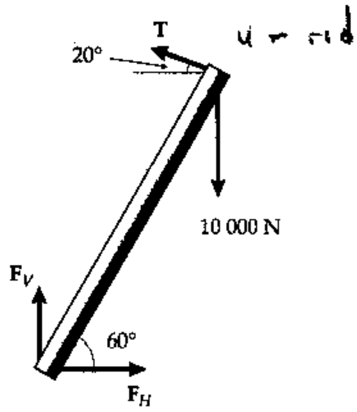


FIG. P12.49

P12.50 Choosing the origin at **R**,

(1) $\sum F_x = +R\sin 15.0° - T\sin\theta = 0$

(2) $\sum F_y = 700 - R\cos 15.0° + T\cos\theta = 0$

(3) $\sum \tau = -700\cos\theta(0.180) + T(0.070\,0) = 0$

Solve the equations for θ

from (3), $T = 1\,800\cos\theta$ from (1), $R = \dfrac{1\,800\sin\theta\cos\theta}{\sin 15.0°}$

Then (2) gives $700 - \dfrac{1\,800\sin\theta\cos\theta\cos 15.0°}{\sin 15.0°} + 1\,800\cos^2\theta = 0$

or $\cos^2\theta + 0.388\,9 - 3.732\sin\theta\cos\theta = 0$

Squaring, $\cos^4\theta - 0.880\,9\cos^2\theta + 0.010\,13 = 0$

Let $u = \cos^2\theta$ then using the quadratic equation,
$u = 0.011\,65$ or $0.869\,3$

Only the second root is physically possible,

$$\therefore \theta = \cos^{-1}\sqrt{0.869\,3} = \boxed{21.2°}$$

$$\therefore T = \boxed{1.68 \times 10^3\text{ N}} \quad \text{and} \quad R = \boxed{2.34 \times 10^3\text{ N}}$$

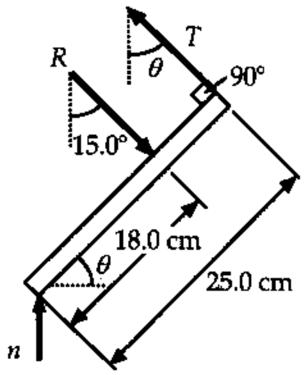


FIG. P12.50

P12.51 Choosing torques about **R**, with $\sum \tau = 0$

$$-\frac{L}{2}(350\text{ N}) + (T\sin 12.0°)\left(\frac{2L}{3}\right) - (200\text{ N})L = 0.$$

From which, $T = \boxed{2.71\text{ kN}}$.
Let R_x = compression force along spine, and from $\sum F_x = 0$

$$R_x = T_x = T\cos 12.0° = \boxed{2.65\text{ kN}}.$$

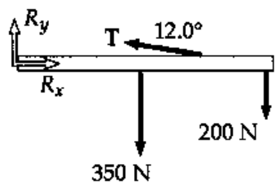


FIG. P12.51

P12.52 (a) Just three forces act on the rod: forces perpendicular to the sides of the trough at A and B, and its weight. The lines of action of A and B will intersect at a point above the rod. They will have no torque about this point. The rod's weight will cause a torque about the point of intersection as in Figure 12.52(a), and the rod will not be in equilibrium unless the center of the rod lies vertically below the intersection point, as in Figure 12.52(b). All three forces must be concurrent. Then the line of action of the weight is a diagonal of the rectangle formed by the trough and the normal forces, and the rod's center of gravity is vertically above the bottom of the trough.

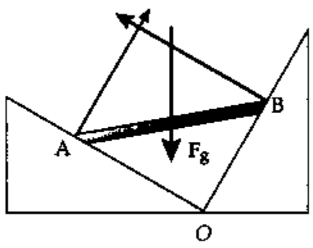


FIG. P12.52(a)

(b) In Figure (b), $\overline{AO}\cos 30.0° = \overline{BO}\cos 60.0°$ and

$$L^2 = \overline{AO}^2 + \overline{BO}^2 = \overline{AO}^2 + \overline{AO}^2\left(\frac{\cos^2 30.0°}{\cos^2 60.0°}\right)$$

$$\overline{AO} = \frac{L}{\sqrt{1+\frac{\cos^2 30.0°}{\cos^2 60.0°}}} = \frac{L}{2}$$

So $\cos\theta = \frac{\overline{AO}}{L} = \frac{1}{2}$ and $\theta = \boxed{60.0°}$.

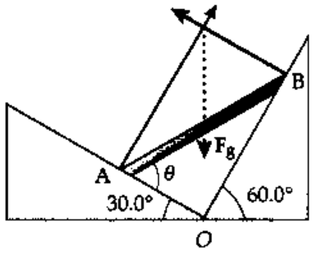


FIG. P12.52(b)

P12.53 (a) Locate the origin at the bottom left corner of the cabinet and let $x =$ distance between the *resultant normal force* and the *front of the cabinet*. Then we have

$$\sum F_x = 200\cos 37.0° - \mu n = 0 \quad (1)$$

$$\sum F_y = 200\sin 37.0° + n - 400 = 0 \quad (2)$$

$$\sum \tau = n(0.600 - x) - 400(0.300) + 200\sin 37.0°(0.600)$$
$$-200\cos 37.0°(0.400) = 0 \quad (3)$$

From (2), $n = 400 - 200\sin 37.0° = 280\text{ N}$

From (3), $x = \frac{72.2 - 120 + 280(0.600) - 64.0}{280}$

$x = \boxed{20.1\text{ cm}}$ to the left of the front edge

From (1), $\mu_k = \frac{200\cos 37.0°}{280} = \boxed{0.571}$

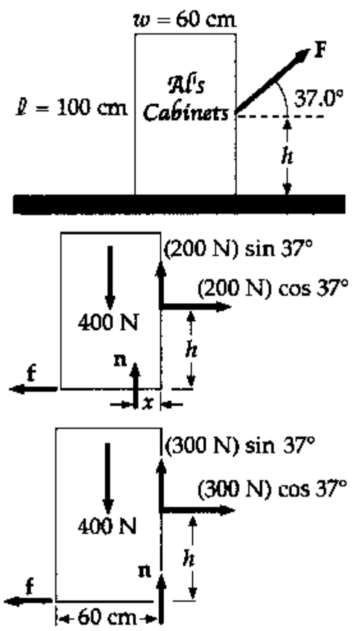


FIG. P12.53

(b) In this case, locate the origin $x = 0$ at the bottom right corner of the cabinet. Since the cabinet is about to tip, we can use $\sum \tau = 0$ to find h:

$$\sum \tau = 400(0.300) - (300\cos 37.0°)h = 0 \qquad h = \frac{120}{300\cos 37.0°} = \boxed{0.501\text{ m}}$$

P12.54 (a), (b) Use the first diagram and sum the torques about the lower front corner of the cabinet.

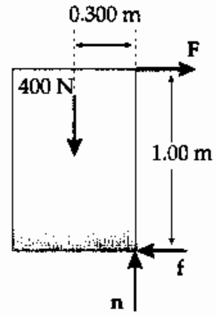
$$\sum \tau = 0 \Rightarrow -F(1.00 \text{ m}) + (400 \text{ N})(0.300 \text{ m}) = 0$$

yielding $F = \dfrac{(400 \text{ N})(0.300 \text{ m})}{1.00 \text{ m}} = \boxed{120 \text{ N}}$

$$\sum F_x = 0 \Rightarrow -f + 120 \text{ N} = 0, \quad \text{or} \quad f = 120 \text{ N}$$

$$\sum F_y = 0 \Rightarrow -400 \text{ N} + n = 0, \quad \text{so} \quad n = 400 \text{ N}$$

Thus, $\mu_s = \dfrac{f}{n} = \dfrac{120 \text{ N}}{400 \text{ N}} = \boxed{0.300}$.



(c) Apply F' at the upper rear corner and directed so $\theta + \phi = 90.0°$ to obtain the largest possible lever arm.

$$\theta = \tan^{-1}\left(\frac{1.00 \text{ m}}{0.600 \text{ m}}\right) = 59.0°$$

Thus, $\phi = 90.0° - 59.0° = 31.0°$.

Sum the torques about the lower front corner of the cabinet:

$$-F'\sqrt{(1.00 \text{ m})^2 + (0.600 \text{ m})^2} + (400 \text{ N})(0.300 \text{ m}) = 0$$

so $\quad F' = \dfrac{120 \text{ N} \cdot \text{m}}{1.17 \text{ m}} = 103 \text{ N}$.

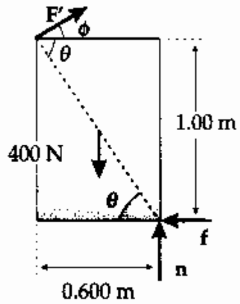


FIG. P12.54

Therefore, the minimum force required to tip the cabinet is

$\boxed{\text{103 N applied at 31.0° above the horizontal at the upper left corner}}$.

P12.55 (a) We can use $\sum F_x = \sum F_y = 0$ and $\sum \tau = 0$ with pivot point at the contact on the floor.

Then $\sum F_x = T - \mu_s n = 0$,

$\sum F_y = n - Mg - mg = 0$, and

$$\sum \tau = Mg(L\cos\theta) + mg\left(\frac{L}{2}\cos\theta\right) - T(L\sin\theta) = 0$$

Solving the above equations gives

$$M = \boxed{\frac{m}{2}\left(\frac{2\mu_s \sin\theta - \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)}$$

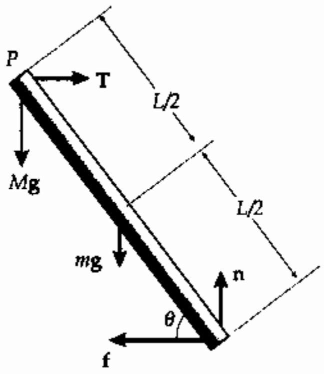


FIG. P12.55

This answer is the maximum vaue for M if $\mu_s < \cot\theta$. If $\mu_s \geq \cot\theta$, the mass M can increase without limit. It has no maximum value, and part (b) cannot be answered as stated either. In the case $\mu_s < \cot\theta$, we proceed.

(b) At the floor, we have the normal force in the y-direction and frictional force in the x-direction. The reaction force then is

$$R = \sqrt{n^2 + (\mu_s n)^2} = \boxed{(M+m)g\sqrt{1+\mu_s^2}}.$$

At point P, the force of the beam on the rope is

$$F = \sqrt{T^2 + (Mg)^2} = \boxed{g\sqrt{M^2 + \mu_s^2(M+m)^2}}.$$

P12.56 (a) The height of pin B is

$$(10.0\text{ m})\sin 30.0° = 5.00\text{ m}.$$

The length of bar BC is then

$$\overline{BC} = \frac{5.00\text{ m}}{\sin 45.0°} = 7.07\text{ m}.$$

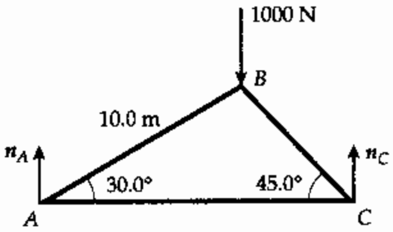


FIG. P12.56(a)

Consider the entire truss:

$$\sum F_y = n_A - 1\,000\text{ N} + n_C = 0$$
$$\sum \tau_A = -(1\,000\text{ N})10.0\cos 30.0° + n_C[10.0\cos 30.0° + 7.07\cos 45.0°] = 0$$

Which gives $\boxed{n_C = 634\text{ N}}$.

Then, $\boxed{n_A = 1\,000\text{ N} - n_C = 366\text{ N}}$.

(b) Suppose that a bar exerts on a pin a force not along the length of the bar. Then, the pin exerts on the bar a force with a component perpendicular to the bar. The only other force on the bar is the pin force on the other end. For $\sum \mathbf{F} = 0$, this force must also have a component perpendicular to the bar. Then, the total torque on the bar is not zero. The contradiction proves that the bar can only exert forces along its length.

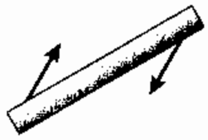


FIG. P12.56(b)

(c) Joint A:

$$\sum F_y = 0: -C_{AB}\sin 30.0° + 366\text{ N} = 0,$$

so $C_{AB} = \boxed{732\text{ N}}$

$$\sum F_x = 0: -C_{AB}\cos 30.0° + T_{AC} = 0$$
$$T_{AC} = (732\text{ N})\cos 30.0° = \boxed{634\text{ N}}$$

Joint B:

$$\sum F_x = 0: (732\text{ N})\cos 30.0° - C_{BC}\cos 45.0° = 0$$
$$C_{BC} = \frac{(732\text{ N})\cos 30.0°}{\cos 45.0°} = \boxed{897\text{ N}}$$

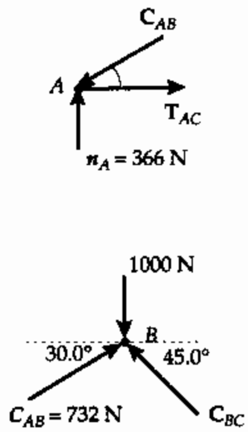


FIG. P12.56(c)

P12.57 From geometry, observe that

$$\cos\theta = \frac{1}{4} \qquad \text{and} \qquad \theta = 75.5°$$

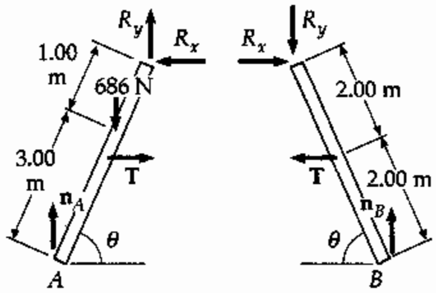


FIG. P12.57

For the left half of the ladder, we have

$$\sum F_x = T - R_x = 0 \qquad (1)$$
$$\sum F_y = R_y + n_A - 686 \text{ N} = 0 \qquad (2)$$
$$\sum \tau_{\text{top}} = 686 \text{ N}(1.00\cos 75.5°) + T(2.00\sin 75.5°) - n_A(4.00\cos 75.5°) = 0 \qquad (3)$$

For the right half of the ladder we have

$$\sum F_x = R_x - T = 0$$
$$\sum F_y = n_B - R_y = 0 \qquad (4)$$
$$\sum \tau_{\text{top}} = n_B(4.00\cos 75.5°) - T(2.00\sin 75.5°) = 0 \qquad (5)$$

Solving equations 1 through 5 simultaneously yields:

(a) $\boxed{T = 133 \text{ N}}$

(b) $\boxed{n_A = 429 \text{ N}}$ and $\boxed{n_B = 257 \text{ N}}$

(c) $\boxed{R_x = 133 \text{ N}}$ and $\boxed{R_y = 257 \text{ N}}$

The force exerted by the left half of the ladder on the right half is to the right and downward.

P12.58 (a)

$$x_{CG} = \frac{\sum m_i x_i}{\sum m_i}$$
$$= \frac{(1\,000 \text{ kg})10.0 \text{ m} + (125 \text{ kg})0 + (125 \text{ kg})0 + (125 \text{ kg})20.0 \text{ m}}{1\,375 \text{ kg}} = \boxed{9.09 \text{ m}}$$
$$y_{CG} = \frac{(1\,000 \text{ kg})10.0 \text{ m} + (125 \text{ kg})20.0 \text{ m} + (125 \text{ kg})20.0 \text{ m} + (125 \text{ kg})0}{1\,375 \text{ kg}}$$
$$= \boxed{10.9 \text{ m}}$$

(b) By symmetry, $x_{CG} = \boxed{10.0 \text{ m}}$

There is no change in $y_{CG} = \boxed{10.9 \text{ m}}$

(c) $$v_{CG} = \left(\frac{10.0 \text{ m} - 9.09 \text{ m}}{8.00 \text{ s}}\right) = \boxed{0.114 \text{ m/s}}$$

P12.59 Considering the torques about the point at the bottom of the bracket yields:

$$(0.050\,0 \text{ m})(80.0 \text{ N}) - F(0.060\,0 \text{ m}) = 0 \text{ so } \boxed{F = 66.7 \text{ N}}.$$

P12.60 When it is on the verge of slipping, the cylinder is in equilibrium.

$\sum F_x = 0$: $\quad f_1 = n_2 = \mu_s n_1 \quad$ and $\quad f_2 = \mu_s n_2$

$\sum F_y = 0$: $\quad P + n_1 + f_2 = F_g$

$\sum \tau = 0$: $\quad P = f_1 + f_2$

As P grows so do f_1 and f_2

Therefore, since $\mu_s = \frac{1}{2}$, $\quad f_1 = \frac{n_1}{2} \quad$ and $\quad f_2 = \frac{n_2}{2} = \frac{n_1}{4}$

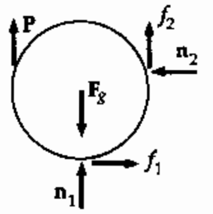


FIG. P12.60

then $\quad P + n_1 + \frac{n_1}{4} = F_g \quad (1) \quad$ and $\quad P = \frac{n_1}{2} + \frac{n_1}{4} = \frac{3}{4}n_1 \quad (2)$

So $\quad P + \frac{5}{4}n_1 = F_g \quad$ becomes $\quad P + \frac{5}{4}\left(\frac{4}{3}P\right) = F_g \quad$ or $\quad \frac{8}{3}P = F_g$

Therefore, $P = \boxed{\frac{3}{8}F_g}$

P12.61 (a) $|F| = k(\Delta L)$, Young's modulus is $Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L_i}} = \frac{FL_i}{A(\Delta L)}$

Thus, $Y = \frac{kL_i}{A}$ and $k = \boxed{\frac{YA}{L_i}}$

(b) $W = -\int_0^{\Delta L} F dx = -\int_0^{\Delta L} (-kx) dx = \frac{YA}{L_i}\int_0^{\Delta L} x dx = \boxed{YA\frac{(\Delta L)^2}{2L_i}}$

P12.62 (a) Take both balls together. Their weight is 3.33 N and their CG is at their contact point.

$\sum F_x = 0: +P_3 - P_1 = 0$

$\sum F_y = 0: +P_2 - 3.33\text{ N} = 0 \qquad P_2 = \boxed{3.33\text{ N}}$

$\sum \tau_A = 0: -P_3 R + P_2 R - 3.33\text{ N}(R + R\cos 45.0°)$
$\quad +P_1(R + 2R\cos 45.0°) = 0$

Substituting,

$-P_1 R + (3.33\text{ N})R - (3.33\text{ N})R(1 + \cos 45.0°)$
$\quad +P_1 R(1 + 2\cos 45.0°) = 0$

$(3.33\text{ N})\cos 45.0° = 2P_1 \cos 45.0°$

$P_1 = \boxed{1.67\text{ N}}$ so $P_3 = \boxed{1.67\text{ N}}$

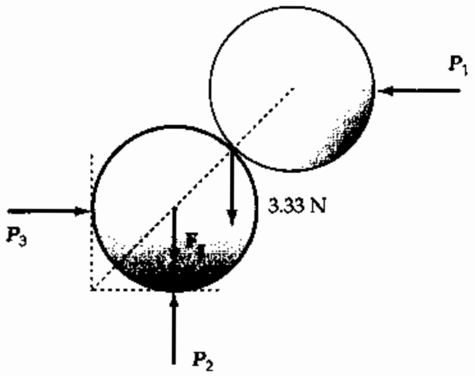


FIG. P12.62(a)

(b) Take the upper ball. The lines of action of its weight, of P_1, and of the normal force n exerted by the lower ball all go through its center, so for rotational equilibrium there can be no frictional force.

$\sum F_x = 0: n\cos 45.0° - P_1 = 0$

$n = \frac{1.67\text{ N}}{\cos 45.0°} = \boxed{2.36\text{ N}}$

$\sum F_y = 0: n\sin 45.0° - 1.67\text{ N} = 0$ gives the same result

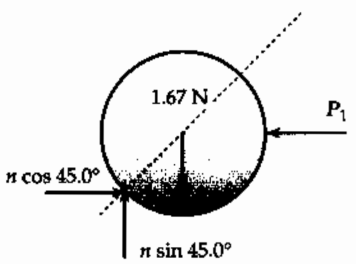


FIG. P12.62(b)

P12.63 $\sum F_y = 0$: $+380\text{ N} - F_g + 320\text{ N} = 0$

$F_g = 700\text{ N}$

Take torques about her feet:

$\sum \tau = 0$: $-380\text{ N}(2.00\text{ m}) + (700\text{ N})x + (320\text{ N})0 = 0$

$x = \boxed{1.09\text{ m}}$

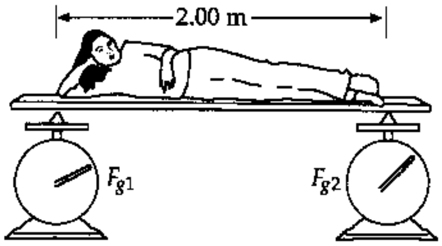


FIG. P12.63

P12.64 The tension in this cable is not uniform, so this becomes a fairly difficult problem.

$$\frac{dL}{L} = \frac{F}{YA}$$

At any point in the cable, F is the weight of cable below that point. Thus, $F = \mu g y$ where μ is the mass per unit length of the cable.

Then, $\Delta y = \int_0^{L_i} \left(\frac{dL}{L}\right) dy = \frac{\mu g}{YA} \int_0^{L_i} y\,dy = \frac{1}{2} \frac{\mu g L_i^2}{YA}$

$$\Delta y = \frac{1}{2} \frac{(2.40)(9.80)(500)^2}{(2.00 \times 10^{11})(3.00 \times 10^{-4})} = 0.049\,0\text{ m} = \boxed{4.90\text{ cm}}$$

P12.65 (a) $F = m\left(\frac{\Delta v}{\Delta t}\right) = (1.00\text{ kg})\frac{(10.0 - 1.00)\text{ m/s}}{0.002\text{ s}} = \boxed{4\,500\text{ N}}$

(b) $\text{stress} = \frac{F}{A} = \frac{4\,500\text{ N}}{(0.010\text{ m})(0.100\text{ m})} = \boxed{4.50 \times 10^6\text{ N/m}^2}$

(c) $\boxed{\text{Yes}}$. This is more than sufficient to break the board.

P12.66 The CG lies above the center of the bottom. Consider a disk of water at height y above the bottom. Its radius is

$$25.0\text{ cm}+(35.0-25.0\text{ cm})\left(\frac{y}{30.0\text{ cm}}\right)=25.0\text{ cm}+\frac{y}{3}$$

Its area is $\pi\left(25.0\text{ cm}+\frac{y}{3}\right)^2$. Its volume is $\pi\left(25.0\text{ cm}+\frac{y}{3}\right)^2 dy$ and its mass is $\pi\rho\left(25.0\text{ cm}+\frac{y}{3}\right)^2 dy$. The whole mass of the water is

$$M=\int_{y=0}^{30.0\text{ cm}} dm=\int_0^{30.0\text{ cm}}\pi\rho\left(625+\frac{50.0y}{3}+\frac{y^2}{9}\right)dy$$

$$M=\pi\rho\left[625y+\frac{50.0y^2}{6}+\frac{y^3}{27}\right]_0^{30.0}$$

$$M=\pi\rho\left[625(30.0)+\frac{50.0(30.0)^2}{6}+\frac{(30.0)^3}{27}\right]$$

$$M=\pi\left(10^{-3}\text{ kg/cm}^3\right)\left(27\,250\text{ cm}^3\right)=85.6\text{ kg}$$

The height of the center of gravity is

$$y_{CG}=\int_{y=0}^{30.0\text{ cm}}\frac{y\,dm}{M}$$

$$=\pi\rho\int_0^{30.0\text{ cm}}\left(625y+\frac{50.0y^2}{3}+\frac{y^3}{9}\right)\frac{dy}{M}$$

$$=\frac{\pi\rho}{M}\left[\frac{625y^2}{2}+\frac{50.0y^3}{9}+\frac{y^4}{36}\right]_0^{30.0\text{ cm}}$$

$$=\frac{\pi\rho}{M}\left[\frac{625(30.0)^2}{2}+\frac{50.0(30.0)^3}{9}+\frac{(30.0)^4}{36}\right]$$

$$=\frac{\pi\left(10^{-3}\text{ kg/cm}^3\right)}{M}\left[453\,750\text{ cm}^4\right]$$

$$y_{CG}=\frac{1.43\times10^3\text{ kg}\cdot\text{cm}}{85.6\text{ kg}}=\boxed{16.7\text{ cm}}$$

P12.67 Let θ represent the angle of the wire with the vertical. The radius of the circle of motion is $r = (0.850\text{ m})\sin\theta$.

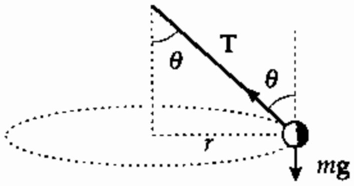


FIG. P12.67

For the mass:

$$\sum F_r = ma_r = m\frac{v^2}{r} = mr\omega^2$$

$$T\sin\theta = m\left[(0.850\text{ m})\sin\theta\right]\omega^2$$

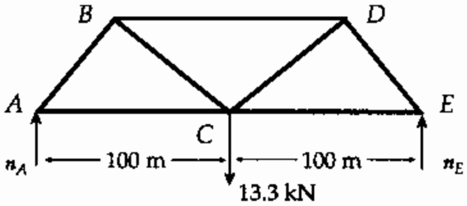
Further, $\frac{T}{A} = Y\cdot(\text{strain})$ or $T = AY\cdot(\text{strain})$

Thus, $AY\cdot(\text{strain}) = m(0.850\text{ m})\omega^2$, giving

$$\omega = \sqrt{\frac{AY\cdot(\text{strain})}{m(0.850\text{ m})}} = \sqrt{\frac{\pi\left(3.90\times10^{-4}\text{ m}\right)^2\left(7.00\times10^{10}\text{ N/m}^2\right)\left(1.00\times10^{-3}\right)}{(1.20\text{ kg})(0.850\text{ m})}}$$

or $\omega = \boxed{5.73\text{ rad/s}}$.

P12.68 For the bridge as a whole:



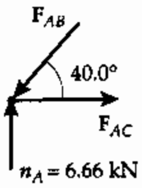
$$\sum\tau_A = n_A(0) - (13.3\text{ kN})(100\text{ m}) + n_E(200\text{ m}) = 0$$

so $n_E = \frac{(13.3\text{ kN})(100\text{ m})}{200\text{ m}} = \boxed{6.66\text{ kN}}$

$\sum F_y = n_A - 13.3\text{ kN} + n_E = 0$ gives

$n_A = 13.3\text{ kN} - n_E = \boxed{6.66\text{ kN}}$

At Pin A:



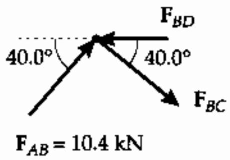
$$\sum F_y = -F_{AB}\sin 40.0° + 6.66\text{ kN} = 0 \text{ or}$$

$$F_{AB} = \frac{6.66\text{ kN}}{\sin 40.0°} = \boxed{10.4\text{ kN (compression)}}$$

$$\sum F_x = F_{AC} - (10.4\text{ kN})\cos 40.0° = 0 \text{ so}$$

$$F_{AC} = (10.4\text{ kN})\cos 40.0° = \boxed{7.94\text{ kN (tension)}}$$

At Pin B:



$$\sum F_y = (10.4\text{ kN})\sin 40.0° - F_{BC}\sin 40.0° = 0$$

Thus, $F_{BC} = \boxed{10.4\text{ kN (tension)}}$

$$\sum F_x = F_{AB}\cos 40.0° + F_{BC}\cos 40.0° - F_{BD} = 0$$

$$F_{BD} = 2(10.4\text{ kN})\cos 40.0° = \boxed{15.9\text{ kN (compression)}}$$

By symmetry: $\boxed{F_{DE} = F_{AB} = 10.4\text{ kN (compression)}}$

$\boxed{F_{DC} = F_{BC} = 10.4\text{ kN (tension)}}$

and $\boxed{F_{EC} = F_{AC} = 7.94\text{ kN (tension)}}$

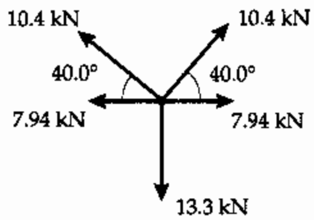


FIG. P12.68

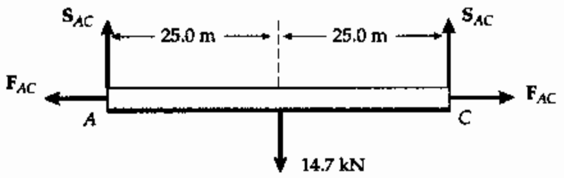
We can check by analyzing Pin C:

$$\sum F_x = +7.94\text{ kN} - 7.94\text{ kN} = 0 \text{ or } 0 = 0$$

$$\sum F_y = 2(10.4\text{ kN})\sin 40.0° - 13.3\text{ kN} = 0$$

which yields $0 = 0$.

P12.69 Member AC is not in pure compression or tension. It also has shear forces present. It exerts a downward force S_{AC} and a tension force F_{AC} on Pin A and on Pin C. Still, this member is in equilibrium.



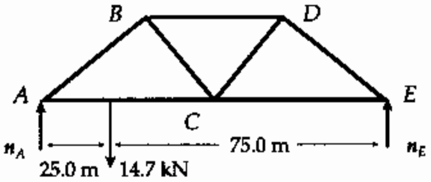
$\sum F_x = F_{AC} - F'_{AC} = 0 \Rightarrow F_{AC} = F'_{AC}$

$\sum \tau_A = 0$: $\quad -(14.7\text{ kN})(25.0\text{ m}) + S'_{AC}(50.0\text{ m}) = 0$

or $\quad S'_{AC} = 7.35\text{ kN}$

$\sum F_y = S_{AC} - 14.7\text{ kN} + 7.35\text{ kN} = 0 \Rightarrow S_{AC} = 7.35\text{ kN}$

Then $S_{AC} = S'_{AC}$ and we have proved that the loading by the car is equivalent to one-half the weight of the car pulling down on each of pins A and C, so far as the rest of the truss is concerned.



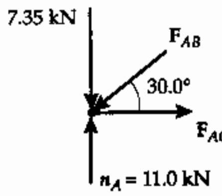
For the Bridge as a whole: $\sum \tau_A = 0$:

$$-(14.7\text{ kN})(25.0\text{ m}) + n_E(100\text{ m}) = 0$$

$$\boxed{n_E = 3.67\text{ kN}}$$

$$\sum F_y = n_A - 14.7\text{ kN} + 3.67\text{ kN} = 0$$

$$\boxed{n_A = 11.0\text{ kN}}$$



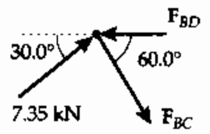
At Pin A:

$$\sum F_y = -7.35\text{ kN} + 11.0\text{ kN} - F_{AB}\sin 30.0° = 0$$

$$\boxed{F_{AB} = 7.35\text{ kN (compression)}}$$

$$\sum F_x = F_{AC} - (7.35\text{ kN})\cos 30.0° = 0$$

$$\boxed{F_{AC} = 6.37\text{ kN (tension)}}$$



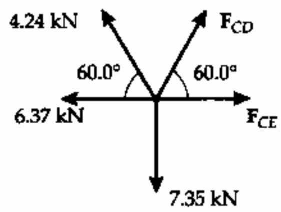
At Pin B:

$$\sum F_y = -(7.35\text{ kN})\sin 30.0° - F_{BC}\sin 60.0° = 0$$

$$\boxed{F_{BC} = 4.24\text{ kN (tension)}}$$

$$\sum F_x = (7.35\text{ kN})\cos 30.0° + (4.24\text{ kN})\cos 60.0° - F_{BD} = 0$$

$$\boxed{F_{BD} = 8.49\text{ kN (compression)}}$$



At Pin C:

$$\sum F_y = (4.24\text{ kN})\sin 60.0° + F_{CD}\sin 60.0° - 7.35\text{ kN} = 0$$

$$\boxed{F_{CD} = 4.24\text{ kN (tension)}}$$

$$\sum F_x = -6.37\text{ kN} - (4.24\text{ kN})\cos 60.0° + (4.24\text{ kN})\cos 60.0° + F_{CE} = 0$$

$$\boxed{F_{CE} = 6.37\text{ kN (tension)}}$$

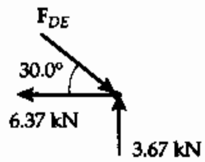


FIG. P12.69

At Pin E:

$$\sum F_y = -F_{DE}\sin 30.0° + 3.67\text{ kN} = 0$$

$$\boxed{F_{DE} = 7.35\text{ kN (compression)}}$$

or $\sum F_x = -6.37\text{ kN} - F_{DE}\cos 30.0° = 0$

which gives $F_{DE} = 7.35\text{ kN}$ as before.

P12.70 (1) $ph = I\omega$

(2) $p = Mv_{CM}$

If the ball rolls without slipping, $R\omega = v_{CM}$

So, $h = \frac{I\omega}{p} = \frac{I\omega}{Mv_{CM}} = \frac{I}{MR} = \boxed{\frac{2}{5}R}$

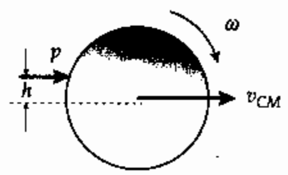


FIG. P12.70

P12.71 (a) If the acceleration is a, we have $P_x = ma$ and $P_y + n - F_g = 0$. Taking the origin at the center of gravity, the torque equation gives

$$P_y(L-d) + P_x h - nd = 0.$$

Solving these equations, we find

$$P_y = \boxed{\frac{F_g}{L}\left(d - \frac{ah}{g}\right)}.$$

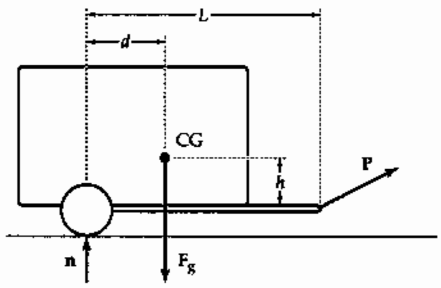


FIG. P12.71

(b) If $P_y = 0$, then $d = \frac{ah}{g} = \frac{(2.00\ \text{m/s}^2)(1.50\ \text{m})}{9.80\ \text{m/s}^2} = \boxed{0.306\ \text{m}}$.

(c) Using the given data, $P_x = -306\ \text{N}$ and $P_y = 553\ \text{N}$.

Thus, $\boxed{\mathbf{P} = \left(-306\hat{\mathbf{i}} + 553\hat{\mathbf{j}}\right)\ \text{N}}$.

***P12.72** When the cyclist is on the point of tipping over forward, the normal force on the rear wheel is zero. Parallel to the plane we have $f_1 - mg\sin\theta = ma$. Perpendicular to the plane, $n_1 - mg\cos\theta = 0$. Torque about the center of mass:

$$mg(0) - f_1(1.05\ \text{m}) + n_1(0.65\ \text{m}) = 0.$$

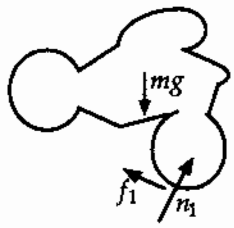


FIG. P12.72

Combining by substitution,

$$ma = f_1 - mg\sin\theta = \frac{n_1 0.65\ \text{m}}{1.05\ \text{m}} - mg\sin\theta = mg\cos\theta\frac{0.65\ \text{m}}{1.05\ \text{m}} - mg\sin\theta$$

$$a = g\left(\cos 20°\frac{0.65}{1.05} - \sin 20°\right) = \boxed{2.35\ \text{m/s}^2}$$

***P12.73** When the car is on the point of rolling over, the normal force on its inside wheels is zero.

$\sum F_y = ma_y$: $\quad n - mg = 0$

$\sum F_x = ma_x$: $\quad f = \frac{mv^2}{R}$

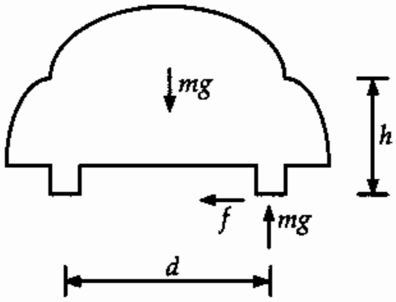


FIG. P12.73

Take torque about the center of mass: $fh - n\frac{d}{2} = 0$.

Then by substitution $\frac{mv_{max}^2}{R}h - \frac{mgd}{2} = 0 \qquad v_{max} = \sqrt{\frac{gdR}{2h}}$

A wider wheelbase (larger d) and a lower center of mass (smaller h) will reduce the risk of rollover.

P12.2 $F_y + R_y - F_g = 0$; $F_x - R_x = 0$;
$F_y \ell \cos\theta - F_g\left(\frac{\ell}{2}\right)\cos\theta - F_x \ell \sin\theta = 0$

P12.4 see the solution

P12.6 0.750 m

P12.8 (2.54 m, 4.75 m)

P12.10 (a) 9.00 g; (b) 52.5 g; (c) 49.0 g

P12.12 (a) 392 N; (b) $(339\hat{\mathbf{i}} + 0\hat{\mathbf{j}})$ N

P12.14 (a) $f = \left[\frac{m_1 g}{2} + \frac{m_2 g x}{L}\right]\cot\theta$;
$n_g = (m_1 + m_2)g$; (b) $\mu = \frac{\left(\frac{m_1}{2} + \frac{m_2 d}{L}\right)\cot\theta}{m_1 + m_2}$

P12.16 see the solution; 0.643 m

P12.18 36.7 N to the left; 31.2 N to the right

P12.20 (a) 35.5 kN; (b) 11.5 kN to the right; (c) 4.19 kN down

P12.22 (a) 859 N; (b) 104 kN at 36.9° above the horizontal to the left

P12.24 $\frac{3L}{4}$

P12.26 (a) see the solution; (b) θ decreases; (c) R decreases

P12.28 (a) 73.6 kN; (b) 2.50 mm

P12.30 ~1 cm

P12.32 9.85×10^{-5}

P12.34 0.029 3 mm

P12.36 (a) $-0.053\,8\ \text{m}^3$; (b) $1.09 \times 10^3\ \text{kg/m}^3$; (c) Yes, in most practical circumstances

P12.38 (a) 53.1°; (b) 1.04 kN; (c) 0.126 m, 51.2°; (d) 1.07 kN; (e) 0.129 m, 51.1°; (f) 51.1°

P12.40 (a) 0.400 mm; (b) 40.0 kN; (c) 2.00 mm; (d) 2.40 mm; (e) 48.0 kN

P12.42 at A: $Mg\frac{\sin\beta}{\sin(\alpha+\beta)}$; at B: $Mg\frac{\sin\alpha}{\sin(\alpha+\beta)}$

P12.44 (a) 160 N to the right; (b) 13.2 N to the right; (c) 292 N up; (d) 192 N

P12.46 1.46 kN; $(1.33\hat{\mathbf{i}} + 2.58\hat{\mathbf{j}})$ kN

P12.48 0.789

P12.50 $T = 1.68$ kN; $R = 2.34$ kN; $\theta = 21.2°$

P12.52 (a) see the solution; (b) 60.0°

P12.54 (a) 120 N; (b) 0.300; (c) 103 N at 31.0° above the horizontal to the right

P12.56 (a), (b) see the solution; (c) $C_{AB} = 732$ N; $T_{AC} = 634$ N; $C_{BC} = 897$ N

P12.58 (a) (9.09 m, 10.9 m); (b) (10.0 m, 10.9 m); (c) 0.114 m/s to the right

P12.60 $\frac{3}{8}F_g$

P12.62 (a) $P_1 = 1.67$ N; $P_2 = 3.33$ N; $P_3 = 1.67$ N; (b) 2.36 N

P12.64 4.90 cm

P12.66 16.7 cm above the center of the bottom

P12.68 $C_{AB} = 10.4$ kN; $T_{AC} = 7.94$ kN; $T_{BC} = 10.4$ kN; $C_{BD} = 15.9$ kN; $C_{DE} = 10.4$ kN; $T_{DC} = 10.4$ kN; $T_{EC} = 7.94$ kN

P12.70 $\frac{2}{5}R$

P12.72 $2.35\ \text{m/s}^2$

Universal Gravitation

ANSWERS TO QUESTIONS

Q13.1 Because g is the same for all objects near the Earth's surface. The larger mass needs a larger force to give it just the same acceleration.

Q13.2 To a good first approximation, your bathroom scale reading is unaffected because you, the Earth, and the scale are all in free fall in the Sun's gravitational field, in orbit around the Sun. To a precise second approximation, you weigh slightly less at noon and at midnight than you do at sunrise or sunset. The Sun's gravitational field is a little weaker at the center of the Earth than at the surface subsolar point, and a little weaker still on the far side of the planet. When the Sun is high in your sky, its gravity pulls up on you a little more strongly than on the Earth as a whole. At midnight the Sun pulls down on you a little less strongly than it does on the Earth below you. So you can have another doughnut with lunch, and your bedsprings will still last a little longer.

Q13.3 Kepler's second law states that the angular momentum of the Earth is constant as the Earth orbits the sun. Since $L = m\omega r$, as the orbital radius decreases from June to December, then the orbital speed must increase accordingly.

Q13.4 Because both the Earth and Moon are moving in orbit about the Sun. As described by $F_{\text{gravitational}} = ma_{\text{centripetal}}$, the gravitational force of the Sun merely keeps the Moon (and Earth) in a nearly circular orbit of radius 150 million kilometers. Because of its velocity, the Moon is kept in its orbit about the Earth by the gravitational force of the Earth. There is no imbalance of these forces, at new moon or full moon.

Q13.5 Air resistance causes a decrease in the energy of the satellite-Earth system. This reduces the diameter of the orbit, bringing the satellite closer to the surface of the Earth. A satellite in a smaller orbit, however, must travel faster. Thus, the effect of air resistance is to speed up the satellite!

Q13.6 Kepler's third law, which applies to all planets, tells us that the period of a planet is proportional to $r^{3/2}$. Because Saturn and Jupiter are farther from the Sun than Earth, they have longer periods. The Sun's gravitational field is much weaker at a distant Jovian planet. Thus, an outer planet experiences much smaller centripetal acceleration than Earth and has a correspondingly longer period.

Q13.7 Ten terms are needed in the potential energy:

$$U = U_{12} + U_{13} + U_{14} + U_{15} + U_{23} + U_{24} + U_{25} + U_{34} + U_{35} + U_{45}.$$

With N particles, you need $\sum_{i=1}^{N}(i-1) = \frac{N^2 - N}{2}$ terms.

Q13.8 No, the escape speed does not depend on the mass of the rocket. If a rocket is launched at escape speed, then the total energy of the rocket-Earth system will be zero. When the separation distance becomes infinite ($U = 0$) the rocket will stop ($K = 0$). In the expression $\frac{1}{2}mv^2 - \frac{GM_E m}{r} = 0$, the mass m of the rocket divides out.

Q13.9 It takes 100 times more energy for the 10^5 kg spacecraft to reach the moon than the 10^3 kg spacecraft. Ideally, each spacecraft can reach the moon with zero velocity, so the only term that need be analyzed is the change in gravitational potential energy. U is proportional to the mass of the spacecraft.

Q13.10 The escape speed from the Earth is 11.2 km/s and that from the Moon is 2.3 km/s, smaller by a factor of 5. The energy required—and fuel—would be proportional to v^2, or 25 times more fuel is required to leave the Earth versus leaving the Moon.

Q13.11 The satellites used for TV broadcast are in geosynchronous orbits. The centers of their orbits are the center of the Earth, and their orbital planes are the Earth's equatorial plane extended. This is the plane of the celestial equator. The communication satellites are so far away that they appear quite close to the celestial equator, from any location on the Earth's surface.

Q13.12 For a satellite in orbit, one focus of an elliptical orbit, or the center of a circular orbit, must be located at the center of the Earth. If the satellite is over the northern hemisphere for half of its orbit, it must be over the southern hemisphere for the other half. We could share with Easter Island a satellite that would look straight down on Arizona each morning and vertically down on Easter Island each evening.

Q13.13 The absolute value of the gravitational potential energy of the Earth-Moon system is twice the kinetic energy of the moon relative to the Earth.

Q13.14 In a circular orbit each increment of displacement is perpendicular to the force applied. The dot product of force and displacement is zero. The work done by the gravitational force on a planet in an elliptical orbit speeds up the planet at closest approach, but negative work is done by gravity and the planet slows as it sweeps out to its farthest distance from the Sun. Therefore, net work in one complete orbit is zero.

Q13.15 Every point q on the sphere that does not lie along the axis connecting the center of the sphere and the particle will have companion point q' for which the components of the gravitational force perpendicular to the axis will cancel. Point q' can be found by rotating the sphere through 180° about the axis. The forces will not necessarily cancel if the mass is not uniformly distributed, unless the center of mass of the non-uniform sphere still lies along the axis.

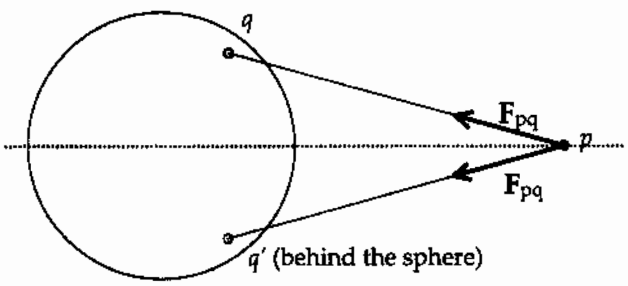


FIG. Q13.15

Q13.16 Speed is maximum at closest approach. Speed is minimum at farthest distance.

Q13.17 Set the universal description of the gravitational force, $F_g = \frac{GM_X m}{R_X^2}$, equal to the local description, $F_g = ma_{\text{gravitational}}$, where M_X and R_X are the mass and radius of planet X, respectively, and m is the mass of a "test particle." Divide both sides by m.

Q13.18 The gravitational force of the Earth on an extra particle at its center must be zero, not infinite as one interpretation of Equation 13.1 would suggest. All the bits of matter that make up the Earth will pull in different outward directions on the extra particle.

Q13.19 Cavendish determined G. Then from $g = \frac{GM}{R^2}$, one may determine the mass of the Earth.

Q13.20 The gravitational force is conservative. An encounter with a stationary mass cannot permanently speed up a spacecraft. Jupiter is moving. A spacecraft flying across its orbit just behind the planet will gain kinetic energy as the planet's gravity does net positive work on it.

Q13.21 **Method one:** Take measurements from an old kinescope of *Apollo* astronauts on the moon. From the motion of a freely falling object or from the period of a swinging pendulum you can find the acceleration of gravity on the moon's surface and calculate its mass. **Method two:** One could determine the approximate mass of the moon using an object hanging from an extremely sensitive balance, with knowledge of the position and distance of the moon and the radius of the Earth. First weigh the object when the moon is directly overhead. Then weigh of the object when the moon is just rising or setting. The slight difference between the measured weights reveals the cause of tides in the Earth's oceans, which is a difference in the strength of the moon's gravity between different points on the Earth. **Method three:** Much more precisely, from the motion of a spacecraft in orbit around the moon, its mass can be determined from Kepler's third law.

Q13.22 The spacecraft did not have enough fuel to stop dead in its high-speed course for the Moon.

SOLUTIONS TO PROBLEMS

Section 13.1 Newton's Law of Universal Gravitation

P13.1 For two 70-kg persons, modeled as spheres,

$$F_g = \frac{Gm_1 m_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(70\ \text{kg})(70\ \text{kg})}{(2\ \text{m})^2}\ \boxed{\sim 10^{-7}\ \text{N}}.$$

P13.2 $F = m_1 g = \frac{Gm_1 m_2}{r^2}$

$$g = \frac{Gm_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(4.00\times10^4\times10^3\ \text{kg}\right)}{(100\ \text{m})^2} = \boxed{2.67\times10^{-7}\ \text{m/s}^2}$$

P13.3 (a) At the midpoint between the two objects, the forces exerted by the 200-kg and 500-kg objects are oppositely directed,

and from $F_g = \dfrac{Gm_1m_2}{r^2}$

we have $\sum F = \dfrac{G(50.0\text{ kg})(500\text{ kg} - 200\text{ kg})}{(0.200\text{ m})^2} = \boxed{2.50\times10^{-5}\text{ N}}$ toward the 500-kg object.

(b) At a point between the two objects at a distance d from the 500-kg objects, the net force on the 50.0-kg object will be zero when

$$\frac{G(50.0\text{ kg})(200\text{ kg})}{(0.400\text{ m}-d)^2} = \frac{G(50.0\text{ kg})(500\text{ kg})}{d^2}$$

or $d = \boxed{0.245\text{ m}}$

P13.4 $m_1 + m_2 = 5.00\text{ kg}$ $\qquad m_2 = 5.00\text{ kg} - m_1$

$$F = G\frac{m_1m_2}{r^2} \Rightarrow 1.00\times10^{-8}\text{ N} = \left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\frac{m_1(5.00\text{ kg}-m_1)}{(0.200\text{ m})^2}$$

$$(5.00\text{ kg})m_1 - m_1^2 = \frac{\left(1.00\times10^{-8}\text{ N}\right)\left(0.040\,0\text{ m}^2\right)}{6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2} = 6.00\text{ kg}^2$$

Thus, $m_1^2 - (5.00\text{ kg})m_1 + 6.00\text{ kg} = 0$

or $(m_1 - 3.00\text{ kg})(m_1 - 2.00\text{ kg}) = 0$

giving $\boxed{m_1 = 3.00\text{ kg, so } m_2 = 2.00\text{ kg}}$. The answer $m_1 = 2.00\text{ kg}$ and $m_2 = 3.00\text{ kg}$ is physically equivalent.

P13.5 The force exerted on the 4.00-kg mass by the 2.00-kg mass is directed upward and given by

$$\mathbf{F}_{24} = G\frac{m_4m_2}{r_{24}^2}\hat{\mathbf{j}} = \left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\frac{(4.00\text{ kg})(2.00\text{ kg})}{(3.00\text{ m})^2}\hat{\mathbf{j}}$$
$$= 5.93\times10^{-11}\,\hat{\mathbf{j}}\text{N}$$

The force exerted on the 4.00-kg mass by the 6.00-kg mass is directed to the left

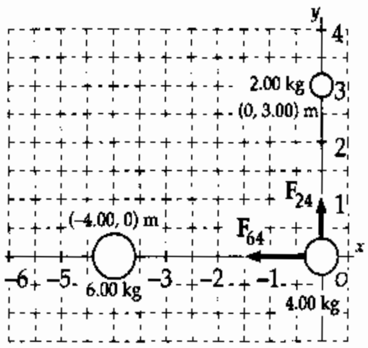


FIG. P13.5

$$\mathbf{F}_{64} = G\frac{m_4m_6}{r_{64}^2}\left(-\hat{\mathbf{i}}\right) = \left(-6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\frac{(4.00\text{ kg})(6.00\text{ kg})}{(4.00\text{ m})^2}\hat{\mathbf{i}}$$
$$= -10.0\times10^{-11}\,\hat{\mathbf{i}}\text{N}$$

Therefore, the resultant force on the 4.00-kg mass is $\mathbf{F}_4 = \mathbf{F}_{24} + \mathbf{F}_{64} = \boxed{\left(-10.0\hat{\mathbf{i}} + 5.93\hat{\mathbf{j}}\right)\times10^{-11}\text{ N}}$.

P13.6 (a) The Sun-Earth distance is 1.496×10^{11} m and the Earth-Moon distance is 3.84×10^{8} m, so the distance from the Sun to the Moon during a solar eclipse is

$$1.496 \times 10^{11}\ \text{m} - 3.84 \times 10^{8}\ \text{m} = 1.492 \times 10^{11}\ \text{m}$$

The mass of the Sun, Earth, and Moon are $M_S = 1.99 \times 10^{30}$ kg

$M_E = 5.98 \times 10^{24}$ kg

and $M_M = 7.36 \times 10^{22}$ kg

We have $F_{SM} = \dfrac{Gm_1m_2}{r^2} = \dfrac{(6.67 \times 10^{-11})(1.99 \times 10^{30})(7.36 \times 10^{22})}{(1.492 \times 10^{11})^2} = \boxed{4.39 \times 10^{20}\ \text{N}}$

(b) $$F_{EM} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24})(7.36 \times 10^{22})}{(3.84 \times 10^{8})^2} = \boxed{1.99 \times 10^{20}\ \text{N}}$$

(c) $$F_{SE} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99 \times 10^{30})(5.98 \times 10^{24})}{(1.496 \times 10^{11})^2} = \boxed{3.55 \times 10^{22}\ \text{N}}$$

Note that the force exerted by the Sun on the Moon is much stronger than the force of the Earth on the Moon. In a sense, the Moon orbits the Sun more than it orbits the Earth. The Moon's path is everywhere concave toward the Sun. Only by subtracting out the solar orbital motion of the Earth-Moon system do we see the Moon orbiting the center of mass of this system.

Section 13.2 Measuring the Gravitational Constant

P13.7 $$F = \frac{GMm}{r^2} = (6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)\frac{(1.50\ \text{kg})(15.0 \times 10^{-3}\ \text{kg})}{(4.50 \times 10^{-2}\ \text{m})^2} = \boxed{7.41 \times 10^{-10}\ \text{N}}$$

P13.8 Let θ represent the angle each cable makes with the vertical, L the cable length, x the distance each ball scrunches in, and $d = 1$ m the original distance between them. Then $r = d - 2x$ is the separation of the balls. We have

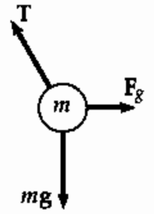


FIG. P13.8

$$\sum F_y = 0: \qquad T\cos\theta - mg = 0$$

$$\sum F_x = 0: \qquad T\sin\theta - \frac{Gmm}{r^2} = 0$$

Then $\tan\theta = \dfrac{Gmm}{r^2 mg}$ $\qquad \dfrac{x}{\sqrt{L^2 - x^2}} = \dfrac{Gm}{g(d-2x)^2}$ $\qquad x(d-2x)^2 = \dfrac{Gm}{g}\sqrt{L^2 - x^2}$.

The factor $\dfrac{Gm}{g}$ is numerically small. There are two possibilities: either x is small or else $d - 2x$ is small.

Possibility one: We can ignore x in comparison to d and L, obtaining

$$x(1\text{ m})^2 = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)(100\text{ kg})}{\left(9.8\text{ m/s}^2\right)}45\text{ m} \qquad x = 3.06\times10^{-8}\text{ m}.$$

The separation distance is $r = 1\text{ m} - 2\left(3.06\times10^{-8}\text{ m}\right) = \boxed{1.000\text{ m} - 61.3\text{ nm}}$.

Possibility two: If $d - 2x$ is small, $x \approx 0.5$ m and the equation becomes

$$(0.5\text{ m})r^2 = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)(100\text{ kg})}{(9.8\text{ N/kg})}\sqrt{(45\text{ m})^2 - (0.5\text{ m})^2} \qquad r = \boxed{2.74\times10^{-4}\text{ m}}.$$

For this answer to apply, the spheres would have to be compressed to a density like that of the nucleus of atom.

Section 13.3 Free-Fall Acceleration and the Gravitational Force

P13.9 $a = \dfrac{MG}{(4R_E)^2} = \dfrac{9.80\text{ m/s}^2}{16.0} = \boxed{0.613\text{ m/s}^2}$ toward the Earth.

P13.10 $g = \dfrac{GM}{R^2} = \dfrac{G\rho\left(\frac{4\pi R^3}{3}\right)}{R^2} = \dfrac{4}{3}\pi G\rho R$

If $\dfrac{g_M}{g_E} = \dfrac{1}{6} = \dfrac{\frac{4\pi G\rho_M R_M}{3}}{\frac{4\pi G\rho_E R_E}{3}}$

then $\dfrac{\rho_M}{\rho_E} = \left(\dfrac{g_M}{g_E}\right)\left(\dfrac{R_E}{R_M}\right) = \left(\dfrac{1}{6}\right)(4) = \boxed{\dfrac{2}{3}}$.

P13.11 (a) At the zero-total field point, $\frac{GmM_E}{r_E^2} = \frac{GmM_M}{r_M^2}$

so $r_M = r_E\sqrt{\frac{M_M}{M_E}} = r_E\sqrt{\frac{7.36\times10^{22}}{5.98\times10^{24}}} = \frac{r_E}{9.01}$

$$r_E + r_M = 3.84\times10^8\text{ m} = r_E + \frac{r_E}{9.01}$$

$$r_E = \frac{3.84\times10^8\text{ m}}{1.11} = \boxed{3.46\times10^8\text{ m}}$$

(b) At this distance the acceleration due to the Earth's gravity is

$$g_E = \frac{GM_E}{r_E^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})}{(3.46\times10^8\text{ m})^2}$$

$$g_E = \boxed{3.34\times10^{-3}\text{ m/s}^2\text{ directed toward the Earth}}$$

Section 13.4 Kepler's Laws and the Motion of Planets

P13.12 (a) $v = \frac{2\pi r}{T} = \frac{2\pi(384\,400)\times10^3\text{ m}}{27.3\times(86\,400\text{ s})} = \boxed{1.02\times10^3\text{ m/s}}$.

(b) In one second, the Moon falls a distance

$$x = \frac{1}{2}at^2 = \frac{1}{2}\frac{v^2}{r}t^2 = \frac{1}{2}\frac{(1.02\times10^3)^2}{(3.844\times10^8)}\times(1.00)^2 = 1.35\times10^{-3}\text{ m} = \boxed{1.35\text{ mm}}.$$

The Moon only moves inward 1.35 mm for every 1020 meters it moves along a straight-line path.

P13.13 Applying Newton's 2nd Law, $\sum F = ma$ yields $F_g = ma_c$ for each star:

$$\frac{GMM}{(2r)^2} = \frac{Mv^2}{r} \qquad \text{or} \qquad M = \frac{4v^2r}{G}.$$

We can write r in terms of the period, T, by considering the time and distance of one complete cycle. The distance traveled in one orbit is the circumference of the stars' common orbit, so $2\pi r = vT$. Therefore

$$M = \frac{4v^2r}{G} = \frac{4v^2}{G}\left(\frac{vT}{2\pi}\right)$$

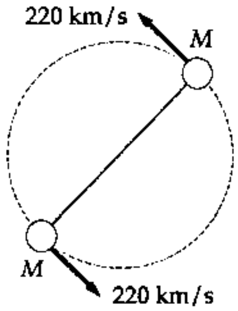


FIG. P13.13

so, $M = \frac{2v^3T}{\pi G} = \frac{2(220\times10^3\text{ m/s})^3(14.4\text{ d})(86\,400\text{ s/d})}{\pi(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)} = \boxed{1.26\times10^{32}\text{ kg} = 63.3\text{ solar masses}}$

P13.14 Since speed is constant, the distance traveled between t_1 and t_2 is equal to the distance traveled between t_3 and t_4. The area of a triangle is equal to one-half its (base) width across one side times its (height) dimension perpendicular to that side.

So $\frac{1}{2}bv(t_2 - t_1) = \frac{1}{2}bv(t_4 - t_3)$

states that the particle's radius vector sweeps out equal areas in equal times.

P13.15 $T^2 = \frac{4\pi^2 a^3}{GM}$ (Kepler's third law with $m << M$)

$$M = \frac{4\pi^2 a^3}{GT^2} = \frac{4\pi^2 \left(4.22 \times 10^8 \text{ m}\right)^3}{\left(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.77 \times 86\,400 \text{ s}\right)^2} = \boxed{1.90 \times 10^{27} \text{ kg}}$$

(Approximately 316 Earth masses)

P13.16 By conservation of angular momentum for the satellite,

$$r_p v_p = r_a v_a \qquad \frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{2\,289 \text{ km} + 6.37 \times 10^3 \text{ km}}{459 \text{ km} + 6.37 \times 10^3 \text{ km}} = \frac{8\,659 \text{ km}}{6\,829 \text{ km}} = \boxed{1.27}.$$

We do not need to know the period.

P13.17 By Kepler's Third Law, $T^2 = ka^3$ ($a =$ semi-major axis)

For any object orbiting the Sun, with T in years and a in A.U., $k = 1.00$. Therefore, for Comet Halley

$$(75.6)^2 = (1.00)\left(\frac{0.570 + y}{2}\right)^3$$

The farthest distance the comet gets from the Sun is

$y = 2(75.6)^{2/3} - 0.570 = \boxed{35.2 \text{ A.U.}}$ (out around the orbit of Pluto)

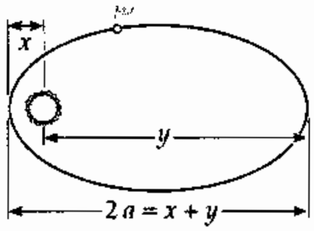


FIG. P13.17

P13.18 $\sum F = ma$: $\quad \frac{Gm_{\text{planet}}M_{\text{star}}}{r^2} = \frac{m_{\text{planet}}v^2}{r}$

$$\frac{GM_{\text{star}}}{r} = v^2 = r^2\omega^2$$

$$GM_{\text{star}} = r^3\omega^3 = r_x^3\omega_x^2 = r_y^3\omega_y^2$$

$$\omega_y = \omega_x\left(\frac{r_x}{r_y}\right)^{3/2} \qquad \omega_y = \left(\frac{90.0°}{5.00 \text{ yr}}\right)3^{3/2} = \frac{468°}{5.00 \text{ yr}}$$

So $\boxed{\text{planet Y has turned through 1.30 revolutions}}$.

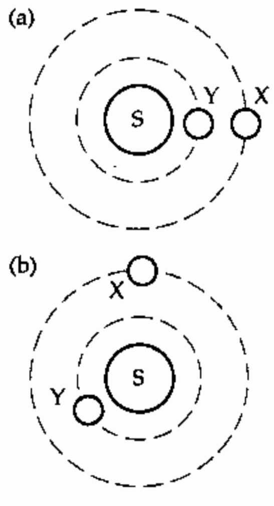


FIG. P13.18

P13.19 $\dfrac{GM_J}{(R_J+d)^2} = \dfrac{4\pi^2(R_J+d)}{T^2}$

$$GM_J T^2 = 4\pi^2 (R_J+d)^3$$

$$\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.90\times10^{27}\ \text{kg}\right)(9.84\times3\,600)^2 = 4\pi^2\left(6.99\times10^7+d\right)^3$$

$d = \boxed{8.92\times10^7\ \text{m}} = \boxed{89\,200\ \text{km}}$ above the planet

P13.20 The gravitational force on a small parcel of material at the star's equator supplies the necessary centripetal force:

$$\frac{GM_s m}{R_s^2} = \frac{mv^2}{R_s} = mR_s\omega^2$$

so $\omega = \sqrt{\dfrac{GM_s}{R_s^3}} = \sqrt{\dfrac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left[2\left(1.99\times10^{30}\ \text{kg}\right)\right]}{\left(10.0\times10^3\ \text{m}\right)^3}}$

$\omega = \boxed{1.63\times10^4\ \text{rad/s}}$

***P13.21** The speed of a planet in a circular orbit is given by

$\sum F = ma$: $\quad \dfrac{GM_{\text{sun}}m}{r^2} = \dfrac{mv^2}{r} \qquad v = \sqrt{\dfrac{GM_{\text{sun}}}{r}}$.

For Mercury the speed is $\quad v_M = \sqrt{\dfrac{\left(6.67\times10^{-11}\right)\left(1.99\times10^{30}\right)\text{m}^2}{\left(5.79\times10^{10}\right)\text{s}^2}} = 4.79\times10^4\ \text{m/s}$

and for Pluto, $\quad v_P = \sqrt{\dfrac{\left(6.67\times10^{-11}\right)\left(1.99\times10^{30}\right)\text{m}^2}{\left(5.91\times10^{12}\right)\text{s}^2}} = 4.74\times10^3\ \text{m/s}$.

With greater speed, Mercury will eventually move farther from the Sun than Pluto. With original distances r_P and r_M perpendicular to their lines of motion, they will be equally far from the Sun after time t where

$$\sqrt{r_P^2+v_P^2t^2} = \sqrt{r_M^2+v_M^2t^2}$$

$$r_P^2 - r_M^2 = \left(v_M^2 - v_P^2\right)t^2$$

$$t = \sqrt{\frac{\left(5.91\times10^{12}\ \text{m}\right)^2 - \left(5.79\times10^{10}\ \text{m}\right)^2}{\left(4.79\times10^4\ \text{m/s}\right)^2 - \left(4.74\times10^3\ \text{m/s}\right)^2}} = \sqrt{\frac{3.49\times10^{25}\ \text{m}^2}{2.27\times10^9\ \text{m}^2/\text{s}^2}} = 1.24\times10^8\ \text{s} = \boxed{393\ \text{yr}}.$$

***P13.22** For the Earth, $\sum F = ma$: $\quad \frac{GM_s m}{r^2} = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2$.

Then $\quad GM_s T^2 = 4\pi^2 r^3$.

Also the angular momentum $\quad L = mvr = m\frac{2\pi r}{T}r$ is a constant for the Earth.

We eliminate $\quad r = \sqrt{\frac{LT}{2\pi m}}$ between the equations:

$$GM_s T^2 = 4\pi^2\left(\frac{LT}{2\pi m}\right)^{3/2} \qquad GM_s T^{1/2} = 4\pi^2\left(\frac{L}{2\pi m}\right)^{3/2}.$$

Now the rate of change is described by

$$GM_s\left(\frac{1}{2}T^{-1/2}\frac{dT}{dt}\right) + G\left(1\frac{dM_s}{dt}T^{1/2}\right) = 0 \qquad \frac{dT}{dt} = -\frac{dM_s}{dt}\left(2\frac{T}{M_s}\right) \approx \frac{\Delta T}{T}$$

$$\Delta T \approx -\Delta t\frac{dM_s}{dt}\left(2\frac{T}{M_s}\right) = -5\,000\text{ yr}\left(\frac{3.16\times10^7\text{ s}}{1\text{ yr}}\right)\left(-3.64\times10^9\text{ kg/s}\right)\left(2\frac{1\text{ yr}}{1.991\times10^{30}\text{ kg}}\right)$$

$$\Delta T = \boxed{1.82\times10^{-2}\text{ s}}$$

Section 13.5 The Gravitational Field

P13.23 $\quad \mathbf{g} = \frac{Gm}{l^2}\hat{\mathbf{i}} + \frac{Gm}{l^2}\hat{\mathbf{j}} + \frac{Gm}{2l^2}\left(\cos 45.0°\,\hat{\mathbf{i}} + \sin 45.0\hat{\mathbf{j}}\right)$

so $\quad \mathbf{g} = \frac{GM}{l^2}\left(1 + \frac{1}{2\sqrt{2}}\right)\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right)$ or

$$\mathbf{g} = \boxed{\frac{Gm}{l^2}\left(\sqrt{2} + \frac{1}{2}\right)\text{ toward the opposite corner}}$$

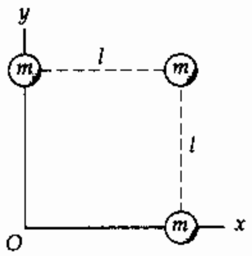


FIG. P13.23

P13.24 (a) $\quad F = \frac{GMm}{r^2} = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left[100\left(1.99\times10^{30}\text{ kg}\right)\left(10^3\text{ kg}\right)\right]}{\left(1.00\times10^4\text{ m} + 50.0\text{ m}\right)^2} = \boxed{1.31\times10^{17}\text{ N}}$

(b) $\quad \Delta F = \frac{GMm}{r_{\text{front}}^2} - \frac{GMm}{r_{\text{back}}^2}$

$$\Delta g = \frac{\Delta F}{m} = \frac{GM\left(r_{\text{back}}^2 - r_{\text{front}}^2\right)}{r_{\text{front}}^2 r_{\text{back}}^2}$$

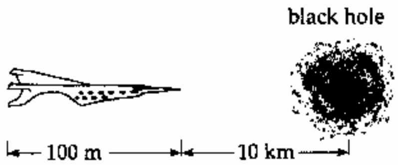


FIG. P13.24

$$\Delta g = \frac{\left(6.67\times10^{-11}\right)\left[100\left(1.99\times10^{30}\right)\right]\left[\left(1.01\times10^4\text{ m}\right)^2 - \left(1.00\times10^4\text{ m}\right)^2\right]}{\left(1.00\times10^4\text{ m}\right)^2\left(1.01\times10^4\text{ m}\right)^2}$$

$$\Delta g = \boxed{2.62\times10^{12}\text{ N/kg}}$$

P13.25 $g_1 = g_2 = \frac{MG}{r^2 + a^2}$

$g_{1y} = -g_{2y}$ $\qquad g_y = g_{1y} + g_{2y}$

$g_{1x} = g_{2x} = g_2 \cos\theta$ $\qquad \cos\theta = \frac{r}{\left(a^2 + r^2\right)^{1/2}}$

$$\mathbf{g} = 2g_{2x}\left(-\hat{\mathbf{i}}\right)$$

or $\mathbf{g} = \boxed{\frac{2MGr}{\left(r^2 + a^2\right)^{3/2}} \text{ toward the center of mass}}$

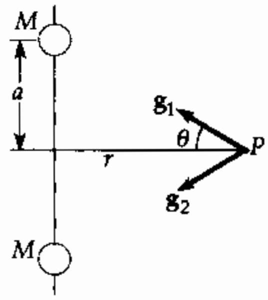


FIG. P13.25

Section 13.6 Gravitational Potential Energy

P13.26 (a) $U = -\frac{GM_E m}{r} = -\frac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)(100 \text{ kg})}{(6.37 + 2.00) \times 10^6 \text{ m}} = \boxed{-4.77 \times 10^9 \text{ J}}$.

(b), (c) Planet and satellite exert forces of equal magnitude on each other, directed downward on the satellite and upward on the planet.

$$F = \frac{GM_E m}{r^2} = \frac{\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24} \text{ kg}\right)(100 \text{ kg})}{\left(8.37 \times 10^6 \text{ m}\right)^2} = \boxed{569 \text{ N}}$$

P13.27 $U = -G\frac{Mm}{r}$ and $g = \frac{GM_E}{R_E^2}$

so that $\Delta U = -GMm\left(\frac{1}{3R_E} - \frac{1}{R_E}\right) = \frac{2}{3}mgR_E$

$$\Delta U = \frac{2}{3}(1\,000 \text{ kg})\left(9.80 \text{ m/s}^2\right)\left(6.37 \times 10^6 \text{ m}\right) = \boxed{4.17 \times 10^{10} \text{ J}}.$$

P13.28 The height attained is not small compared to the radius of the Earth, so $U = mgy$ does not apply; $U = -\frac{GM_1M_2}{r}$ does. From launch to apogee at height h,

$K_i + U_i + \Delta E_{\text{mch}} = K_f + U_f$: $\quad \frac{1}{2}M_p v_i^2 - \frac{GM_E M_p}{R_E} + 0 = 0 - \frac{GM_E M_p}{R_E + h}$

$$\frac{1}{2}\left(10.0 \times 10^3 \text{ m/s}\right)^2 - \left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(\frac{5.98 \times 10^{24} \text{ kg}}{6.37 \times 10^6 \text{ m}}\right)$$

$$= -\left(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2\right)\left(\frac{5.98 \times 10^{24} \text{ kg}}{6.37 \times 10^6 \text{ m} + h}\right)$$

$$\left(5.00 \times 10^7 \text{ m}^2/\text{s}^2\right) - \left(6.26 \times 10^7 \text{ m}^2/\text{s}^2\right) = \frac{-3.99 \times 10^{14} \text{ m}^3/\text{s}^2}{6.37 \times 10^6 \text{ m} + h}$$

$$6.37 \times 10^6 \text{ m} + h = \frac{3.99 \times 10^{14} \text{ m}^3/\text{s}^2}{1.26 \times 10^7 \text{ m}^2/\text{s}^2} = 3.16 \times 10^7 \text{ m}$$

$$\boxed{h = 2.52 \times 10^7 \text{ m}}$$

P13.29 (a) $\rho = \dfrac{M_S}{\frac{4}{3}\pi r_E^2} = \dfrac{3\left(1.99\times10^{30}\text{ kg}\right)}{4\pi\left(6.37\times10^{6}\text{ m}\right)^3} = \boxed{1.84\times10^{9}\text{ kg/m}^3}$

(b) $g = \dfrac{GM_S}{r_E^2} = \dfrac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\text{ kg}\right)}{\left(6.37\times10^{6}\text{ m}\right)^2} = \boxed{3.27\times10^{6}\text{ m/s}^2}$

(c) $U_g = -\dfrac{GM_S m}{r_E} = -\dfrac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\text{ kg}\right)(1.00\text{ kg})}{6.37\times10^{6}\text{ m}} = \boxed{-2.08\times10^{13}\text{ J}}$

P13.30 $W = -\Delta U = -\left(\dfrac{-Gm_1m_2}{r} - 0\right)$

$W = \dfrac{\left(+6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.36\times10^{22}\text{ kg}\right)\left(1.00\times10^{3}\text{ kg}\right)}{1.74\times10^{6}\text{ m}} = \boxed{2.82\times10^{9}\text{ J}}$

P13.31 (a) $U_{\text{Tot}} = U_{12} + U_{13} + U_{23} = 3U_{12} = 3\left(-\dfrac{Gm_1m_2}{r_{12}}\right)$

$U_{\text{Tot}} = -\dfrac{3\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.00\times10^{-3}\text{ kg}\right)^2}{0.300\text{ m}} = \boxed{-1.67\times10^{-14}\text{ J}}$

(b) $\boxed{\text{At the center}}$ of the equilateral triangle

***P13.32** (a) Energy conservation of the object-Earth system from release to radius r:

$\left(K + U_g\right)_{\text{altitude } h} = \left(K + U_g\right)_{\text{radius } r}$

$0 - \dfrac{GM_E m}{R_E + h} = \dfrac{1}{2}mv^2 - \dfrac{GM_E m}{r}$

$v = \left(2GM_E\left(\dfrac{1}{r} - \dfrac{1}{R_E + h}\right)\right)^{1/2} = -\dfrac{dr}{dt}$

(b) $\int_i^f dt = \int_i^f -\dfrac{dr}{v} = \int_f^i \dfrac{dr}{v}$. The time of fall is

$\Delta t = \int_{R_E}^{R_E+h}\left(2GM_E\left(\dfrac{1}{r} - \dfrac{1}{R_E + h}\right)\right)^{-1/2} dr$

$\Delta t = \int_{6.37\times10^6\text{ m}}^{6.87\times10^6\text{ m}}\left[2\times6.67\times10^{-11}\times5.98\times10^{24}\left(\dfrac{1}{r} - \dfrac{1}{6.87\times10^{6}\text{ m}}\right)\right]^{-1/2} dr$

We can enter this expression directly into a mathematical calculation program.

Alternatively, to save typing we can change variables to $u = \dfrac{r}{10^6}$. Then

$\Delta t = \left(7.977\times10^{14}\right)^{-1/2}\int_{6.37}^{6.87}\left(\dfrac{1}{10^6 u} - \dfrac{1}{6.87\times10^6}\right)^{-1/2}10^6\,du = 3.541\times10^{-8}\dfrac{10^6}{\left(10^6\right)^{-1/2}}\int_{6.37}^{6.87}\left(\dfrac{1}{u} - \dfrac{1}{6.87}\right)^{-1/2}du$

A mathematics program returns the value 9.596 for this integral, giving for the time of fall $\Delta t = 3.541\times10^{-8}\times10^{9}\times9.596 = 339.8 = \boxed{340\text{ s}}$.

Section 13.7 **Energy Considerations in Planetary and Satellite Motion**

P13.33 $\frac{1}{2}mv_i^2+GM_Em\left(\frac{1}{r_f}-\frac{1}{r_i}\right)=\frac{1}{2}mv_f^2$ $\qquad$ $\frac{1}{2}v_i^2+GM_E\left(0-\frac{1}{R_E}\right)=\frac{1}{2}v_f^2$

or $\qquad$ $v_f^2=v_1^2-\frac{2GM_E}{R_E}$

and $\qquad$ $v_f=\left(v_1^2-\frac{2GM_E}{R_E}\right)^{1/2}$

$$v_f=\left[\left(2.00\times10^4\right)^2-1.25\times10^8\right]^{1/2}=\boxed{1.66\times10^4\ \text{m/s}}$$

P13.34 (a) $\qquad v_{\text{solar escape}}=\sqrt{\frac{2M_{\text{Sun}}G}{R_{E\text{-Sun}}}}=\boxed{42.1\ \text{km/s}}$

(b) Let $r=R_{E\text{-}S}x$ represent variable distance from the Sun, with x in astronomical units.

$$v=\sqrt{\frac{2M_{\text{Sun}}G}{R_{E\text{-}S}x}}=\frac{42.1}{\sqrt{x}}$$

If $v=\frac{125\,000\text{ km}}{3\,600\text{ s}}$, then $x=1.47\text{ A.U.}=\boxed{2.20\times10^{11}\ \text{m}}$

(at or beyond the orbit of Mars, 125 000 km/h is sufficient for escape).

P13.35 To obtain the orbital velocity, we use $\qquad \sum F=\frac{mMG}{R^2}=\frac{mv^2}{R}$

or $\qquad v=\sqrt{\frac{MG}{R}}$

We can obtain the escape velocity from $\qquad \frac{1}{2}mv_{\text{esc}}^2=\frac{mMG}{R}$

or $\qquad v_{\text{esc}}=\sqrt{\frac{2MG}{R}}=\boxed{\sqrt{2}v}$

P13.36 $\frac{v_i^2}{R_E+h}=\frac{GM_E}{(R_E+h)^2}$

$$K_i=\frac{1}{2}mv_i^2=\frac{1}{2}\left(\frac{GM_Em}{R_E+h}\right)=\frac{1}{2}\left[\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(500\ \text{kg})}{\left(6.37\times10^6\ \text{m}\right)+\left(0.500\times10^6\ \text{m}\right)}\right]=1.45\times10^{10}\ \text{J}$$

The change in gravitational potential energy of the satellite-Earth system is

$$\Delta U=\frac{GM_Em}{R_i}-\frac{GM_Em}{R_f}=GM_Em\left(\frac{1}{R_i}-\frac{1}{R_f}\right)$$

$$=\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)(500\ \text{kg})\left(-1.14\times10^{-8}\ \text{m}^{-1}\right)=-2.27\times10^9\ \text{J}$$

Also, $\qquad K_f=\frac{1}{2}mv_f^2=\frac{1}{2}(500\ \text{kg})\left(2.00\times10^3\ \text{m/s}\right)^2=1.00\times10^9\ \text{J}.$

The energy transformed due to friction is

$$\Delta E_{\text{int}}=K_i-K_f-\Delta U=(14.5-1.00+2.27)\times10^9\ \text{J}=\boxed{1.58\times10^{10}\ \text{J}}.$$

P13.37 $F_c = F_G$ gives $\frac{mv^2}{r} = \frac{GmM_E}{r^2}$

which reduces to $v = \sqrt{\frac{GM_E}{r}}$

and period $= \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{GM_E}}$.

(a) $r = R_E + 200\text{ km} = 6\,370\text{ km} + 200\text{ km} = 6\,570\text{ km}$

Thus,

$$\text{period} = 2\pi\left(6.57\times10^6\text{ m}\right)\sqrt{\frac{\left(6.57\times10^6\text{ m}\right)}{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)}}$$

$$T = 5.30\times10^3\text{ s} = 88.3\text{ min} = \boxed{1.47\text{ h}}$$

(b) $$v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)}{\left(6.57\times10^6\text{ m}\right)}} = \boxed{7.79\text{ km/s}}$$

(c) $K_f + U_f = K_i + U_i +$ energy input, gives

$$\text{input} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + \left(\frac{-GM_Em}{r_f}\right) - \left(\frac{-GM_Em}{r_i}\right) \qquad (1)$$

$$r_i = R_E = 6.37\times10^6\text{ m}$$

$$v_i = \frac{2\pi R_E}{86\,400\text{ s}} = 4.63\times10^2\text{ m/s}$$

Substituting the appropriate values into (1) yields the

$$\text{minimum energy input} = \boxed{6.43\times10^9\text{ J}}$$

P13.38 The gravitational force supplies the needed centripetal acceleration.

Thus, $\frac{GM_E m}{(R_E+h)^2}=\frac{mv^2}{(R_E+h)}$ or $v^2=\frac{GM_E}{R_E+h}$

(a) $T=\frac{2\pi r}{v}=\frac{2\pi(R_E+h)}{\sqrt{\frac{GM_E}{(R_E+h)}}}$ $\boxed{T=2\pi\sqrt{\frac{(R_E+h)^3}{GM_E}}}$

(b) $\boxed{v=\sqrt{\frac{GM_E}{R_E+h}}}$

(c) Minimum energy input is $\Delta E_{\min}=\left(K_f+U_{gf}\right)-\left(K_i-U_{gi}\right)$.

It is simplest to

$\boxed{\text{launch the satellite from a location on the equator, and launch it toward the east.}}$

This choice has the object starting with energy $K_i=\frac{1}{2}mv_i^2$

with $v_i=\frac{2\pi R_E}{1.00\text{ day}}=\frac{2\pi R_E}{86\,400\text{ s}}$ and $U_{gi}=-\frac{GM_E m}{R_E}$.

Thus, $\Delta E_{\min}=\frac{1}{2}m\left(\frac{GM_E}{R_E+h}\right)-\frac{GM_E m}{R_E+h}-\frac{1}{2}m\left[\frac{4\pi^2R_E^2}{(86\,400\text{ s})^2}\right]+\frac{GM_E m}{R_E}$

or $\Delta E_{\min}=\boxed{GM_E m\left[\frac{R_E+2h}{2R_E(R_E+h)}\right]-\frac{2\pi^2R_E^2m}{(86\,400\text{ s})^2}}$

P13.39 $E_{\text{tot}}=-\frac{GMm}{2r}$

$\Delta E=\frac{GMm}{2}\left(\frac{1}{r_i}-\frac{1}{r_f}\right)=\frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)}{2}\frac{10^3\text{ kg}}{10^3\text{ m}}\left(\frac{1}{6\,370+100}-\frac{1}{6\,370+200}\right)$

$\Delta E=4.69\times10^8\text{ J}=\boxed{469\text{ MJ}}$

P13.40 $g_E=\frac{Gm_E}{r_E^2}$ $g_U=\frac{Gm_U}{r_U^2}$

(a) $\frac{g_U}{g_E}=\frac{m_Ur_E^2}{m_Er_U^2}=14.0\left(\frac{1}{3.70}\right)^2=1.02$ $g_U=(1.02)\left(9.80\text{ m/s}^2\right)=\boxed{10.0\text{ m/s}^2}$

(b) $v_{esc,E}=\sqrt{\frac{2Gm_E}{r_E}}$; $v_{esc,U}=\sqrt{\frac{2Gm_U}{r_U}}$: $\frac{v_{esc,E}}{v_{esc,U}}=\sqrt{\frac{m_Ur_E}{m_Er_U}}=\sqrt{\frac{14.0}{3.70}}=1.95$

For the Earth, from the text's table of escape speeds, $v_{esc,E}=11.2\text{ km/s}$

$\therefore v_{esc,U}=(1.95)(11.2\text{ km/s})=\boxed{21.8\text{ km/s}}$

P13.41 The rocket is in a potential well at Ganymede's surface with energy

$$U_1 = -\frac{Gm_1m_2}{r} = -\frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2 m_2\left(1.495\times10^{23}\ \text{kg}\right)}{\text{kg}^2\left(2.64\times10^6\ \text{m}\right)}$$

$$U_1 = -3.78\times10^6 m_2\ \text{m}^2/\text{s}^2$$

The potential well from Jupiter at the distance of Ganymede is

$$U_2 = -\frac{Gm_1m_2}{r} = -\frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2 m_2\left(1.90\times10^{27}\ \text{kg}\right)}{\text{kg}^2\left(1.071\times10^9\ \text{m}\right)}$$

$$U_2 = -1.18\times10^8 m_2\ \text{m}^2/\text{s}^2$$

To escape from both requires

$$\frac{1}{2}m_2v_{\text{esc}}^2 = +\left(3.78\times10^6 + 1.18\times10^8\right)m_2\ \text{m}^2/\text{s}^2$$

$$v_{\text{esc}} = \sqrt{2\times1.22\times10^8\ \text{m}^2/\text{s}^2} = \boxed{15.6\ \text{km/s}}$$

P13.42 We interpret "lunar escape speed" to be the escape speed from the surface of a stationary moon alone in the Universe:

$$\frac{1}{2}mv_{\text{esc}}^2 = \frac{GM_mm}{R_m}$$

$$v_{\text{esc}} = \sqrt{\frac{2GM_m}{R_m}}$$

$$v_{\text{launch}} = 2\sqrt{\frac{2GM_m}{R_m}}$$

Now for the flight from moon to Earth

$$(K+U)_i = (K+U)_f$$

$$\frac{1}{2}mv_{\text{launch}}^2 - \frac{GmM_m}{R_m} - \frac{GmM_E}{r_{\text{el}}} = \frac{1}{2}mv_{\text{impact}}^2 - \frac{GmM_m}{r_{m_2}} - \frac{GmM_E}{R_E}$$

$$\frac{4GM_m}{R_m} - \frac{GM_m}{R_m} - \frac{GM_E}{r_{\text{el}}} = \frac{1}{2}v_{\text{impact}}^2 - \frac{GM_m}{r_{m_2}} - \frac{GM_E}{R_E}$$

$$v_{\text{impact}} = \left[2G\left(\frac{3M_m}{R_m} + \frac{M_m}{r_{m_2}} + \frac{M_E}{R_E} - \frac{M_E}{r_{\text{el}}}\right)\right]^{1/2}$$

$$= \left[2G\left(\frac{3\times7.36\times10^{22}\ \text{kg}}{1.74\times10^6\ \text{m}} + \frac{7.36\times10^{22}\ \text{kg}}{3.84\times10^8\ \text{m}} + \frac{5.98\times10^{24}\ \text{kg}}{6.37\times10^6\ \text{m}} - \frac{5.98\times10^{24}\ \text{kg}}{3.84\times10^8\ \text{m}}\right)\right]^{1/2}$$

$$= \left[2G\left(1.27\times10^{17} + 1.92\times10^{14} + 9.39\times10^{17} - 1.56\times10^{16}\right)\ \text{kg/m}\right]^{1/2}$$

$$= \left[2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)10.5\times10^{17}\ \text{kg/m}\right]^{1/2} = \boxed{11.8\ \text{km/s}}$$

***P13.43** (a) Energy conservation for the object-Earth system from firing to apex:

$$\left(K+U_g\right)_i=\left(K+U_g\right)_f$$
$$\frac{1}{2}mv_i^2-\frac{GmM_E}{R_E}=0-\frac{GmM_E}{R_E+h}$$

where $\frac{1}{2}mv_{\text{esc}}^2=\frac{GmM_E}{R_E}$. Then

$$\frac{1}{2}v_i^2-\frac{1}{2}v_{\text{esc}}^2=-\frac{1}{2}v_{\text{esc}}^2\frac{R_E}{R_E+h}$$
$$v_{\text{esc}}^2-v_i^2=\frac{v_{\text{esc}}^2R_E}{R_E+h}$$
$$\frac{1}{v_{\text{esc}}^2-v_i^2}=\frac{R_E+h}{v_{\text{esc}}^2R_E}$$
$$h=\frac{v_{\text{esc}}^2R_E}{v_{\text{esc}}^2-v_i^2}-R_E=\frac{v_{\text{esc}}^2R_E-v_{\text{esc}}^2R_E+v_i^2R_E}{v_{\text{esc}}^2-v_i^2}$$
$$h=\frac{R_Ev_i^2}{v_{\text{esc}}^2-v_i^2}$$

(b) $$h=\frac{6.37\times10^6\text{ m}\,(8.76)^2}{(11.2)^2-(8.76)^2}=\boxed{1.00\times10^7\text{ m}}$$

(c) The fall of the meteorite is the time-reversal of the upward flight of the projectile, so it is described by the same energy equation

$$v_i^2=v_{\text{esc}}^2\left(1-\frac{R_E}{R_E+h}\right)=v_{\text{esc}}^2\left(\frac{h}{R_E+h}\right)=\left(11.2\times10^3\text{ m/s}\right)^2\left(\frac{2.51\times10^7\text{ m}}{6.37\times10^6\text{ m}+2.51\times10^7\text{ m}}\right)$$
$$=1.00\times10^8\text{ m}^2/\text{s}^2$$
$$v_i=\boxed{1.00\times10^4\text{ m/s}}$$

(d) With $v_i<<v_{\text{esc}}$, $h\approx\frac{R_Ev_i^2}{v_{\text{esc}}^2}=\frac{R_Ev_i^2R_E}{2GM_E}$. But $g=\frac{GM_E}{R_E^2}$, so $h=\frac{v_i^2}{2g}$, in agreement with $0^2=v_i^2+2(-g)(h-0)$.

P13.44 For a satellite in an orbit of radius r around the Earth, the total energy of the satellite-Earth system is $E=-\frac{GM_E}{2r}$. Thus, in changing from a circular orbit of radius $r=2R_E$ to one of radius $r=3R_E$, the required work is

$$W=\Delta E=-\frac{GM_Em}{2r_f}+\frac{GM_Em}{2r_i}=GM_Em\left[\frac{1}{4R_E}-\frac{1}{6R_E}\right]=\boxed{\frac{GM_Em}{12R_E}}.$$

***P13.45** (a) The major axis of the orbit is $2a = 50.5$ AU so $a = 25.25$ AU

Further, in Figure 13.5, $a + c = 50$ AU so $c = 24.75$ AU

Then $e = \frac{c}{a} = \frac{24.75}{25.25} = \boxed{0.980}$

(b) In $T^2 = K_s a^3$ for objects in solar orbit, the Earth gives us

$$(1\text{ yr})^2 = K_s(1\text{ AU})^3 \qquad K_s = \frac{(1\text{ yr})^2}{(1\text{ AU})^3}$$

Then $$T^2 = \frac{(1\text{ yr})^2}{(1\text{ AU})^3}(25.25\text{ AU})^3 \qquad T = \boxed{127\text{ yr}}$$

(c) $$U = -\frac{GMm}{r} = -\frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\text{ kg}\right)\left(1.2\times10^{10}\text{ kg}\right)}{50\left(1.496\times10^{11}\text{ m}\right)} = \boxed{-2.13\times10^{17}\text{ J}}$$

***P13.46** (a) For the satellite $\sum F = ma$ $\qquad \frac{GmM_E}{r^2} = \frac{mv_0^2}{r}$

$$\boxed{v_0 = \left(\frac{GM_E}{r}\right)^{1/2}}$$

(b) Conservation of momentum in the forward direction for the exploding satellite:

$$\left(\sum mv\right)_i = \left(\sum mv\right)_f$$
$$5mv_0 = 4mv_i + m0$$
$$v_i = \frac{5}{4}v_0 = \boxed{\frac{5}{4}\left(\frac{GM_E}{r}\right)^{1/2}}$$

(c) With velocity perpendicular to radius, the orbiting fragment is at perigee. Its apogee distance and speed are related to r and v_i by $4mrv_i = 4mr_f v_f$ and $\frac{1}{2}4mv_i^2 - \frac{GM_E 4m}{r} = \frac{1}{2}4mv_f^2 - \frac{GM_E 4m}{r_f}$. Substituting $v_f = \frac{v_i r}{r_f}$ we have $\frac{1}{2}v_i^2 - \frac{GM_E}{r} = \frac{1}{2}\frac{v_i^2 r^2}{r_f^2} - \frac{GM_E}{r_f}$. Further, substituting $v_i^2 = \frac{25}{16}\frac{GM_E}{r}$ gives

$$\frac{25}{32}\frac{GM_E}{r} - \frac{GM_E}{r} = \frac{25}{32}\frac{GM_E r}{r_f^2} - \frac{GM_E}{r_f}$$
$$\frac{-7}{32r} = \frac{25r}{32r_f^2} - \frac{1}{r_f}$$

Clearing of fractions, $-7r_f^2 = 25r^2 - 32rr_f$ or $7\left(\frac{r_f}{r}\right)^2 - 32\left(\frac{r_f}{r}\right) + 25 = 0$ giving

$\frac{r_f}{r} = \frac{+32 \pm \sqrt{32^2 - 4(7)(25)}}{14} = \frac{50}{14}$ or $\frac{14}{14}$. The latter root describes the starting point. The outer end of the orbit has $\frac{r_f}{r} = \frac{25}{7}$; $\boxed{r_f = \frac{25r}{7}}$.

Additional Problems

P13.47 Let m represent the mass of the spacecraft, r_E the radius of the Earth's orbit, and x the distance from Earth to the spacecraft.

The Sun exerts on the spacecraft a radial inward force of $$F_s = \frac{GM_s m}{(r_E - x)^2}$$

while the Earth exerts on it a radial outward force of $$F_E = \frac{GM_E m}{x^2}$$

The net force on the spacecraft must produce the correct centripetal acceleration for it to have an orbital period of 1.000 year.

Thus, $$F_S - F_E = \frac{GM_S m}{(r_E - x)^2} - \frac{GM_E m}{x^2} = \frac{mv^2}{(r_E - x)} = \frac{m}{(r_E - x)}\left[\frac{2\pi(r_E - x)}{T}\right]^2$$

which reduces to $$\frac{GM_S}{(r_E - x)^2} - \frac{GM_E}{x^2} = \frac{4\pi^2(r_E - x)}{T^2}. \qquad (1)$$

Cleared of fractions, this equation would contain powers of x ranging from the fifth to the zeroth. We do not solve it algebraically. We may test the assertion that x is between 1.47×10^9 m and 1.48×10^9 m by substituting both of these as trial solutions, along with the following data: $M_S = 1.991 \times 10^{30}$ kg, $M_E = 5.983 \times 10^{24}$ kg, $r_E = 1.496 \times 10^{11}$ m, and $T = 1.000 \text{ yr} = 3.156 \times 10^7$ s.

With $x = 1.47 \times 10^9$ m substituted into equation (1), we obtain

$$6.052 \times 10^{-3} \text{ m/s}^2 - 1.85 \times 10^{-3} \text{ m/s}^2 \approx 5.871 \times 10^{-3} \text{ m/s}^2$$

or $$5.868 \times 10^{-3} \text{ m/s}^2 \approx 5.871 \times 10^{-3} \text{ m/s}^2$$

With $x = 1.48 \times 10^9$ m substituted into the same equation, the result is

$$6.053 \times 10^{-3} \text{ m/s}^2 - 1.82 \times 10^{-3} \text{ m/s}^2 \approx 5.870\,8 \times 10^{-3} \text{ m/s}^2$$

or $$5.870\,9 \times 10^{-3} \text{ m/s}^2 \approx 5.870\,8 \times 10^{-3} \text{ m/s}^2.$$

Since the first trial solution makes the left-hand side of equation (1) slightly less than the right hand side, and the second trial solution does the opposite, the true solution is determined as between the trial values. To three-digit precision, it is 1.48×10^9 m.

As an equation of fifth degree, equation (1) has five roots. The Sun-Earth system has five Lagrange points, all revolving around the Sun synchronously with the Earth. The SOHO and ACE satellites are at one. Another is beyond the far side of the Sun. Another is beyond the night side of the Earth. Two more are on the Earth's orbit, ahead of the planet and behind it by 60°. Plans are under way to gain perspective on the Sun by placing a spacecraft at one of these two co-orbital Lagrange points. The Greek and Trojan asteroids are at the co-orbital Lagrange points of the Jupiter-Sun system.

P13.48 The acceleration of an object at the center of the Earth due to the gravitational force of the Moon is given by

$$a = G\frac{M_{\text{Moon}}}{d^2}$$

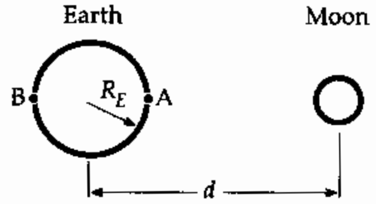


FIG. P13.48

At the point A nearest the Moon, $a_+ = G\frac{M_M}{(d-r)^2}$

At the point B farthest from the Moon, $a_- = G\frac{M_M}{(d+r)^2}$

$$\Delta a = a_+ - a = GM_M\left[\frac{1}{(d-r)^2} - \frac{1}{d^2}\right]$$

For $d >> r$, $\Delta a = \frac{2GM_M r}{d^3} = 1.11\times10^{-6}\ \text{m/s}^2$

Across the planet, $\frac{\Delta g}{g} = \frac{2\Delta a}{g} = \frac{2.22\times10^{-6}\ \text{m/s}^2}{9.80\ \text{m/s}^2} = \boxed{2.26\times10^{-7}}$

***P13.49** Energy conservation for the two-sphere system from release to contact:

$$-\frac{Gmm}{R} = -\frac{Gmm}{2r} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$Gm\left(\frac{1}{2r} - \frac{1}{R}\right) = v^2 \qquad v = \left(Gm\left[\frac{1}{2r} - \frac{1}{R}\right]\right)^{1/2}$$

(a) The injected impulse is the final momentum of each sphere,

$$mv = m^{2/2}\left(Gm\left[\frac{1}{2r} - \frac{1}{R}\right]\right)^{1/2} = \left[Gm^3\left(\frac{1}{2r} - \frac{1}{R}\right)\right]^{1/2}.$$

(b) If they now collide elastically each sphere reverses its velocity to receive impulse

$$mv - (-mv) = 2mv = \boxed{2\left[Gm^3\left(\frac{1}{2r} - \frac{1}{R}\right)\right]^{1/2}}$$

P13.50 Momentum is conserved:

$$m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f}$$

$$0 = M\mathbf{v}_{1f} + 2M\mathbf{v}_{2f}$$

$$\mathbf{v}_{2f} = -\frac{1}{2}\mathbf{v}_{1f}$$

Energy is conserved:

$$(K+U)_i + \Delta E = (K+U)_f$$

$$0 - \frac{Gm_1m_2}{r_i} + 0 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 - \frac{Gm_1m_2}{r_f}$$

$$-\frac{GM(2M)}{12R} = \frac{1}{2}Mv_{1f}^2 + \frac{1}{2}(2M)\left(\frac{1}{2}v_{1f}\right)^2 - \frac{GM(2M)}{4R}$$

$$v_{1f} = \boxed{\frac{2}{3}\sqrt{\frac{GM}{R}}} \qquad v_{2f} = \frac{1}{2}v_{1f} = \boxed{\frac{1}{3}\sqrt{\frac{GM}{R}}}$$

P13.51 (a) $a_c = \frac{v^2}{r}$ $\quad a_c = \frac{(1.25\times 10^6\ \text{m/s})^2}{1.53\times 10^{11}\ \text{m}} = \boxed{10.2\ \text{m/s}^2}$

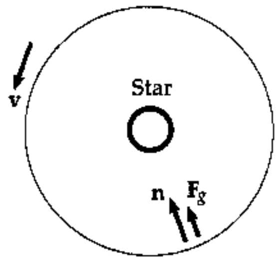


FIG. P13.51

(b) $\text{diff} = 10.2 - 9.90 = 0.312\ \text{m/s}^2 = \frac{GM}{r^2}$

$$M = \frac{(0.312\ \text{m/s}^2)(1.53\times 10^{11}\ \text{m})^2}{6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = \boxed{1.10\times 10^{32}\ \text{kg}}$$

P13.52 (a) The free-fall acceleration produced by the Earth is $g = \frac{GM_E}{r^2} = GM_E r^{-2}$ (directed downward)

Its rate of change is $\frac{dg}{dr} = GM_E(-2)r^{-3} = -2GM_E r^{-3}$.

The minus sign indicates that g decreases with increasing height.

At the Earth's surface, $\boxed{\frac{dg}{dr} = -\frac{2GM_E}{R_E^3}}$.

(b) For small differences,

$\frac{|\Delta g|}{\Delta r} = \frac{|\Delta g|}{h} = \frac{2GM_E}{R_E^3}$ Thus, $\boxed{|\Delta g| = \frac{2GM_E h}{R_E^3}}$

(c) $$|\Delta g| = \frac{2(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times 10^{24}\ \text{kg})(6.00\ \text{m})}{(6.37\times 10^6\ \text{m})^3} = \boxed{1.85\times 10^{-5}\ \text{m/s}^2}$$

***P13.53** (a) Each bit of mass dm in the ring is at the same distance from the object at A. The separate contributions $-\frac{Gmdm}{r}$ to the system energy add up to $-\frac{GmM_{\text{ring}}}{r}$. When the object is at A, this is

$$\frac{-6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2 1\,000\ \text{kg}\ 2.36\times 10^{20}\ \text{kg}}{\text{kg}^2\sqrt{(1\times 10^8\ \text{m})^2 + (2\times 10^8\ \text{m})^2}} = \boxed{-7.04\times 10^4\ \text{J}}.$$

(b) When the object is at the center of the ring, the potential energy is

$$-\frac{6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2\ 1\,000\ \text{kg}\ 2.36\times 10^{20}\ \text{kg}}{\text{kg}^2 1\times 10^8\ \text{m}} = \boxed{-1.57\times 10^5\ \text{J}}.$$

(c) Total energy of the object-ring system is conserved:

$$(K + U_g)_A = (K + U_g)_B$$

$$0 - 7.04\times 10^4\ \text{J} = \frac{1}{2}1\,000\ \text{kg}v_B^2 - 1.57\times 10^5\ \text{J}$$

$$v_B = \left(\frac{2\times 8.70\times 10^4\ \text{J}}{1\,000\ \text{kg}}\right)^{1/2} = \boxed{13.2\ \text{m/s}}$$

P13.54 To approximate the height of the sulfur, set

$$\frac{mv^2}{2}=mg_{Io}h \qquad h=70\,000\text{ m} \qquad g_{Io}=\frac{GM}{r^2}=1.79\text{ m/s}^2$$

$$v=\sqrt{2g_{Io}h} \qquad v=\sqrt{2(1.79)(70\,000)}\approx 500\text{ m/s (over 1 000 mi/h)}$$

A more precise answer is given by

$$\frac{1}{2}mv^2-\frac{GMm}{r_1}=-\frac{GMm}{r_2}$$

$$\frac{1}{2}v^2=\left(6.67\times10^{-11}\right)\left(8.90\times10^{22}\right)\left(\frac{1}{1.82\times10^6}-\frac{1}{1.89\times10^6}\right) \qquad v=\boxed{492\text{ m/s}}$$

P13.55 From the walk, $2\pi r=25\,000$ m. Thus, the radius of the planet is $r=\dfrac{25\,000\text{ m}}{2\pi}=3.98\times10^3$ m

From the drop: $\Delta y=\dfrac{1}{2}gt^2=\dfrac{1}{2}g(29.2\text{ s})^2=1.40$ m

so, $$g=\frac{2(1.40\text{ m})}{(29.2\text{ s})^2}=3.28\times10^{-3}\text{ m/s}^2=\frac{MG}{r^2} \qquad \therefore M=\boxed{7.79\times10^{14}\text{ kg}}$$

***P13.56** The distance between the orbiting stars is $d=2r\cos30°=\sqrt{3}r$ since $\cos30°=\dfrac{\sqrt{3}}{2}$. The net inward force on one orbiting star is

$$\frac{Gmm}{d^2}\cos30°+\frac{GMm}{r^2}+\frac{Gmm}{d^2}\cos30°=\frac{mv^2}{r}$$

$$\frac{Gm2\cos30°}{3r^2}+\frac{GM}{r^2}=\frac{4\pi^2r^2}{rT^2}$$

$$G\left(\frac{m}{\sqrt{3}}+M\right)=\frac{4\pi^2r^3}{T^2}$$

$$T^2=\frac{4\pi^2r^3}{G\left(M+\frac{m}{\sqrt{3}}\right)}$$

$$T=2\pi\left(\frac{r^3}{G\left(M+\frac{m}{\sqrt{3}}\right)}\right)^{1/2}$$

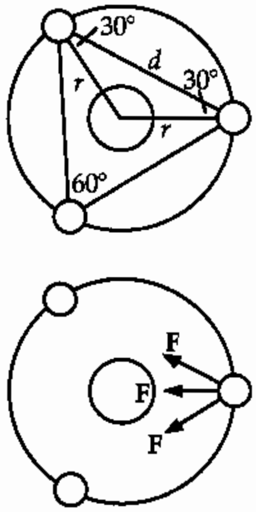


FIG. P13.56

P13.57 For a 6.00 km diameter cylinder, $r=3\,000$ m and to simulate $1g=9.80\text{ m/s}^2$

$$g=\frac{v^2}{r}=\omega^2r$$

$$\omega=\sqrt{\frac{g}{r}}=\boxed{0.057\,2\text{ rad/s}}$$

The required rotation rate of the cylinder is $\boxed{\dfrac{1\text{ rev}}{110\text{ s}}}$

(For a description of proposed cities in space, see Gerard K. O'Neill in *Physics Today*, Sept. 1974.)

P13.58 (a) G has units $\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}=\frac{\text{kg}\cdot\text{m}\cdot\text{m}^2}{\text{s}^2\cdot\text{kg}^2}=\frac{\text{m}^3}{\text{s}^2\cdot\text{kg}}$

and dimensions $[\text{G}]=\frac{\text{L}^3}{\text{T}^2\cdot\text{M}}$.

The speed of light has dimensions of $[c]=\frac{\text{L}}{\text{T}}$, and Planck's constant has the same dimensions as angular momentum or $[h]=\frac{\text{M}\cdot\text{L}^2}{\text{T}}$.

We require $\left[G^p c^q h^r\right]=\text{L}$, or $\text{L}^{3p}\text{T}^{-2p}\text{M}^{-p}\text{L}^q\text{T}^{-q}\text{M}^r\text{L}^{2r}\text{T}^{-r}=\text{L}^1\text{M}^0\text{T}^0$.

Thus, $3p+q+2r=1$

$-2p-q-r=0$

$-p+r=0$

which reduces (using $r=p$) to $3p+q+2p=1$

$-2p-q-p=0$

These equations simplify to $5p+q=1$ and $q=-3p$.

Then, $5p-3p=1$, yielding $p=\frac{1}{2}$, $q=-\frac{3}{2}$, and $r=\frac{1}{2}$.

Therefore, Planck length $=\boxed{G^{1/2}c^{-3/2}h^{1/2}}$.

(b) $\left(6.67\times10^{-11}\right)^{1/2}\left(3\times10^8\right)^{-3/2}\left(6.63\times10^{-34}\right)^{1/2}=\left(1.64\times10^{-69}\right)^{1/2}=4.05\times10^{-35}\text{ m}\boxed{\sim10^{-34}\text{ m}}$

P13.59 $\frac{1}{2}m_0v_{\text{esc}}^2=\frac{Gm_pm_0}{R}$

$v_{\text{esc}}=\sqrt{\frac{2Gm_p}{R}}$

With $m_p=\rho\frac{4}{3}\pi R^3$, we have

$$v_{\text{esc}}=\sqrt{\frac{2G\rho\frac{4}{3}\pi R^3}{R}}$$

$$=\boxed{\sqrt{\frac{8\pi G\rho}{3}}R}$$

So, $v_{\text{esc}}\propto R$.

***P13.60** For both circular orbits,

$$\sum F = ma: \qquad \frac{GM_E m}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM_E}{r}}$$

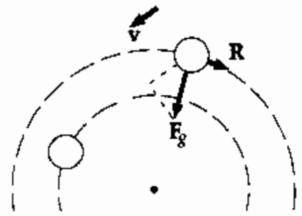


FIG. P13.60

(a) The original speed is $v_i = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(6.37 \times 10^6 \text{ m} + 2 \times 10^5 \text{ m})}} = \boxed{7.79 \times 10^3 \text{ m/s}}$.

(b) The final speed is $v_i = \sqrt{\frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(6.47 \times 10^6 \text{ m})}} = \boxed{7.85 \times 10^3 \text{ m/s}}$.

The energy of the satellite-Earth system is

$$K + U_g = \frac{1}{2}mv^2 - \frac{GM_E m}{r} = \frac{1}{2}m\frac{GM_E}{r} - \frac{GM_E}{r} = -\frac{GM_E m}{2r}$$

(c) Originally $E_i = -\frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(100 \text{ kg})}{2(6.57 \times 10^6 \text{ m})} = \boxed{-3.04 \times 10^9 \text{ J}}$.

(d) Finally $E_f = -\frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(100 \text{ kg})}{2(6.47 \times 10^6 \text{ m})} = \boxed{-3.08 \times 10^9 \text{ J}}$.

(e) Thus the object speeds up as it spirals down to the planet. The loss of gravitational energy is so large that the total energy decreases by

$$E_i - E_f = -3.04 \times 10^9 \text{ J} - (-3.08 \times 10^9 \text{ J}) = \boxed{4.69 \times 10^7 \text{ J}}.$$

(f) The only forces on the object are the backward force of air resistance R, comparatively very small in magnitude, and the force of gravity. Because the spiral path of the satellite is not perpendicular to the gravitational force, [one component of the gravitational force] pulls forward on the satellite to do positive work and make its speed increase.

P13.61 (a) At infinite separation $U = 0$ and at rest $K = 0$. Since energy of the two-planet system is conserved we have,

$$0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} \qquad (1)$$

The initial momentum of the system is zero and momentum is conserved.

Therefore, $0 = m_1v_1 - m_2v_2$ (2)

Combine equations (1) and (2): $\boxed{v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}}$ and $\boxed{v_2 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}}$

Relative velocity $v_r = v_1 - (-v_2) = \boxed{\sqrt{\frac{2G(m_1+m_2)}{d}}}$

(b) Substitute given numerical values into the equation found for v_1 and v_2 in part (a) to find

$v_1 = 1.03\times10^4$ m/s and $v_2 = 2.58\times10^3$ m/s

Therefore, $K_1 = \frac{1}{2}m_1v_1^2 = \boxed{1.07\times10^{32}\ \text{J}}$ and $K_2 = \frac{1}{2}m_2v_2^2 = \boxed{2.67\times10^{31}\ \text{J}}$

P13.62 (a) The net torque exerted on the Earth is zero. Therefore, the angular momentum of the Earth is conserved;

$$mr_av_a = mr_pv_p \text{ and } v_a = v_p\left(\frac{r_p}{r_a}\right) = \left(3.027\times10^4\ \text{m/s}\right)\left(\frac{1.471}{1.521}\right) = \boxed{2.93\times10^4\ \text{m/s}}$$

(b) $K_p = \frac{1}{2}mv_p^2 = \frac{1}{2}\left(5.98\times10^{24}\right)\left(3.027\times10^4\right)^2 = \boxed{2.74\times10^{33}\ \text{J}}$

$$U_p = -\frac{GmM}{r_p} = -\frac{\left(6.673\times10^{-11}\right)\left(5.98\times10^{24}\right)\left(1.99\times10^{30}\right)}{1.471\times10^{11}} = \boxed{-5.40\times10^{33}\ \text{J}}$$

(c) Using the same form as in part (b), $K_a = \boxed{2.57\times10^{33}\ \text{J}}$ and $U_a = \boxed{-5.22\times10^{33}\ \text{J}}$.

Compare to find that $K_p + U_p = \boxed{-2.66\times10^{33}\ \text{J}}$ and $K_a + U_a = \boxed{-2.65\times10^{33}\ \text{J}}$. They agree.

P13.63 (a) The work must provide the increase in gravitational energy

$$W = \Delta U_g = U_{gf} - U_{gi}$$

$$= -\frac{GM_E M_p}{r_f} + \frac{GM_E M_p}{r_i}$$

$$= -\frac{GM_E M_p}{R_E + y} + \frac{GM_E M_p}{R_E}$$

$$= GM_E M_p\left(\frac{1}{R_E} - \frac{1}{R_E + y}\right)$$

$$= \left(\frac{6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.98 \times 10^{24}\ \text{kg})(100\ \text{kg})\left(\frac{1}{6.37 \times 10^6\ \text{m}} - \frac{1}{7.37 \times 10^6\ \text{m}}\right)$$

$$W = \boxed{850\ \text{MJ}}$$

(b) In a circular orbit, gravity supplies the centripetal force:

$$\frac{GM_E M_p}{(R_E + y)^2} = \frac{M_p v^2}{(R_E + y)}$$

Then, $\frac{1}{2}M_p v^2 = \frac{1}{2}\frac{GM_E M_p}{(R_E + y)}$

So, additional work = kinetic energy required

$$= \frac{1}{2}\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2)(5.98 \times 10^{24}\ \text{kg})(100\ \text{kg})}{(\text{kg}^2)(7.37 \times 10^6\ \text{m})}$$

$$\Delta W = \boxed{2.71 \times 10^9\ \text{J}}$$

P13.64 Centripetal acceleration comes from gravitational acceleration.

$$\frac{v^2}{r} = \frac{M_c G}{r^2} = \frac{4\pi^2 r^2}{T^2 r}$$

$$GM_c T^2 = 4\pi^2 r^3$$

$$(6.67 \times 10^{-11})(20)(1.99 \times 10^{30})(5.00 \times 10^{-3})^2 = 4\pi^2 r^3$$

$$r_{\text{orbit}} = \boxed{119\ \text{km}}$$

P13.65 (a) $T = \frac{2\pi r}{v} = \frac{2\pi(30\,000 \times 9.46 \times 10^{15}\ \text{m})}{2.50 \times 10^5\ \text{m/s}} = 7 \times 10^{15}\ \text{s} = \boxed{2 \times 10^8\ \text{yr}}$

(b) $M = \frac{4\pi^2 a^3}{GT^2} = \frac{4\pi^2(30\,000 \times 9.46 \times 10^{15}\ \text{m})^3}{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.13 \times 10^{15}\ \text{s})^2} = 2.66 \times 10^{41}\ \text{kg}$

$M = 1.34 \times 10^{11}$ solar masses $\boxed{\sim 10^{11}\ \text{solar masses}}$

The number of stars is $\boxed{\text{on the order of } 10^{11}}$.

P13.66 (a) From the data about perigee, the energy of the satellite-Earth system is

$$E = \frac{1}{2}mv_p^2 - \frac{GM_E m}{r_p} = \frac{1}{2}(1.60)\left(8.23\times10^3\right)^2 - \frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)(1.60)}{7.02\times10^6}$$

or $E = \boxed{-3.67\times10^7\ \text{J}}$

(b) $L = mvr\sin\theta = mv_p r_p \sin 90.0° = (1.60\ \text{kg})\left(8.23\times10^3\ \text{m/s}\right)\left(7.02\times10^6\ \text{m}\right)$

$= \boxed{9.24\times10^{10}\ \text{kg}\cdot\text{m}^2/\text{s}}$

(c) Since both the energy of the satellite-Earth system and the angular momentum of the Earth are conserved,

at apogee we must have $\frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = E$

and $mv_a r_a \sin 90.0° = L$.

Thus, $\frac{1}{2}(1.60)v_a^2 - \frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)(1.60)}{r_s} = -3.67\times10^7\ \text{J}$

and $(1.60\ \text{kg})v_a r_a = 9.24\times10^{10}\ \text{kg}\cdot\text{m}^2/\text{s}$.

Solving simultaneously, $\frac{1}{2}(1.60)v_a^2 - \frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)(1.60)(1.60)v_a}{9.24\times10^{10}} = -3.67\times10^7$

which reduces to $0.800v_a^2 - 11\,046v_a + 3.672\,3\times10^7 = 0$

so $v_a = \frac{11\,046 \pm \sqrt{(11\,046)^2 - 4(0.800)\left(3.672\,3\times10^7\right)}}{2(0.800)}$.

This gives $v_a = 8\,230$ m/s or $\boxed{5\,580\ \text{m/s}}$. The smaller answer refers to the velocity at the apogee while the larger refers to perigee.

Thus, $r_a = \frac{L}{mv_a} = \frac{9.24\times10^{10}\ \text{kg}\cdot\text{m}^2/\text{s}}{(1.60\ \text{kg})\left(5.58\times10^3\ \text{m/s}\right)} = \boxed{1.04\times10^7\ \text{m}}$.

(d) The major axis is $2a = r_p + r_a$, so the semi-major axis is

$$a = \frac{1}{2}\left(7.02\times10^6\ \text{m} + 1.04\times10^7\ \text{m}\right) = \boxed{8.69\times10^6\ \text{m}}$$

(e) $T = \sqrt{\frac{4\pi^2 a^3}{GM_E}} = \sqrt{\frac{4\pi^2\left(8.69\times10^6\ \text{m}\right)^3}{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}}$

$T = 8\,060\ \text{s} = \boxed{134\ \text{min}}$

***P13.67** Let m represent the mass of the meteoroid and v_i its speed when far away. No torque acts on the meteoroid, so its angular momentum is conserved as it moves between the distant point and the point where it grazes the Earth, moving perpendicular to the radius:

FIG. P13.67

$L_i = L_f$: $\quad m\mathbf{r}_i \times \mathbf{v}_i = m\mathbf{r}_f \times \mathbf{v}_f$

$$m(3R_E v_i) = mR_E v_f$$

$$v_f = 3v_i$$

Now energy of the meteoroid-Earth system is also conserved:

$(K+U_g)_i = (K+U_g)_f$: $\quad \frac{1}{2}mv_i^2 + 0 = \frac{1}{2}mv_f^2 - \frac{GM_E m}{R_E}$

$$\frac{1}{2}v_i^2 = \frac{1}{2}(9v_i^2) - \frac{GM_E}{R_E}$$

$\frac{GM_E}{R_E} = 4v_i^2$: $\quad \boxed{v_i = \sqrt{\frac{GM_E}{4R_E}}}$

***P13.68** From Kepler's third law, minimum period means minimum orbit size. The "treetop satellite" in Figure P13.35 has minimum period. The radius of the satellite's circular orbit is essentially equal to the radius R of the planet.

$\sum F = ma$: $\quad \frac{GMm}{R^2} = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$

$$G\rho V = \frac{R^2(4\pi^2 R^2)}{RT^2}$$

$$G\rho\left(\frac{4}{3}\pi R^3\right) = \frac{4\pi^2 R^3}{T^2}$$

The radius divides out: $T^2 G\rho = 3\pi$ $\quad \boxed{T = \sqrt{\frac{3\pi}{G\rho}}}$

P13.69 If we choose the coordinate of the center of mass at the origin, then

$$0 = \frac{(Mr_2 - mr_1)}{M+m} \quad \text{and} \quad Mr_2 = mr_1$$

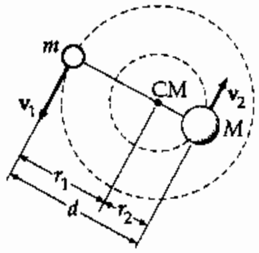


FIG. P13.69

(Note: this is equivalent to saying that the net torque must be zero and the two experience no angular acceleration.) For each mass $F = ma$ so

$$mr_1\omega_1^2 = \frac{MGm}{d^2} \quad \text{and} \quad Mr_2\omega_2^2 = \frac{MGm}{d^2}$$

Combining these two equations and using $d = r_1 + r_2$ gives $(r_1 + r_2)\omega^2 = \frac{(M+m)G}{d^2}$

with $\omega_1 = \omega_2 = \omega$

and $T = \frac{2\pi}{\omega}$

we find $\boxed{T^2 = \frac{4\pi^2 d^3}{G(M+m)}}$.

P13.70 (a) The gravitational force exerted on m_2 by the Earth (mass m_1) accelerates m_2 according to: $m_2 g_2 = \frac{Gm_1 m_2}{r^2}$. The equal magnitude force exerted on the Earth by m_2 produces negligible acceleration of the Earth. The acceleration of relative approach is then

$$g_2 = \frac{Gm_1}{r^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})}{(1.20\times10^7\ \text{m})^2} = \boxed{2.77\ \text{m/s}^2}.$$

(b) Again, m_2 accelerates toward the center of mass with $g_2 = 2.77\ \text{m/s}^2$. Now the Earth accelerates toward m_2 with an acceleration given as

$$m_1 g_1 = \frac{Gm_1 m_2}{r^2}$$

$$g_1 = \frac{Gm_2}{r^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(2.00\times10^{24}\ \text{kg})}{(1.20\times10^7\ \text{m})^2} = 0.926\ \text{m/s}^2$$

The distance between the masses closes with relative acceleration of

$$g_{\text{rel}} = g_1 + g_2 = 0.926\ \text{m/s}^2 + 2.77\ \text{m/s}^2 = \boxed{3.70\ \text{m/s}^2}.$$

P13.71 *Initial Conditions and Constants:*

Mass of planet:	5.98×10^{24} kg
Radius of planet:	6.37×10^6 m
Initial x:	0.0 planet radii
Initial y:	2.0 planet radii
Initial v_x:	+5 000 m/s
Initial v_y:	0.0 m/s
Time interval:	10.9 s

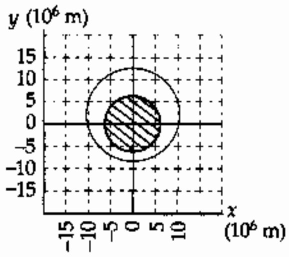


FIG. P13.71

t (s)	x (m)	y (m)	r (m)	v_x (m/s)	v_y (m/s)	a_x (m/s²)	a_y (m/s²)
0.0	0.0	12 740 000.0	12 740 000.0	5 000.0	0.0	0.000 0	−2.457 5
10.9	54 315.3	12 740 000.0	12 740 115.8	4 999.9	−26.7	−0.010 0	−2.457 4
21.7	108 629.4	12 739 710.0	12 740 173.1	4 999.7	−53.4	−0.021 0	−2.457 3
32.6	162 941.1	12 739 130.0	12 740 172.1	4 999.3	−80.1	−0.031 0	−2.457 2
…							
5 431.6	112 843.8	−8 466 816.0	8 467 567.9	−7 523.0	−39.9	−0.074 0	5.562 5
5 442.4	31 121.4	−8 467 249.7	8 467 306.9	−7 523.2	20.5	−0.020 0	5.563 3
5 453.3	−50 603.4	−8 467 026.9	8 467 178.2	−7 522.8	80.9	0.033 0	5.563 4
5 464.1	−132 324.3	−8 466 147.7	8 467 181.7	−7 521.9	141.4	0.087 0	5.562 8
…							
10 841.3	−108 629.0	12 739 134.4	12 739 597.5	4 999.9	53.3	0.021 0	−2.457 5
10 852.2	−54 314.9	12 739 713.4	12 739 829.2	5 000.0	26.6	0.010 0	−2.457 5
10 863.1	0.4	12 740 002.4	12 740 002.4	5 000.0	−0.1	0.000 0	−2.457 5

The $\boxed{\text{object does not hit the Earth}}$; its minimum radius is $\boxed{1.33R_E}$.

Its period is $\boxed{1.09\times10^4\ \text{s}}$. A circular orbit would require a speed of $\boxed{5.60\ \text{km/s}}$.

ANSWERS TO EVEN PROBLEMS

P13.2 $2.67 \times 10^{-7}\ \text{m/s}^2$

P13.4 3.00 kg and 2.00 kg

P13.6 (a) 4.39×10^{20} N toward the Sun; (b) 1.99×10^{20} N toward the Earth; (c) 3.55×10^{22} N toward the Sun

P13.8 see the solution; either 1 m − 61.3 nm or 2.74×10^{-4} m

P13.10 $\frac{2}{3}$

P13.12 (a) 1.02 km/s; (b) 1.35 mm

P13.14 see the solution

P13.16 1.27

P13.18 Planet *Y* has turned through 1.30 revolutions

P13.20 1.63×10^{4} rad/s

P13.22 18.2 ms

P13.24 (a) 1.31×10^{17} N toward the center; (b) 2.62×10^{12} N/kg

P13.26 (a) -4.77×10^{9} J; (b) 569 N down; (c) 569 N up

P13.28 2.52×10^{7} m

P13.30 2.82×10^{9} J

P13.32 (a) see the solution; (b) 340 s

P13.34 (a) 42.1 km/s; (b) 2.20×10^{11} m

P13.36 1.58×10^{10} J

P13.38 (a) $2\pi(R_E + h)^{3/2}(GM_E)^{-1/2}$; (b) $(GM_E)^{1/2}(R_E + h)^{-1/2}$; (c) $GM_E m\left[\dfrac{R_E + 2h}{2R_E(R_E + h)}\right] - \dfrac{2\pi^2 R_E^2 m}{(86\,400\text{ s})^2}$ The satellite should be launched from the Earth's equator toward the east.

P13.40 (a) $10.0\ \text{m/s}^2$; (b) 21.8 km/s

P13.42 11.8 km/s

P13.44 $\dfrac{GM_E m}{12R_E}$

P13.46 (a) $v_0 = \left(\dfrac{GM_E}{r}\right)^{1/2}$; (b) $v_i = \dfrac{5\left(\frac{GM_E}{r}\right)^{1/2}}{4}$; (c) $r_f = \dfrac{25r}{7}$

P13.48 2.26×10^{-7}

P13.50 $\dfrac{2}{3}\sqrt{\dfrac{GM}{R}}$; $\dfrac{1}{3}\sqrt{\dfrac{GM}{R}}$

P13.52 (a), (b) see the solution; (c) $1.85 \times 10^{-5}\ \text{m/s}^2$

P13.54 492 m/s

P13.56 see the solution

P13.58 (a) $G^{1/2}c^{-3/2}h^{1/2}$; (b) $\sim 10^{-34}$ m

P13.60 (a) 7.79 km/s; (b) 7.85 km/s; (c) −3.04 GJ; (d) −3.08 GJ; (e) loss = 46.9 MJ; (f) A component of the Earth's gravity pulls forward on the satellite in its downward banking trajectory.

P13.62 (a) 29.3 km/s; (b) $K_p = 2.74 \times 10^{33}$ J; $U_p = -5.40 \times 10^{33}$ J; (c) $K_a = 2.57 \times 10^{33}$ J; $U_a = -5.22 \times 10^{33}$ J; yes

P13.64 119 km

P13.66 (a) −36.7 MJ; (b) $9.24 \times 10^{10}\ \text{kg} \cdot \text{m}^2/\text{s}$; (c) 5.58 km/s; 10.4 Mm; (d) 8.69 Mm; (e) 134 min

P13.68 see the solution

P13.70 (a) $2.77\ \text{m/s}^2$; (b) $3.70\ \text{m/s}^2$

14

Fluid Mechanics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q14.1 The weight depends upon the total volume of glass. The pressure depends only on the depth.

Q14.2 Both must be built the same. The force on the back of each dam is the average pressure of the water times the area of the dam. If both reservoirs are equally deep, the force is the same.

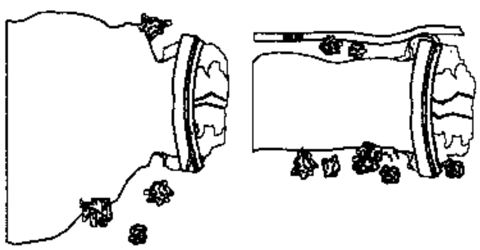


FIG. Q14.2

Q14.3 If the tube were to fill up to the height of several stories of the building, the pressure at the bottom of the depth of the tube of fluid would be very large according to Equation 14.4. This pressure is much larger than that originally exerted by inward elastic forces of the rubber on the water. As a result, water is pushed into the bottle from the tube. As more water is added to the tube, more water continues to enter the bottle, stretching it thin. For a typical bottle, the pressure at the bottom of the tube can become greater than the pressure at which the rubber material will rupture, so the bottle will simply fill with water and expand until it bursts. Blaise Pascal splintered strong barrels by this method.

Q14.4 About 1 000 N: that's about 250 pounds.

Q14.5 The submarine would stop if the density of the surrounding water became the same as the average density of the submarine. Unfortunately, because the water is almost incompressible, this will be much deeper than the crush depth of the submarine.

Q14.6 Yes. The propulsive force of the fish on the water causes the scale reading to fluctuate. Its average value will still be equal to the total weight of bucket, water, and fish.

Q14.7 The boat floats higher in the ocean than in the inland lake. According to Archimedes's principle, the magnitude of buoyant force on the ship is equal to the weight of the water displaced by the ship. Because the density of salty ocean water is greater than fresh lake water, less ocean water needs to be displaced to enable the ship to float.

Q14.8 In the ocean, the ship floats due to the buoyant force from *salt water*. Salt water is denser than fresh water. As the ship is pulled up the river, the buoyant force from the fresh water in the river is not sufficient to support the weight of the ship, and it sinks.

Q14.9 Exactly the same. Buoyancy equals density of water times volume displaced.

Q14.10 At lower elevation the water pressure is greater because pressure increases with increasing depth below the water surface in the reservoir (or water tower). The penthouse apartment is not so far below the water surface. The pressure behind a closed faucet is weaker there and the flow weaker from an open faucet. Your fire department likely has a record of the precise elevation of every fire hydrant.

Q14.11 As the wind blows over the chimney, it creates a lower pressure at the top of the chimney. The smoke flows from the relatively higher pressure in front of the fireplace to the low pressure outside. Science doesn't suck; the smoke is pushed from below.

Q14.12 The rapidly moving air above the ball exerts less pressure than the atmospheric pressure below the ball. This can give substantial lift to balance the weight of the ball.

Q14.13 The ski–jumper gives her body the shape of an airfoil. She deflects downward the air stream as it rushes past and it deflects her upward by Newton's third law. The air exerts on her a lift force, giving her a higher and longer trajectory. To say it in different words, the pressure on her back is less than the pressure on her front.

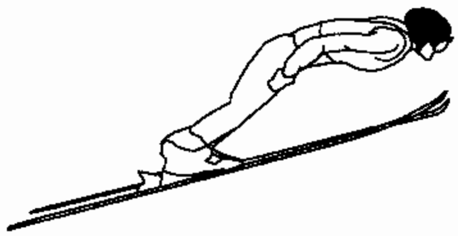


FIG. Q14.13

Q14.14 The horizontal force exerted by the outside fluid, on an area element of the object's side wall, has equal magnitude and opposite direction to the horizontal force the fluid exerts on another element diametrically opposite the first.

Q14.15 The glass may have higher density than the liquid, but the air inside has lower density. The total weight of the bottle can be less than the weight of an equal volume of the liquid.

Q14.16 Breathing in makes your volume greater and increases the buoyant force on you. You instinctively take a deep breath if you fall into the lake.

Q14.17 No. The somewhat lighter barge will float higher in the water.

Q14.18 The level of the pond falls. This is because the anchor displaces more water while in the boat. A floating object displaces a volume of water whose weight is equal to the weight of the object. A submerged object displaces a volume of water equal to the volume of the object. Because the density of the anchor is greater than that of water, a volume of water that weighs the same as the anchor will be greater than the volume of the anchor.

Q14.19 The metal is more dense than water. If the metal is sufficiently thin, it can float like a ship, with the lip of the dish above the water line. Most of the volume below the water line is filled with air. The mass of the dish divided by the volume of the part below the water line is just equal to the density of water. Placing a bar of soap into this space to replace the air raises the average density of the compound object and the density can become greater than that of water. The dish sinks with its cargo.

Q14.20 The excess pressure is transmitted undiminished throughout the container. It will compress air inside the wood. The water driven into the wood raises its average density and makes if float lower in the water. Add some thumbtacks to reach neutral buoyancy and you can make the wood sink or rise at will by subtly squeezing a large clear–plastic soft–drink bottle. Bored with graph paper and proving his own existence, René Descartes invented this toy or trick.

Q14.21 The plate must be horizontal. Since the pressure of a fluid increases with increasing depth, other orientations of the plate will give a non-uniform pressure on the flat faces.

Q14.22 The air in your lungs, the blood in your arteries and veins, and the protoplasm in each cell exert nearly the same pressure, so that the wall of your chest can be in equilibrium.

Q14.23 Use a balance to determine its mass. Then partially fill a graduated cylinder with water. Immerse the rock in the water and determine the volume of water displaced. Divide the mass by the volume and you have the density.

Q14.24 When taking off into the wind, the increased airspeed over the wings gives a larger lifting force, enabling the pilot to take off in a shorter length of runway.

Q14.25 Like the ball, the balloon will remain in front of you. It will not bob up to the ceiling. Air pressure will be no higher at the floor of the sealed car than at the ceiling. The balloon will experience no buoyant force. You might equally well switch off gravity.

Q14.26 Styrofoam is a little more dense than air, so the first ship floats lower in the water.

Q14.27 We suppose the compound object floats. In both orientations it displaces its own weight of water, so it displaces equal volumes of water. The water level in the tub will be unchanged when the object is turned over. Now the steel is underwater and the water exerts on the steel a buoyant force that was not present when the steel was on top surrounded by air. Thus, slightly less wood will be below the water line on the block. It will appear to float higher.

Q14.28 A breeze from any direction speeds up to go over the mound and the air pressure drops. Air then flows through the burrow from the lower entrance to the upper entrance.

Q14.29 Regular cola contains a considerable mass of dissolved sugar. Its density is higher than that of water. Diet cola contains a very small mass of artificial sweetener and has nearly the same density as water. The low–density air in the can has a bigger effect than the thin aluminum shell, so the can of diet cola floats.

Q14.30 (a) Lowest density: oil; highest density: mercury

(b) The density must increase from top to bottom.

Q14.31 (a) Since the velocity of the air in the right-hand section of the pipe is lower than that in the middle, the pressure is higher.

(b) The equation that predicts the same pressure in the far right and left-hand sections of the tube assumes laminar flow without viscosity. Internal friction will cause some loss of mechanical energy and turbulence will also progressively reduce the pressure. If the pressure at the left were not higher than at the right, the flow would stop.

Q14.32 Clap your shoe or wallet over the hole, or a seat cushion, or your hand. Anything that can sustain a force on the order of 100 N is strong enough to cover the hole and greatly slow down the escape of the cabin air. You need not worry about the air rushing out instantly, or about your body being "sucked" through the hole, or about your blood boiling or your body exploding. If the cabin pressure drops a lot, your ears will pop and the saliva in your mouth may boil—at body temperature—but you will still have a couple of minutes to plug the hole and put on your emergency oxygen mask. Passengers who have been drinking carbonated beverages may find that the carbon dioxide suddenly comes out of solution in their stomachs, distending their vests, making them belch, and all but frothing from their ears; so you might warn them of this effect.

SOLUTIONS TO PROBLEMS

Section 14.1 **Pressure**

P14.1 $M = \rho_{\text{iron}} V = \left(7\,860 \text{ kg/m}^3\right)\left[\frac{4}{3}\pi(0.015\,0 \text{ m})^3\right]$

$M = \boxed{0.111 \text{ kg}}$

P14.2 The density of the nucleus is of the same order of magnitude as that of one proton, according to the assumption of close packing:

$$\rho = \frac{m}{V} \sim \frac{1.67 \times 10^{-27} \text{ kg}}{\frac{4}{3}\pi\left(10^{-15} \text{ m}\right)^3} \boxed{\sim 10^{18} \text{ kg/m}^3}.$$

With vastly smaller average density, a macroscopic chunk of matter or an atom must be mostly empty space.

P14.3 $P = \frac{F}{A} = \frac{50.0(9.80)}{\pi\left(0.500 \times 10^{-2}\right)^2} = \boxed{6.24 \times 10^6 \text{ N/m}^2}$

P14.4 Let F_g be its weight. Then each tire supports $\frac{F_g}{4}$,

so $P = \frac{F}{A} = \frac{F_g}{4A}$

yielding $F_g = 4AP = 4\left(0.024\,0 \text{ m}^2\right)\left(200 \times 10^3 \text{ N/m}^2\right) = \boxed{1.92 \times 10^4 \text{ N}}$

P14.5 The Earth's surface area is $4\pi R^2$. The force pushing inward over this area amounts to

$$F = P_0 A = P_0\left(4\pi R^2\right).$$

This force is the weight of the air:

$$F_g = mg = P_0\left(4\pi R^2\right)$$

so the mass of the air is

$$m = \frac{P_0\left(4\pi R^2\right)}{g} = \frac{\left(1.013 \times 10^5 \text{ N/m}^2\right)\left[4\pi\left(6.37 \times 10^6 \text{ m}\right)^2\right]}{9.80 \text{ m/s}^2} = \boxed{5.27 \times 10^{18} \text{ kg}}.$$

Section 14.2 Variation of Pressure with Depth

P14.6 (a) $P = P_0 + \rho g h = 1.013\times10^5\ \text{Pa} + \left(1\,024\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(1\,000\ \text{m}\right)$

$P = \boxed{1.01\times10^7\ \text{Pa}}$

(b) The gauge pressure is the difference in pressure between the water outside and the air inside the submarine, which we suppose is at 1.00 atmosphere.

$$P_{\text{gauge}} = P - P_0 = \rho g h = 1.00\times10^7\ \text{Pa}$$

The resultant inward force on the porthole is then

$$F = P_{\text{gauge}}A = 1.00\times10^7\ \text{Pa}\left[\pi(0.150\ \text{m})^2\right] = \boxed{7.09\times10^5\ \text{N}}.$$

P14.7 $F_{el} = F_{\text{fluid}}$ or $kx = \rho g h A$

and $h = \dfrac{kx}{\rho g A}$

$$h = \frac{\left(1\,000\ \text{N/m}^2\right)\left(5.00\times10^{-3}\ \text{m}\right)}{\left(10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left[\pi\left(1.00\times10^{-2}\ \text{m}\right)^2\right]} = \boxed{1.62\ \text{m}}$$

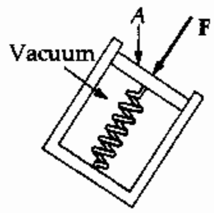


FIG. P14.7

P14.8 Since the pressure is the same on both sides, $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$

In this case, $\dfrac{15\,000}{200} = \dfrac{F_2}{3.00}$ or $F_2 = \boxed{225\ \text{N}}$

P14.9 $F_g = 80.0\ \text{kg}\left(9.80\ \text{m/s}^2\right) = 784\ \text{N}$

When the cup barely supports the student, the normal force of the ceiling is zero and the cup is in equilibrium.

$$F_g = F = PA = \left(1.013\times10^5\ \text{Pa}\right)A$$

$$A = \frac{F_g}{P} = \frac{784}{1.013\times10^5} = \boxed{7.74\times10^{-3}\ \text{m}^2}$$

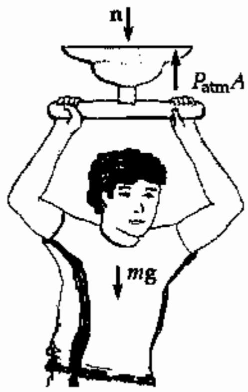


FIG. P14.9

P14.10 (a) Suppose the "vacuum cleaner" functions as a high–vacuum pump. The air below the brick will exert on it a lifting force

$$F = PA = 1.013\times10^5\ \text{Pa}\left[\pi\left(1.43\times10^{-2}\ \text{m}\right)^2\right] = \boxed{65.1\ \text{N}}.$$

(b) The octopus can pull the bottom away from the top shell with a force that could be no larger than

$$F = PA = (P_0 + \rho g h)A = \left[1.013\times10^5\ \text{Pa} + \left(1\,030\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(32.3\ \text{m})\right]\left[\pi\left(1.43\times10^{-2}\ \text{m}\right)^2\right]$$

$F = \boxed{275\ \text{N}}$

P14.11 The excess water pressure (over air pressure) halfway down is

$$P_{\text{gauge}} = \rho g h = (1\,000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1.20 \text{ m}) = 1.18 \times 10^4 \text{ Pa}.$$

The force on the wall due to the water is

$$F = P_{\text{gauge}} A = (1.18 \times 10^4 \text{ Pa})(2.40 \text{ m})(9.60 \text{ m}) = \boxed{2.71 \times 10^5 \text{ N}}$$

horizontally toward the back of the hole.

P14.12 The pressure on the bottom due to the water is $P_b = \rho g z = 1.96 \times 10^4$ Pa

So, $F_b = P_b A = \boxed{5.88 \times 10^6 \text{ N}}$

On each end, $F = \overline{P}A = 9.80 \times 10^3 \text{ Pa}(20.0 \text{ m}^2) = \boxed{196 \text{ kN}}$

On the side, $F = \overline{P}A = 9.80 \times 10^3 \text{ Pa}(60.0 \text{ m}^2) = \boxed{588 \text{ kN}}$

P14.13 In the reference frame of the fluid, the cart's acceleration causes a fictitious force to act backward, as if the acceleration of gravity were $\sqrt{g^2 + a^2}$ directed downward and backward at $\theta = \tan^{-1}\left(\frac{a}{g}\right)$ from the vertical. The center of the spherical shell is at depth $\frac{d}{2}$ below the air bubble and the pressure there is

$$P = P_0 + \rho g_{\text{eff}} h = \boxed{P_0 + \frac{1}{2}\rho d\sqrt{g^2 + a^2}}.$$

P14.14 The air outside and water inside both exert atmospheric pressure, so only the excess water pressure $\rho g h$ counts for the net force. Take a strip of hatch between depth h and $h + dh$. It feels force

$$dF = P\,dA = \rho g h (2.00 \text{ m})dh.$$

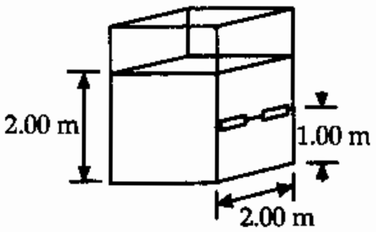


FIG. P14.14

(a) The total force is

$$F = \int dF = \int_{h=1.00 \text{ m}}^{2.00 \text{ m}} \rho g h (2.00 \text{ m})dh$$

$$F = \rho g (2.00 \text{ m})\frac{h^2}{2}\Bigg|_{1.00 \text{ m}}^{2.00 \text{ m}} = (1\,000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)\frac{(2.00 \text{ m})}{2}\left[(2.00 \text{ m})^2 - (1.00 \text{ m})^2\right]$$

$$F = \boxed{29.4 \text{ kN (to the right)}}$$

(b) The lever arm of dF is the distance $(h - 1.00 \text{ m})$ from hinge to strip:

$$\tau = \int d\tau = \int_{h=1.00 \text{ m}}^{2.00 \text{ m}} \rho g h (2.00 \text{ m})(h - 1.00 \text{ m})dh$$

$$\tau = \rho g (2.00 \text{ m})\left[\frac{h^3}{3} - (1.00 \text{ m})\frac{h^2}{2}\right]_{1.00 \text{ m}}^{2.00 \text{ m}}$$

$$\tau = (1\,000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(2.00 \text{ m})\left(\frac{7.00 \text{ m}^3}{3} - \frac{3.00 \text{ m}^3}{2}\right)$$

$$\tau = \boxed{16.3 \text{ kN}\cdot\text{m counterclockwise}}$$

P14.15 The bell is uniformly compressed, so we can model it with any shape. We choose a sphere of diameter 3.00 m.
The pressure on the ball is given by: $P = P_{atm} + \rho_w gh$ so the change in pressure on the ball from when it is on the surface of the ocean to when it is at the bottom of the ocean is $\Delta P = \rho_w gh$.
In addition:

$$\Delta V = \frac{-V\Delta P}{B} = -\frac{\rho_w ghV}{B} = -\frac{4\pi\rho_w ghr^3}{3B}, \text{ where } B \text{ is the Bulk Modulus.}$$

$$\Delta V = -\frac{4\pi\left(1\,030 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)(10\,000 \text{ m})(1.50 \text{ m})^3}{(3)\left(14.0\times10^{10} \text{ Pa}\right)} = -0.010\,2 \text{ m}^3$$

Therefore, the volume of the ball at the bottom of the ocean is

$$V - \Delta V = \frac{4}{3}\pi(1.50 \text{ m})^3 - 0.010\,2 \text{ m}^3 = 14.137 \text{ m}^3 - 0.010\,2 \text{ m}^3 = 14.127 \text{ m}^3.$$

This gives a radius of 1.499 64 m and a new diameter of 2.999 3 m. Therefore the diameter decreases by $\boxed{0.722 \text{ mm}}$.

Section 14.3 Pressure Measurements

P14.16 (a) We imagine the superhero to produce a perfect vacuum in the straw. Take point 1 at the water surface in the basin and point 2 at the water surface in the straw:

$$P_1 + \rho g y_1 = P_2 + \rho g y_2$$

$$1.013\times10^5 \text{ N/m}^2 + 0 = 0 + \left(1\,000 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)y_2 \qquad y_2 = \boxed{10.3 \text{ m}}$$

(b) No atmosphere can lift the water in the straw through $\boxed{\text{zero}}$ height difference.

P14.17 $P_0 = \rho g h$

$$h = \frac{P_0}{\rho g} = \frac{10.13\times10^5 \text{ Pa}}{\left(0.984\times10^3 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)} = \boxed{10.5 \text{ m}}$$

$\boxed{\text{No. Some alcohol and water will evaporate.}}$ The equilibrium vapor pressures of alcohol and water are higher than the vapor pressure of mercury.

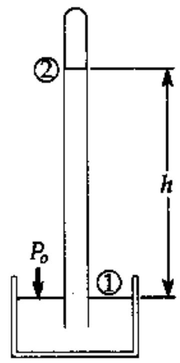


FIG. P14.17

P14.18 (a) Using the definition of density, we have

$$h_w = \frac{m_{\text{water}}}{A_2 \rho_{\text{water}}} = \frac{100\text{ g}}{5.00\text{ cm}^2\left(1.00\text{ g/cm}^3\right)} = \boxed{20.0\text{ cm}}$$

(b) Sketch (b) at the right represents the situation after the water is added. A volume $(A_2 h_2)$ of mercury has been displaced by water in the right tube. The additional volume of mercury now in the left tube is $A_1 h$. Since the total volume of mercury has not changed,

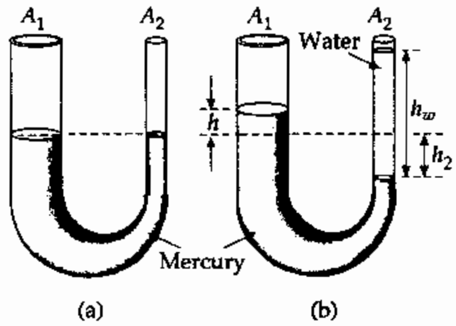


FIG. P14.18

$$A_2 h_2 = A_1 h \qquad \text{or} \qquad h_2 = \frac{A_1}{A_2} h \qquad (1)$$

At the level of the mercury–water interface in the right tube, we may write the absolute pressure as:

$$P = P_0 + \rho_{\text{water}} g h_w$$

The pressure at this same level in the left tube is given by

$$P = P_0 + \rho_{\text{Hg}} g\left(h + h_2\right) = P_0 + \rho_{\text{water}} g h_w$$

which, using equation (1) above, reduces to

$$\rho_{\text{Hg}} h\left[1 + \frac{A_1}{A_2}\right] = \rho_{\text{water}} h_w$$

or $h = \dfrac{\rho_{\text{water}} h_w}{\rho_{\text{Hg}}\left(1 + \frac{A_1}{A_2}\right)}$.

Thus, the level of mercury has risen a distance of $h = \dfrac{\left(1.00\text{ g/cm}^3\right)(20.0\text{ cm})}{\left(13.6\text{ g/cm}^3\right)\left(1 + \frac{10.0}{5.00}\right)} = \boxed{0.490\text{ cm}}$

above the original level.

P14.19 $\Delta P_0 = \rho g \Delta h = -2.66 \times 10^3\text{ Pa}$: $\quad P = P_0 + \Delta P_0 = (1.013 - 0.026\,6) \times 10^5\text{ Pa} = \boxed{0.986 \times 10^5\text{ Pa}}$

P14.20 Let h be the height of the water column added to the right side of the U-tube. Then when equilibrium is reached, the situation is as shown in the sketch at right. Now consider two points, A and B shown in the sketch, at the level of the water–mercury interface. By Pascal's Principle, the absolute pressure at B is the same as that at A. But,

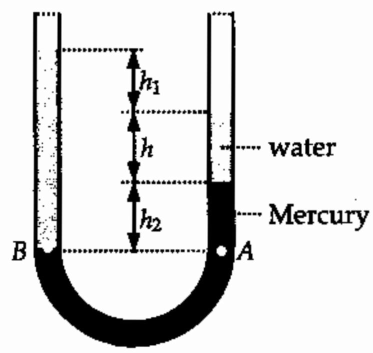


FIG. P14.20

$$P_A = P_0 + \rho_w g h + \rho_{\text{Hg}} g h_2 \text{ and}$$
$$P_B = P_0 + \rho_w g\left(h_1 + h + h_2\right).$$

Thus, from $P_A = P_B$, $\rho_w h_1 + \rho_w h + \rho_w h_2 = \rho_w h + \rho_{\text{Hg}} h_2$, or

$$h_1 = \left[\frac{\rho_{\text{Hg}}}{\rho_w} - 1\right] h_2 = (13.6 - 1)(1.00\text{ cm}) = \boxed{12.6\text{ cm}}.$$

***P14.21** (a) $P = P_0 + \rho g h$

The gauge pressure is

$$P - P_0 = \rho g h = 1\,000\text{ kg}\left(9.8\text{ m/s}^2\right)(0.160\text{ m}) = \boxed{1.57\text{ kPa}} = 1.57\times10^3\text{ Pa}\left(\frac{1\text{ atm}}{1.013\times10^5\text{ Pa}}\right)$$

$$= \boxed{0.015\,5\text{ atm}}.$$

It would lift a mercury column to height

$$h = \frac{P - P_0}{\rho g} = \frac{1\,568\text{ Pa}}{\left(13\,600\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)} = \boxed{11.8\text{ mm}}.$$

(b) **Increased pressure of the cerebrospinal fluid will raise the level of the fluid in the spinal tap.**

(c) **Blockage of the fluid within the spinal column or between the skull and the spinal column would prevent the fluid level from rising.**

Section 14.4 Buoyant Forces and Archimede's Principle

P14.22 (a) The balloon is nearly in equilibrium:

$$\sum F_y = ma_y \Rightarrow B - \left(F_g\right)_{\text{helium}} - \left(F_g\right)_{\text{payload}} = 0$$

or $\rho_{\text{air}} g V - \rho_{\text{helium}} g V - m_{\text{payload}} g = 0$

This reduces to

$$m_{\text{payload}} = (\rho_{\text{air}} - \rho_{\text{helium}})V = \left(1.29\text{ kg/m}^3 - 0.179\text{ kg/m}^3\right)\left(400\text{ m}^3\right)$$

$$m_{\text{payload}} = \boxed{444\text{ kg}}$$

(b) Similarly,

$$m_{\text{payload}} = \left(\rho_{\text{air}} - \rho_{\text{hydrogen}}\right)V = \left(1.29\text{ kg/m}^3 - 0.089\,9\text{ kg/m}^3\right)\left(400\text{ m}^3\right)$$

$$m_{\text{payload}} = \boxed{480\text{ kg}}$$

The air does the lifting, nearly the same for the two balloons.

P14.23 At equilibrium $\sum F = 0$ or $F_{app} + mg = B$

where B is the buoyant force.

The applied force, $F_{app} = B - mg$

where $B = \text{Vol}(\rho_{\text{water}})g$

and $m = (\text{Vol})\rho_{\text{ball}}$.

So, $F_{app} = (\text{Vol})g(\rho_{\text{water}} - \rho_{\text{ball}}) = \frac{4}{3}\pi r^3 g(\rho_{\text{water}} - \rho_{\text{ball}})$

$$F_{app} = \frac{4}{3}\pi\left(1.90\times10^{-2}\text{ m}\right)^3\left(9.80\text{ m/s}^2\right)\left(10^3\text{ kg/m}^3 - 84.0\text{ kg/m}^3\right) = \boxed{0.258\text{ N}}$$

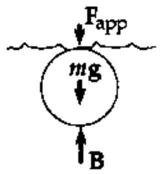


FIG. P14.23

P14.24 $F_g = (m + \rho_s V)g$ must be equal to $F_b = \rho_w V g$

Since $V = Ah$, $m + \rho_s Ah = \rho_w Ah$

and $A = \boxed{\dfrac{m}{(\rho_w - \rho_s)h}}$

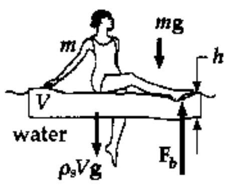


FIG. P14.24

P14.25 (a) Before the metal is immersed:

$\sum F_y = T_1 - Mg = 0$ or

$T_1 = Mg = (1.00 \text{ kg})(9.80 \text{ m/s}^2)$

$= \boxed{9.80 \text{ N}}$

(b) After the metal is immersed:

$\sum F_y = T_2 + B - Mg = 0$ or

$T_2 = Mg - B = Mg - (\rho_w V)g$

$V = \frac{M}{\rho} = \frac{1.00 \text{ kg}}{2\,700 \text{ kg/m}^3}$

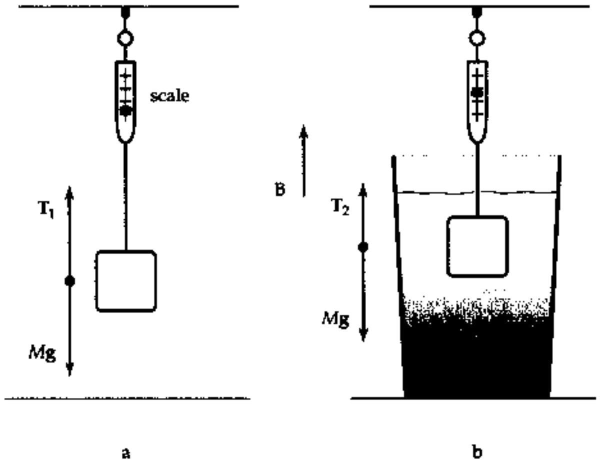


FIG. P14.25

Thus,

$$T_2 = Mg - B = 9.80 \text{ N} - (1\,000 \text{ kg/m}^3)\left(\frac{1.00 \text{ kg}}{2\,700 \text{ kg/m}^3}\right)(9.80 \text{ m/s}^2) = \boxed{6.17 \text{ N}}.$$

***P14.26** (a)

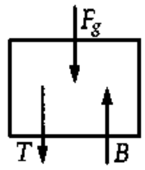


FIG. P14.26(a)

(b) $\sum F_y = 0$: $-15 \text{ N} - 10 \text{ N} + B = 0$

$\boxed{B = 25.0 \text{ N}}$

(c) The oil pushes $\boxed{\text{horizontally inward}}$ on each side of the block.

(d) $\boxed{\text{String tension increases}}$. The oil causes the water below to be under greater pressure, and the water pushes up more strongly on the bottom of the block.

(e) Consider the equilibrium just before the string breaks:

$$-15 \text{ N} - 60 \text{ N} + 25 \text{ N} + B_{\text{oil}} = 0$$

$$B_{\text{oil}} = 50 \text{ N}$$

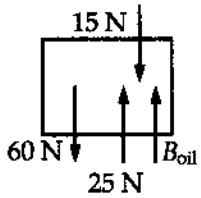


FIG. P14.26(e)

For the buoyant force of the water we have

$B = \rho V g \qquad 25 \text{ N} = (1\,000 \text{ kg/m}^3)(0.25 V_{\text{block}})9.8 \text{ m/s}^2$

$V_{\text{block}} = 1.02 \times 10^{-2} \text{ m}^3$

For the buoyant force of the oil

$$50 \text{ N} = (800 \text{ kg/m}^3) f_e (1.02 \times 10^{-2} \text{ m}^3) 9.8 \text{ m/s}^2$$

$$f_e = 0.625 = \boxed{62.5\%}$$

(f) $-15 \text{ N} + (800 \text{ kg/m}^3) f_f (1.02 \times 10^{-2} \text{ m}^3) 9.8 \text{ m/s}^2 = 0$

$f_f = 0.187 = \boxed{18.7\%}$

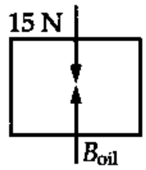


FIG. P14.26(f)

P14.27 (a) $P = P_0 + \rho g h$

Taking $P_0 = 1.013 \times 10^5 \text{ N/m}^2$ and $h = 5.00$ cm

we find $P_{top} = 1.0179 \times 10^5 \text{ N/m}^2$

For $h = 17.0$ cm, we get $P_{bot} = 1.0297 \times 10^5 \text{ N/m}^2$

Since the areas of the top and bottom are $A = (0.100 \text{ m})^2 = 10^{-2} \text{ m}^2$

we find $F_{top} = P_{top} A = \boxed{1.0179 \times 10^3 \text{ N}}$

and $F_{bot} = \boxed{1.0297 \times 10^3 \text{ N}}$

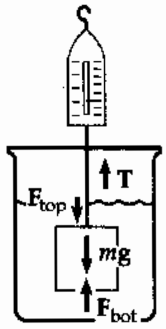


FIG. P14.27

(b) $T + B - Mg = 0$

where $B = \rho_w V g = (10^3 \text{ kg/m}^3)(1.20 \times 10^{-3} \text{ m}^3)(9.80 \text{ m/s}^2) = 11.8 \text{ N}$

and $Mg = 10.0(9.80) = 98.0 \text{ N}$

Therefore, $T = Mg - B = 98.0 - 11.8 = \boxed{86.2 \text{ N}}$

(c) $F_{bot} - F_{top} = (1.0297 - 1.0179) \times 10^3 \text{ N} = \boxed{11.8 \text{ N}}$

which is equal to B found in part (b).

P14.28 Consider spherical balloons of radius 12.5 cm containing helium at STP and immersed in air at 0°C and 1 atm. If the rubber envelope has mass 5.00 g, the upward force on each is

$$B - F_{g,\text{He}} - F_{g,env} = \rho_{\text{air}} V g - \rho_{\text{He}} V g - m_{env} g$$

$$F_{up} = (\rho_{\text{air}} - \rho_{\text{He}})\left(\frac{4}{3}\pi r^3\right) g - m_{env} g$$

$$F_{up} = \left[(1.29 - 0.179) \text{ kg/m}^3\right]\left[\frac{4}{3}\pi (0.125 \text{ m})^3\right](9.80 \text{ m/s}^2) - 5.00 \times 10^{-3} \text{ kg}(9.80 \text{ m/s}^2) = 0.0401 \text{ N}$$

If your weight (including harness, strings, and submarine sandwich) is

$$70.0 \text{ kg}(9.80 \text{ m/s}^2) = 686 \text{ N}$$

you need this many balloons: $\dfrac{686 \text{ N}}{0.0401 \text{ N}} = 17\,000 \boxed{\sim 10^4}$.

P14.29 (a) According to Archimedes, $B = \rho_{\text{water}} V_{\text{water}} g = (1.00 \text{ g/cm}^3)[20.0 \times 20.0 \times (20.0 - h)]g$

But $B = \text{Weight of block} = mg = \rho_{\text{wood}} V_{\text{wood}} g = (0.650 \text{ g/cm}^3)(20.0 \text{ cm})^3 g$

$$0.650(20.0)^3 g = 1.00(20.0)(20.0)(20.0 - h)g$$

$20.0 - h = 20.0(0.650)$ so $h = 20.0(1 - 0.650) = \boxed{7.00 \text{ cm}}$

(b) $B = F_g + Mg$ where M = mass of lead

$$1.00(20.0)^3 g = 0.650(20.0)^3 g + Mg$$

$M = (1.00 - 0.650)(20.0)^3 = 0.350(20.0)^3 = 2\,800 \text{ g} = \boxed{2.80 \text{ kg}}$

***P14.30** (a) The weight of the ball must be equal to the buoyant force of the water:

$$1.26\ \text{kg}g = \rho_{\text{water}} \frac{4}{3} \pi r_{\text{outer}}^3 g$$

$$r_{\text{outer}} = \left(\frac{3 \times 1.26\ \text{kg}}{4\pi 1\,000\ \text{kg/m}^3} \right)^{1/3} = \boxed{6.70\ \text{cm}}$$

(b) The mass of the ball is determined by the density of aluminum:

$$m = \rho_{\text{Al}} V = \rho_{\text{Al}} \left(\frac{4}{3} \pi r_0^3 - \frac{4}{3} \pi r_i^3 \right)$$

$$1.26\ \text{kg} = 2\,700\ \text{kg/m}^3 \left(\frac{4}{3} \pi \right) \left((0.067\ \text{m})^3 - r_i^3 \right)$$

$$1.11 \times 10^{-4}\ \text{m}^3 = 3.01 \times 10^{-4}\ \text{m}^3 - r_i^3$$

$$r_i = \left(1.89 \times 10^{-4}\ \text{m}^3 \right)^{1/3} = \boxed{5.74\ \text{cm}}$$

***P14.31** Let A represent the horizontal cross-sectional area of the rod, which we presume to be constant. The rod is in equilibrium:

$\sum F_y = 0$: $\quad -mg + B = 0 = -\rho_0 V_{\text{whole rod}} g + \rho_{\text{fluid}} V_{\text{immersed}} g$

$$\rho_0 ALg = \rho A(L-h)g$$

The density of the liquid is $\rho = \dfrac{\rho_0 L}{L-h}$.

***P14.32** We use the result of Problem 14.31. For the rod floating in a liquid of density $0.98\ \text{g/cm}^3$,

$$\rho = \rho_0 \frac{L}{L-h}$$

$$0.98\ \text{g/cm}^3 = \frac{\rho_0 L}{(L - 0.2\ \text{cm})}$$

$$0.98\ \text{g/cm}^3 L - \left(0.98\ \text{g/cm}^3 \right) 0.2\ \text{cm} = \rho_0 L$$

For floating in the dense liquid,

$$1.14\ \text{g/cm}^3 = \frac{\rho_0 L}{(L - 1.8\ \text{cm})}$$

$$1.14\ \text{g/cm}^3 - \left(1.14\ \text{g/cm}^3 \right) 1.8\ \text{cm} = \rho_0 L$$

(a) By substitution,

$$1.14L - 1.14(1.8\ \text{cm}) = 0.98L - 0.2(0.98)$$

$$0.16L = 1.856\ \text{cm}$$

$$L = \boxed{11.6\ \text{cm}}$$

(b) Substituting back,

$$0.98\ \text{g/cm}^3 (11.6\ \text{cm} - 0.2\ \text{cm}) = \rho_0 11.6\ \text{cm}$$

$$\rho_0 = \boxed{0.963\ \text{g/cm}^3}$$

(c) The marks are not equally spaced. Because $\rho = \dfrac{\rho_0 L}{L-h}$ is not of the form $\rho = a + bh$, equal-size steps of ρ do not correspond to equal-size steps of h.

P14.33 The balloon stops rising when $(\rho_{air} - \rho_{He})gV = Mg$ and $(\rho_{air} - \rho_{He})V = M$,

Therefore, $V = \frac{M}{\rho_{air} - \rho_{He}} = \frac{400}{1.25e^{-1} - 0.180}$ $V = \boxed{1\,430\ \text{m}^3}$

P14.34 Since the frog floats, the buoyant force = the weight of the frog. Also, the weight of the displaced water = weight of the frog, so

$$\rho_{ooze} Vg = m_{frog} g$$

or $$m_{frog} = \rho_{ooze} V = \rho_{ooze} \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \left(1.35 \times 10^3\ \text{kg/m}^3\right)\frac{2\pi}{3}\left(6.00 \times 10^{-2}\ \text{m}\right)^3$$

Hence, $m_{frog} = \boxed{0.611\ \text{kg}}$.

P14.35 $B = F_g$

$$\rho_{H_2O} g \frac{V}{2} = \rho_{sphere} gV$$

$$\rho_{sphere} = \frac{1}{2}\rho_{H_2O} = \boxed{500\ \text{kg/m}^3}$$

$$\rho_{glycerin} g\left(\frac{4}{10}V\right) - \rho_{sphere} gV = 0$$

$$\rho_{glycerin} = \frac{10}{4}\left(500\ \text{kg/m}^3\right) = \boxed{1\,250\ \text{kg/m}^3}$$

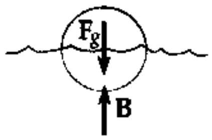


FIG. P14.35

P14.36 Constant velocity implies zero acceleration, which means that the submersible is in equilibrium under the gravitational force, the upward buoyant force, and the upward resistance force:

$\sum F_y = ma_y = 0$ $\quad -\left(1.20 \times 10^4\ \text{kg} + m\right)g + \rho_w gV + 1\,100\ \text{N} = 0$

where m is the mass of the added water and V is the sphere's volume.

$$1.20 \times 10^4\ \text{kg} + m = 1.03 \times 10^3\left[\frac{4}{3}\pi(1.50)^3\right] + \frac{1\,100\ \text{N}}{9.8\ \text{m/s}^2}$$

so $$m = \boxed{2.67 \times 10^3\ \text{kg}}$$

P14.37 By Archimedes's principle, the weight of the fifty planes is equal to the weight of a horizontal slice of water 11.0 cm thick and circumscribed by the water line:

$$\Delta B = \rho_{water} g(\Delta V)$$

$$50\left(2.90 \times 10^4\ \text{kg}\right)g = \left(1\,030\ \text{kg/m}^3\right)g(0.110\ \text{m})A$$

giving $A = \boxed{1.28 \times 10^4\ \text{m}^2}$. The acceleration of gravity does not affect the answer.

Section 14.5 Fluid Dynamics

Section 14.6 Bernoulli's Equation

P14.38 By Bernoulli's equation,

$$8.00\times10^4\ \text{N/m}^2+\frac{1}{2}(1\,000)v^2=6.00\times10^4\ \text{N/m}^2+\frac{1}{2}(1\,000)16v^2$$

$$2.00\times10^4\ \text{N/m}^2=\frac{1}{2}(1\,000)15v^2$$

$$v=1.63\ \text{m/s}$$

$$\frac{dm}{dt}=\rho Av=1\,000\pi\left(5.00\times10^{-2}\right)^2(1.63\ \text{m/s})=\boxed{12.8\ \text{kg/s}}$$

FIG. P14.38

P14.39 Assuming the top is open to the atmosphere, then

$P_1=P_0$.

Note $P_2=P_0$.

$$\text{Flow rate}=2.50\times10^{-3}\ \text{m}^3/\text{min}=4.17\times10^{-5}\ \text{m}^3/\text{s}.$$

(a) $A_1>>A_2$ so $v_1<<v_2$

Assuming $v_1=0$,

$$P_1+\frac{\rho v_1^2}{2}+\rho gy_1=P_2+\frac{\rho v_2^2}{2}+\rho gy_2$$

$$v_2=(2gy_1)^{1/2}=[2(9.80)(16.0)]^{1/2}=\boxed{17.7\ \text{m/s}}$$

(b) $\text{Flow rate}=A_2v_2=\left(\frac{\pi d^2}{4}\right)(17.7)=4.17\times10^{-5}\ \text{m}^3/\text{s}$

$$d=\boxed{1.73\times10^{-3}\ \text{m}}=1.73\ \text{mm}$$

***P14.40** Take point ① at the free surface of the water in the tank and ② inside the nozzle.

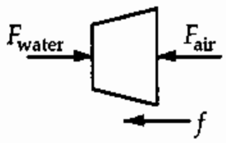


FIG. P14.40

(a) With the cork in place $P_1+\rho gy_1+\frac{1}{2}\rho v_1^2=P_2+\rho gy_2+\frac{1}{2}\rho v_2^2$ becomes

$P_0+1\,000\ \text{kg/m}^3\ 9.8\ \text{m/s}^2\ 7.5\ \text{m}+0=P_2+0+0$; $P_2-P_0=7.35\times10^4\ \text{Pa}$.

For the stopper $\sum F_x=0$

$$F_{water}-F_{air}-f=0$$

$$P_2A-P_0A=f$$

$$f=7.35\times10^4\ \text{Pa}\,\pi(0.011\ \text{m})^2=\boxed{27.9\ \text{N}}$$

(b) Now Bernoulli's equation gives

$$P_0+7.35\times10^4\ \text{Pa}+0=P_0+0+\frac{1}{2}\left(1\,000\ \text{kg/m}^3\right)v_2^2$$

$$v_2=12.1\ \text{m/s}$$

The quantity leaving the nozzle in 2 h is

$$\rho V=\rho Av_2t=\left(1\,000\ \text{kg/m}^3\right)\pi(0.011\ \text{m})^2(12.1\ \text{m/s})7\,200\ \text{s}=\boxed{3.32\times10^4\ \text{kg}}.$$

continued on next page

(c) Take point 1 in the wide hose and 2 just outside the nozzle. Continuity:

$$A_1 v_1 = A_2 v_2$$
$$\pi\left(\frac{6.6\text{ cm}}{2}\right)^2 v_1 = \pi\left(\frac{2.2\text{ cm}}{2}\right)^2 12.1\text{ m/s}$$
$$v_1 = \frac{12.1\text{ m/s}}{9} = 1.35\text{ m/s}$$
$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$
$$P_1 + 0 + \frac{1}{2}\left(1\,000\text{ kg/m}^3\right)(1.35\text{ m/s})^2 = P_0 + 0 + \frac{1}{2}\left(1\,000\text{ kg/m}^3\right)(12.1\text{ m/s})^2$$
$$P_1 - P_0 = 7.35\times10^4\text{ Pa} - 9.07\times10^2\text{ Pa} = \boxed{7.26\times10^4\text{ Pa}}$$

P14.41 Flow rate $Q = 0.012\,0\text{ m}^3/\text{s} = v_1 A_1 = v_2 A_2$

$$v_2 = \frac{Q}{A_2} = \frac{0.012\,0}{A_2} = \boxed{31.6\text{ m/s}}$$

***P14.42** (a) $\mathcal{P} = \frac{\Delta E}{\Delta t} = \frac{\Delta mgh}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)gh = Rgh$

(b) $\mathcal{P}_{EL} = 0.85\left(8.5\times10^5\right)(9.8)(87) = \boxed{616\text{ MW}}$

***P14.43** The volume flow rate is

$$\frac{125\text{ cm}^3}{16.3\text{ s}} = Av_1 = \pi\left(\frac{0.96\text{ cm}}{2}\right)^2 v_1.$$

The speed at the top of the falling column is

$$v_1 = \frac{7.67\text{ cm}^3/\text{s}}{0.724\text{ cm}^2} = 10.6\text{ cm/s}.$$

Take point 2 at 13 cm below:

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$
$$P_0 + \left(1\,000\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)0.13\text{ m} + \frac{1}{2}\left(1\,000\text{ kg/m}^3\right)(0.106\text{ m/s})^2$$
$$= P_0 + 0 + \frac{1}{2}\left(1\,000\text{ kg/m}^3\right)v_2^2$$
$$v_2 = \sqrt{2\left(9.8\text{ m/s}^2\right)0.13\text{ m} + (0.106\text{ m/s})^2} = 1.60\text{ m/s}$$

The volume flow rate is constant:

$$7.67\text{ cm}^3/\text{s} = \pi\left(\frac{d}{2}\right)^2 160\text{ cm/s}$$
$$d = \boxed{0.247\text{ cm}}$$

***P14.44** (a) Between sea surface and clogged hole: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

$1\text{ atm} + 0 + (1\,030\text{ kg/m}^3)(9.8\text{ m/s}^2)(2\text{ m}) = P_2 + 0 + 0 \qquad P_2 = 1\text{ atm} + 20.2\text{ kPa}$

The air on the back of his hand pushes opposite the water, so the net force on his hand is

$F = PA = (20.2\times10^3\text{ N/m}^2)\left(\frac{\pi}{4}\right)(1.2\times10^{-2}\text{ m})^2 \qquad F = \boxed{2.28\text{ N}}$

(b) Now, Bernoulli's theorem is

$1\text{ atm} + 0 + 20.2\text{ kPa} = 1\text{ atm} + \frac{1}{2}(1\,030\text{ kg/m}^3)v_2^2 + 0 \qquad v_2 = 6.26\text{ m/s}$

The volume rate of flow is $A_2 v_2 = \frac{\pi}{4}(1.2\times10^{-2}\text{ m})^2(6.26\text{ m/s}) = 7.08\times10^{-4}\text{ m}^3/\text{s}$

One acre–foot is $4\,047\text{ m}^2 \times 0.304\,8\text{ m} = 1\,234\text{ m}^3$

Requiring $\frac{1\,234\text{ m}^3}{7.08\times10^{-4}\text{ m}^3/\text{s}} = \boxed{1.74\times10^6\text{ s}} = 20.2\text{ days}$

P14.45 (a) Suppose the flow is very slow: $\left(P + \frac{1}{2}\rho v^2 + \rho g y\right)_{\text{river}} = \left(P + \frac{1}{2}\rho v^2 + \rho g y\right)_{\text{rim}}$

$$P + 0 + \rho g(564\text{ m}) = 1\text{ atm} + 0 + \rho g(2\,096\text{ m})$$

$$P = 1\text{ atm} + (1\,000\text{ kg/m}^3)(9.8\text{ m/s}^2)(1\,532\text{ m}) = \boxed{1\text{ atm} + 15.0\text{ MPa}}$$

(b) The volume flow rate is $4\,500\text{ m}^3/\text{d} = Av = \frac{\pi d^2 v}{4}$

$$v = (4\,500\text{ m}^3/\text{d})\left(\frac{1\text{ d}}{86\,400\text{ s}}\right)\left(\frac{4}{\pi(0.150\text{ m})^2}\right) = \boxed{2.95\text{ m/s}}$$

(c) Imagine the pressure as applied to stationary water at the bottom of the pipe:

$$\left(P + \frac{1}{2}\rho v^2 + \rho g y\right)_{\text{bottom}} = \left(P + \frac{1}{2}\rho v^2 + \rho g y\right)_{\text{top}}$$

$$P + 0 = 1\text{ atm} + \frac{1}{2}(1\,000\text{ kg/m}^3)(2.95\text{ m/s})^2 + 1\,000\text{ kg}(9.8\text{ m/s}^2)(1\,532\text{ m})$$

$$P = 1\text{ atm} + 15.0\text{ MPa} + 4.34\text{ kPa}$$

The additional pressure is $\boxed{4.34\text{ kPa}}$.

P14.46 (a) For upward flight of a water-drop projectile from geyser vent to fountain–top, $v_{yf}^2 = v_{yi}^2 + 2a_y\Delta y$

Then $0 = v_i^2 + 2\left(-9.80\ \text{m/s}^2\right)(+40.0\ \text{m})$ and $v_i = \boxed{28.0\ \text{m/s}}$

(b) Between geyser vent and fountain–top: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

Air is so low in density that very nearly $P_1 = P_2 = 1\ \text{atm}$

Then, $\frac{1}{2}v_i^2 + 0 = 0 + \left(9.80\ \text{m/s}^2\right)(40.0\ \text{m})$

$v_1 = \boxed{28.0\ \text{m/s}}$

(c) Between the chamber and the fountain-top: $P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$

$$P_1 + 0 + \left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(-175\ \text{m}) = P_0 + 0 + \left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(+40.0\ \text{m})$$

$$P_1 - P_0 = \left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(215\ \text{m}) = \boxed{2.11\ \text{MPa}}$$

P14.47 $P_1 + \frac{\rho v_1^2}{2} = P_2 + \frac{\rho_2^2}{2}$ (Bernoulli equation), $v_1 A_1 = v_2 A_2$ where $\frac{A_1}{A_2} = 4$

$$\Delta P = P_1 - P_2 = \frac{\rho}{2}\left(v_2^2 - v_1^2\right) = \frac{\rho}{2}v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right) \text{ and } \Delta P = \frac{\rho v_1^2}{2}15 = 21\,000\ \text{Pa}$$

$v_1 = 2.00\ \text{m/s}$; $v_2 = 4v_1 = 8.00\ \text{m/s}$:

The volume flow rate is $v_1 A_1 = \boxed{2.51 \times 10^{-3}\ \text{m}^3/\text{s}}$

Section 14.7 **Other Applications of Fluid Dynamics**

P14.48 $Mg = (P_1 - P_2)A$ for a balanced condition $\frac{16\,000(9.80)}{A} = 7.00 \times 10^4 - P_2$

where $A = 80.0\ \text{m}^2$ $\therefore P_2 = 7.0 \times 10^4 - 0.196 \times 10^4 = \boxed{6.80 \times 10^4\ \text{Pa}}$

P14.49 $\rho_{\text{air}}\frac{v^2}{2} = \Delta P = \rho_{\text{Hg}} g \Delta h$

$$v = \sqrt{\frac{2\rho_{\text{Hg}} g \Delta h}{\rho_{\text{air}}}} = \boxed{103\ \text{m/s}}$$

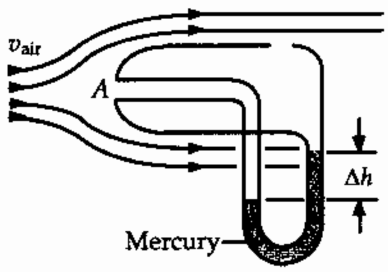


FIG. P14.49

P14.50 The assumption of incompressibility is surely unrealistic, but allows an estimate of the speed:

$$P_1+\rho g y_1+\frac{1}{2}\rho v_1^2=P_2+\rho g y_2+\frac{1}{2}\rho v_2^2$$

$$1.00\text{ atm}+0+0=0.287\text{ atm}+0+\frac{1}{2}\left(1.20\text{ kg/m}^3\right)v_2^2$$

$$v_2=\sqrt{\frac{2(1.00-0.287)\left(1.013\times10^5\text{ N/m}^2\right)}{1.20\text{ kg/m}^3}}=\boxed{347\text{ m/s}}$$

P14.51 (a) $P_0+\rho g h+0=P_0+0+\frac{1}{2}\rho v_3^2$ $\quad v_3=\sqrt{2gh}$

If $h=1.00$ m, $\quad v_3=\boxed{4.43\text{ m/s}}$

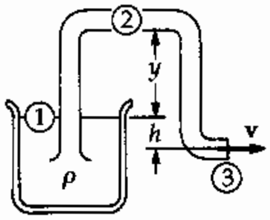


FIG. P14.51

(b) $P+\rho g y+\frac{1}{2}\rho v_2^2=P_0+0+\frac{1}{2}\rho v_3^2$

Since $v_2=v_3$, $\quad P=P_0-\rho g y$

Since $P\geq 0$ $\quad y\leq\frac{P_0}{\rho g}=\frac{1.013\times10^5\text{ Pa}}{\left(10^3\text{ kg/m}^3\right)\left(9.8\text{ m/s}^2\right)}=\boxed{10.3\text{ m}}$

***P14.52** Take points 1 and 2 in the air just inside and outside the window pane.

$$P_1+\frac{1}{2}\rho v_1^2+\rho g y_1=P_2+\frac{1}{2}\rho v_2^2+\rho g y_2$$

$$P_0+0=P_2+\frac{1}{2}\left(1.30\text{ kg/m}^3\right)(11.2\text{ m/s})^2 \qquad P_2=P_0-81.5\text{ Pa}$$

(a) The total force exerted by the air is outward,

$$P_1A-P_2A=P_0A-P_0A+\left(81.5\text{ N/m}^2\right)(4\text{ m})(1.5\text{ m})=\boxed{489\text{ N outward}}$$

(b) $P_1A-P_2A=\frac{1}{2}\rho v_2^2A=\frac{1}{2}\left(1.30\text{ kg/m}^3\right)(22.4\text{ m/s})^2(4\text{ m})(1.5\text{ m})=\boxed{1.96\text{ kN outward}}$

P14.53 In the reservoir, the gauge pressure is $\quad \Delta P=\frac{2.00\text{ N}}{2.50\times10^{-5}\text{ m}^2}=8.00\times10^4\text{ Pa}$

From the equation of continuity: $\quad A_1v_1=A_2v_2$

$\left(2.50\times10^{-5}\text{ m}^2\right)v_1=\left(1.00\times10^{-8}\text{ m}^2\right)v_2 \quad v_1=\left(4.00\times10^{-4}\right)v_2$

Thus, v_1^2 is negligible in comparison to v_2^2.

Then, from Bernoulli's equation: $\quad (P_1-P_2)+\frac{1}{2}\rho v_1^2+\rho g y_1=\frac{1}{2}\rho v_2^2+\rho g y_2$

$$8.00\times10^4\text{ Pa}+0+0=0+\frac{1}{2}\left(1\,000\text{ kg/m}^3\right)v_2^2$$

$$v_2=\sqrt{\frac{2\left(8.00\times10^4\text{ Pa}\right)}{1\,000\text{ kg/m}^3}}=\boxed{12.6\text{ m/s}}$$

Additional Problems

P14.54 Consider the diagram and apply Bernoulli's equation to points A and B, taking $y = 0$ at the level of point B, and recognizing that v_A is approximately zero. This gives:

$$P_A + \frac{1}{2}\rho_w(0)^2 + \rho_w g(h - L\sin\theta)$$
$$= P_B + \frac{1}{2}\rho_w v_B^2 + \rho_w g(0)$$

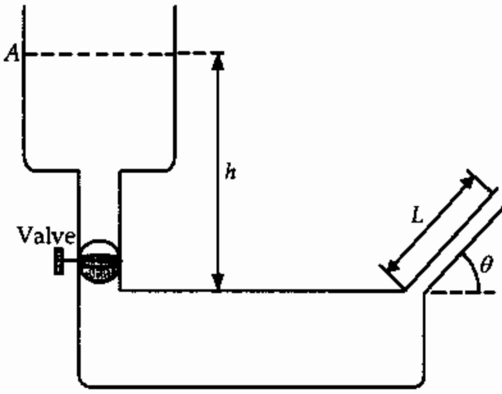


FIG. P14.54

Now, recognize that $P_A = P_B = P_{\text{atmosphere}}$ since both points are open to the atmosphere (neglecting variation of atmospheric pressure with altitude). Thus, we obtain

$$v_B = \sqrt{2g(h - L\sin\theta)} = \sqrt{2(9.80\ \text{m/s}^2)[10.0\ \text{m} - (2.00\ \text{m})\sin 30.0°]}$$
$$v_B = 13.3\ \text{m/s}$$

Now the problem reduces to one of projectile motion with $v_{yi} = v_B \sin 30.0° = 6.64\ \text{m/s}$. Then, $v_{yf}^2 = v_{yi}^2 + 2a(\Delta y)$ gives at the top of the arc (where $y = y_{\max}$ and $v_{yf} = 0$)

$$0 = (6.64\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(y_{\max} - 0)$$

or $y_{\max} = \boxed{2.25\ \text{m (above the level where the water emerges)}}$.

P14.55 When the balloon comes into equilibrium, we must have

$$\sum F_y = B - F_{g,\,\text{balloon}} - F_{g,\,\text{He}} - F_{g,\,\text{string}} = 0$$

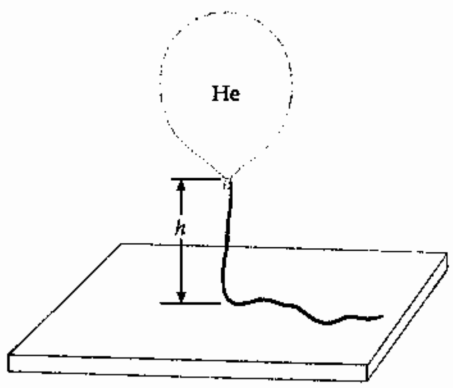


FIG. P14.55

$F_{g,\,\text{string}}$ is the weight of the string above the ground, and B is the buoyant force. Now

$$F_{g,\,\text{balloon}} = m_{\text{balloon}} g$$
$$F_{g,\,\text{He}} = \rho_{\text{He}} V g$$
$$B = \rho_{\text{air}} V g$$

and $F_{g,\,\text{string}} = m_{\text{string}} \frac{h}{L} g$

Therefore, we have

$$\rho_{\text{air}} V g - m_{\text{balloon}} g - \rho_{\text{He}} V g - m_{\text{string}} \frac{h}{L} g = 0$$

or $$h = \frac{(\rho_{\text{air}} - \rho_{\text{He}})V - m_{\text{balloon}}}{m_{\text{string}}} L$$

giving,

$$h = \frac{(1.29 - 0.179)(\text{kg/m}^3)\left(\frac{4\pi(0.400\ \text{m})^3}{3}\right) - 0.250\ \text{kg}}{0.050\,0\ \text{kg}}(2.00\ \text{m}) = \boxed{1.91\ \text{m}}.$$

P14.56 Assume $v_{\text{inside}} \approx 0$

$$P + 0 + 0 = 1\text{ atm} + \frac{1}{2}(1\,000)(30.0)^2 + 1\,000(9.80)(0.500)$$

$$P_{\text{gauge}} = P - 1\text{ atm} = 4.50\times10^5 + 4.90\times10^3 = \boxed{455\text{ kPa}}$$

P14.57 The "balanced" condition is one in which the apparent weight of the body equals the apparent weight of the weights. This condition can be written as:

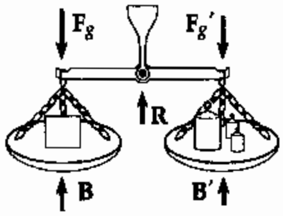


FIG. P14.57

$$F_g - B = F_g' - B'$$

where B and B' are the buoyant forces on the body and weights respectively. The buoyant force experienced by an object of volume V in air equals:

$$\text{Buoyant force} = (\text{Volume of object})\rho_{\text{air}} g$$

so we have $B = V\rho_{\text{air}} g$ and $B' = \left(\frac{F_g'}{\rho g}\right)\rho_{\text{air}} g$.

Therefore, $F_g = F_g' + \left(V - \frac{F_g'}{\rho g}\right)\rho_{\text{air}} g$.

P14.58 The cross-sectional area above water is

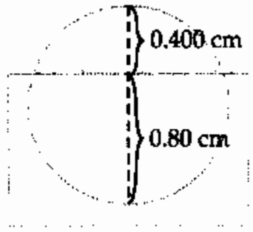


FIG. P14.58

$$\frac{2.46\text{ rad}}{2\pi}\pi(0.600\text{ cm})^2 - (0.200\text{ cm})(0.566\text{ cm}) = 0.330\text{ cm}^2$$

$$A_{\text{all}} = \pi(0.600)^2 = 1.13\text{ cm}^2$$

$$\rho_{\text{water}} g A_{\text{under}} = \rho_{\text{wood}} A_{\text{all}} g$$

$$\rho_{\text{wood}} = \frac{1.13 - 0.330}{1.13} = 0.709\text{ g/cm}^3 = \boxed{709\text{ kg/m}^3}$$

P14.59 At equilibrium, $\sum F_y = 0$: $B - F_{\text{spring}} - F_{g,\text{ He}} - F_{g,\text{ balloon}} = 0$

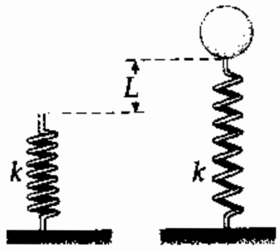


FIG. P14.59

giving $F_{\text{spring}} = kL = B - (m_{\text{He}} + m_{\text{balloon}})g$.

But $B = \text{weight of displaced air} = \rho_{\text{air}} V g$

and $m_{\text{He}} = \rho_{\text{He}} V$.

Therefore, we have: $kL = \rho_{\text{air}} V g - \rho_{\text{He}} V g - m_{\text{balloon}} g$

or $L = \frac{(\rho_{\text{air}} - \rho_{\text{He}})V - m_{\text{balloon}}}{k} g$.

From the data given, $L = \frac{(1.29\text{ kg/m}^3 - 0.180\text{ kg/m}^3)5.00\text{ m}^3 - 2.00\times10^{-3}\text{ kg}}{90.0\text{ N/m}}(9.80\text{ m/s}^2)$.

Thus, this gives $L = \boxed{0.604\text{ m}}$.

P14.60 $P = \rho g h$ $\quad 1.013\times10^5 = 1.29(9.80)h$

$h = \boxed{8.01\text{ km}}$ For Mt. Everest, $29\,300\text{ ft} = 8.88\text{ km}$ $\boxed{\text{Yes}}$

P14.61 The torque is $\tau = \int d\tau = \int r dF$

From the figure $\tau = \int_0^H y[\rho g(H-y)w dy] = \boxed{\frac{1}{6}\rho g w H^3}$

The total force is given as $\frac{1}{2}\rho g w H^2$

If this were applied at a height y_{eff} such that the torque remains unchanged, we have

$$\frac{1}{6}\rho g w H^3 = y_{eff}\left[\frac{1}{2}\rho g w H^2\right] \quad \text{and} \quad y_{eff} = \boxed{\frac{1}{3}H}.$$

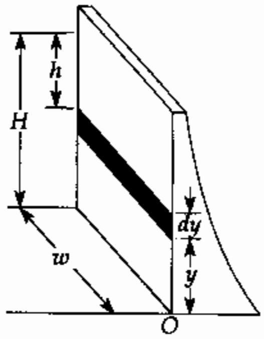


FIG. P14.61

P14.62 (a) The pressure on the surface of the two hemispheres is constant at all points, and the force on each element of surface area is directed along the radius of the hemispheres. The applied force along the axis must balance the force on the "effective" area, which is the projection of the actual surface onto a plane perpendicular to the x axis,

$$A = \pi R^2$$

Therefore, $F = \boxed{(P_0 - P)\pi R^2}$

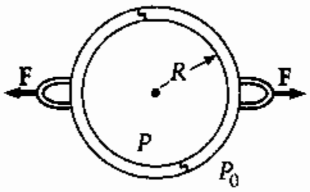


FIG. P14.62

(b) For the values given $F = (P_0 - 0.100P_0)\left[\pi(0.300\text{ m})^2\right] = 0.254P_0 = \boxed{2.58\times10^4\text{ N}}$

P14.63 Looking first at the top scale and the iron block, we have:

$$T_1 + B = F_{g,\text{ iron}}$$

where T_1 is the tension in the spring scale, B is the buoyant force, and $F_{g,\text{ iron}}$ is the weight of the iron block. Now if m_{iron} is the mass of the iron block, we have

$$m_{\text{iron}} = \rho_{\text{iron}}V \qquad \text{so} \qquad V = \frac{m_{\text{iron}}}{\rho_{\text{iron}}} = V_{\text{displaced oil}}$$

Then, $B = \rho_{\text{oil}}V_{\text{iron}}g$

Therefore, $T_1 = F_{g,\text{ iron}} - \rho_{\text{oil}}V_{\text{iron}}g = m_{\text{iron}}g - \rho_{\text{oil}}\frac{m_{\text{iron}}}{\rho_{\text{iron}}}g$

or $T_1 = \left(1 - \frac{\rho_{\text{oil}}}{\rho_{\text{iron}}}\right)m_{\text{iron}}g = \left(1 - \frac{916}{7\,860}\right)(2.00)(9.80) = \boxed{17.3\text{ N}}$

Next, we look at the bottom scale which reads T_2 (i.e., exerts an upward force T_2 on the system). Consider the external vertical forces acting on the beaker–oil–iron combination.

$\sum F_y = 0$ gives

$$T_1 + T_2 - F_{g,\text{ beaker}} - F_{g,\text{ oil}} - F_{g,\text{ iron}} = 0$$

or $T_2 = (m_{\text{beaker}} + m_{\text{oil}} + m_{\text{iron}})g - T_1 = (5.00\text{ kg})(9.80\text{ m/s}^2) - 17.3\text{ N}$

Thus, $T_2 = \boxed{31.7\text{ N}}$ is the lower scale reading.

P14.64 Looking at the top scale and the iron block:

$T_1 + B = F_{g,\text{Fe}}$ where $B = \rho_0 V_{\text{Fe}} g = \rho_0 \left(\frac{m_{\text{Fe}}}{\rho_{\text{Fe}}}\right) g$

is the buoyant force exerted on the iron block by the oil.

Thus, $T_1 = F_{g,\text{Fe}} - B = m_{\text{Fe}} g - \rho_0 \left(\frac{m_{\text{Fe}}}{\rho_{\text{Fe}}}\right) g$

or $T_1 = \boxed{\left(1 - \frac{\rho_0}{\rho_{\text{Fe}}}\right) m_{\text{Fe}} g}$ is the reading on the top scale.

Now, consider the bottom scale, which exerts an upward force of T_2 on the beaker–oil–iron combination.

$\sum F_y = 0$: $T_1 + T_2 - F_{g,\text{beaker}} - F_{g,\text{oil}} - F_{g,\text{Fe}} = 0$

$$T_2 = F_{g,\text{beaker}} + F_{g,\text{oil}} + F_{g,\text{Fe}} - T_1 = (m_b + m_0 + m_{\text{Fe}})g - \left(1 - \frac{\rho_0}{\rho_{\text{Fe}}}\right) m_{\text{Fe}} g$$

or $T_2 = \boxed{\left[m_b + m_0 + \left(\frac{\rho_0}{\rho_{\text{Fe}}}\right) m_{\text{Fe}}\right] g}$ is the reading on the bottom scale.

P14.65 $\rho_{\text{Cu}} V = 3.083 \text{ g}$

$\rho_{\text{Zn}}(xV) + \rho_{\text{Cu}}(1-x)V = 2.517 \text{ g}$

$\rho_{\text{Zn}}\left(\frac{3.083}{\rho_{\text{Cu}}}\right)x + 3.083(1-x) = 2.517$

$\left(1 - \frac{7.133}{8.960}\right)x = \left(1 - \frac{2.517}{3.083}\right)$

$x = 0.900\,4$

$\%\text{Zn} = \boxed{90.04\%}$

P14.66 (a) From $\sum F = ma$

$$B - m_{\text{shell}} g - m_{\text{He}} g = m_{\text{total}} a = (m_{\text{shell}} + m_{\text{He}})a \qquad (1)$$

Where $B = \rho_{\text{water}} V g$ and $m_{\text{He}} = \rho_{\text{He}} V$

Also, $V = \frac{4}{3}\pi r^3 = \frac{\pi d^3}{6}$

Putting these into equation (1) above,

$$\left(m_{\text{shell}} + \rho_{\text{He}} \frac{\pi d^3}{6}\right) a = \left(\rho_{\text{water}} \frac{\pi d^3}{6} - m_{\text{shell}} - \rho_{\text{He}} \frac{\pi d^3}{6}\right) g$$

which gives

$$a = \frac{(\rho_{\text{water}} - \rho_{\text{He}})\frac{\pi d^3}{6} - m_{\text{shell}}}{m_{\text{shell}} + \rho_{\text{He}}\frac{\pi d^3}{6}} g$$

or

$$a = \frac{(1\,000 - 0.180)(\text{kg/m}^3)\frac{\pi(0.200 \text{ m})^3}{6} - 4.00 \text{ kg}}{4.00 \text{ kg} + (0.180 \text{ kg/m}^3)\frac{\pi(0.200 \text{ m})^3}{6}} 9.80 \text{ m/s}^2 = \boxed{0.461 \text{ m/s}^2}$$

(b) $$t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(h-d)}{a}} = \sqrt{\frac{2(4.00 \text{ m} - 0.200 \text{ m})}{0.461 \text{ m/s}^2}} = \boxed{4.06 \text{ s}}$$

P14.67 Inertia of the disk: $I = \frac{1}{2}MR^2 = \frac{1}{2}(10.0\text{ kg})(0.250\text{ m})^2 = 0.312\text{ kg}\cdot\text{m}^2$

Angular acceleration: $\omega_f = \omega_i + \alpha t$

$$\alpha = \left(\frac{0 - 300\text{ rev/min}}{60.0\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = -0.524\text{ rad/s}^2$$

Braking torque: $\sum \tau = I\alpha \Rightarrow -fd = I\alpha$, so $f = \frac{-I\alpha}{d}$

Friction force: $f = \frac{(0.312\text{ kg}\cdot\text{m}^2)(0.524\text{ rad/s}^2)}{0.220\text{ m}} = 0.744\text{ N}$

Normal force: $f = \mu_k n \Rightarrow n = \frac{f}{\mu_k} = \frac{0.744\text{ N}}{0.500} = 1.49\text{ N}$

gauge pressure: $P = \frac{n}{A} = \frac{1.49\text{ N}}{\pi(2.50\times 10^{-2}\text{ m})^2} = \boxed{758\text{ Pa}}$

P14.68 The incremental version of $P - P_0 = \rho g y$ is $\quad dP = -\rho g dy$

We assume that the density of air is proportional to pressure, or $\quad \frac{P}{\rho} = \frac{P_0}{\rho_0}$

Combining these two equations we have $\quad dP = -P\frac{\rho_0}{P_0}g dy$

$$\int_{P_0}^{P}\frac{dP}{P} = -g\frac{\rho_0}{P_0}\int_0^h dy$$

and integrating gives $\quad \ln\left(\frac{P}{P_0}\right) = -\frac{\rho_0 g h}{P_0}$

so where $\alpha = \frac{\rho_0 g}{P_0}$, $\quad P = P_0 e^{-\alpha h}$

P14.69 Energy for the fluid-Earth system is conserved.

$(K+U)_i + \Delta E_{\text{mech}} = (K+U)_f$: $\quad 0 + \frac{mgL}{2} + 0 = \frac{1}{2}mv^2 + 0$

$$v = \sqrt{gL} = \sqrt{2.00\text{ m}(9.8\text{ m/s}^2)} = \boxed{4.43\text{ m/s}}$$

P14.70 Let s stand for the edge of the cube, h for the depth of immersion, ρ_{ice} stand for the density of the ice, ρ_w stand for density of water, and ρ_a stand for density of the alcohol.

(a) According to Archimedes's principle, at equilibrium we have

$$\rho_{\text{ice}} g s^3 = \rho_w g h s^2 \Rightarrow h = s\frac{\rho_{\text{ice}}}{\rho_w}$$

With $\rho_{\text{ice}} = 0.917 \times 10^3 \text{ kg/m}^3$

$\rho_w = 1.00 \times 10^3 \text{ kg/m}^3$

and $s = 20.0 \text{ mm}$

we get $h = 20.0(0.917) = 18.34 \text{ mm} \approx \boxed{18.3 \text{ mm}}$

(b) We assume that the top of the cube is still above the alcohol surface. Letting h_a stand for the thickness of the alcohol layer, we have

$$\rho_a g s^2 h_a + \rho_w g s^2 h_w = \rho_{\text{ice}} g s^3 \qquad \text{so} \qquad h_w = \left(\frac{\rho_{\text{ice}}}{\rho_w}\right)s - \left(\frac{\rho_a}{\rho_w}\right)h_a$$

With $\rho_a = 0.806 \times 10^3 \text{ kg/m}^3$

and $h_a = 5.00 \text{ mm}$

we obtain $h_w = 18.34 - 0.806(5.00) = 14.31 \text{ mm} \approx \boxed{14.3 \text{ mm}}$

(c) Here $h'_w = s - h'_a$, so Archimedes's principle gives

$$\rho_a g s^2 h'_a + \rho_w g s^2 (s - h'_a) = \rho_{\text{ice}} g s^3 \Rightarrow \rho_a h'_a + \rho_w (s - h'_a) = \rho_{\text{ice}} s$$

$$h'_a = s\frac{(\rho_w - \rho_{\text{ice}})}{(\rho_w - \rho_a)} = 20.0\frac{(1.000 - 0.917)}{(1.000 - 0.806)} = 8.557 \approx \boxed{8.56 \text{ mm}}$$

P14.71 **Note**: Variation of atmospheric pressure with altitude is included in this solution. Because of the small distances involved, this effect is unimportant in the final answers.

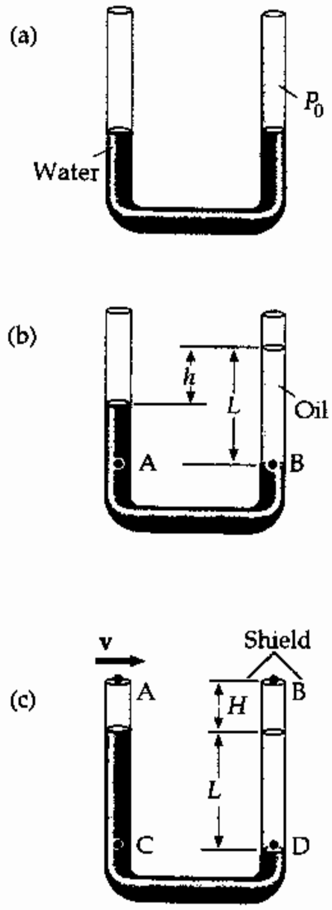


FIG. P14.71

(a) Consider the pressure at points A and B in part (b) of the figure:

Using the left tube: $P_A = P_{atm} + \rho_a gh + \rho_w g(L-h)$ where the second term is due to the variation of air pressure with altitude.

Using the right tube: $P_B = P_{atm} + \rho_0 gL$

But Pascal's principle says that $P_A = P_B$.

Therefore, $P_{atm} + \rho_0 gL = P_{atm} + \rho_a gh + \rho_w g(L-h)$

or $(\rho_w - \rho_a)h = (\rho_w - \rho_0)L$, giving

$$h = \left(\frac{\rho_w - \rho_0}{\rho_w - \rho_a}\right)L = \left(\frac{1\,000 - 750}{1\,000 - 1.29}\right)5.00\text{ cm} = \boxed{1.25\text{ cm}}$$

(b) Consider part (c) of the diagram showing the situation when the air flow over the left tube equalizes the fluid levels in the two tubes. First, apply Bernoulli's equation to points A and B ($y_A = y_B$, $v_A = v$, and $v_B = 0$)

This gives: $P_A + \frac{1}{2}\rho_a v^2 + \rho_a g y_A = P_B + \frac{1}{2}\rho_a (0)^2 + \rho_a g y_B$

and since $y_A = y_B$, this reduces to: $P_B - P_A = \frac{1}{2}\rho_a v^2$ (1)

Now consider points C and D, both at the level of the oil–water interface in the right tube. Using the variation of pressure with depth in static fluids, we have:

$P_C = P_A + \rho_a gH + \rho_w gL$ and $P_D = P_B + \rho_a gH + \rho_0 gL$

But Pascal's principle says that $P_C = P_D$. Equating these two gives:

$P_B + \rho_a gH + \rho_0 gL = P_A + \rho_a gH + \rho_w gL$ or $P_B - P_A = (\rho_w - \rho_0)gL$ (2)

Substitute equation (1) for $P_B - P_A$ into (2) to obtain $\frac{1}{2}\rho_a v^2 = (\rho_w - \rho_0)gL$

or $$v = \sqrt{\frac{2gL(\rho_w - \rho_0)}{\rho_a}} = \sqrt{2(9.80\text{ m/s}^2)(0.050\,0\text{ m})\left(\frac{1\,000 - 750}{1.29}\right)}$$

$$v = \boxed{13.8\text{ m/s}}$$

P14.72 (a) The flow rate, Av, as given may be expressed as follows:

$$\frac{25.0\text{ liters}}{30.0\text{ s}}=0.833\text{ liters/s}=833\text{ cm}^3/\text{s}.$$

The area of the faucet tap is $\pi\text{ cm}^2$, so we can find the velocity as

$$v=\frac{\text{flow rate}}{A}=\frac{833\text{ cm}^3/\text{s}}{\pi\text{ cm}^2}=265\text{ cm/s}=\boxed{2.65\text{ m/s}}.$$

(b) We choose point 1 to be in the entrance pipe and point 2 to be at the faucet tap. $A_1v_1=A_2v_2$ gives $v_1=0.295\text{ m/s}$. Bernoulli's equation is:

$$P_1-P_2=\frac{1}{2}\rho\left(v_2^2-v_1^2\right)+\rho g\left(y_2-y_1\right)$$

and gives

$$P_1-P_2=\frac{1}{2}\left(10^3\text{ kg/m}^3\right)\left[(2.65\text{ m/s})^2-(0.295\text{ m/s})^2\right]+\left(10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)(2.00\text{ m})$$

or $P_{\text{gauge}}=P_1-P_2=\boxed{2.31\times10^4\text{ Pa}}$.

P14.73 (a) Since the upward buoyant force is balanced by the weight of the sphere,

$$m_1g=\rho Vg=\rho\left(\frac{4}{3}\pi R^3\right)g.$$

In this problem, $\rho=0.789\,45\text{ g/cm}^3$ at 20.0°C, and $R=1.00\text{ cm}$ so we find:

$$m_1=\rho\left(\frac{4}{3}\pi R^3\right)=\left(0.789\,45\text{ g/cm}^3\right)\left[\frac{4}{3}\pi(1.00\text{ cm})^3\right]=\boxed{3.307\text{ g}}.$$

(b) Following the same procedure as in part (a), with $\rho'=0.780\,97\text{ g/cm}^3$ at 30.0°C, we find:

$$m_2=\rho'\left(\frac{4}{3}\pi R^3\right)=\left(0.780\,97\text{ g/cm}^3\right)\left[\frac{4}{3}\pi(1.00\text{ cm})^3\right]=\boxed{3.271\text{ g}}.$$

(c) When the first sphere is resting on the bottom of the tube,

$n+B=F_{g1}=m_1g$, where n is the normal force.

Since $B=\rho'Vg$

$$n=m_1g-\rho'Vg=\left[3.307\text{ g}-\left(0.780\,97\text{ g/cm}^3\right)(1.00\text{ cm})^3\right]980\text{ cm/s}^2$$

$$n=34.8\text{ g}\cdot\text{cm/s}^2=\boxed{3.48\times10^{-4}\text{ N}}$$

***P14.74** (a) Take point ① at the free water surface in the tank and point ② at the bottom end of the tube:

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$
$$P_0 + \rho g d + 0 = P_0 + 0 + \frac{1}{2}\rho v_2^2$$
$$v_2 = \sqrt{2gd}$$

The volume flow rate is $\frac{V}{t} = \frac{Ah}{t} = v_2 A'$. Then $t = \frac{Ah}{v_2 A'} = \frac{Ah}{A'\sqrt{2gd}}$.

(b) $t = \dfrac{(0.5\text{ m})^2 0.5\text{ m}}{2\times 10^{-4}\text{ m}^2\sqrt{2(9.8\text{ m/s}^2)10\text{ m}}} = \boxed{44.6\text{ s}}$

***P14.75** (a) For diverging stream lines that pass just above and just below the hydrofoil we have

$$P_t + \rho g y_t + \frac{1}{2}\rho v_t^2 = P_b + \rho g y_b + \frac{1}{2}\rho v_b^2.$$

Ignoring the buoyant force means taking $y_t \approx y_b$

$$P_t + \frac{1}{2}\rho (n v_b)^2 = P_b + \frac{1}{2}\rho v_b^2$$
$$P_b - P_t = \frac{1}{2}\rho v_b^2 (n^2 - 1)$$

The lift force is $(P_b - P_t)A = \frac{1}{2}\rho v_b^2 (n^2 - 1)A$.

(b) For liftoff,

$$\frac{1}{2}\rho v_b^2 (n^2 - 1)A = Mg$$
$$v_b = \left(\frac{2Mg}{\rho (n^2 - 1)A}\right)^{1/2}$$

The speed of the boat relative to the shore must be nearly equal to this speed of the water below the hydrofoil relative to the boat.

(c) $v^2(n^2 - 1)A\rho = 2Mg$

$A = \dfrac{2(800\text{ kg})9.8\text{ m/s}^2}{(9.5\text{ m/s})^2(1.05^2 - 1)1\,000\text{ kg/m}^3} = \boxed{1.70\text{ m}^2}$

ANSWERS TO EVEN PROBLEMS

P14.2 $\sim 10^{18}$ kg/m^3; matter is mostly empty space

P14.4 1.92×10^4 N

P14.6 (a) 1.01×10^7 Pa; (b) 7.09×10^5 N outward

P14.8 255 N

P14.10 (a) 65.1 N; (b) 275 N

P14.12 5.88×10^6 N down; 196 kN outward; 588 kN outward

P14.14 (a) 29.4 kN to the right; (b) 16.3 kN·m counterclockwise

P14.16 (a) 10.3 m; (b) zero

P14.18 (a) 20.0 cm; (b) 0.490 cm

P14.20 12.6 cm

P14.22 (a) 444 kg; (b) 480 kg

P14.24 $\dfrac{m}{(\rho_w - \rho_s)h}$

P14.26 (a) see the solution; (b) 25.0 N up; (c) horizontally inward; (d) tension increases; see the solution; (e) 62.5%; (f) 18.7%

P14.28 $\sim 10^4$ balloons of 25-cm diameter

P14.30 (a) 6.70 cm; (b) 5.74 cm

P14.32 (a) 11.6 cm; (b) 0.963 g/cm^3; (c) no; see the solution

P14.34 0.611 kg

P14.36 2.67×10^3 kg

P14.38 12.8 kg/s

P14.40 (a) 27.9 N; (b) 3.32×10^4 kg; (c) 7.26×10^4 Pa

P14.42 (a) see the solution; (b) 616 MW

P14.44 (a) 2.28 N toward Holland; (b) 1.74×10^6 s

P14.46 (a), (b) 28.0 m/s; (c) 2.11 MPa

P14.48 6.80×10^4 Pa

P14.50 347 m/s

P14.52 (a) 489 N outward; (b) 1.96 kN outward

P14.54 2.25 m above the level where the water emerges

P14.56 455 kPa

P14.58 709 kg/m^3

P14.60 8.01 km; yes

P14.62 (a) see the solution; (b) 2.58×10^4 N

P14.64 top scale: $\left(1 - \dfrac{\rho_0}{\rho_{Fe}}\right) m_{Fe} g$;

bottom scale: $\left(m_b + m_0 + \dfrac{\rho_0 m_{Fe}}{\rho_{Fe}}\right) g$

P14.66 (a) 0.461 m/s^2; (b) 4.06 s

P14.68 see the solution

P14.70 (a) 18.3 mm; (b) 14.3 mm; (c) 8.56 mm

P14.72 (a) 2.65 m/s; (b) 2.31×10^4 Pa

P14.74 (a) see the solution; (b) 44.6 s

15

Oscillatory Motion

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q15.1 Neither are examples of simple harmonic motion, although they are both periodic motion. In neither case is the acceleration proportional to the position. Neither motion is so smooth as SHM. The ball's acceleration is very large when it is in contact with the floor, and the student's when the dismissal bell rings.

Q15.2 You can take $\phi = \pi$, or equally well, $\phi = -\pi$. At $t = 0$, the particle is at its turning point on the negative side of equilibrium, at $x = -A$.

Q15.3 The two will be equal if and only if the position of the particle at time zero is its equilibrium position, which we choose as the origin of coordinates.

Q15.4 (a) In simple harmonic motion, one-half of the time, the velocity is in the same direction as the displacement away from equilibrium.

(b) Velocity and acceleration are in the same direction half the time.

(c) Acceleration is always opposite to the position vector, and never in the same direction.

Q15.5 No. It is necessary to know both the position and velocity at time zero.

Q15.6 The motion will still be simple harmonic motion, but the period of oscillation will be a bit larger. The effective mass of the system in $\omega = \left(\frac{k}{m_{eff}}\right)^{1/2}$ will need to include a certain fraction of the mass of the spring.

Q15.7 We assume that the coils of the spring do not hit one another. The frequency will be higher than f by the factor $\sqrt{2}$. When the spring with two blocks is set into oscillation in space, the coil in the center of the spring does not move. We can imagine clamping the center coil in place without affecting the motion. We can effectively duplicate the motion of each individual block in space by hanging a single block on a half-spring here on Earth. The half-spring with its center coil clamped—or its other half cut off—has twice the spring constant as the original uncut spring, because an applied force of the same size would produce only one-half the extension distance. Thus the oscillation frequency in space is $\left(\frac{1}{2\pi}\right)\left(\frac{2k}{m}\right)^{1/2} = \sqrt{2}f$. The absence of a force required to support the vibrating system in orbital free fall has no effect on the frequency of its vibration.

Q15.8 No; Kinetic, Yes; Potential, No. For constant amplitude, the total energy $\frac{1}{2}kA^2$ stays constant. The kinetic energy $\frac{1}{2}mv^2$ would increase for larger mass if the speed were constant, but here the greater mass causes a decrease in frequency and in the average and maximum speed, so that the kinetic and potential energies at every point are unchanged.

Q15.9 Since the acceleration is not constant in simple harmonic motion, none of the equations in Table 2.2 are valid.

Equation	*Information given by equation*
$x(t) = A\cos(\omega t + \phi)$	position as a function of time
$v(t) = -\omega A\sin(\omega t + \phi)$	velocity as a function of time
$v(x) = \pm\omega\left(A^2 - x^2\right)^{1/2}$	velocity as a function of position
$a(t) = -\omega^2 A\cos(\omega t + \phi)$	acceleration as a function of time
$a(t) = -\omega^2 x(t)$	acceleration as a function of position

The angular frequency ω appears in every equation. It is a good idea to figure out the value of angular frequency early in the solution to a problem about vibration, and to store it in calculator memory.

Q15.10 We have $T_i = \sqrt{\frac{L_i}{g}}$ and $T_f = \sqrt{\frac{L_f}{g}} = \sqrt{\frac{2L_i}{g}} = \sqrt{2}T_i$. The period gets larger by $\sqrt{2}$ times. Changing the mass has no effect on the period of a simple pendulum.

Q15.11 (a) Period decreases. (b) Period increases. (c) No change.

Q15.12 No, the equilibrium position of the pendulum will be shifted (angularly) towards the back of the car. The period of oscillation will increase slightly, since the restoring force (in the reference frame of the accelerating car) is reduced.

Q15.13 The motion will be periodic—that is, it will repeat. The period is nearly constant as the angular amplitude increases through small values; then the period becomes noticeably larger as θ increases farther.

Q15.14 Shorten the pendulum to decrease the period between ticks.

Q15.15 No. If the resistive force is greater than the restoring force of the spring (in particular, if $b^2 > 4mk$), the system will be overdamped and will not oscillate.

Q15.16 Yes. An oscillator with damping can vibrate at resonance with amplitude that remains constant in time. Without damping, the amplitude would increase without limit at resonance.

Q15.17 The phase constant must be π rad.

Q15.18 Higher frequency. When it supports your weight, the center of the diving board flexes down less than the end does when it supports your weight. Thus the stiffness constant describing the center of the board is greater than the stiffness constant describing the end. And then $f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}}$ is greater for you bouncing on the center of the board.

Q15.19 The release of air from one side of the parachute can make the parachute turn in the opposite direction, causing it to release air from the opposite side. This behavior will result in a periodic driving force that can set the parachute into side-to-side oscillation. If the amplitude becomes large enough, the parachute will not supply the needed air resistance to slow the fall of the unfortunate skydiver.

Q15.20 An imperceptibly slight breeze may be blowing past the leaves in tiny puffs. As a leaf twists in the wind, the fibers in its stem provide a restoring torque. If the frequency of the breeze matches the natural frequency of vibration of one particular leaf as a torsional pendulum, that leaf can be driven into a large-amplitude resonance vibration. Note that it is not the *size* of the driving force that sets the leaf into resonance, but the *frequency* of the driving force. If the frequency changes, another leaf will be set into resonant oscillation.

Q15.21 We assume the diameter of the bob is not very small compared to the length of the cord supporting it. As the water leaks out, the center of mass of the bob moves down, increasing the effective length of the pendulum and slightly lowering its frequency. As the last drops of water dribble out, the center of mass of the bob hops back up to the center of the sphere, and the pendulum frequency quickly increases to its original value.

SOLUTIONS TO PROBLEMS

Section 15.1 Motion of an Object Attached to a Spring

P15.1 (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m and then repeat the motion over and over again. Thus, the $\boxed{\text{motion is periodic}}$.

(b) To determine the period, we use: $x = \frac{1}{2}gt^2$.

The time for the ball to hit the ground is $t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(4.00\text{ m})}{9.80\text{ m/s}^2}} = 0.909\text{ s}$

This equals one-half the period, so $T = 2(0.909\text{ s}) = \boxed{1.82\text{ s}}$.

(c) $\boxed{\text{No}}$. The net force acting on the ball is a constant given by $F = -mg$ (except when it is in contact with the ground), which is not in the form of Hooke's law.

Section 15.2 **Mathematical Representation of Simple Harmonic Motion**

P15.2 (a) $x = (5.00\text{ cm})\cos\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $x = (5.00\text{ cm})\cos\left(\frac{\pi}{6}\right) = \boxed{4.33\text{ cm}}$

(b) $v = \frac{dx}{dt} = -(10.0\text{ cm/s})\sin\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $v = \boxed{-5.00\text{ cm/s}}$

(c) $a = \frac{dv}{dt} = -\left(20.0\text{ cm/s}^2\right)\cos\left(2t + \frac{\pi}{6}\right)$ At $t = 0$, $a = \boxed{-17.3\text{ cm/s}^2}$

(d) $A = \boxed{5.00\text{ cm}}$ and $T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \boxed{3.14\text{ s}}$

P15.3 $x = (4.00\text{ m})\cos(3.00\pi t + \pi)$ Compare this with $x = A\cos(\omega t + \phi)$ to find

(a) $\omega = 2\pi f = 3.00\pi$

or $\boxed{f = 1.50\text{ Hz}}$ $T = \frac{1}{f} = \boxed{0.667\text{ s}}$

(b) $A = \boxed{4.00\text{ m}}$

(c) $\phi = \boxed{\pi\text{ rad}}$

(d) $x(t = 0.250\text{ s}) = (4.00\text{ m})\cos(1.75\pi) = \boxed{2.83\text{ m}}$

***P15.4** (a) The spring constant of this spring is

$$k = \frac{F}{x} = \frac{0.45\text{ kg } 9.8\text{ m/s}^2}{0.35\text{ m}} = 12.6\text{ N/m}$$

we take the x-axis pointing downward, so $\phi = 0$

$$x = A\cos\omega t = 18.0\text{ cm}\cos\sqrt{\frac{12.6\text{ kg}}{0.45\text{ kg}\cdot\text{s}^2}}84.4\text{ s} = 18.0\text{ cm}\cos 446.6\text{ rad} = \boxed{15.8\text{ cm}}$$

(d) Now $446.6\text{ rad} = 71\times 2\pi + 0.497\text{ rad}$. In each cycle the object moves $4(18) = 72\text{ cm}$, so it has moved $71(72\text{ cm}) + (18 - 15.8)\text{ cm} = \boxed{51.1\text{ m}}$.

(b) By the same steps, $k = \frac{0.44\text{ kg } 9.8\text{ m/s}^2}{0.355\text{ m}} = 12.1\text{ N/m}$

$$x = A\cos\sqrt{\frac{k}{m}}t = 18.0\text{ cm}\cos\sqrt{\frac{12.1}{0.44}}84.4 = 18.0\text{ cm }\cos 443.5\text{ rad} = \boxed{-15.9\text{ cm}}$$

(e) $443.5\text{ rad} = 70(2\pi) + 3.62\text{ rad}$

$$\text{Distance moved } = 70(72\text{ cm}) + 18 + 15.9\text{ cm} = \boxed{50.7\text{ m}}$$

(c) The answers to (d) and (e) are not very different given the difference in the data about the two vibrating systems. But when we ask about details of the future, the imprecision in our knowledge about the present makes it impossible to make precise predictions. The two oscillations start out in phase but get completely out of phase.

P15.5 (a) At $t=0$, $x=0$ and v is positive (to the right). Therefore, this situation corresponds to $x = A\sin\omega t$

and $\qquad v = v_i\cos\omega t$

Since $f = 1.50$ Hz, $\qquad \omega = 2\pi f = 3.00\pi$

Also, $A = 2.00$ cm, so that $\qquad \boxed{x = (2.00\text{ cm})\sin 3.00\pi t}$

(b) $v_{\max} = v_i = A\omega = 2.00(3.00\pi) = 6.00\pi\text{ cm/s} = \boxed{18.8\text{ cm/s}}$

The particle has this speed at $t=0$ and next at $\qquad t = \dfrac{T}{2} = \boxed{\dfrac{1}{3}\text{ s}}$

(c) $a_{\max} = A\omega^2 = 2.00(3.00\pi)^2 = 18.0\pi^2\text{ cm/s}^2 = \boxed{178\text{ cm/s}^2}$

This positive value of acceleration first occurs at $\qquad t = \dfrac{3}{4}T = \boxed{0.500\text{ s}}$

(d) Since $T = \dfrac{2}{3}$ s and $A = 2.00$ cm, the particle will travel 8.00 cm in this time.

Hence, in 1.00 s$\left(=\dfrac{3}{2}T\right)$, the particle will travel $\qquad 8.00\text{ cm} + 4.00\text{ cm} = \boxed{12.0\text{ cm}}$.

P15.6 The proposed solution $\qquad x(t) = x_i\cos\omega t + \left(\dfrac{v_i}{\omega}\right)\sin\omega t$

implies velocity $\qquad v = \dfrac{dx}{dt} = -x_i\omega\sin\omega t + v_i\cos\omega t$

and acceleration $\qquad a = \dfrac{dv}{dt} = -x_i\omega^2\cos\omega t - v_i\omega\sin\omega t = -\omega^2\left(x_i\cos\omega t + \left(\dfrac{v_i}{\omega}\right)\sin\omega t\right) = -\omega^2 x$

(a) The acceleration being a negative constant times position means we do have SHM, and its angular frequency is ω. At $t=0$ the equations reduce to $x = x_i$ and $v = v_i$ so they satisfy all the requirements.

(b) $v^2 - ax = \left(-x_i\omega\sin\omega t + v_i\cos\omega t\right)^2 - \left(-x_i\omega^2\cos\omega t - v_i\sin\omega t\right)\left(x_i\cos\omega t + \left(\dfrac{v_i}{\omega}\right)\sin\omega t\right)$

$$v^2 - ax = x_i^2\omega^2\sin^2\omega t - 2x_iv_i\omega\sin\omega t\cos\omega t + v_i^2\cos^2\omega t$$
$$+x_i^2\omega^2\cos^2\omega t + x_iv_i\omega\cos\omega t\sin\omega t + x_iv_i\omega\sin\omega t\cos\omega t + v_i^2\sin^2\omega t = x_i^2\omega^2 + v_i^2$$

So this expression is constant in time. On one hand, it must keep its original value $v_i^2 - a_ix_i$. On the other hand, if we evaluate it at a turning point where $v=0$ and $x=A$, it is $A^2\omega^2 + 0^2 = A^2\omega^2$. Thus it is proved.

P15.7 (a) $T = \dfrac{12.0\text{ s}}{5} = \boxed{2.40\text{ s}}$

(b) $f = \dfrac{1}{T} = \dfrac{1}{2.40} = \boxed{0.417\text{ Hz}}$

(c) $\omega = 2\pi f = 2\pi(0.417) = \boxed{2.62\text{ rad/s}}$

***P15.8** The mass of the cube is

$$m = \rho V = \left(2.7\times10^{3}\ \text{kg/m}^{3}\right)(0.015\ \text{m})^{3} = 9.11\times10^{-3}\ \text{kg}$$

The spring constant of the strip of steel is

$$k = \frac{F}{x} = \frac{14.3\ \text{N}}{0.0275\ \text{m}} = 52.0\ \text{N/m}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{52\ \text{kg}}{\text{s}^2 9.11\times10^{-3}\ \text{kg}}} = \boxed{12.0\ \text{Hz}}$$

P15.9 $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ or $T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$

Solving for k, $k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(7.00\ \text{kg})}{(2.60\ \text{s})^2} = \boxed{40.9\ \text{N/m}}$.

***P15.10** $x = A\cos\omega t$ $A = 0.05$ m $v = -A\omega\sin\omega t$ $a = -A\omega^2\cos\omega t$

If $f = 3\,600$ rev/min $= 60$ Hz, then $\omega = 120\pi\ \text{s}^{-1}$

$v_{max} = 0.05(120\pi)\ \text{m/s} = \boxed{18.8\ \text{m/s}}$ $a_{max} = 0.05(120\pi)^2\ \text{m/s}^2 = \boxed{7.11\ \text{km/s}^2}$

P15.11 (a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{8.00\ \text{N/m}}{0.500\ \text{kg}}} = 4.00\ \text{s}^{-1}$ so position is given by $x = 10.0\sin(4.00t)$ cm.

From this we find that $v = 40.0\cos(4.00t)$ cm/s $v_{max} = \boxed{40.0\ \text{cm/s}}$

$a = -160\sin(4.00t)\ \text{cm/s}^2$ $a_{max} = \boxed{160\ \text{cm/s}^2}$.

(b) $t = \left(\frac{1}{4.00}\right)\sin^{-1}\left(\frac{x}{10.0}\right)$ and when $x = 6.00$ cm, $t = 0.161$ s.

We find $v = 40.0\cos[4.00(0.161)] = \boxed{32.0\ \text{cm/s}}$

$a = -160\sin[4.00(0.161)] = \boxed{-96.0\ \text{cm/s}^2}$.

(c) Using $t = \left(\frac{1}{4.00}\right)\sin^{-1}\left(\frac{x}{10.0}\right)$

when $x = 0$, $t = 0$ and when $x = 8.00$ cm, $t = 0.232$ s.

Therefore, $\Delta t = \boxed{0.232\ \text{s}}$.

P15.12 $m = 1.00$ kg, $k = 25.0$ N/m, and $A = 3.00$ cm. At $t = 0$, $x = -3.00$ cm

(a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{25.0}{1.00}} = 5.00$ rad/s

so that, $T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00} = \boxed{1.26 \text{ s}}$

(b) $v_{max} = A\omega = 3.00 \times 10^{-2}$ m$(5.00$ rad/s$) = \boxed{0.150 \text{ m/s}}$

$a_{max} = A\omega^2 = 3.00 \times 10^{-2}$ m$(5.00$ rad/s$)^2 = \boxed{0.750 \text{ m/s}^2}$

(c) Because $x = -3.00$ cm and $v = 0$ at $t = 0$, the required solution is $x = -A\cos\omega t$

or $\boxed{x = -3.00\cos(5.00t) \text{ cm}}$

$$v = \frac{dx}{dt} = \boxed{15.0\sin(5.00t) \text{ cm/s}}$$

$$a = \frac{dv}{dt} = \boxed{75.0\cos(5.00t) \text{ cm/s}^2}$$

P15.13 The 0.500 s must elapse between one turning point and the other. Thus the period is 1.00 s.

$$\omega = \frac{2\pi}{T} = 6.28/\text{s}$$

and $v_{max} = \omega A = (6.28/\text{s})(0.100 \text{ m}) = \boxed{0.628 \text{ m/s}}$.

P15.14 (a) $v_{max} = \omega A$

$$A = \frac{v_{max}}{\omega} = \boxed{\frac{v}{\omega}}$$

(b) $x = -A\sin\omega t = \boxed{-\left(\frac{v}{\omega}\right)\sin\omega t}$

Section 15.3 Energy of the Simple Harmonic Oscillator

P15.15 (a) Energy is conserved for the block-spring system between the maximum-displacement and the half-maximum points:

$(K+U)_i = (K+U)_f$ $\qquad 0 + \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$

$$\frac{1}{2}(6.50 \text{ N/m})(0.100 \text{ m})^2 = \frac{1}{2}m(0.300 \text{ m/s})^2 + \frac{1}{2}(6.50 \text{ N/m})(5.00 \times 10^{-2} \text{ m})^2$$

$32.5 \text{ mJ} = \frac{1}{2}m(0.300 \text{ m/s})^2 + 8.12 \text{ mJ}$ $\qquad m = \frac{2(24.4 \text{ mJ})}{9.0 \times 10^{-2} \text{ m}^2/\text{s}^2} = \boxed{0.542 \text{ kg}}$

(b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{6.50 \text{ N/m}}{0.542 \text{ kg}}} = 3.46$ rad/s $\qquad \therefore T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{3.46 \text{ rad/s}} = \boxed{1.81 \text{ s}}$

(c) $a_{max} = A\omega^2 = 0.100$ m$(3.46$ rad/s$)^2 = \boxed{1.20 \text{ m/s}^2}$

P15.16 $m = 200$ g, $T = 0.250$ s, $E = 2.00$ J; $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{0.250} = 25.1$ rad/s

(a) $k = m\omega^2 = 0.200\text{ kg}(25.1\text{ rad/s})^2 = \boxed{126\text{ N/m}}$

(b) $E = \dfrac{kA^2}{2} \Rightarrow A = \sqrt{\dfrac{2E}{k}} = \sqrt{\dfrac{2(2.00)}{126}} = \boxed{0.178\text{ m}}$

P15.17 Choose the car with its shock-absorbing bumper as the system; by conservation of energy,

$\dfrac{1}{2}mv^2 = \dfrac{1}{2}kx^2$: $\quad v = x\sqrt{\dfrac{k}{m}} = \left(3.16\times10^{-2}\text{ m}\right)\sqrt{\dfrac{5.00\times10^6}{10^3}} = \boxed{2.23\text{ m/s}}$

P15.18 (a) $E = \dfrac{kA^2}{2} = \dfrac{250\text{ N/m}\left(3.50\times10^{-2}\text{ m}\right)^2}{2} = \boxed{0.153\text{ J}}$

(b) $v_{\max} = A\omega$ where $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{250}{0.500}} = 22.4\text{ s}^{-1}$ $\quad v_{\max} = \boxed{0.784\text{ m/s}}$

(c) $a_{\max} = A\omega^2 = 3.50\times10^{-2}\text{ m}\left(22.4\text{ s}^{-1}\right)^2 = \boxed{17.5\text{ m/s}^2}$

P15.19 (a) $E = \dfrac{1}{2}kA^2 = \dfrac{1}{2}(35.0\text{ N/m})\left(4.00\times10^{-2}\text{ m}\right)^2 = \boxed{28.0\text{ mJ}}$

(b) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{\dfrac{k}{m}}\sqrt{A^2 - x^2}$

$|v| = \sqrt{\dfrac{35.0}{50.0\times10^{-3}}}\sqrt{\left(4.00\times10^{-2}\right)^2 - \left(1.00\times10^{-2}\right)^2} = \boxed{1.02\text{ m/s}}$

(c) $\dfrac{1}{2}mv^2 = \dfrac{1}{2}kA^2 - \dfrac{1}{2}kx^2 = \dfrac{1}{2}(35.0)\left[\left(4.00\times10^{-2}\right)^2 - \left(3.00\times10^{-2}\right)^2\right] = \boxed{12.2\text{ mJ}}$

(d) $\dfrac{1}{2}kx^2 = E - \dfrac{1}{2}mv^2 = \boxed{15.8\text{ mJ}}$

P15.20 (a) $k = \dfrac{|F|}{x} = \dfrac{20.0\text{ N}}{0.200\text{ m}} = \boxed{100\text{ N/m}}$

(b) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{50.0}$ rad/s so $\quad f = \dfrac{\omega}{2\pi} = \boxed{1.13\text{ Hz}}$

(c) $v_{\max} = \omega A = \sqrt{50.0}(0.200) = \boxed{1.41\text{ m/s}}$ at $x = 0$

(d) $a_{\max} = \omega^2 A = 50.0(0.200) = \boxed{10.0\text{ m/s}^2}$ at $x = \pm A$

(e) $E = \dfrac{1}{2}kA^2 = \dfrac{1}{2}(100)(0.200)^2 = \boxed{2.00\text{ J}}$

(f) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{50.0}\sqrt{\dfrac{8}{9}(0.200)^2} = \boxed{1.33\text{ m/s}}$

(g) $|a| = \omega^2 x = 50.0\left(\dfrac{0.200}{3}\right) = \boxed{3.33\text{ m/s}^2}$

P15.21 (a) $E = \frac{1}{2}kA^2$, so if $A' = 2A$, $E' = \frac{1}{2}k(A')^2 = \frac{1}{2}k(2A)^2 = 4E$

Therefore $\boxed{E \text{ increases by factor of 4}}$.

(b) $v_{\max} = \sqrt{\frac{k}{m}}A$, so if A is doubled, $\boxed{v_{\max} \text{ is doubled}}$.

(c) $a_{\max} = \frac{k}{m}A$, so if A is doubled, $\boxed{a_{\max} \text{ also doubles}}$.

(d) $T = 2\pi\sqrt{\frac{m}{k}}$ is independent of A, so $\boxed{\text{the period is unchanged}}$.

***P15.22** (a)
$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$-11 \text{ m} = 0 + 0 + \frac{1}{2}\left(-9.8 \text{ m/s}^2\right)t^2$$
$$t = \sqrt{\frac{22 \text{ m}\cdot\text{s}^2}{9.8 \text{ m}}} = \boxed{1.50 \text{ s}}$$

(b) Take the initial point where she steps off the bridge and the final point at the bottom of her motion.

$$\left(K + U_g + U_s\right)_i = \left(K + U_g + U_s\right)_f$$
$$0 + mgy + 0 = 0 + 0 + \frac{1}{2}kx^2$$
$$65 \text{ kg } 9.8 \text{ m/s}^2 \ 36 \text{ m} = \frac{1}{2}k(25 \text{ m})^2$$
$$k = \boxed{73.4 \text{ N/m}}$$

(c) The spring extension at equilibrium is $x = \frac{F}{k} = \frac{65 \text{ kg } 9.8 \text{ m/s}^2}{73.4 \text{ N/m}} = 8.68 \text{ m}$, so this point is $11 + 8.68 \text{ m} = \boxed{19.7 \text{ m below the bridge}}$ and the amplitude of her oscillation is $36 - 19.7 = 16.3 \text{ m}$.

(d) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{73.4 \text{ N/m}}{65 \text{ kg}}} = \boxed{1.06 \text{ rad/s}}$

(e) Take the phase as zero at maximum downward extension. We find what the phase was 25 m higher when $x = -8.68$ m:

In $x = A\cos\omega t$, $16.3 \text{ m} = 16.3 \text{ m}\cos 0$

$-8.68 \text{ m} = 16.3 \text{ m}\cos\left(1.06 \ \frac{t}{\text{s}}\right)$ $1.06\frac{t}{\text{s}} = -122° = -2.13 \text{ rad}$

$t = -2.01$ s

Then $\boxed{+2.01 \text{ s}}$ is the time over which the spring stretches.

(f) total time $= 1.50 \text{ s} + 2.01 \text{ s} = \boxed{3.50 \text{ s}}$

P15.23 Model the oscillator as a block-spring system.

From energy considerations, $v^2 + \omega^2 x^2 = \omega^2 A^2$

$v_{max} = \omega A$ and $v = \frac{\omega A}{2}$ so $\left(\frac{\omega A}{2}\right)^2 + \omega^2 x^2 = \omega^2 A^2$

From this we find $x^2 = \frac{3}{4}A^2$ and $x = \frac{\sqrt{3}}{2}A = \boxed{\pm 2.60 \text{ cm}}$ where $A = 3.00$ cm

P15.24 The potential energy is

$$U_s = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t).$$

The rate of change of potential energy is

$$\frac{dU_s}{dt} = \frac{1}{2}kA^2 2\cos(\omega t)\left[-\omega\sin(\omega t)\right] = -\frac{1}{2}kA^2\omega\sin 2\omega t.$$

(a) This rate of change is maximal and negative at

$$2\omega t = \frac{\pi}{2},\ 2\omega t = 2\pi + \frac{\pi}{2}, \text{ or in general, } 2\omega t = 2n\pi + \frac{\pi}{2} \text{ for integer } n.$$

Then, $t = \frac{\pi}{4\omega}(4n+1) = \frac{\pi(4n+1)}{4\left(3.60 \text{ s}^{-1}\right)}$

For $n = 0$, this gives $t = \boxed{0.218 \text{ s}}$ while $n = 1$ gives $t = \boxed{1.09 \text{ s}}$.

All other values of n yield times outside the specified range.

(b) $$\left|\frac{dU_s}{dt}\right|_{max} = \frac{1}{2}kA^2\omega = \frac{1}{2}(3.24 \text{ N/m})\left(5.00\times10^{-2} \text{ m}\right)^2\left(3.60 \text{ s}^{-1}\right) = \boxed{14.6 \text{ mW}}$$

Section 15.4 **Comparing Simple Harmonic Motion with Uniform Circular Motion**

P15.25 (a) The motion is simple harmonic because the tire is rotating with constant velocity and you are looking at the motion of the bump projected in a plane perpendicular to the tire.

(b) Since the car is moving with speed $v = 3.00$ m/s, and its radius is 0.300 m, we have:

$$\omega = \frac{3.00 \text{ m/s}}{0.300 \text{ m}} = 10.0 \text{ rad/s}.$$

Therefore, the period of the motion is:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{(10.0 \text{ rad/s})} = \boxed{0.628 \text{ s}}.$$

P15.26 The angle of the crank pin is $\theta = \omega t$. Its x-coordinate is

$$\boxed{x = A\cos\theta = A\cos\omega t}$$

where A is the distance from the center of the wheel to the crank pin. This is of the form $x = A\cos(\omega t + \phi)$, so the yoke and piston rod move with simple harmonic motion.

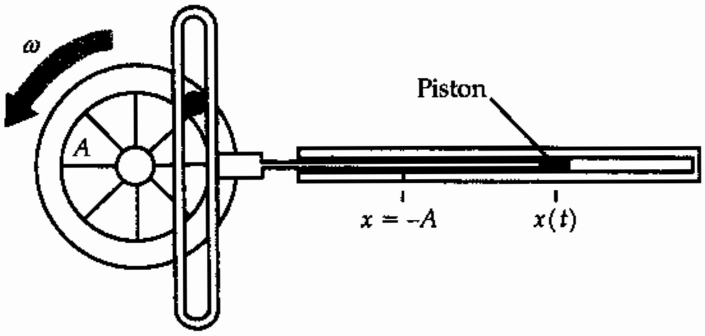


FIG. P15.26

Section 15.5 The Pendulum

P15.27 (a) $T = 2\pi\sqrt{\dfrac{L}{g}}$

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80\ \text{m/s}^2)(12.0\ \text{s})^2}{4\pi^2} = \boxed{35.7\ \text{m}}$$

(b) $$T_{\text{moon}} = 2\pi\sqrt{\frac{L}{g_{\text{moon}}}} = 2\pi\sqrt{\frac{35.7\ \text{m}}{1.67\ \text{m/s}^2}} = \boxed{29.1\ \text{s}}$$

P15.28 The period in Tokyo is $T_T = 2\pi\sqrt{\dfrac{L_T}{g_T}}$

and the period in Cambridge is $T_C = 2\pi\sqrt{\dfrac{L_C}{g_C}}$

We know $T_T = T_C = 2.00\ \text{s}$

For which, we see $\dfrac{L_T}{g_T} = \dfrac{L_C}{g_C}$

or $\dfrac{g_C}{g_T} = \dfrac{L_C}{L_T} = \dfrac{0.994\,2}{0.992\,7} = \boxed{1.001\,5}$

P15.29 The swinging box is a physical pendulum with period $T = 2\pi\sqrt{\dfrac{I}{mgd}}$.

The moment of inertia is given approximately by

$$I = \frac{1}{3}mL^2 \text{ (treating the box as a rod suspended from one end).}$$

Then, with $L \approx 1.0\ \text{m}$ and $d \approx \dfrac{L}{2}$,

$$T \approx 2\pi\sqrt{\frac{\frac{1}{3}mL^2}{mg\left(\frac{L}{2}\right)}} = 2\pi\sqrt{\frac{2L}{3g}} = 2\pi\sqrt{\frac{2(1.0\ \text{m})}{3(9.8\ \text{m/s}^2)}} = 1.6\ \text{s or } T \sim \boxed{10^0\ \text{s}}.$$

P15.30 $\omega = \frac{2\pi}{T}$: $\quad T = \frac{2\pi}{\omega} = \frac{2\pi}{4.43} = \boxed{1.42 \text{ s}}$

$\omega = \sqrt{\frac{g}{L}}$: $\quad L = \frac{g}{\omega^2} = \frac{9.80}{(4.43)^2} = \boxed{0.499 \text{ m}}$

P15.31 Using the simple harmonic motion model:

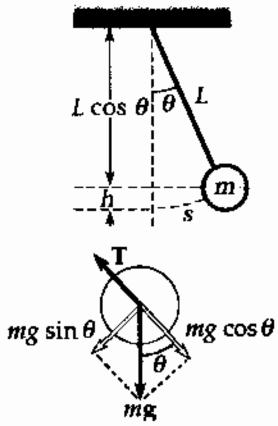


FIG. P15.31

$$A = r\theta = 1 \text{ m } 15° \frac{\pi}{180°} = 0.262 \text{ m}$$

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8 \text{ m/s}^2}{1 \text{ m}}} = 3.13 \text{ rad/s}$$

(a) $v_{\max} = A\omega = 0.262 \text{ m } 3.13/\text{s} = \boxed{0.820 \text{ m/s}}$

(b) $a_{\max} = A\omega^2 = 0.262 \text{ m}(3.13/\text{s})^2 = 2.57 \text{ m/s}^2$

$a_{\tan} = r\alpha \qquad \alpha = \frac{a_{\tan}}{r} = \frac{2.57 \text{ m/s}^2}{1 \text{ m}} = \boxed{2.57 \text{ rad/s}^2}$

(c) $F = ma = 0.25 \text{ kg } 2.57 \text{ m/s}^2 = \boxed{0.641 \text{ N}}$

More precisely,

(a) $mgh = \frac{1}{2}mv^2 \qquad$ and $\qquad h = L(1 - \cos\theta)$

$\therefore v_{\max} = \sqrt{2gL(1-\cos\theta)} = \boxed{0.817 \text{ m/s}}$

(b) $I\alpha = mgL\sin\theta$

$\alpha_{\max} = \frac{mgL\sin\theta}{mL^2} = \frac{g}{L}\sin\theta_i = \boxed{2.54 \text{ rad/s}^2}$

(c) $F_{\max} = mg\sin\theta_i = 0.250(9.80)(\sin 15.0°) = \boxed{0.634 \text{ N}}$

P15.32 (a) The string tension must support the weight of the bob, accelerate it upward, and also provide the restoring force, just as if the elevator were at rest in a gravity field $(9.80 + 5.00) \text{ m/s}^2$

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{5.00 \text{ m}}{14.8 \text{ m/s}^2}}$$

$$T = \boxed{3.65 \text{ s}}$$

(b) $T = 2\pi\sqrt{\frac{5.00 \text{ m}}{(9.80 \text{ m/s}^2 - 5.00 \text{ m/s}^2)}} = \boxed{6.41 \text{ s}}$

(c) $g_{eff} = \sqrt{(9.80 \text{ m/s}^2)^2 + (5.00 \text{ m/s}^2)^2} = 11.0 \text{ m/s}^2$

$T = 2\pi = \sqrt{\frac{5.00 \text{ m}}{11.0 \text{ m/s}^2}} = \boxed{4.24 \text{ s}}$

P15.33 Referring to the sketch we have

$F = -mg\sin\theta$ and $\tan\theta = \frac{x}{R}$

For small displacements, $\tan\theta \approx \sin\theta$

and $F = -\frac{mg}{R}x = -kx$

Since the restoring force is proportional to the displacement from equilibrium, the motion is simple harmonic motion.

Comparing to $F = -m\omega^2 x$ shows $\boxed{\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{R}}}$.

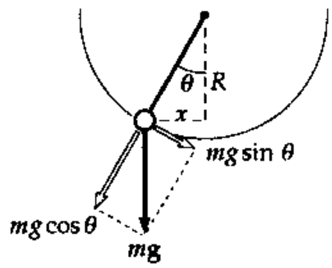


FIG. P15.33

P15.34 (a) $T = \frac{\text{total measured time}}{50}$

The measured periods are:

Length, L (m)	1.000	0.750	0.500
Period, T (s)	1.996	1.732	1.422

(b) $T = 2\pi\sqrt{\frac{L}{g}}$ so $g = \frac{4\pi^2 L}{T^2}$

The calculated values for g are:

Period, T (s)	1.996	1.732	1.422
$g\left(\text{m/s}^2\right)$	9.91	9.87	9.76

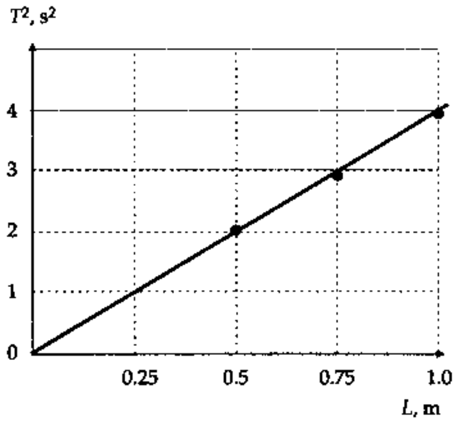


FIG. P15.34

Thus, $g_{\text{ave}} = \boxed{9.85 \text{ m/s}^2}$ this agrees with the accepted value of $g = 9.80 \text{ m/s}^2$ within 0.5%.

(c) From $T^2 = \left(\frac{4\pi^2}{g}\right)L$, the slope of T^2 versus L graph $= \frac{4\pi^2}{g} = 4.01 \text{ s}^2/\text{m}$.

Thus, $g = \frac{4\pi^2}{\text{slope}} = \boxed{9.85 \text{ m/s}^2}$. This is the same as the value in (b).

P15.35 $f = 0.450 \text{ Hz}$, $d = 0.350 \text{ m}$, and $m = 2.20 \text{ kg}$

$T = \frac{1}{f}$;

$T = 2\pi\sqrt{\frac{I}{mgd}}$; $T^2 = \frac{4\pi^2 I}{mgd}$

$$I = T^2\frac{mgd}{4\pi^2} = \left(\frac{1}{f}\right)^2\frac{mgd}{4\pi^2} = \frac{2.20(9.80)(0.350)}{4\pi^2\left(0.450 \text{ s}^{-1}\right)^2} = \boxed{0.944 \text{ kg}\cdot\text{m}^2}$$

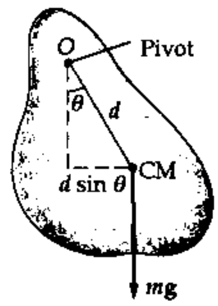


FIG. P15.35

P15.36 (a) The parallel-axis theorem:

$$I = I_{CM} + Md^2 = \frac{1}{12}ML^2 + Md^2 = \frac{1}{12}M(1.00\text{ m})^2 + M(1.00\text{ m})^2$$

$$= M\left(\frac{13}{12}\text{ m}^2\right)$$

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{M(13\text{ m}^2)}{12Mg(1.00\text{ m})}} = 2\pi\sqrt{\frac{13\text{ m}}{12(9.80\text{ m/s}^2)}} = \boxed{2.09\text{ s}}$$

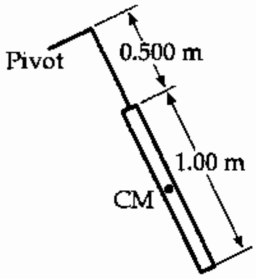


FIG. P15.36

(b) For the simple pendulum

$$T = 2\pi\sqrt{\frac{1.00\text{ m}}{9.80\text{ m/s}^2}} = 2.01\text{ s} \qquad \text{difference} = \frac{2.09\text{ s} - 2.01\text{ s}}{2.01\text{ s}} = \boxed{4.08\%}$$

P15.37 (a) The parallel axis theorem says directly $I = I_{CM} + md^2$

so $T = 2\pi\sqrt{\frac{I}{mgd}} = \boxed{2\pi\sqrt{\frac{(I_{CM} + md^2)}{mgd}}}$

(b) When d is very large $T \to 2\pi\sqrt{\frac{d}{g}}$ gets large.

When d is very small $T \to 2\pi\sqrt{\frac{I_{CM}}{mgd}}$ gets large.

So there must be a minimum, found by

$$\frac{dT}{dd} = 0 = \frac{d}{dd}2\pi\left(I_{CM} + md^2\right)^{1/2}(mgd)^{-1/2}$$

$$= 2\pi\left(I_{CM} + md^2\right)^{1/2}\left(-\frac{1}{2}\right)(mgd)^{-3/2}mg + 2\pi(mgd)^{-1/2}\left(\frac{1}{2}\right)\left(I_{CM} + md^2\right)^{-1/2}2md$$

$$= \frac{-\pi\left(I_{CM} + md^2\right)mg}{\left(I_{CM} + md^2\right)^{1/2}(mgd)^{3/2}} + \frac{2\pi md\,mgd}{\left(I_{CM} + md^2\right)^{1/2}(mgd)^{3/2}} = 0$$

This requires

$$-I_{CM} - md^2 + 2md^2 = 0$$

or $\boxed{I_{CM} = md^2}$.

P15.38 We suppose the stick moves in a horizontal plane. Then,

$$I = \frac{1}{12}mL^2 = \frac{1}{12}(2.00\text{ kg})(1.00\text{ m})^2 = 0.167\text{ kg}\cdot\text{m}^2$$

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

$$\kappa = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2(0.167\text{ kg}\cdot\text{m}^2)}{(180\text{ s})^2} = \boxed{203\ \mu\text{N}\cdot\text{m}}$$

P15.39 $T = 0.250\text{ s}$, $I = mr^2 = \left(20.0\times10^{-3}\text{ kg}\right)\left(5.00\times10^{-3}\text{ m}\right)^2$

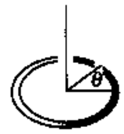


FIG. P15.39

(a) $I = \boxed{5.00\times10^{-7}\text{ kg}\cdot\text{m}^2}$

(b) $I\dfrac{d^2\theta}{dt^2} = -\kappa\theta;\ \sqrt{\dfrac{\kappa}{I}} = \omega = \dfrac{2\pi}{T}$

$$\kappa = I\omega^2 = \left(5.00\times10^{-7}\right)\left(\frac{2\pi}{0.250}\right)^2 = \boxed{3.16\times10^{-4}\ \frac{\text{N}\cdot\text{m}}{\text{rad}}}$$

Section 15.6 Damped Oscillations

P15.40 The total energy is $E = \dfrac{1}{2}mv^2 + \dfrac{1}{2}kx^2$

Taking the time-derivative, $\dfrac{dE}{dt} = mv\dfrac{d^2x}{dt^2} + kxv$

Use Equation 15.31: $\dfrac{md^2x}{dt^2} = -kx - bv$

$$\frac{dE}{dt} = v(-kx - bv) + kvx$$

Thus, $\boxed{\dfrac{dE}{dt} = -bv^2 < 0}$

P15.41 $\theta_i = 15.0°$ $\qquad \theta(t = 1\,000) = 5.50°$

$x = Ae^{-bt/2m}$ $\qquad \dfrac{x_{1\,000}}{x_i} = \dfrac{Ae^{-bt/2m}}{A} = \dfrac{5.50}{15.0} = e^{-b(1\,000)/2m}$

$$\ln\left(\frac{5.50}{15.0}\right) = -1.00 = \frac{-b(1\,000)}{2m}$$

$$\therefore \frac{b}{2m} = \boxed{1.00\times10^{-3}\text{ s}^{-1}}$$

P15.42 Show that $x = Ae^{-bt/2m}\cos(\omega t + \phi)$

is a solution of $-kx - b\dfrac{dx}{dt} = m\dfrac{d^2x}{dt^2}$ (1)

where $\omega = \sqrt{\dfrac{k}{m} - \left(\dfrac{b}{2m}\right)^2}$. (2)

$$x = Ae^{-bt/2m}\cos(\omega t + \phi) \tag{3}$$

$$\frac{dx}{dt} = Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\omega\sin(\omega t + \phi) \tag{4}$$

$$\frac{d^2x}{dt^2} = -\frac{b}{2m}\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t + \phi) - Ae^{-bt/2m}\omega\sin(\omega t + \phi)\right] - \left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\omega\sin(\omega t + \phi) + Ae^{-bt/2m}\omega^2\cos(\omega t + \phi)\right] \tag{5}$$

continued on next page

Substitute (3), (4) into the left side of (1) and (5) into the right side of (1);

$$-kAe^{-bt/2m}\cos(\omega t+\phi)+\frac{b^2}{2m}Ae^{-bt/2m}\cos(\omega t+\phi)+b\omega Ae^{-bt/2m}\sin(\omega t+\phi)$$

$$=-\frac{b}{2}\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos(\omega t+\phi)-Ae^{-bt/2m}\omega\sin(\omega t+\phi)\right]$$

$$+\frac{b}{2}Ae^{-bt/2m}\omega\sin(\omega t+\phi)-m\omega^2Ae^{-bt/2m}\cos(\omega t+\phi)$$

Compare the coefficients of $Ae^{-bt/2m}\cos(\omega t+\phi)$ and $Ae^{-bt/2m}\sin(\omega t+\phi)$:

cosine-term: $-k+\dfrac{b^2}{2m}=-\dfrac{b}{2}\left(-\dfrac{b}{2m}\right)-m\omega^2=\dfrac{b^2}{4m}-m\left(\dfrac{k}{m}-\dfrac{b^2}{4m^2}\right)=-k+\dfrac{b^2}{2m}$

sine-term: $b\omega=+\dfrac{b}{2}(\omega)+\dfrac{b}{2}(\omega)=b\omega$

Since the coefficients are equal, $x=Ae^{-bt/2m}\cos(\omega t+\phi)$ is a solution of the equation.

***P15.43** The frequency if undamped would be $\omega_0=\sqrt{\dfrac{k}{m}}=\sqrt{\dfrac{2.05\times10^4\text{ N/m}}{10.6\text{ kg}}}=44.0/\text{s}$.

(a) With damping

$$\omega=\sqrt{\omega_0^2-\left(\frac{b}{2m}\right)^2}=\sqrt{\left(44\ \frac{1}{\text{s}}\right)^2-\left(\frac{3\text{ kg}}{\text{s }2\ 10.6\text{ kg}}\right)^2}$$

$$=\sqrt{1\,933.96-0.02}=44.0\ \frac{1}{\text{s}}$$

$$f=\frac{\omega}{2\pi}=\frac{44.0}{2\pi\text{ s}}=\boxed{7.00\text{ Hz}}$$

(b) In $x=A_0e^{-bt/2m}\cos(\omega t+\phi)$ over one cycle, a time $T=\dfrac{2\pi}{\omega}$, the amplitude changes from A_0 to $A_0e^{-b2\pi/2m\omega}$ for a fractional decrease of

$$\frac{A_0-A_0e^{-\pi b/m\omega}}{A_0}=1-e^{-\pi 3/(10.6\cdot 44.0)}=1-e^{-0.020\,2}=1-0.979\,98=0.020\,0=\boxed{2.00\%}.$$

(c) The energy is proportional to the square of the amplitude, so its fractional rate of decrease is twice as fast:

$$E=\frac{1}{2}kA^2=\frac{1}{2}kA_0^2e^{-2bt/2m}=E_0e^{-bt/m}.$$

We specify

$$0.05E_0=E_0e^{-3t/10.6}$$

$$0.05=e^{-3t/10.6}$$

$$e^{+3t/10.6}=20$$

$$\frac{3t}{10.6}=\ln 20=3.00$$

$$t=\boxed{10.6\text{ s}}$$

Section 15.7 Forced Oscillations

P15.44 (a) For resonance, her frequency must match

$$f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.30\times 10^3\ \text{N/m}}{12.5\ \text{kg}}} = \boxed{2.95\ \text{Hz}}.$$

(b) From $x = A\cos\omega t$, $v = \frac{dx}{dt} = -A\omega\sin\omega t$, and $a = \frac{dv}{dt} = -A\omega^2\cos\omega t$, the maximum acceleration is $A\omega^2$. When this becomes equal to the acceleration due to gravity, the normal force exerted on her by the mattress will drop to zero at one point in the cycle:

$$A\omega^2 = g \quad \text{or} \quad A = \frac{g}{\omega^2} = \frac{g}{\frac{k}{m}} = \frac{gm}{k} \qquad A = \frac{(9.80\ \text{m/s}^2)(12.5\ \text{kg})}{4.30\times 10^3\ \text{N/m}} = \boxed{2.85\ \text{cm}}$$

P15.45 $F = 3.00\cos(2\pi t)$ N and $k = 20.0$ N/m

(a) $\omega = \frac{2\pi}{T} = 2\pi$ rad/s so $T = \boxed{1.00\ \text{s}}$

(b) In this case, $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{20.0}{2.00}} = 3.16$ rad/s

The equation for the amplitude of a driven oscillator,

with $b = 0$, gives $A = \left(\frac{F_0}{m}\right)\left(\omega^2 - \omega_0^2\right)^{-1} = \frac{3}{2}\left[4\pi^2 - (3.16)^2\right]^{-1}$

Thus $A = 0.050\,9\ \text{m} = \boxed{5.09\ \text{cm}}$.

P15.46 $F_0\cos\omega t - kx = m\frac{d^2x}{dt^2} \qquad \omega_0 = \sqrt{\frac{k}{m}}$ (1)

$x = A\cos(\omega t + \phi)$ (2)

$\frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$ (3)

$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi)$ (4)

Substitute (2) and (4) into (1): $F_0\cos\omega t - kA\cos(\omega t + \phi) = m\left(-A\omega^2\right)\cos(\omega t + \phi)$

Solve for the amplitude: $\left(kA - mA\omega^2\right)\cos(\omega t + \phi) = F_0\cos\omega t$

These will be equal, provided only that ϕ must be zero and $kA - mA\omega^2 = F_0$

Thus, $A = \frac{\frac{F_0}{m}}{\left(\frac{k}{m}\right) - \omega^2}$

P15.47 From the equation for the amplitude of a driven oscillator with no damping,

$$A=\frac{F_0/m}{\sqrt{\left(\omega^2-\omega_0^2\right)^2}}$$

$$\omega=2\pi f=\left(20.0\pi\ \text{s}^{-1}\right) \qquad \omega_0^2=\frac{k}{m}=\frac{200}{\left(\frac{40.0}{9.80}\right)}=49.0\ \text{s}^{-2}$$

$$F_0=mA\left(\omega^2-\omega_0^2\right)$$

$$F_0=\left(\frac{40.0}{9.80}\right)\left(2.00\times10^{-2}\right)(3\,950-49.0)=\boxed{318\ \text{N}}$$

P15.48 $$A=\frac{F_{ext}/m}{\sqrt{\left(\omega^2-\omega_0^2\right)^2+(b\omega/m)^2}}$$

With $b=0$, $$A=\frac{F_{ext}/m}{\sqrt{\left(\omega^2-\omega_0^2\right)^2}}=\frac{F_{ext}/m}{\pm\left(\omega^2-\omega_0^2\right)}=\pm\frac{F_{ext}/m}{\omega^2-\omega_0^2}$$

Thus, $$\omega^2=\omega_0^2\pm\frac{F_{ext}/m}{A}=\frac{k}{m}\pm\frac{F_{ext}}{mA}=\frac{6.30\ \text{N/m}}{0.150\ \text{kg}}\pm\frac{1.70\ \text{N}}{(0.150\ \text{kg})(0.440\ \text{m})}$$

This yields $\omega=8.23\ \text{rad/s}$ or $\omega=4.03\ \text{rad/s}$

Then, $f=\frac{\omega}{2\pi}$ gives either $f=\boxed{1.31\ \text{Hz}}$ or $f=\boxed{0.641\ \text{Hz}}$

P15.49 The beeper must resonate at the frequency of a simple pendulum of length 8.21 cm:

$$f=\frac{1}{2\pi}\sqrt{\frac{g}{L}}=\frac{1}{2\pi}\sqrt{\frac{9.80\ \text{m/s}^2}{0.082\,1\ \text{m}}}=\boxed{1.74\ \text{Hz}}.$$

***P15.50** For the resonance vibration with the occupants in the car, we have for the spring constant of the suspension

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad k=4\pi^2f^2m=4\pi^2\left(1.8\ \text{s}^{-1}\right)^2\left(1\,130\ \text{kg}+4(72.4\ \text{kg})\right)=1.82\times10^5\ \text{kg/s}^2$$

Now as the occupants exit $$x=\frac{F}{k}=\frac{4(72.4\ \text{kg})\left(9.8\ \text{m/s}^2\right)}{1.82\times10^5\ \text{kg/s}^2}=\boxed{1.56\times10^{-2}\ \text{m}}$$

Additional Problems

P15.51 Let F represent the tension in the rod.

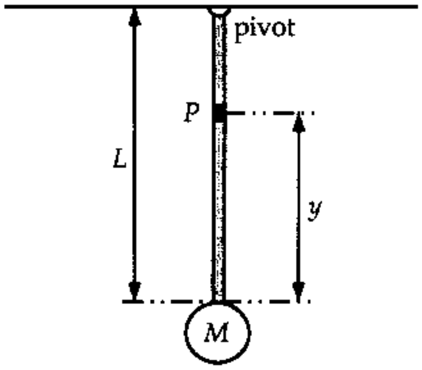


FIG. P15.51

(a) At the pivot, $F = Mg + Mg = \boxed{2Mg}$

A fraction of the rod's weight $Mg\left(\frac{y}{L}\right)$ as well as the weight of the ball pulls down on point P. Thus, the tension in the rod at point P is

$$F = Mg\left(\frac{y}{L}\right) + Mg = \boxed{Mg\left(1+\frac{y}{L}\right)}.$$

(b) Relative to the pivot, $I = I_{\text{rod}} + I_{\text{ball}} = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2$

For the physical pendulum, $T = 2\pi\sqrt{\frac{I}{mgd}}$ where $m = 2M$ and d is the distance from the pivot to the center of mass of the rod and ball combination. Therefore,

$$d = \frac{M\left(\frac{L}{2}\right) + ML}{M+M} = \frac{3L}{4} \text{ and } T = 2\pi\sqrt{\frac{\frac{4}{3}ML^2}{(2M)g\left(\frac{3L}{4}\right)}} = \boxed{\frac{4\pi}{3}\sqrt{\frac{2L}{g}}}.$$

For $L = 2.00$ m, $T = \frac{4\pi}{3}\sqrt{\frac{2(2.00 \text{ m})}{9.80 \text{ m/s}^2}} = \boxed{2.68 \text{ s}}$.

P15.52 (a) Total energy $= \frac{1}{2}kA^2 = \frac{1}{2}(100 \text{ N/m})(0.200 \text{ m})^2 = 2.00 \text{ J}$

At equilibrium, the total energy is:

$$\frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}(16.0 \text{ kg})v^2 = (8.00 \text{ kg})v^2.$$

Therefore,

$$(8.00 \text{ kg})v^2 = 2.00 \text{ J, and } v = \boxed{0.500 \text{ m/s}}.$$

This is the speed of m_1 and m_2 at the equilibrium point. Beyond this point, the mass m_2 moves with the constant speed of 0.500 m/s while mass m_1 starts to slow down due to the restoring force of the spring.

continued on next page

(b) The energy of the m_1-spring system at equilibrium is:

$$\frac{1}{2}m_1v^2 = \frac{1}{2}(9.00\text{ kg})(0.500\text{ m/s})^2 = 1.125\text{ J}.$$

This is also equal to $\frac{1}{2}k(A')^2$, where A' is the amplitude of the m_1-spring system. Therefore,

$$\frac{1}{2}(100)(A')^2 = 1.125 \text{ or } A' = 0.150\text{ m}.$$

The period of the m_1-spring system is $T = 2\pi\sqrt{\frac{m_1}{k}} = 1.885\text{ s}$

and it takes $\frac{1}{4}T = 0.471\text{ s}$ after it passes the equilibrium point for the spring to become fully stretched the first time. The distance separating m_1 and m_2 at this time is:

$$D = v\left(\frac{T}{4}\right) - A' = 0.500\text{ m/s}(0.471\text{ s}) - 0.150\text{ m} = 0.085\,6 = \boxed{8.56\text{ cm}}.$$

P15.53 $\left(\frac{d^2x}{dt^2}\right)_{\max} = A\omega^2$

$f_{\max} = \mu_s n = \mu_s mg = mA\omega^2$

$A = \frac{\mu_s g}{\omega^2} = \boxed{6.62\text{ cm}}$

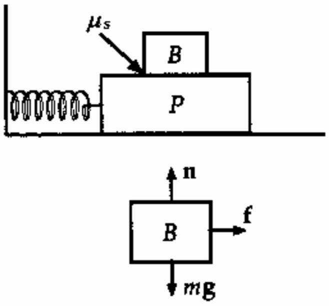


FIG. P15.53

P15.54 The maximum acceleration of the oscillating system is $a_{\max} = A\omega^2 = 4\pi^2Af^2$. The friction force exerted between the two blocks must be capable of accelerating block B at this rate. Thus, if Block B is about to slip,

$$f = f_{\max} = \mu_s n = \mu_s mg = m\left(4\pi^2Af^2\right) \quad \text{or} \quad A = \boxed{\frac{\mu_s g}{4\pi^2 f^2}}.$$

P15.55 Deuterium is the isotope of the element hydrogen with atoms having nuclei consisting of one proton and one neutron. For brevity we refer to the molecule formed by two deuterium atoms as D and to the diatomic molecule of hydrogen-1 as H.

$$M_D = 2M_H \qquad \frac{\omega_D}{\omega_H} = \frac{\sqrt{\frac{k}{M_D}}}{\sqrt{\frac{k}{M_H}}} = \sqrt{\frac{M_H}{M_D}} = \sqrt{\frac{1}{2}} \qquad f_D = \frac{f_H}{\sqrt{2}} = \boxed{0.919 \times 10^{14}\text{ Hz}}$$

P15.56 The kinetic energy of the ball is $K=\frac{1}{2}mv^2+\frac{1}{2}I\Omega^2$, where Ω is the rotation rate of the ball about its center of mass. Since the center of the ball moves along a circle of radius $4R$, its displacement from equilibrium is $s=(4R)\theta$ and its speed is $v=\frac{ds}{dt}=4R\left(\frac{d\theta}{dt}\right)$. Also, since the ball rolls without slipping,

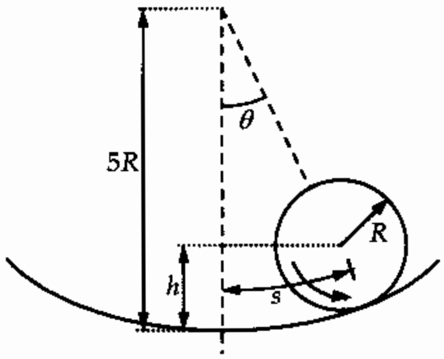


FIG. P15.56

$$v=\frac{ds}{dt}=R\Omega \qquad \text{so} \qquad \Omega=\frac{v}{R}=4\left(\frac{d\theta}{dt}\right)$$

The kinetic energy is then

$$K=\frac{1}{2}m\left(4R\frac{d\theta}{dt}\right)^2+\frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(4\frac{d\theta}{dt}\right)^2$$
$$=\frac{112mR^2}{10}\left(\frac{d\theta}{dt}\right)^2$$

When the ball has an angular displacement θ, its center is distance $h=4R(1-\cos\theta)$ higher than when at the equilibrium position. Thus, the potential energy is $U_g=mgh=4mgR(1-\cos\theta)$. For small angles, $(1-\cos\theta)\approx\frac{\theta^2}{2}$ (see Appendix B). Hence, $U_g\approx 2mgR\theta^2$, and the total energy is

$$E=K+U_g=\frac{112mR^2}{10}\left(\frac{d\theta}{dt}\right)^2+2mgR\theta^2.$$

Since $E=$ constant in time, $\frac{dE}{dt}=0=\frac{112mR^2}{5}\left(\frac{d\theta}{dt}\right)\frac{d^2\theta}{dt^2}+4mgR\theta\left(\frac{d\theta}{dt}\right)$.

This reduces to $\frac{28R}{5}\frac{d^2\theta}{dt^2}+g\theta=0$, or $\frac{d^2\theta}{dt^2}=-\left(\frac{5g}{28R}\right)\theta$.

With the angular acceleration equal to a negative constant times the angular position, this is in the defining form of a simple harmonic motion equation with $\omega=\sqrt{\frac{5g}{28R}}$.

The period of the simple harmonic motion is then $T=\frac{2\pi}{\omega}=\boxed{2\pi\sqrt{\frac{28R}{5g}}}$.

P15.57 (a)

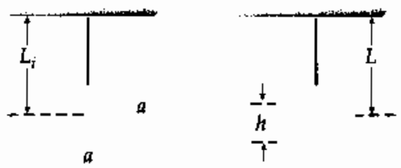


FIG. P15.57(a)

(b) $T = 2\pi\sqrt{\dfrac{L}{g}}$ $\qquad \dfrac{dT}{dt} = \dfrac{\pi}{\sqrt{g}}\dfrac{1}{\sqrt{L}}\dfrac{dL}{dt}$ (1)

We need to find $L(t)$ and $\dfrac{dL}{dt}$. From the diagram in (a),

$$L = L_i + \frac{a}{2} - \frac{h}{2};\ \frac{dL}{dt} = -\left(\frac{1}{2}\right)\frac{dh}{dt}.$$

But $\dfrac{dM}{dt} = \rho\dfrac{dV}{dt} = -\rho A\dfrac{dh}{dt}$. Therefore,

$$\frac{dh}{dt} = -\frac{1}{\rho A}\frac{dM}{dt};\ \frac{dL}{dt} = \left(\frac{1}{2\rho A}\right)\frac{dM}{dt} \tag{2}$$

Also, $\displaystyle\int_{L_i}^{L} dL = \left(\frac{1}{2\rho A}\right)\left(\frac{dM}{dt}\right)t = L - L_i$ (3)

Substituting Equation (2) and Equation (3) into Equation (1):

$$\frac{dT}{dt} = \boxed{\frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\frac{1}{\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}}}.$$

(c) Substitute Equation (3) into the equation for the period.

$$T = \boxed{\frac{2\pi}{\sqrt{g}}\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}}$$

Or one can obtain T by integrating (b):

$$\int_{T}^{T_i} dT = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\int_0^t \frac{dt}{\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}}$$

$$T - T_i = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\left[\frac{2}{\frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)}\right]\left[\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t} - \sqrt{L_i}\right]$$

But $T_i = 2\pi\sqrt{\dfrac{L_i}{g}}$, so $T = \dfrac{2\pi}{\sqrt{g}}\sqrt{L_i + \dfrac{1}{2\rho a^2}\left(\dfrac{dM}{dt}\right)t}$.

P15.58 $\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T}$

(a) $k = \omega^2 m = \boxed{\frac{4\pi^2 m}{T^2}}$ (b) $m' = \frac{k(T')^2}{4\pi^2} = \boxed{m\left(\frac{T'}{T}\right)^2}$

P15.59 We draw a free-body diagram of the pendulum. The force **H** exerted by the hinge causes no torque about the axis of rotation.

$\tau = I\alpha$ and $\frac{d^2\theta}{dt^2} = -\alpha$

$\tau = MgL\sin\theta + kxh\cos\theta = -I\frac{d^2\theta}{dt^2}$

For small amplitude vibrations, use the approximations: $\sin\theta \approx \theta$, $\cos\theta \approx 1$, and $x \approx s = h\theta$.

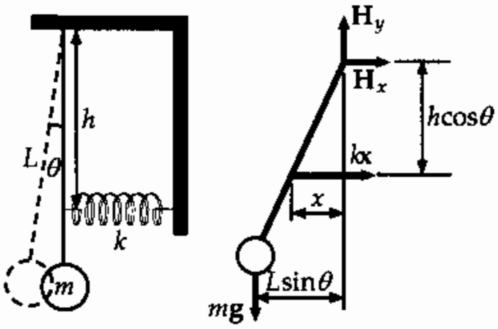


FIG. P15.59

Therefore, $\frac{d^2\theta}{dt^2} = -\left(\frac{MgL + kh^2}{I}\right)\theta = -\omega^2\theta$ $\omega = \sqrt{\frac{MgL + kh^2}{ML^2}} = 2\pi f$

$f = \boxed{\frac{1}{2\pi}\sqrt{\frac{MgL + kh^2}{ML^2}}}$

***P15.60** (a) In $x = A\cos(\omega t + \phi)$, $v = -\omega A\sin(\omega t + \phi)$

we have at $t = 0$ $v = -\omega A\sin\phi = -v_{max}$

This requires $\phi = 90°$, so $x = A\cos(\omega t + 90°)$

And this is equivalent to $x = -A\sin\omega t$

Numerically we have $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50\text{ N/m}}{0.5\text{ kg}}} = 10\text{ s}^{-1}$

and $v_{max} = \omega A$ $20\text{ m/s} = (10\text{ s}^{-1})A$ $A = 2\text{ m}$

So $\boxed{x = (-2\text{ m})\sin[(10\text{ s}^{-1})t]}$

(b) In $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$, $\frac{1}{2}kx^2 = 3\left(\frac{1}{2}mv^2\right)$

implies $\frac{1}{3}\frac{1}{2}kx^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$ $\frac{4}{3}x^2 = A^2$

$x = \pm\sqrt{\frac{3}{4}}A = \pm 0.866A = \boxed{\pm 1.73\text{ m}}$

continued on next page

(c) $\omega = \sqrt{\frac{g}{L}}$ $\qquad L = \frac{g}{\omega^2} = \frac{9.8\ \text{m/s}^2}{\left(10\ \text{s}^{-1}\right)^2} = \boxed{0.098\,0\ \text{m}}$

(d) In $\qquad x = (-2\ \text{m})\sin\left[\left(10\ \text{s}^{-1}\right)t\right]$

the particle is at $x = 0$ at $t = 0$, at $10t = \pi$ s, and so on.

The particle is at $\qquad x = 1$ m

when $\qquad -\frac{1}{2} = \sin\left[\left(10\ \text{s}^{-1}\right)t\right]$

with solutions $\qquad \left(10\ \text{s}^{-1}\right)t = -\frac{\pi}{6}$

$$\left(10\ \text{s}^{-1}\right)t = \pi + \frac{\pi}{6}, \text{ and so on.}$$

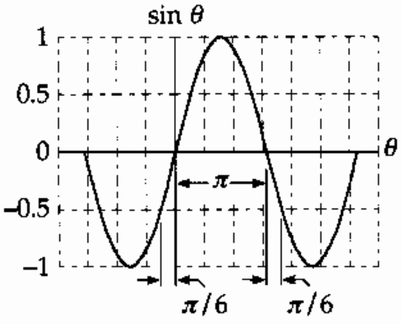


FIG. P15.60(d)

The minimum time for the motion is Δt in $10\Delta t = \left(\frac{\pi}{6}\right)$ s

$$\Delta t = \left(\frac{\pi}{60}\right)\text{s} = \boxed{0.052\,4\ \text{s}}$$

P15.61 (a) At equilibrium, we have

$$\sum \tau = 0 - mg\left(\frac{L}{2}\right) + kx_0 L$$

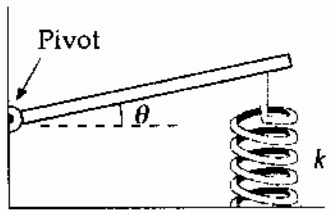


FIG. P15.61

where x_0 is the equilibrium compression.

After displacement by a small angle,

$$\sum \tau = -mg\left(\frac{L}{2}\right) + kxL = -mg\left(\frac{L}{2}\right) + k(x_0 - L\theta)L = -k\theta L^2$$

But, $\sum \tau = I\alpha = \frac{1}{3}mL^2\frac{d^2\theta}{dt^2}$. $\qquad$ So $\frac{d^2\theta}{dt^2} = -\frac{3k}{m}\theta$.

The angular acceleration is opposite in direction and proportional to the displacement, so we have simple harmonic motion with $\boxed{\omega^2 = \frac{3k}{m}}$.

(b) $f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}} = \frac{1}{2\pi}\sqrt{\frac{3(100\ \text{N/m})}{5.00\ \text{kg}}} = \boxed{1.23\ \text{Hz}}$

***P15.62** As it passes through equilibrium, the 4-kg object has speed

$$v_{\max} = \omega A = \sqrt{\frac{k}{m}}A = \sqrt{\frac{100\ \text{N/m}}{4\ \text{kg}}}2\ \text{m} = 10.0\ \text{m/s}.$$

In the completely inelastic collision momentum of the two-object system is conserved. So the new 10-kg object starts its oscillation with speed given by

$$4\ \text{kg}(10\ \text{m/s}) + (6\ \text{kg})0 = (10\ \text{kg})v_{\max}$$
$$v_{\max} = 4.00\ \text{m/s}$$

(a) The new amplitude is given by $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$

$$10\ \text{kg}(4\ \text{m/s})^2 = (100\ \text{N/m})A^2$$
$$A = 1.26\ \text{m}$$

Thus the amplitude has $\boxed{\text{decreased by}}$ $2.00\ \text{m} - 1.26\ \text{m} = \boxed{0.735\ \text{m}}$

(b) The old period was $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4\ \text{kg}}{100\ \text{N/m}}} = 1.26\ \text{s}$

The new period is $T = 2\pi\sqrt{\frac{10}{100}\ \text{s}^2} = 1.99\ \text{s}$

The period has $\boxed{\text{increased by}}$ $1.99\ \text{m} - 1.26\ \text{m} = \boxed{0.730\ \text{s}}$

(c) The old energy was $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}(4\ \text{kg})(10\ \text{m/s})^2 = 200\ \text{J}$

The new mechanical energy is $\frac{1}{2}(10\ \text{kg})(4\ \text{m/s})^2 = 80\ \text{J}$

The energy has $\boxed{\text{decreased by 120 J}}$.

(d) The missing mechanical energy has turned into internal energy in the completely inelastic collision.

P15.63 (a) $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} = \boxed{3.00\ \text{s}}$

(b) $E = \frac{1}{2}mv^2 = \frac{1}{2}(6.74)(2.06)^2 = \boxed{14.3\ \text{J}}$

(c) At maximum angular displacement $mgh = \frac{1}{2}mv^2$ $h = \frac{v^2}{2g} = 0.217\ \text{m}$

$h = L - L\cos\theta = L(1 - \cos\theta)$ $\cos\theta = 1 - \frac{h}{L}$ $\boxed{\theta = 25.5°}$

P15.64 One can write the following equations of motion:

$T - kx = 0$ (describes the spring)

$mg - T' = ma = m\dfrac{d^2x}{dt^2}$ (for the hanging object)

$R(T' - T) = I\dfrac{d^2\theta}{dt^2} = \dfrac{I}{R}\dfrac{d^2x}{dt^2}$ (for the pulley)

with $I = \dfrac{1}{2}MR^2$

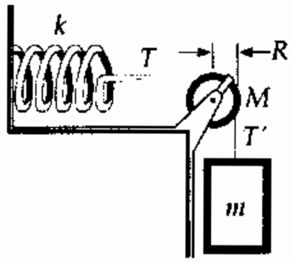


FIG. P15.64

Combining these equations gives the equation of motion

$$\left(m + \frac{1}{2}M\right)\frac{d^2x}{dt^2} + kx = mg.$$

The solution is $x(t) = A\sin\omega t + \dfrac{mg}{k}$ (where $\dfrac{mg}{k}$ arises because of the extension of the spring due to the weight of the hanging object), with frequency

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m + \frac{1}{2}M}} = \frac{1}{2\pi}\sqrt{\frac{100\text{ N/m}}{0.200\text{ kg} + \frac{1}{2}M}}.$$

(a) For $M = 0$ $f = \boxed{3.56\text{ Hz}}$

(b) For $M = 0.250$ kg $f = \boxed{2.79\text{ Hz}}$

(c) For $M = 0.750$ kg $f = \boxed{2.10\text{ Hz}}$

P15.65 Suppose a 100-kg biker compresses the suspension 2.00 cm.

Then, $k = \dfrac{F}{x} = \dfrac{980\text{ N}}{2.00 \times 10^{-2}\text{ m}} = 4.90 \times 10^4\text{ N/m}$

If total mass of motorcycle and biker is 500 kg, the frequency of free vibration is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.90 \times 10^4\text{ N/m}}{500\text{ kg}}} = 1.58\text{ Hz}$$

If he encounters washboard bumps at the same frequency, resonance will make the motorcycle bounce a lot. Assuming a speed of 20.0 m/s, we find these ridges are separated by

$$\frac{20.0\text{ m/s}}{1.58\text{ s}^{-1}} = 12.7\text{ m}\ \boxed{\sim 10^1\text{ m}}.$$

In addition to this vibration mode of bouncing up and down as one unit, the motorcycle can also vibrate at higher frequencies by rocking back and forth between front and rear wheels, by having just the front wheel bounce inside its fork, or by doing other things. Other spacing of bumps will excite all of these other resonances.

P15.66 (a) For each segment of the spring

$$dK = \frac{1}{2}(dm)v_x^2.$$

Also, $v_x = \frac{x}{\ell}v$ and $dm = \frac{m}{\ell}dx$.

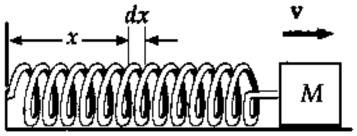


FIG. P15.66

Therefore, the total kinetic energy of the block-spring system is

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}\int_0^{\ell}\left(\frac{x^2v^2}{\ell^2}\right)\frac{m}{\ell}dx = \boxed{\frac{1}{2}\left(M + \frac{m}{3}\right)v^2}.$$

(b) $\omega = \sqrt{\frac{k}{m_{eff}}}$ and $\frac{1}{2}m_{eff}v^2 = \frac{1}{2}\left(M + \frac{m}{3}\right)v^2$

Therefore, $T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{M + \frac{m}{3}}{k}}}$.

P15.67 (a) $\sum \mathbf{F} = -2T\sin\theta\hat{\mathbf{j}}$ where $\theta = \tan^{-1}\left(\frac{y}{L}\right)$

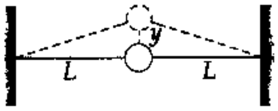


FIG. P15.67

Therefore, for a small displacement

$\sin\theta \approx \tan\theta = \frac{y}{L}$ and $\boxed{\sum \mathbf{F} = \frac{-2Ty}{L}\hat{\mathbf{j}}}$

(b) The total force exerted on the ball is opposite in direction and proportional to its displacement from equilibrium, so the ball moves with simple harmonic motion. For a spring system,

$$\sum \mathbf{F} = -k\mathbf{x} \quad \text{becomes here} \quad \sum \mathbf{F} = -\frac{2T}{L}\mathbf{y}.$$

Therefore, the effective spring constant is $\frac{2T}{L}$ and $\boxed{\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2T}{mL}}}$.

P15.68 (a) Assuming a Hooke's Law type spring,

$$F = Mg = kx$$

and empirically

$$Mg = 1.74x - 0.113$$

so $k = \boxed{1.74 \text{ N/m} \pm 6\%}$.

M, kg	x, m	Mg, N
0.020 0	0.17	0.196
0.040 0	0.293	0.392
0.050 0	0.353	0.49
0.060 0	0.413	0.588
0.070 0	0.471	0.686
0.080 0	0.493	0.784

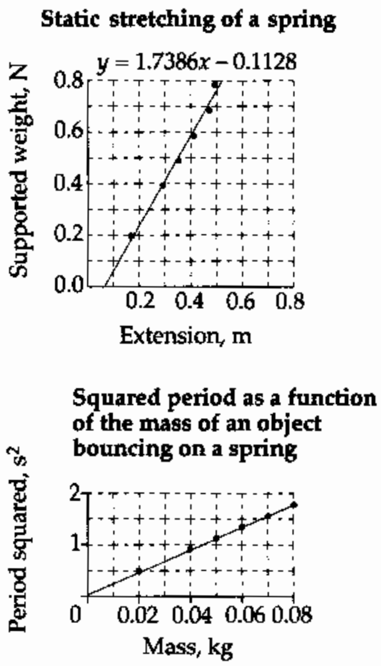


FIG. P15.68

(b) We may write the equation as theoretically

$$T^2 = \frac{4\pi^2}{k}M + \frac{4\pi^2}{3k}m_s$$

and empirically

$$T^2 = 21.7M + 0.058\,9$$

so $k = \dfrac{4\pi^2}{21.7} = \boxed{1.82 \text{ N/m} \pm 3\%}$

Time, s	T, s	M, kg	T^2, s^2
7.03	0.703	0.020 0	0.494
9.62	0.962	0.040 0	0.925
10.67	1.067	0.050 0	1.138
11.67	1.167	0.060 0	1.362
12.52	1.252	0.070 0	1.568
13.41	1.341	0.080 0	1.798

The k values 1.74 N/m ± 6%

and 1.82 N/m ± 3% differ by 4%

so $\boxed{\text{they agree.}}$

(c) Utilizing the axis-crossing point, $m_s = 3\left(\dfrac{0.058\,9}{21.7}\right)\text{kg} = \boxed{8 \text{ grams} \pm 12\%}$

$\boxed{\text{in agreement}}$ with 7.4 grams.

P15.69 (a) $\Delta K + \Delta U = 0$

Thus, $K_{\text{top}} + U_{\text{top}} = K_{\text{bot}} + U_{\text{bot}}$

where $K_{\text{top}} = U_{\text{bot}} = 0$

Therefore, $mgh = \frac{1}{2} I\omega^2$, but

$$h = R - R\cos\theta = R(1-\cos\theta)$$

$$\omega = \frac{v}{R}$$

and $I = \frac{MR^2}{2} + \frac{mr^2}{2} + mR^2$

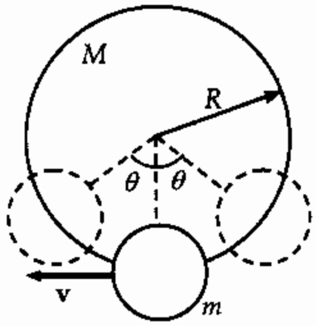


FIG. P15.69

Substituting we find

$$mgR(1-\cos\theta) = \frac{1}{2}\left(\frac{MR^2}{2} + \frac{mr^2}{2} + mR^2\right)\frac{v^2}{R^2}$$

$$mgR(1-\cos\theta) = \left[\frac{M}{4} + \frac{mr^2}{4R^2} + \frac{m}{2}\right]v^2$$

and $v^2 = 4gR\frac{(1-\cos\theta)}{\left(\frac{M}{m} + \frac{r^2}{R^2} + 2\right)}$

so $\boxed{v = 2\sqrt{\frac{Rg(1-\cos\theta)}{\frac{M}{m} + \frac{r^2}{R^2} + 2}}}$

(b) $T = 2\pi\sqrt{\frac{I}{m_T g d_{\text{CM}}}}$

$m_T = m + M$ $\qquad d_{\text{CM}} = \frac{mR + M(0)}{m+M}$

$T = \boxed{2\pi\sqrt{\frac{\frac{1}{2}MR^2 + \frac{1}{2}mr^2 + mR^2}{mgR}}}$

P15.70 (a) We require $Ae^{-bt/2m} = \frac{A}{2}$ $\qquad e^{+bt/2m} = 2$

or $\frac{bt}{2m} = \ln 2$ $\qquad$ or $\frac{0.100\ \text{kg/s}}{2(0.375\ \text{kg})}t = 0.693$ $\qquad \therefore t = \boxed{5.20\ \text{s}}$

The spring constant is irrelevant.

(b) We can evaluate the energy at successive turning points, where $\cos(\omega t + \phi) = \pm 1$ and the energy is $\frac{1}{2}kx^2 = \frac{1}{2}kA^2 e^{-bt/2m}$. We require $\frac{1}{2}kA^2 e^{-bt/2m} = \frac{1}{2}\left(\frac{1}{2}kA^2\right)$

or $e^{+bt/m} = 2$ $\qquad \therefore t = \frac{m\ln 2}{b} = \frac{0.375\ \text{kg}(0.693)}{0.100\ \text{kg/s}} = \boxed{2.60\ \text{s}}$.

(c) From $E = \frac{1}{2}kA^2$, the fractional rate of change of energy over time is

$$\frac{\frac{dE}{dt}}{E} = \frac{\frac{d}{dt}\left(\frac{1}{2}kA^2\right)}{\frac{1}{2}kA^2} = \frac{\frac{1}{2}k(2A)\frac{dA}{dt}}{\frac{1}{2}kA^2} = 2\frac{\frac{dA}{dt}}{A}$$

two times faster than the fractional rate of change in amplitude.

P15.71 (a) When the mass is displaced a distance x from equilibrium, spring 1 is stretched a distance x_1 and spring 2 is stretched a distance x_2.

By Newton's third law, we expect

$$k_1x_1 = k_2x_2.$$

When this is combined with the requirement that

$$x = x_1 + x_2,$$

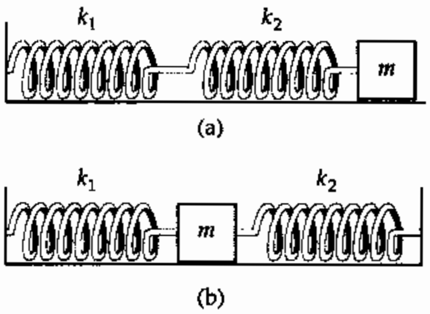


FIG. P15.71

we find $$x_1 = \left[\frac{k_2}{k_1+k_2}\right]x$$

The force on either spring is given by $$F_1 = \left[\frac{k_1k_2}{k_1+k_2}\right]x = ma$$

where a is the acceleration of the mass m.

This is in the form $$F = k_{eff}x = ma$$

and $$T = 2\pi\sqrt{\frac{m}{k_{eff}}} = \boxed{2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}}$$

(b) In this case each spring is distorted by the distance x which the mass is displaced. Therefore, the restoring force is

$$F = -(k_1+k_2)x \qquad \text{and} \qquad k_{eff} = k_1 + k_2$$

so that $$T = \boxed{2\pi\sqrt{\frac{m}{(k_1+k_2)}}}.$$

P15.72 Let ℓ represent the length below water at equilibrium and M the tube's mass:

$$\sum F_y = 0 \Rightarrow -Mg + \rho\pi r^2\ell g = 0.$$

Now with any excursion x from equilibrium

$$-Mg + \rho\pi r^2(\ell - x)g = Ma.$$

Subtracting the equilibrium equation gives

$$-\rho\pi r^2 gx = Ma$$

$$a = -\left(\frac{\rho\pi r^2 g}{M}\right)x = -\omega^2 x$$

The opposite direction and direct proportionality of a to x imply SHM with angular frequency

$$\omega = \sqrt{\frac{\rho\pi r^2 g}{M}}$$

$$T = \frac{2\pi}{\omega} = \boxed{\left(\frac{2}{r}\right)\sqrt{\frac{\pi M}{\rho g}}}$$

P15.73 For $\theta_{max} = 5.00°$, the motion calculated by the Euler method agrees quite precisely with the prediction of $\theta_{max} \cos \omega t$. The period is $T = 2.20$ s.

Time, t (s)	Angle, θ (°)	Ang. speed (°/s)	Ang. Accel. $(°/s^2)$	$\theta_{max} \cos \omega t$
0.000	5.000 0	0.000 0	−40.781 5	5.000 0
0.004	4.999 3	−0.163 1	−40.776 2	4.999 7
0.008	4.998 0	−0.326 2	−40.765 6	4.998 7
…				
0.544	0.056 0	−14.282 3	−0.457 6	0.081 0
0.548	−0.001 1	−14.284 2	0.009 0	0.023 9
0.552	−0.058 2	−14.284 1	0.475 6	−0.033 3
…				
1.092	−4.999 4	−0.319 9	40.776 5	−4.998 9
1.096	−5.000 0	−0.156 8	40.781 6	−4.999 8
1.100	−5.000 0	0.006 3	40.781 4	−5.000 0
1.104	−4.999 3	0.169 4	40.775 9	−4.999 6
…				
1.644	−0.063 8	14.282 4	0.439 7	−0.071 6
1.648	0.003 3	14.284 2	−0.027 0	−0.014 5
1.652	0.060 4	14.284 1	−0.493 6	0.042 7
…				
2.192	4.999 4	0.313 7	−40.776 8	4.999 1
2.196	5.000 0	0.150 6	−40.781 7	4.999 9
2.200	5.000 0	−0.012 6	−40.781 3	5.000 0
2.204	4.999 3	−0.175 7	−40.775 6	4.999 4

For $\theta_{max} = 100°$, the simple harmonic motion approximation $\theta_{max} \cos \omega t$ diverges greatly from the Euler calculation. The period is $T = 2.71$ s, larger than the small-angle period by 23%.

Time, t (s)	Angle, θ (°)	Ang. speed (°/s)	Ang. Accel. $(°/s^2)$	$\theta_{max} \cos \omega t$
0.000	100.000 0	0.000 0	−460.606 6	100.000 0
0.004	99.992 6	−1.843 2	−460.817 3	99.993 5
0.008	99.977 6	−3.686 5	−460.838 2	99.973 9
…				
1.096	−84.744 9	−120.191 0	465.948 8	−99.995 4
1.100	−85.218 2	−118.327 2	466.286 9	−99.999 8
1.104	−85.684 0	−116.462 0	466.588 6	−99.991 1
…				
1.348	−99.996 0	−3.053 3	460.812 5	−75.797 9
1.352	−100.000 8	−1.210 0	460.805 7	−75.047 4
1.356	−99.998 3	0.633 2	460.809 3	−74.287 0
…				
2.196	40.150 9	224.867 7	−301.713 2	99.997 1
2.200	41.045 5	223.660 9	−307.260 7	99.999 3
2.204	41.935 3	222.431 8	−312.703 5	99.988 5
…				
2.704	99.998 5	2.420 0	−460.809 0	12.642 2
2.708	100.000 8	0.576 8	−460.805 7	11.507 5
2.712	99.995 7	−1.266 4	−460.812 9	10.371 2

Motion of a Simple Pendulum

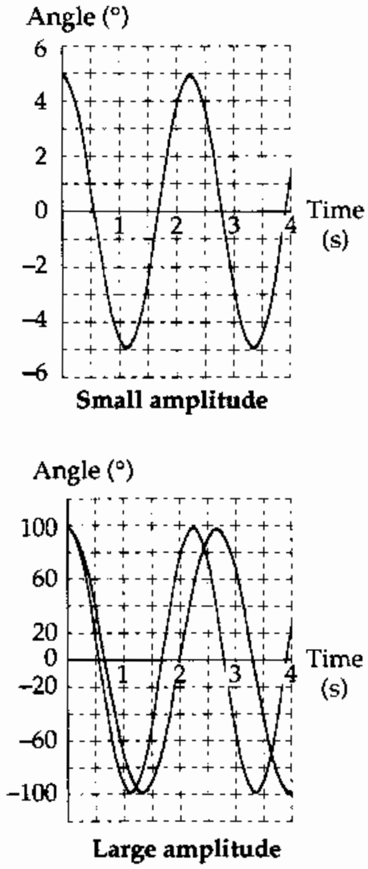


FIG. P15.73

***P15.74** (a) The block moves with the board in what we take as the positive x direction, stretching the spring until the spring force $-kx$ is equal in magnitude to the maximum force of static friction $\mu_s n = \mu_s mg$. This occurs at $x = \dfrac{\mu_s mg}{k}$.

(b) Since v is small, the block is nearly at the rest at this break point. It starts almost immediately to move back to the left, the forces on it being $-kx$ and $+\mu_k mg$. While it is sliding the net force exerted on it can be written as

$$-kx + \mu_k mg = -kx + \frac{k\mu_k mg}{k} = -k\left(x - \frac{\mu_k mg}{k}\right) = -kx_{rel}$$

where x_{rel} is the excursion of the block away from the point $\dfrac{\mu_k mg}{k}$.

Conclusion: the block goes into simple harmonic motion centered about the equilibrium position where the spring is stretched by $\dfrac{\mu_k mg}{k}$.

(d) The amplitude of its motion is its original displacement, $A = \dfrac{\mu_s mg}{k} - \dfrac{\mu_k mg}{k}$. It first comes to rest at spring extension $\dfrac{\mu_k mg}{k} - A = \dfrac{(2\mu_k - \mu_s)mg}{k}$. Almost immediately at this point it latches onto the slowly-moving board to move with the board. The board exerts a force of static friction on the block, and the cycle continues.

(c) The graph of the motion looks like this:

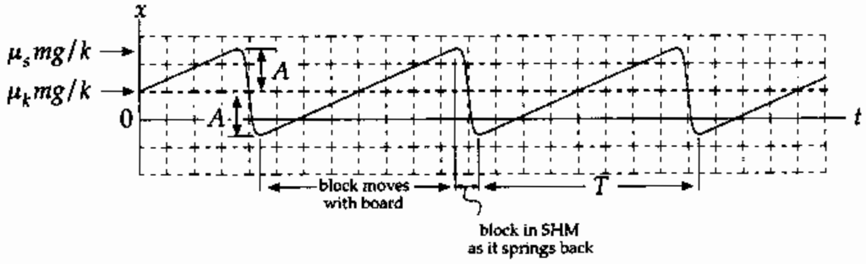


FIG. P15.74(c)

(e) The time during each cycle when the block is moving with the board is $\dfrac{2A}{v} = \dfrac{2(\mu_s - \mu_k)mg}{kv}$.

The time for which the block is springing back is one half a cycle of simple harmonic motion, $\dfrac{1}{2}\left(2\pi\sqrt{\dfrac{m}{k}}\right) = \pi\sqrt{\dfrac{m}{k}}$. We ignore the times at the end points of the motion when the speed of the block changes from v to 0 and from 0 to v. Since v is small compared to $\dfrac{2A}{\pi\sqrt{\frac{m}{k}}}$, these times are negligible. Then the period is

$$T = \frac{2(\mu_s - \mu_k)mg}{kv} + \pi\sqrt{\frac{m}{k}}.$$

continued on next page

(f) $$T = \frac{2(0.4-0.25)(0.3\text{ kg})\left(9.8\text{ m/s}^2\right)}{(0.024\text{ m/s})(12\text{ N/m})} + \pi\sqrt{\frac{0.3\text{ kg}}{12\text{ N/m}}} = 3.06\text{ s} + 0.497\text{ s} = 3.56\text{ s}$$

Then $f = \frac{1}{T} = \boxed{0.281\text{ Hz}}$.

(g) $T = \frac{2(\mu_s - \mu_k)mg}{kv} + \pi\sqrt{\frac{m}{k}}$ increases as m increases, so the frequency $\boxed{\text{decreases}}$.

(h) As k increases, T decreases and f $\boxed{\text{increases}}$.

(i) As v increases, T decreases and f $\boxed{\text{increases}}$.

(j) As $(\mu_s - \mu_k)$ increases, T increases and f $\boxed{\text{decreases}}$.

***P15.75** (a) Newton's law of universal gravitation is $F = -\frac{GMm}{r^2} = -\frac{Gm}{r^2}\left(\frac{4}{3}\pi r^3\right)\rho$

Thus, $F = -\left(\frac{4}{3}\pi\rho Gm\right)r$

Which is of Hooke's law form with $k = \frac{4}{3}\pi\rho Gm$

(b) The sack of mail moves without friction according to $-\left(\frac{4}{3}\right)\pi\rho Gmr = ma$

$$a = -\left(\frac{4}{3}\right)\pi\rho Gr = -\omega^2 r$$

Since acceleration is a negative constant times excursion from equilibrium, it executes SHM with

$\omega = \sqrt{\frac{4\pi\rho G}{3}}$ and period $T = \frac{2\pi}{\omega} = \sqrt{\frac{3\pi}{\rho G}}$

The time for a one-way trip through the earth is $\frac{T}{2} = \sqrt{\frac{3\pi}{4\rho G}}$

We have also $g = \frac{GM_e}{R_e^2} = \frac{G4\pi R_e^3\rho}{3R_e^2} = \frac{4}{3}\pi\rho GR_e$

so $\frac{4\rho G}{3} = \frac{g}{(\pi R_e)}$ and $\frac{T}{2} = \pi\sqrt{\frac{R_e}{g}} = \pi\sqrt{\frac{6.37\times10^6\text{ m}}{9.8\text{ m/s}^2}} = 2.53\times10^3\text{ s} = \boxed{42.2\text{ min}}$.

ANSWERS TO EVEN PROBLEMS

P15.2 (a) 4.33 cm; (b) −5.00 cm/s; (c) −17.3 cm/s^2; (d) 3.14 s; 5.00 cm

P15.4 (a) 15.8 cm; (b) −15.9 cm; (c) see the solution; (d) 51.1 m; (e) 50.7 m

P15.6 see the solution

P15.8 12.0 Hz

P15.10 18.8 m/s; 7.11 km/s^2

P15.12 (a) 1.26 s; (b) 0.150 m/s; 0.750 m/s^2;
(c) $x = -3\text{ cm}\cos 5t$; $v = \left(\frac{15\text{ cm}}{\text{s}}\right)\sin 5t$;
$a = \left(\frac{75\text{ cm}}{\text{s}^2}\right)\cos 5t$

P15.14 (a) $\frac{v}{\omega}$; (b) $x = -\left(\frac{v}{\omega}\right)\sin\omega t$

P15.16 (a) 126 N/m; (b) 0.178 m

P15.18 (a) 0.153 J; (b) 0.784 m/s; (c) 17.5 m/s^2

P15.20 (a) 100 N/m; (b) 1.13 Hz;
(c) 1.41 m/s at $x = 0$;
(d) 10.0 m/s^2 at $x = \pm A$; (e) 2.00 J;
(f) 1.33 m/s; (g) 3.33 m/s^2

P15.22 (a) 1.50 s; (b) 73.4 N/m;
(c) 19.7 m below the bridge; (d) 1.06 rad/s;
(e) 2.01 s; (f) 3.50 s

P15.24 (a) 0.218 s and 1.09 s; (b) 14.6 mW

P15.26 The position of the piston is given by $x = A\cos\omega t$.

P15.28 $\frac{g_C}{g_T} = 1.0015$

P15.30 1.42 s; 0.499 m

P15.32 (a) 3.65 s; (b) 6.41 s; (c) 4.24 s

P15.34 (a) see the solution;
(b), (c) 9.85 m/s^2; agreeing with the accepted value within 0.5%

P15.36 (a) 2.09 s; (b) 4.08%

P15.38 203 μN·m

P15.40 see the solution

P15.42 see the solution

P15.44 (a) 2.95 Hz; (b) 2.85 cm

P15.46 see the solution

P15.48 either 1.31 Hz or 0.641 Hz

P15.50 1.56 cm

P15.52 (a) 0.500 m/s; (b) 8.56 cm

P15.54 $A = \frac{\mu_s g}{4\pi^2 f^2}$

P15.56 see the solution

P15.58 (a) $k = \frac{4\pi^2 m}{T^2}$; (b) $m' = m\left(\frac{T'}{T}\right)^2$

P15.60 (a) $x = (-2\text{ m})\sin(10t)$; (b) at $x \pm 1.73$ m;
(c) 98.0 mm; (d) 52.4 ms

P15.62 (a) decreased by 0.735 m;
(b) increased by 0.730 s;
(c) decreased by 120 J; (d) see the solution

P15.64 (a) 3.56 Hz; (b) 2.79 Hz; (c) 2.10 Hz

P15.66 (a) $\frac{1}{2}\left(M + \frac{m}{3}\right)v^2$; (b) $T = 2\pi\sqrt{\frac{M + \frac{m}{3}}{k}}$

P15.68 see the solution; (a) $k = 1.74\text{ N/m} \pm 6\%$;
(b) $1.82\text{ N/m} \pm 3\%$; they agree;
(c) $8\text{ g} \pm 12\%$; it agrees

P15.70 (a) 5.20 s; (b) 2.60 s; (c) see the solution

P15.72 see the solution; $T = \left(\frac{2}{r}\right)\sqrt{\frac{\pi M}{\rho g}}$

P15.74 see the solution; (f) 0.281 Hz;
(g) decreases; (h) increases; (i) increases;
(j) decreases

16

Wave Motion

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q16.1 As the pulse moves down the string, the particles of the string itself move side to side. Since the medium—here, the string—moves perpendicular to the direction of wave propagation, the wave is transverse by definition.

Q16.2 To use a slinky to create a longitudinal wave, pull a few coils back and release. For a transverse wave, jostle the end coil side to side.

Q16.3 From $v=\sqrt{\frac{T}{\mu}}$, we must increase the tension by a factor of 4.

Q16.4 It depends on from what the wave reflects. If reflecting from a less dense string, the reflected part of the wave will be right side up.

Q16.5 Yes, among other things it depends on. $v_{\max}=\omega A=2\pi fA=\frac{2\pi vA}{\lambda}$. Here v is the speed of the wave.

Q16.6 Since the frequency is 3 cycles per second, the period is $\frac{1}{3}$ second = 333 ms.

Q16.7 Amplitude is increased by a factor of $\sqrt{2}$. The wave speed does not change.

Q16.8 The section of rope moves up and down in SHM. Its speed is always changing. The wave continues on with constant speed in one direction, setting further sections of the rope into up-and-down motion.

Q16.9 Each element of the rope must support the weight of the rope below it. The tension increases with height. (It increases linearly, if the rope does not stretch.) Then the wave speed $v=\sqrt{\frac{T}{\mu}}$ increases with height.

Q16.10 The difference is in the direction of motion of the elements of the medium. In longitudinal waves, the medium moves back and forth parallel to the direction of wave motion. In transverse waves, the medium moves perpendicular to the direction of wave motion.

Q16.11 Slower. Wave speed is inversely proportional to the square root of linear density.

Q16.12 As the wave passes from the massive string to the less massive string, the wave speed will increase according to $v=\sqrt{\frac{T}{\mu}}$. The frequency will remain unchanged. Since $v=f\lambda$, the wavelength must increase.

Q16.13 Higher tension makes wave speed higher. Greater linear density makes the wave move more slowly.

Q16.14 The wave speed is independent of the maximum particle speed. The source determines the maximum particle speed, through its frequency and amplitude. The wave speed depends instead on properties of the medium.

Q16.15 Longitudinal waves depend on the compressibility of the fluid for their propagation. Transverse waves require a restoring force in response to sheer strain. Fluids do not have the underlying structure to supply such a force. A fluid cannot support static sheer. A viscous fluid can temporarily be put under sheer, but the higher its viscosity the more quickly it converts input work into internal energy. A local vibration imposed on it is strongly damped, and not a source of wave propagation.

Q16.16 Let $\Delta t=t_s-t_p$ represent the difference in arrival times of the two waves at a station at distance $d=v_st_s=v_pt_p$ from the hypocenter. Then $d=\Delta t\left(\frac{1}{v_s}-\frac{1}{v_p}\right)^{-1}$. Knowing the distance from the first station places the hypocenter on a sphere around it. A measurement from a second station limits it to another sphere, which intersects with the first in a circle. Data from a third non-collinear station will generally limit the possibilities to a point.

Q16.17 The speed of a wave on a "massless" string would be infinite!

SOLUTIONS TO PROBLEMS

Section 16.1 Propagation of a Disturbance

P16.1 Replace x by $x-vt=x-4.5t$

to get $\boxed{y=\dfrac{6}{\left[(x-4.5t)^2+3\right]}}$

P16.2

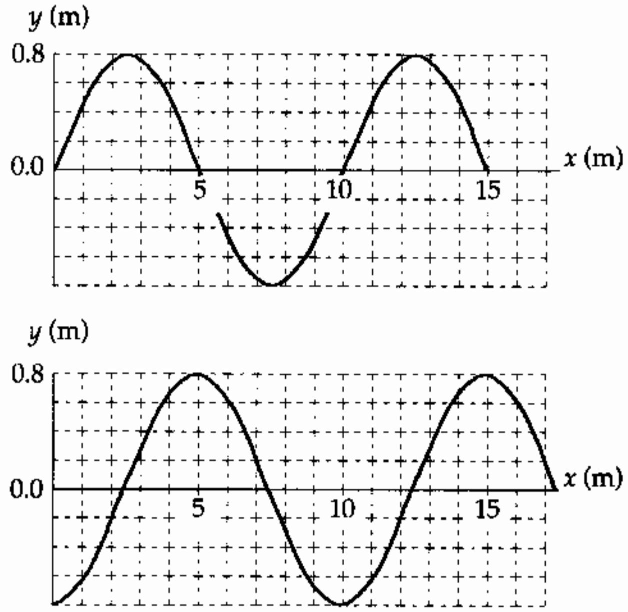


FIG. P16.2

P16.3 $5.00e^{-(x+5t)^2}$ is of the form $f(x+vt)$

so it describes a wave moving to the $\boxed{\text{left}}$ at $v = \boxed{5.00\text{ m/s}}$.

P16.4 (a) The $\boxed{\text{longitudinal}}$ wave travels a shorter distance and is moving faster, so it will arrive at point B first.

(b) The wave that travels through the Earth must travel

a distance of $2R\sin 30.0° = 2(6.37\times10^6\text{ m})\sin 30.0° = 6.37\times10^6\text{ m}$

at a speed of 7 800 m/s

Therefore, it takes $\dfrac{6.37\times10^6\text{ m}}{7\,800\text{ m/s}} = 817\text{ s}$

The wave that travels along the Earth's surface must travel

a distance of $s = R\theta = R\left(\dfrac{\pi}{3}\text{ rad}\right) = 6.67\times10^6\text{ m}$

at a speed of 4 500 m/s

Therefore, it takes $\dfrac{6.67\times10^6}{4\,500} = 1\,482\text{ s}$

The time difference is $\boxed{665\text{ s}} = 11.1\text{ min}$

P16.5 The distance the waves have traveled is $d = (7.80\ \text{km/s})t = (4.50\ \text{km/s})(t + 17.3\ \text{s})$

where t is the travel time for the faster wave.

Then, $(7.80 - 4.50)(\text{km/s})t = (4.50\ \text{km/s})(17.3\ \text{s})$

or $t = \dfrac{(4.50\ \text{km/s})(17.3\ \text{s})}{(7.80 - 4.50)\ \text{km/s}} = 23.6\ \text{s}$

and the distance is $d = (7.80\ \text{km/s})(23.6\ \text{s}) = \boxed{184\ \text{km}}$.

Section 16.2 **Sinusoidal Waves**

P16.6 Using data from the observations, we have $\lambda = 1.20\ \text{m}$

and $f = \dfrac{8.00}{12.0\ \text{s}}$

Therefore, $v = \lambda f = (1.20\ \text{m})\left(\dfrac{8.00}{12.0\ \text{s}}\right) = \boxed{0.800\ \text{m/s}}$

P16.7 $f = \dfrac{40.0\ \text{vibrations}}{30.0\ \text{s}} = \dfrac{4}{3}\ \text{Hz}$ $\qquad v = \dfrac{425\ \text{cm}}{10.0\ \text{s}} = 42.5\ \text{cm/s}$

$\lambda = \dfrac{v}{f} = \dfrac{42.5\ \text{cm/s}}{\frac{4}{3}\ \text{Hz}} = 31.9\ \text{cm} = \boxed{0.319\ \text{m}}$

P16.8 $v = f\lambda = (4.00\ \text{Hz})(60.0\ \text{cm}) = 240\ \text{cm/s} = \boxed{2.40\ \text{m/s}}$

P16.9 $y = (0.020\,0\ \text{m})\sin(2.11x - 3.62t)$ in SI units $\qquad A = \boxed{2.00\ \text{cm}}$

$k = 2.11\ \text{rad/m}$ $\qquad \lambda = \dfrac{2\pi}{k} = \boxed{2.98\ \text{m}}$

$\omega = 3.62\ \text{rad/s}$ $\qquad f = \dfrac{\omega}{2\pi} = \boxed{0.576\ \text{Hz}}$

$v = f\lambda = \dfrac{\omega}{2\pi}\dfrac{2\pi}{k} = \dfrac{3.62}{2.11} = \boxed{1.72\ \text{m/s}}$

P16.10 $y = (0.005\,1\ \text{m})\sin(310x - 9.30t)$ SI units

$v = \dfrac{\omega}{k} = \dfrac{9.30}{310} = 0.030\,0\ \text{m/s}$

$s = vt = \boxed{0.300\ \text{m in positive } x\text{-direction}}$

***P16.11** From $y = (12.0\text{ cm})\sin\left((1.57\text{ rad/m})x - (31.4\text{ rad/s})t\right)$

(a) The transverse velocity is $\frac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$

Its maximum magnitude is $A\omega = 12\text{ cm}(31.4\text{ rad/s}) = \boxed{3.77\text{ m/s}}$

(b) $a_y = \frac{\partial v_y}{\partial t} = \frac{\partial}{\partial t}\left(-A\omega\cos(kx - \omega t)\right) = -A\omega^2\sin(kx - \omega t)$

The maximum value is $A\omega^2 = (0.12\text{ m})\left(31.4\text{ s}^{-1}\right)^2 = \boxed{118\text{ m/s}^2}$

P16.12 At time t, the phase of $y = (15.0\text{ cm})\cos(0.157x - 50.3t)$ at coordinate x is

$\phi = (0.157\text{ rad/cm})x - (50.3\text{ rad/s})t$. Since $60.0° = \frac{\pi}{3}$ rad, the requirement for point B is that

$\phi_B = \phi_A \pm \frac{\pi}{3}$ rad, or (since $x_A = 0$),

$$(0.157\text{ rad/cm})x_B - (50.3\text{ rad/s})t = 0 - (50.3\text{ rad/s})t \pm \frac{\pi}{3}\text{ rad}.$$

This reduces to $x_B = \frac{\pm\pi\text{ rad}}{3(0.157\text{ rad/cm})} = \boxed{\pm 6.67\text{ cm}}$.

P16.13 $y = 0.250\sin(0.300x - 40.0t)$ m

Compare this with the general expression $y = A\sin(kx - \omega t)$

(a) $A = \boxed{0.250\text{ m}}$

(b) $\omega = \boxed{40.0\text{ rad/s}}$

(c) $k = \boxed{0.300\text{ rad/m}}$

(d) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.300\text{ rad/m}} = \boxed{20.9\text{ m}}$

(e) $v = f\lambda = \left(\frac{\omega}{2\pi}\right)\lambda = \left(\frac{40.0\text{ rad/s}}{2\pi}\right)(20.9\text{ m}) = \boxed{133\text{ m/s}}$

(f) The wave moves to the right, $\boxed{\text{in } +x \text{ direction}}$.

P16.14 (a) See figure at right.

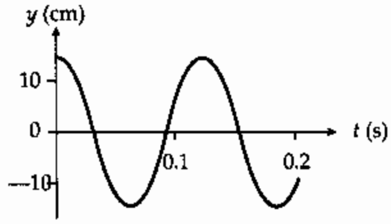


FIG. P16.14

(b) $T = \frac{2\pi}{\omega} = \frac{2\pi}{50.3} = \boxed{0.125\text{ s}}$

This agrees with the period found in the example in the text.

P16.15 (a) $A = y_{max} = 8.00\text{ cm} = 0.080\,0\text{ m}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.800\text{ m})} = 7.85\text{ m}^{-1}$

$\omega = 2\pi f = 2\pi(3.00) = 6.00\pi\text{ rad/s}$

Therefore, $y = A\sin(kx + \omega t)$

Or (where $y(0, t) = 0$ at $t = 0$) $\boxed{y = (0.080\,0)\sin(7.85x + 6\pi t)\text{ m}}$

(b) In general, $y = 0.080\,0\sin(7.85x + 6\pi t + \phi)$

Assuming $y(x, 0) = 0$ at $x = 0.100$ m

then we require that $0 = 0.080\,0\sin(0.785 + \phi)$

or $\phi = -0.785$

Therefore, $\boxed{y = 0.080\,0\sin(7.85x + 6\pi t - 0.785)\text{ m}}$

P16.16 (a)

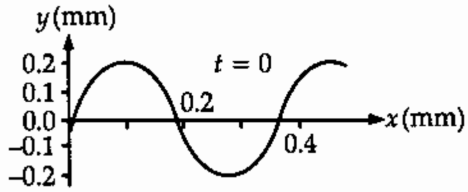


FIG. P16.16(a)

(b) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.350\text{ m}} = \boxed{18.0\text{ rad/m}}$

$T = \frac{1}{f} = \frac{1}{12.0/\text{s}} = \boxed{0.083\,3\text{ s}}$

$\omega = 2\pi f = 2\pi 12.0/\text{s} = \boxed{75.4\text{ rad/s}}$

$|v| = f\lambda = (12.0/\text{s})(0.350\text{ m}) = \boxed{4.20\text{ m/s}}$

(c) $y = A\sin(kx + \omega t + \phi)$ specializes to

$y = 0.200\text{ m}\sin(18.0x/\text{m} + 75.4t/\text{s} + \phi)$

at $x = 0$, $t = 0$ we require

$-3.00\times10^{-2}\text{ m} = 0.200\text{ m}\sin(+\phi)$

$\phi = -8.63° = -0.151\text{ rad}$

so $y(x, t) = \boxed{(0.200\text{ m})\sin(18.0x/\text{m} + 75.4t/\text{s} - 0.151\text{ rad})}$

P16.17 $y = (0.120\text{ m})\sin\left(\frac{\pi}{8}x + 4\pi t\right)$

(a) $v = \frac{dy}{dt}$: $\quad x = (0.120)(4\pi)\cos\left(\frac{\pi}{8}x + 4\pi t\right)$

$$v(0.200\text{ s},\ 1.60\text{ m}) = \boxed{-1.51\text{ m/s}}$$

$a = \frac{dv}{dt}$: $\quad a = (-0.120\text{ m})(4\pi)^2\sin\left(\frac{\pi}{8}x + 4\pi t\right)$

$$a(0.200\text{ s},\ 1.60\text{ m}) = \boxed{0}$$

(b) $k = \frac{\pi}{8} = \frac{2\pi}{\lambda}$: $\quad \lambda = \boxed{16.0\text{ m}}$

$\omega = 4\pi = \frac{2\pi}{T}$: $\quad T = \boxed{0.500\text{ s}}$

$$v = \frac{\lambda}{T} = \frac{16.0\text{ m}}{0.500\text{ s}} = \boxed{32.0\text{ m/s}}$$

P16.18 (a) Let us write the wave function as $y(x,\ t) = A\sin(kx + \omega t + \phi)$

$$y(0,\ 0) = A\sin\phi = 0.020\,0\text{ m}$$

$$\left.\frac{dy}{dt}\right|_{0,\ 0} = A\omega\cos\phi = -2.00\text{ m/s}$$

Also, $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.025\,0\text{ s}} = 80.0\,\pi/\text{s}$

$$A^2 = x_i^2 + \left(\frac{v_i}{\omega}\right)^2 = (0.020\,0\text{ m})^2 + \left(\frac{2.00\text{ m/s}}{80.0\,\pi/\text{s}}\right)^2$$

$$A = \boxed{0.021\,5\text{ m}}$$

(b) $\frac{A\sin\phi}{A\cos\phi} = \frac{0.020\,0}{\frac{-2}{80.0\pi}} = -2.51 = \tan\phi$

Your calculator's answer $\tan^{-1}(-2.51) = -1.19$ rad has a negative sine and positive cosine, just the reverse of what is required. You must look beyond your calculator to find

$\phi = \pi - 1.19\text{ rad} = \boxed{1.95\text{ rad}}$

(c) $v_{y,\max} = A\omega = 0.021\,5\text{ m}(80.0\pi/\text{s}) = \boxed{5.41\text{ m/s}}$

(d) $\lambda = v_xT = (30.0\text{ m/s})(0.025\,0\text{ s}) = 0.750\text{ m}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.750\text{ m}} = 8.38/\text{m}$ $\qquad \omega = 80.0\pi/\text{s}$

$$\boxed{y(x,\ t) = (0.021\,5\text{ m})\sin(8.38x\text{ rad/m} + 80.0\pi t\text{ rad/s} + 1.95\text{ rad})}$$

P16.19 (a) $f = \frac{v}{\lambda} = \frac{(1.00 \text{ m/s})}{2.00 \text{ m}} = \boxed{0.500 \text{ Hz}}$

$\omega = 2\pi f = 2\pi(0.500/\text{s}) = \boxed{3.14 \text{ rad/s}}$

(b) $k = \frac{2\pi}{\lambda} = \frac{2\pi}{2.00 \text{ m}} = \boxed{3.14 \text{ rad/m}}$

(c) $y = A\sin(kx - \omega t + \phi)$ becomes

$y = \boxed{(0.100 \text{ m})\sin(3.14x/\text{m} - 3.14t/\text{s} + 0)}$

(d) For $x = 0$ the wave function requires

$\boxed{y = (0.100 \text{ m})\sin(-3.14t/\text{s})}$

(e) $\boxed{y = (0.100 \text{ m})\sin(4.71 \text{ rad} - 3.14t/\text{s})}$

(f) $v_y = \frac{\partial y}{\partial t} = 0.100 \text{ m}(-3.14/\text{s})\cos(3.14x/\text{m} - 3.14t/\text{s})$

The cosine varies between +1 and −1, so

$v_y \le (0.314 \text{ m/s})$

P16.20 (a) at $x = 2.00 \text{ m}$, $y = \boxed{(0.100 \text{ m})\sin(1.00 \text{ rad} - 20.0t)}$

(b) $y = (0.100 \text{ m})\sin(0.500x - 20.0t) = A\sin(kx - \omega t)$

so $\omega = 20.0 \text{ rad/s}$ and $f = \frac{\omega}{2\pi} = \boxed{3.18 \text{ Hz}}$

Section 16.3 **The Speed of Waves on Strings**

P16.21 The down and back distance is $4.00 \text{ m} + 4.00 \text{ m} = 8.00 \text{ m}$.

The speed is then $v = \frac{d_{\text{total}}}{t} = \frac{4(8.00 \text{ m})}{0.800 \text{ s}} = 40.0 \text{ m/s} = \sqrt{\frac{T}{\mu}}$

Now, $\mu = \frac{0.200 \text{ kg}}{4.00 \text{ m}} = 5.00 \times 10^{-2} \text{ kg/m}$

So $T = \mu v^2 = \left(5.00 \times 10^{-2} \text{ kg/m}\right)(40.0 \text{ m/s})^2 = \boxed{80.0 \text{ N}}$

P16.22 The mass per unit length is: $\mu = \frac{0.0600 \text{ kg}}{5.00 \text{ m}} = 1.20 \times 10^{-2} \text{ kg/m}$.

The required tension is: $T = \mu v^2 = (0.0120 \text{ kg/m})(50.0 \text{ m/s})^2 = \boxed{30.0 \text{ N}}$.

P16.23 $v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{1\,350\text{ kg}\cdot\text{m/s}^2}{5.00\times10^{-3}\text{ kg/m}}}=\boxed{520\text{ m/s}}$

P16.24 (a) $\omega=2\pi f=2\pi(500)=3\,140\text{ rad/s}$, $k=\frac{\omega}{v}=\frac{3\,140}{196}=16.0\text{ rad/m}$

$$\boxed{y=\left(2.00\times10^{-4}\text{ m}\right)\sin(16.0x-3\,140t)}$$

(b) $v=196\text{ m/s}=\sqrt{\frac{T}{4.10\times10^{-3}\text{ kg/m}}}$

$T=\boxed{158\text{ N}}$

P16.25 $T=Mg$ is the tension; $v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{Mg}{\frac{m}{L}}}=\sqrt{\frac{MgL}{m}}=\frac{L}{t}$ is the wave speed.

Then, $\frac{MgL}{m}=\frac{L^2}{t^2}$

and $g=\frac{Lm}{Mt^2}=\frac{1.60\text{ m}\left(4.00\times10^{-3}\text{ kg}\right)}{3.00\text{ kg}\left(3.61\times10^{-3}\text{ s}\right)^2}=\boxed{1.64\text{ m/s}^2}$

P16.26 $v=\sqrt{\frac{T}{\mu}}$

$T=\mu v^2=\rho Av^2=\rho\pi r^2v^2$

$T=\left(8\,920\text{ kg/m}^3\right)(\pi)\left(7.50\times10^{-4}\text{ m}\right)^2(200\text{ m/s})^2$

$T=\boxed{631\text{ N}}$

P16.27 Since μ is constant, $\mu=\frac{T_2}{v_2^2}=\frac{T_1}{v_1^2}$ and

$$T_2=\left(\frac{v_2}{v_1}\right)^2T_1=\left(\frac{30.0\text{ m/s}}{20.0\text{ m/s}}\right)^2(6.00\text{ N})=\boxed{13.5\text{ N}}.$$

P16.28 The period of the pendulum is $T=2\pi\sqrt{\frac{L}{g}}$

Let F represent the tension in the string (to avoid confusion with the period) when the pendulum is vertical and stationary. The speed of waves in the string is then:

$$v=\sqrt{\frac{F}{\mu}}=\sqrt{\frac{Mg}{\frac{m}{L}}}=\sqrt{\frac{MgL}{m}}$$

Since it might be difficult to measure L precisely, we eliminate $\sqrt{L}=\frac{T\sqrt{g}}{2\pi}$

so $v=\sqrt{\frac{Mg}{m}}\frac{T\sqrt{g}}{2\pi}=\boxed{\frac{Tg}{2\pi}\sqrt{\frac{M}{m}}}$.

P16.29 If the tension in the wire is T, the tensile stress is

$$\text{Stress} = \frac{T}{A} \quad \text{so} \quad T = A(\text{stress}).$$

The speed of transverse waves in the wire is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{A(\text{Stress})}{\frac{m}{L}}} = \sqrt{\frac{\text{Stress}}{\frac{m}{AL}}} = \sqrt{\frac{\text{Stress}}{\frac{m}{\text{Volume}}}} = \sqrt{\frac{\text{Stress}}{\rho}}$$

where ρ is the density. The maximum velocity occurs when the stress is a maximum:

$$v_{\max} = \sqrt{\frac{2.70 \times 10^8 \text{ Pa}}{7\,860 \text{ kg/m}^3}} = \boxed{185 \text{ m/s}}.$$

P16.30 From the free-body diagram $mg = 2T\sin\theta$

$$T = \frac{mg}{2\sin\theta}$$

The angle θ is found from $\cos\theta = \frac{\frac{3L}{8}}{\frac{L}{2}} = \frac{3}{4}$

$$\therefore \theta = 41.4°$$

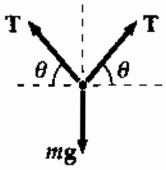


FIG. P16.30

(a) $v = \sqrt{\frac{T}{\mu}}$ $\quad v = \sqrt{\frac{mg}{2\mu\sin 41.4°}} = \left(\sqrt{\frac{9.80 \text{ m/s}^2}{2(8.00 \times 10^{-3} \text{ kg/m})\sin 41.4°}}\right)\sqrt{m}$

or $\quad v = \boxed{\left(30.4 \frac{\text{m/s}}{\sqrt{\text{kg}}}\right)\sqrt{m}}$

(b) $v = 60.0 = 30.4\sqrt{m}$ and $\boxed{m = 3.89 \text{ kg}}$

P16.31 The total time is the sum of the two times.

In each wire $\quad t = \frac{L}{v} = L\sqrt{\frac{\mu}{T}}$

Let A represent the cross-sectional area of one wire. The mass of one wire can be written both as $m = \rho V = \rho AL$ and also as $m = \mu L$.

Then we have $\quad \mu = \rho A = \frac{\pi\rho d^2}{4}$

Thus, $\quad t = L\left(\frac{\pi\rho d^2}{4T}\right)^{1/2}$

For copper, $\quad t = (20.0)\left[\frac{(\pi)(8\,920)(1.00 \times 10^{-3})^2}{(4)(150)}\right]^{1/2} = 0.137 \text{ s}$

For steel, $\quad t = (30.0)\left[\frac{(\pi)(7\,860)(1.00 \times 10^{-3})^2}{(4)(150)}\right]^{1/2} = 0.192 \text{ s}$

The total time is $\quad 0.137 + 0.192 = \boxed{0.329 \text{ s}}$

P16.32 Refer to the diagrams. From the free-body diagram of point A:

$$\sum F_y = 0 \Rightarrow T_1 \sin\theta = Mg \qquad \text{and} \qquad \sum F_x = 0 \Rightarrow T_1 \cos\theta = T$$

Combining these equations to eliminate T_1 gives the tension in the string connecting points A and B as: $T = \dfrac{Mg}{\tan\theta}$.

The speed of transverse waves in this segment of string is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\frac{Mg}{\tan\theta}}{\frac{m}{L}}} = \sqrt{\frac{MgL}{m\tan\theta}}$$

and the time for a pulse to travel from A to B is

$$t = \frac{\frac{L}{2}}{v} = \boxed{\sqrt{\frac{mL\tan\theta}{4Mg}}}.$$

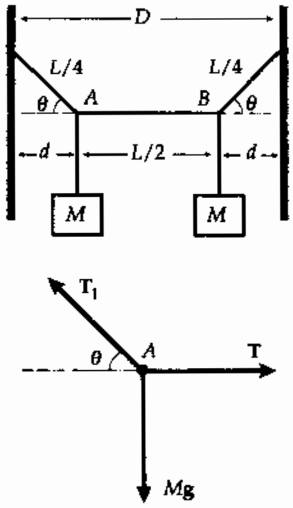


FIG. P16.32

***P16.33** (a) f has units $\text{Hz} = 1/\text{s}$, so $T = \dfrac{1}{f}$ has units of seconds, $\boxed{\text{s}}$. For the other T we have $T = \mu v^2$,

with units $\dfrac{\text{kg}}{\text{m}}\dfrac{\text{m}^2}{\text{s}^2} = \dfrac{\text{kg}\cdot\text{m}}{\text{s}^2} = \boxed{\text{N}}$.

(b) The first T is $\boxed{\text{period}}$ of time; the second is $\boxed{\text{force}}$ of tension.

Section 16.4 Reflection and Transmission

Problem 7 in Chapter 18 can be assigned with this section.

Section 16.5 Rate of Energy Transfer by Sinusoidal Waves on Strings

P16.34 $f = \dfrac{v}{\lambda} = \dfrac{30.0}{0.500} = 60.0 \text{ Hz}$ $\qquad \omega = 2\pi f = 120\pi \text{ rad/s}$

$$\mathcal{P} = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}\left(\frac{0.180}{3.60}\right)(120\pi)^2(0.100)^2(30.0) = \boxed{1.07 \text{ kW}}$$

P16.35 Suppose that no energy is absorbed or carried down into the water. Then a fixed amount of power is spread thinner farther away from the source, spread over the circumference $2\pi r$ of an expanding circle. The power-per-width across the wave front

$$\frac{\mathcal{P}}{2\pi r}$$

is proportional to amplitude squared so amplitude is proportional to

$$\sqrt{\frac{\mathcal{P}}{2\pi r}}.$$

P16.36 $T =$ constant; $v = \sqrt{\frac{T}{\mu}}$; $\mathscr{P} = \frac{1}{2}\mu\omega^2 A^2 v$

(a) If L is doubled, v remains constant and $\boxed{\mathscr{P} \text{ is constant}}$.

(b) If A is doubled and ω is halved, $\mathscr{P} \propto \omega^2 A^2$ $\boxed{\text{remains constant}}$.

(c) If λ and A are doubled, the product $\omega^2 A^2 \propto \frac{A^2}{\lambda^2}$ remains constant, so $\boxed{\mathscr{P} \text{ remains constant}}$.

(d) If L and λ are halved, then $\omega^2 \propto \frac{1}{\lambda^2}$ is quadrupled, so $\boxed{\mathscr{P} \text{ is quadrupled}}$.
(Changing L doesn't affect $\mathscr{P}$).

P16.37 $A = 5.00 \times 10^{-2}$ m $\mu = 4.00 \times 10^{-2}$ kg/m $\mathscr{P} = 300$ W $T = 100$ N

Therefore, $v = \sqrt{\frac{T}{\mu}} = 50.0$ m/s

$\mathscr{P} = \frac{1}{2}\mu\omega^2 A^2 v$: $\omega^2 = \frac{2\mathscr{P}}{\mu A^2 v} = \frac{2(300)}{(4.00 \times 10^{-2})(5.00 \times 10^{-2})^2 (50.0)}$

$\omega = 346$ rad/s

$f = \frac{\omega}{2\pi} = \boxed{55.1 \text{ Hz}}$

P16.38 $\mu = 30.0$ g/m $= 30.0 \times 10^{-3}$ kg/m

$\lambda = 1.50$ m

$f = 50.0$ Hz: $\omega = 2\pi f = 314\ \text{s}^{-1}$

$2A = 0.150$ m: $A = 7.50 \times 10^{-2}$ m

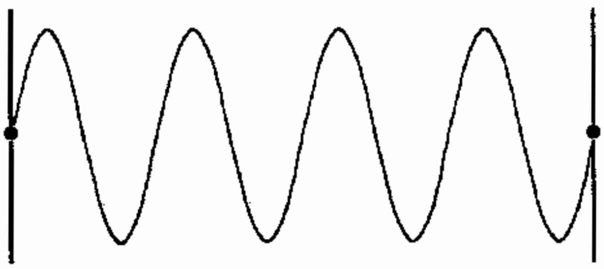


FIG. P16.38

(a) $y = A\sin\left(\frac{2\pi}{\lambda}x - \omega t\right)$

$\boxed{y = (7.50 \times 10^{-2})\sin(4.19x - 314t)}$

(b) $\mathscr{P} = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}(30.0 \times 10^{-3})(314)^2(7.50 \times 10^{-2})^2\left(\frac{314}{4.19}\right)$ W $\boxed{\mathscr{P} = 625 \text{ W}}$

P16.39 (a) $v = f\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{\omega}{k} = \frac{50.0}{0.800}$ m/s $= \boxed{62.5 \text{ m/s}}$

(b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.800}$ m $= \boxed{7.85 \text{ m}}$

(c) $f = \frac{50.0}{2\pi} = \boxed{7.96 \text{ Hz}}$

(d) $\mathscr{P} = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}(12.0 \times 10^{-3})(50.0)^2(0.150)^2(62.5)$ W $= \boxed{21.1 \text{ W}}$

***P16.40** Comparing $y = 0.35\sin\left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$ with $y = A\sin(kx - \omega t + \phi) = A\sin(\omega t - kx - \phi + \pi)$ we have

$k = \frac{3\pi}{m}$, $\omega = 10\pi/\text{s}$, $A = 0.35$ m. Then $v = f\lambda = 2\pi f\frac{\lambda}{2\pi} = \frac{\omega}{k} = \frac{10\pi/\text{s}}{3\pi/\text{m}} = 3.33$ m/s.

(a) The rate of energy transport is

$$\mathcal{P} = \frac{1}{2}\mu\omega^2A^2v = \frac{1}{2}\left(75\times10^{-3}\ \text{kg/m}\right)\left(10\pi/\text{s}\right)^2(0.35\ \text{m})^2 3.33\ \text{m/s} = \boxed{15.1\ \text{W}}.$$

(b) The energy per cycle is

$$E_\lambda = \mathcal{P}T = \frac{1}{2}\mu\omega^2A^2\lambda = \frac{1}{2}\left(75\times10^{-3}\ \text{kg/m}\right)\left(10\pi/\text{s}\right)^2(0.35\ \text{m})^2\frac{2\pi\ \text{m}}{3\pi} = \boxed{3.02\ \text{J}}.$$

P16.41 Originally,

$$\mathcal{P}_0 = \frac{1}{2}\mu\omega^2A^2v$$

$$\mathcal{P}_0 = \frac{1}{2}\mu\omega^2A^2\sqrt{\frac{T}{\mu}}$$

$$\mathcal{P}_0 = \frac{1}{2}\omega^2A^2\sqrt{T\mu}$$

The doubled string will have doubled mass-per-length. Presuming that we hold tension constant, it can carry power larger by $\sqrt{2}$ times.

$$\boxed{\sqrt{2}\mathcal{P}_0} = \frac{1}{2}\omega^2A^2\sqrt{T2\mu}$$

***P16.42** As for a strong wave, the rate of energy transfer is proportional to the square of the amplitude and to the speed. We write $\mathcal{P} = FvA^2$ where F is some constant. With no absorption of energy,

$$Fv_{\text{bedrock}}A^2_{\text{bedrock}} = Fv_{\text{mudfill}}A^2_{\text{mudfill}}$$

$$\sqrt{\frac{v_{\text{bedrock}}}{v_{\text{mudfill}}}} = \frac{A_{\text{mudfill}}}{A_{\text{bedrock}}} = \sqrt{\frac{25v_{\text{mudfill}}}{v_{\text{mudfill}}}} = 5$$

The amplitude increases by 5.00 times.

Section 16.6 The Linear Wave Equation

P16.43 (a) $A=(7.00+3.00)4.00$ yields $\boxed{A=40.0}$

(b) In order for two vectors to be equal, they must have the same magnitude and the same direction in three-dimensional space. All of their components must be equal. Thus, $7.00\hat{\mathbf{i}}+0\hat{\mathbf{j}}+3.00\hat{\mathbf{k}}=A\hat{\mathbf{i}}+B\hat{\mathbf{j}}+C\hat{\mathbf{k}}$ requires $\boxed{A=7.00,\ B=0,\ \text{and}\ C=3.00}$.

(c) In order for two functions to be identically equal, they must be equal for every value of every variable. They must have the same graphs. In

$$A+B\cos(Cx+Dt+E)=0+7.00\ \text{mm}\cos(3.00x+4.00t+2.00),$$

the equality of average values requires that $\boxed{A=0}$. The equality of maximum values requires $\boxed{B=7.00\ \text{mm}}$. The equality for the wavelength or periodicity as a function of x requires $\boxed{C=3.00\ \text{rad/m}}$. The equality of period requires $\boxed{D=4.00\ \text{rad/s}}$, and the equality of zero-crossings requires $\boxed{E=2.00\ \text{rad}}$.

***P16.44** The linear wave equation is $\dfrac{\partial^2 y}{\partial x^2}=\dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$

If $y=e^{b(x-vt)}$

then $\dfrac{\partial y}{\partial t}=-bve^{b(x-vt)}$ and $\dfrac{\partial y}{\partial x}=be^{b(x-vt)}$

$$\frac{\partial^2 y}{\partial t^2}=b^2v^2e^{b(x-vt)}\ \text{and}\ \frac{\partial^2 y}{\partial x^2}=b^2e^{b(x-vt)}$$

Therefore, $\dfrac{\partial^2 y}{\partial t^2}=v^2\dfrac{\partial^2 y}{\partial x^2}$, demonstrating that $e^{b(x-vt)}$ is a solution

P16.45 The linear wave equation is $\dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}=\dfrac{\partial^2 y}{\partial x^2}$

To show that $y=\ln[b(x-vt)]$ is a solution, we find its first and second derivatives with respect to x and t and substitute into the equation.

$$\frac{\partial y}{\partial t}=\frac{1}{b(x-vt)}(-bv) \qquad \frac{\partial^2 y}{\partial t^2}=\frac{-1(-bv)^2}{b^2(x-vt)^2}=-\frac{v^2}{(x-vt)^2}$$

$$\frac{\partial y}{\partial x}=[b(x-vt)]^{-1}b \qquad \frac{\partial^2 y}{\partial x^2}=-\frac{b}{b}(x-vt)^2=-\frac{1}{(x-vt)^2}$$

Then $\dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}=\dfrac{1}{v^2}\dfrac{(-v^2)}{(x-vt)^2}=-\dfrac{1}{(x-vt)^2}=\dfrac{\partial^2 y}{\partial x^2}$ so the given wave function is a solution.

P16.46 (a) From $y = x^2 + v^2t^2$,

evaluate $\frac{\partial y}{\partial x} = 2x$ $\quad \frac{\partial^2 y}{\partial x^2} = 2$

$\frac{\partial y}{\partial t} = v^2 2t$ $\quad \frac{\partial^2 y}{\partial t^2} = 2v^2$

Does $\frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$?

By substitution: $2 = \frac{1}{v^2}2v^2$ and this is true, so the wave function does satisfy the wave equation.

(b) Note $\frac{1}{2}(x+vt)^2 + \frac{1}{2}(x-vt)^2 = \frac{1}{2}x^2 + xvt + \frac{1}{2}v^2t^2 + \frac{1}{2}x^2 - xvt + \frac{1}{2}v^2t^2$
$= x^2 + v^2t^2$ as required.

So $\boxed{f(x+vt) = \frac{1}{2}(x+vt)^2}$ and $\boxed{g(x-vt) = \frac{1}{2}(x-vt)^2}$.

(c) $y = \sin x \cos vt$ makes

$\frac{\partial y}{\partial x} = \cos x \cos vt$ $\quad \frac{\partial^2 y}{\partial x^2} = -\sin x \cos vt$

$\frac{\partial y}{\partial t} = -v \sin x \sin vt$ $\quad \frac{\partial^2 y}{\partial t^2} = -v^2 \sin x \cos vt$

Then $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

becomes $-\sin x \cos vt = \frac{-1}{v^2}v^2 \sin x \cos vt$ which is true as required.

Note $\sin(x+vt) = \sin x \cos vt + \cos x \sin vt$

$\sin(x-vt) = \sin x \cos vt - \cos x \sin vt$.

So $\sin x \cos vt = f(x+vt) + g(x-vt)$ with

$\boxed{f(x+vt) = \frac{1}{2}\sin(x+vt)}$ and $\boxed{g(x-vt) = \frac{1}{2}\sin(x-vt)}$.

Additional Problems

P16.47 Assume a typical distance between adjacent people ~ 1 m.

Then the wave speed is $\quad v = \frac{\Delta x}{\Delta t} \sim \frac{1 \text{ m}}{0.1 \text{ s}} \sim 10 \text{ m/s}$

Model the stadium as a circle with a radius of order 100 m. Then, the time for one circuit around the stadium is

$$T = \frac{2\pi r}{v} \sim \frac{2\pi(10^2)}{10 \text{ m/s}} = 63 \text{ s} \boxed{\sim 1 \text{ min}}.$$

P16.48 Compare the given wave function $y = 4.00\sin(2.00x - 3.00t)$ cm to the general form $y = A\sin(kx - \omega t)$ to find

(a) amplitude $A = 4.00\text{ cm} = \boxed{0.040\,0\text{ m}}$

(b) $k = \dfrac{2\pi}{\lambda} = 2.00\text{ cm}^{-1}$ and $\lambda = \pi\text{ cm} = \boxed{0.031\,4\text{ m}}$

(c) $\omega = 2\pi f = 3.00\text{ s}^{-1}$ and $f = \boxed{0.477\text{ Hz}}$

(d) $T = \dfrac{1}{f} = \boxed{2.09\text{ s}}$

(e) The minus sign indicates that the wave is traveling in the $\boxed{\text{positive } x\text{-direction}}$.

P16.49 (a) Let $u = 10\pi t - 3\pi x + \dfrac{\pi}{4}$ $\qquad \dfrac{du}{dt} = 10\pi - 3\pi\dfrac{dx}{dt} = 0$ at a point of constant phase

$$\frac{dx}{dt} = \frac{10}{3} = \boxed{3.33\text{ m/s}}$$

The velocity is in the $\boxed{\text{positive } x\text{-direction}}$.

(b) $y(0.100,\ 0) = (0.350\text{ m})\sin\left(-0.300\pi + \dfrac{\pi}{4}\right) = -0.054\,8\text{ m} = \boxed{-5.48\text{ cm}}$

(c) $k = \dfrac{2\pi}{\lambda} = 3\pi:\ \lambda = \boxed{0.667\text{ m}}$ $\qquad \omega = 2\pi f = 10\pi:\ f = \boxed{5.00\text{ Hz}}$

(d) $v_y = \dfrac{\partial y}{\partial t} = (0.350)(10\pi)\cos\left(10\pi t - 3\pi x + \dfrac{\pi}{4}\right)$ $\qquad v_{y,\max} = (10\pi)(0.350) = \boxed{11.0\text{ m/s}}$

***P16.50** (a) $0.175\text{ m} = (0.350\text{ m})\sin[(99.6\text{ rad/s})t]$

$\therefore \sin[(99.6\text{ rad/s})t] = 0.5$

The smallest two angles for which the sine function is 0.5 are 30° and 150°, i.e., 0.523 6 rad and 2.618 rad.

$(99.6\text{ rad/s})t_1 = 0.523\,6\text{ rad}$, thus $t_1 = 5.26\text{ ms}$

$(99.6\text{ rad/s})t_2 = 2.618\text{ rad}$, thus $t_2 = 26.3\text{ ms}$

$\Delta t \equiv t_2 - t_1 = 26.3\text{ ms} - 5.26\text{ ms} = \boxed{21.0\text{ ms}}$

(b) Distance traveled by the wave $= \left(\dfrac{\omega}{k}\right)\Delta t = \left(\dfrac{99.6\text{ rad/s}}{1.25\text{ rad/m}}\right)(21.0\times10^{-3}\text{ s}) = \boxed{1.68\text{ m}}$.

P16.51 The equation $v = \lambda f$ is a special case of

speed = (cycle length)(repetition rate).

Thus, $v = (19.0\times10^{-3}\text{ m/frame})(24.0\text{ frames/s}) = \boxed{0.456\text{ m/s}}$.

P16.52 Assuming the incline to be frictionless and taking the positive x-direction to be up the incline:

$\sum F_x = T - Mg\sin\theta = 0$ or the tension in the string is $T = Mg\sin\theta$

The speed of transverse waves in the string is then $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{Mg\sin\theta}{\frac{m}{L}}} = \sqrt{\dfrac{MgL\sin\theta}{m}}$

The time interval for a pulse to travel the string's length is $\Delta t = \dfrac{L}{v} = L\sqrt{\dfrac{m}{MgL\sin\theta}} = \boxed{\sqrt{\dfrac{mL}{Mg\sin\theta}}}$

P16.53 Energy is conserved as the block moves down distance x:

$$\left(K + U_g + U_s\right)_{\text{top}} + \Delta E = \left(K + U_g + U_s\right)_{\text{bottom}}$$

$$0 + Mgx + 0 + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$x = \frac{2Mg}{k}$$

(a) $T = kx = 2Mg = 2(2.00\text{ kg})(9.80\text{ m/s}^2) = \boxed{39.2\text{ N}}$

(b) $L = L_0 + x = L_0 + \dfrac{2Mg}{k}$

$L = 0.500\text{ m} + \dfrac{39.2\text{ N}}{100\text{ N/m}} = \boxed{0.892\text{ m}}$

(c) $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{TL}{m}}$

$v = \sqrt{\dfrac{39.2\text{ N} \times 0.892\text{ m}}{5.0 \times 10^{-3}\text{ kg}}}$

$v = \boxed{83.6\text{ m/s}}$

P16.54 $Mgx = \dfrac{1}{2}kx^2$

(a) $T = kx = \boxed{2Mg}$

(b) $L = L_0 + x = \boxed{L_0 + \dfrac{2Mg}{k}}$

(c) $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{TL}{m}} = \boxed{\sqrt{\dfrac{2Mg}{m}\left(L_0 + \dfrac{2Mg}{k}\right)}}$

P16.55 (a) $v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{80.0\text{ N}}{\left(5.00\times10^{-3}\text{ kg}/2.00\text{ m}\right)}}=\boxed{179\text{ m/s}}$

(b) From Equation 16.21, $\mathcal{P}=\frac{1}{2}\mu v\omega^2A^2$ and $\omega=2\pi\left(\frac{v}{\lambda}\right)$

$$\mathcal{P}=\frac{1}{2}\mu vA^2\left(\frac{2\pi v}{\lambda}\right)^2=\frac{2\pi^2\mu A^2v^3}{\lambda^2}$$

$$\mathcal{P}=\frac{2\pi^2\left(\frac{5.00\times10^{-3}\text{ kg}}{2.00\text{ m}}\right)(0.040\,0\text{ m})^2(179\text{ m/s})^3}{(0.160\text{ m})^2}$$

$$\mathcal{P}=1.77\times10^4\text{ W}=\boxed{17.7\text{ kW}}$$

P16.56 $v=\sqrt{\frac{T}{\mu}}$ and in this case $T=mg$; therefore, $m=\frac{\mu v^2}{g}$.

Now $v=f\lambda$ implies $v=\frac{\omega}{k}$ so that

$$m=\frac{\mu}{g}\left(\frac{\omega}{k}\right)^2=\frac{0.250\text{ kg/m}}{9.80\text{ m/s}^2}\left[\frac{18\pi\text{ s}^{-1}}{0.750\pi\text{ m}^{-1}}\right]^2=\boxed{14.7\text{ kg}}.$$

***P16.57** Let M = mass of block, m = mass of string. For the block, $\sum F=ma$ implies $T=\frac{mv_b^2}{r}=m\omega^2r$. The speed of a wave on the string is then

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{M\omega^2r}{\frac{m}{r}}}=r\omega\sqrt{\frac{M}{m}}$$

$$t=\frac{r}{v}=\frac{1}{\omega}\sqrt{\frac{m}{M}}$$

$$\theta=\omega t=\sqrt{\frac{m}{M}}=\sqrt{\frac{0.003\,2\text{ kg}}{0.450\text{ kg}}}=\boxed{0.084\,3\text{ rad}}$$

P16.58 (a) $\mu=\frac{dm}{dL}=\rho A\frac{dx}{dx}=\rho A$

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{T}{\rho A}}=\sqrt{\frac{T}{\left[\rho(ax+b)\right]}}=\sqrt{\frac{T}{\left[\rho\left(10^{-3}x+10^{-2}\right)\text{cm}^2\right]}}$$

With all SI units, $\boxed{v=\sqrt{\frac{T}{\left[\rho\left(10^{-3}x+10^{-2}\right)10^{-4}\right]}}\text{ m/s}}$

(b) $v\big|_{x=0}=\sqrt{\frac{24.0}{\left[(2\,700)\left(0+10^{-2}\right)\left(10^{-4}\right)\right]}}=\boxed{94.3\text{ m/s}}$

$$v\big|_{x=10.0}=\sqrt{\frac{24.0}{\left[(2\,700)\left(10^{-2}+10^{-2}\right)\left(10^{-4}\right)\right]}}=\boxed{66.7\text{ m/s}}$$

P16.59 $v=\sqrt{\frac{T}{\mu}}$ where $T=\mu x g$, the weight of a length x, of rope.

Therefore, $v=\sqrt{gx}$

But $v=\frac{dx}{dt}$, so that $dt=\frac{dx}{\sqrt{gx}}$

and $$t=\int_0^L \frac{dx}{\sqrt{gx}}=\frac{1}{\sqrt{g}}\frac{\sqrt{x}}{\frac{1}{2}}\Bigg|_0^L=\boxed{2\sqrt{\frac{L}{g}}}$$

P16.60 At distance x from the bottom, the tension is $T=\left(\frac{mxg}{L}\right)+Mg$, so the wave speed is:

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{TL}{m}}=\sqrt{xg+\left(\frac{MgL}{m}\right)}=\frac{dx}{dt}.$$

(a) Then $t=\int_0^t dt=\int_0^L\left[xg+\left(\frac{MgL}{m}\right)\right]^{-1/2}dx$ $\qquad t=\frac{1}{g}\frac{\left[xg+(MgL/m)\right]^{1/2}}{\frac{1}{2}}\Bigg|_{x=0}^{x=L}$

$$t=\frac{2}{g}\left[\left(Lg+\frac{MgL}{m}\right)^{1/2}-\left(\frac{MgL}{m}\right)^{1/2}\right] \qquad \boxed{t=2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{m+M}-\sqrt{M}}{\sqrt{m}}\right)}$$

(b) When $M=0$, as in the previous problem, $t=2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{m}-0}{\sqrt{m}}\right)=\boxed{2\sqrt{\frac{L}{g}}}$

(c) As $m\to 0$ we expand $\sqrt{m+M}=\sqrt{M}\left(1+\frac{m}{M}\right)^{1/2}=\sqrt{M}\left(1+\frac{1}{2}\frac{m}{M}-\frac{1}{8}\frac{m^2}{M^2}+\ldots\right)$

to obtain $$t=2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{M}+\frac{1}{2}\left(m/\sqrt{M}\right)-\frac{1}{8}\left(m^2/M^{3/2}\right)+\ldots-\sqrt{M}}{\sqrt{m}}\right)$$

$$t\approx 2\sqrt{\frac{L}{g}}\left(\frac{1}{2}\sqrt{\frac{m}{M}}\right)=\boxed{\sqrt{\frac{mL}{Mg}}}$$

P16.61 (a) The speed in the lower half of a rope of length L is the same function of distance (from the bottom end) as the speed along the entire length of a rope of length $\left(\frac{L}{2}\right)$.

Thus, the time required $=2\sqrt{\frac{L'}{g}}$ with $L'=\frac{L}{2}$

and the time required $=2\sqrt{\frac{L}{2g}}=\boxed{0.707\left(2\sqrt{\frac{L}{g}}\right)}$.

It takes the pulse more that 70% of the total time to cover 50% of the distance.

(b) By the same reasoning applied in part (a), the distance climbed in τ is given by $d=\frac{g\tau^2}{4}$.

For $\tau=\frac{t}{2}=\sqrt{\frac{L}{g}}$, we find the *distance climbed* $=\boxed{\frac{L}{4}}$.

In half the total trip time, the pulse has climbed $\frac{1}{4}$ of the total length.

P16.62 (a) $v=\frac{\omega}{k}=\frac{15.0}{3.00}=\boxed{5.00\text{ m/s in positive }x\text{-direction}}$

(b) $v=\frac{15.0}{3.00}=\boxed{5.00\text{ m/s in negative }x\text{-direction}}$

(c) $v=\frac{15.0}{2.00}=\boxed{7.50\text{ m/s in negative }x\text{-direction}}$

(d) $v=\frac{12.0}{\frac{1}{2}}=\boxed{24.0\text{ m/s in positive }x\text{-direction}}$

P16.63 Young's modulus for the wire may be written as $Y=\frac{\frac{T}{A}}{\frac{\Delta L}{L}}$, where T is the tension maintained in the wire and ΔL is the elongation produced by this tension. Also, the mass density of the wire may be expressed as $\rho=\frac{\mu}{A}$.

The speed of transverse waves in the wire is then

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{\frac{T}{A}}{\frac{\mu}{A}}}=\sqrt{\frac{Y\left(\frac{\Delta L}{L}\right)}{\rho}}$$

and the strain in the wire is $\frac{\Delta L}{L}=\frac{\rho v^2}{Y}$.

If the wire is aluminum and $v=100$ m/s, the strain is

$$\frac{\Delta L}{L}=\frac{\left(2.70\times10^3\text{ kg/m}^3\right)(100\text{ m/s})^2}{7.00\times10^{10}\text{ N/m}^2}=\boxed{3.86\times10^{-4}}.$$

***P16.64** (a) Consider a short section of chain at the top of the loop. A free-body diagram is shown. Its length is $s=R(2\theta)$ and its mass is $\mu R2\theta$. In the frame of reference of the center of the loop, Newton's second law is

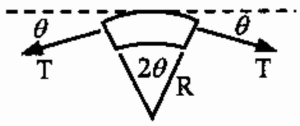


FIG. P16.64(a)

$$\sum F_y=ma_y \qquad 2T\sin\theta\text{ down}=\frac{mv_0^2}{R}\text{ down}=\frac{\mu R2\theta v_0^2}{R}$$

For a very short section, $\sin\theta=\theta$ and $\boxed{T=\mu v_0^2}$.

(b) The wave speed is $v=\sqrt{\frac{T}{\mu}}=\boxed{v_0}$.

(c) In the frame of reference of the center of the loop, each pulse moves with equal speed clockwise and counterclockwise.

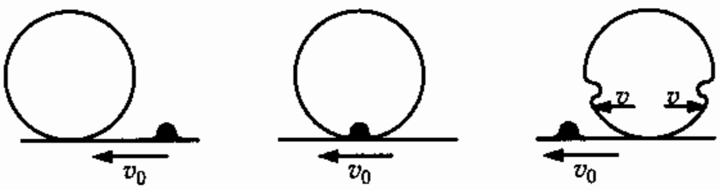


FIG. P16.64(c-1)

continued on next page

In the frame of reference of the ground, once pulse moves backward at speed $v_0 + v = 2v_0$ and the other forward at $v_0 - v = 0$. The one pulse makes two revolutions while the loop makes one revolution and the other pulse does not move around the loop. If it is generated at the six-o'clock position, it will stay at the six-o'clock position.

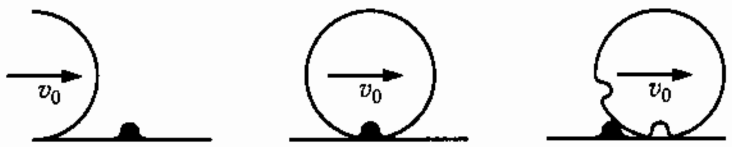


FIG. P16.64(c-2)

P16.65 (a) Assume the spring is originally stationary throughout, extended to have a length L much greater than its equilibrium length. We start moving one end forward with the speed v at which a wave propagates on the spring. In this way we create a single pulse of compression that moves down the length of the spring. For an increment of spring with length dx and mass dm, just as the pulse swallows it up, $\sum F = ma$

becomes $kdx = adm$ or $\frac{k}{\frac{dm}{dx}} = a$.

But $\frac{dm}{dx} = \mu$ so $a = \frac{k}{\mu}$.

Also, $a = \frac{dv}{dt} = \frac{v}{t}$ when $v_i = 0$. But $L = vt$, so $a = \frac{v^2}{L}$.

Equating the two expressions for a, we have $\frac{k}{\mu} = \frac{v^2}{L}$ or $\boxed{v = \sqrt{\frac{kL}{\mu}}}$.

(b) Using the expression from part (a) $v = \sqrt{\frac{kL}{\mu}} = \sqrt{\frac{kL^2}{m}} = \sqrt{\frac{(100\text{ N/m})(2.00\text{ m})^2}{0.400\text{ kg}}} = \boxed{31.6\text{ m/s}}$.

P16.66 (a) $v = \left(\frac{T}{\mu}\right)^{1/2} = \left(\frac{2T_0}{\mu_0}\right)^{1/2} = \boxed{v_0\sqrt{2}}$ where $v_0 \equiv \left(\frac{T_0}{\mu_0}\right)^{1/2}$

$$v' = \left(\frac{T}{\mu'}\right)^{1/2} = \left(\frac{2T_0}{3\mu_0}\right)^{1/2} = \boxed{v_0\sqrt{\frac{2}{3}}}$$

(b) $\Delta t_{\text{left}} = \frac{\frac{L}{2}}{v} = \frac{L}{2v_0\sqrt{2}} = \frac{\Delta t_0}{2\sqrt{2}} = 0.354\Delta t_0$ where $\Delta t_0 \equiv \frac{L}{v_0}$

$$\Delta t_{\text{right}} = \frac{\frac{L}{2}}{v'} = \frac{L}{2v_0\sqrt{\frac{2}{3}}} = \frac{\Delta t_0}{2\sqrt{\frac{2}{3}}} = 0.612\Delta t_0$$

$$\Delta t_{\text{left}} + \Delta t_{\text{right}} = \boxed{0.966\Delta t_0}$$

P16.67 (a) $\mathscr{P}(x)=\frac{1}{2}\mu\omega^2A^2v=\frac{1}{2}\mu\omega^2A_0^2e^{-2bx}\left(\frac{\omega}{k}\right)=\boxed{\frac{\mu\omega^3}{2k}A_0^2e^{-2bx}}$

(b) $\mathscr{P}(0)=\boxed{\frac{\mu\omega^3}{2k}A_0^2}$

(c) $\frac{\mathscr{P}(x)}{\mathscr{P}(0)}=\boxed{e^{-2bx}}$

P16.68 $v=\frac{4\,450\text{ km}}{9.50\text{ h}}=468\text{ km/h}=\boxed{130\text{ m/s}}$

$$\bar{d}=\frac{v^2}{g}=\frac{(130\text{ m/s})^2}{(9.80\text{ m/s}^2)}=\boxed{1\,730\text{ m}}$$

***P16.69** (a) $\mu(x)$ is a linear function, so it is of the form $\mu(x)=mx+b$

To have $\mu(0)=\mu_0$ we require $b=\mu_0$. Then $\mu(L)=\mu_L=mL+\mu_0$

so $m=\frac{\mu_L-\mu_0}{L}$

Then $\boxed{\mu(x)=\frac{(\mu_L-\mu_0)x}{L}+\mu_0}$

(b) From $v=\frac{dx}{dt}$, the time required to move from x to $x+dx$ is $\frac{dx}{v}$. The time required to move from 0 to L is

$$\Delta t=\int_0^L\frac{dx}{v}=\int_0^L\frac{dx}{\sqrt{\frac{T}{\mu}}}=\frac{1}{\sqrt{T}}\int_0^L\sqrt{\mu(x)}dx$$

$$\Delta t=\frac{1}{\sqrt{T}}\int_0^L\left(\frac{(\mu_L-\mu_0)x}{L}+\mu_0\right)^{1/2}\left(\frac{\mu_L-\mu_0}{L}\right)dx\left(\frac{L}{\mu_L-\mu_0}\right)$$

$$\Delta t=\frac{1}{\sqrt{T}}\left(\frac{L}{\mu_L-\mu_0}\right)\left(\frac{(\mu_L-\mu_0)x}{L}+\mu_0\right)^{3/2}\frac{1}{\frac{3}{2}}\Bigg|_0^L$$

$$\Delta t=\frac{2L}{3\sqrt{T}(\mu_L-\mu_0)}\left(\mu_L^{3/2}-\mu_0^{3/2}\right)$$

$$\Delta t=\frac{2L\left(\sqrt{\mu_L}-\sqrt{\mu_0}\right)\left(\mu_L+\sqrt{\mu_L\mu_0}+\mu_0\right)}{3\sqrt{T}\left(\sqrt{\mu_L}-\sqrt{\mu_0}\right)\left(\sqrt{\mu_L}+\sqrt{\mu_0}\right)}$$

$$\Delta t=\frac{2L}{3\sqrt{T}}\left(\frac{\mu_L+\sqrt{\mu_L\mu_0}+\mu_0}{\sqrt{\mu_L}+\sqrt{\mu_0}}\right)$$

ANSWERS TO EVEN PROBLEMS

P16.2 see the solution

P16.4 (a) the P wave; (b) 665 s

P16.6 0.800 m/s

P16.8 2.40 m/s

P16.10 0.300 m in the positive x-direction

P16.12 ±6.67 cm

P16.14 (a) see the solution; (b) 0.125 s; in agreement with the example

P16.16 (a) see the solution; (b) 18.0/m; 83.3 ms; 75.4 rad/s; 4.20 m/s; (c) $(0.2\text{ m})\sin(18x+75.4t-0.151)$

P16.18 (a) 0.021 5 m; (b) 1.95 rad; (c) 5.41 m/s; (d) $y(x,t)=(0.021\,5\text{ m})\sin(8.38x+80.0\pi t+1.95)$

P16.20 (a) see the solution; (b) 3.18 Hz

P16.22 30.0 N

P16.24 (a) $y=(0.2\text{ mm})\sin(16x-3\,140t)$; (b) 158 N

P16.26 631 N

P16.28 $v=\frac{Tg}{2\pi}\sqrt{\frac{M}{m}}$

P16.30 (a) $v=\left(30.4\ \frac{\text{m}}{\text{s}\cdot\sqrt{\text{kg}}}\right)\sqrt{m}$; (b) 3.89 kg

P16.32 $\sqrt{\frac{mL\tan\theta}{4Mg}}$

P16.34 1.07 kW

P16.36 (a), (b), (c) $\mathcal{P}$ is constant; (d) $\mathcal{P}$ is quadrupled

P16.38 (a) $y=(0.075\,0)\sin(4.19x-314t)$; (b) 625 W

P16.40 (a) 15.1 W; (b) 3.02 J

P16.42 The amplitude increases by 5.00 times

P16.44 see the solution

P16.46 (a) see the solution; (b) $\frac{1}{2}(x+vt)^2+\frac{1}{2}(x-vt)^2$; (c) $\frac{1}{2}\sin(x+vt)+\frac{1}{2}\sin(x-vt)$

P16.48 (a) 0.040 0 m; (b) 0.031 4 m; (c) 0.477 Hz; (d) 2.09 s; (e) positive x-direction

P16.50 (a) 21.0 ms; (b) 1.68 m

P16.52 $\Delta t=\sqrt{\frac{mL}{Mg\sin\theta}}$

P16.54 (a) $2Mg$; (b) $L_0+\frac{2Mg}{k}$; (c) $\sqrt{\frac{2Mg}{m}\left(L_0+\frac{2Mg}{k}\right)}$

P16.56 14.7 kg

P16.58 (a) $v=\sqrt{\frac{T}{\rho\left(10^{-7}x+10^{-6}\right)}}$ in SI units; (b) 94.3 m/s; 66.7 m/s

P16.60 see the solution

P16.62 (a) $5.00\hat{\mathbf{i}}$ m/s; (b) $-5.00\hat{\mathbf{i}}$ m/s; (c) $-7.50\hat{\mathbf{i}}$ m/s; (d) $24.0\hat{\mathbf{i}}$ m/s

P16.64 (a) μv_0^2; (b) v_0; (c) One travels 2 rev and the other does not move around the loop.

P16.66 (a) $v = \left(\frac{2T_0}{\mu_0}\right)^{1/2} = v_0\sqrt{2}$;

$v' = \left(\frac{2T_0}{3\mu_0}\right)^{1/2} = v_0\sqrt{\frac{2}{3}}$; (b) $0.966\Delta t_0$

P16.68 130 m/s; 1.73 km

17

Sound Waves

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q17.1 Sound waves are longitudinal because elements of the medium—parcels of air—move parallel and antiparallel to the direction of wave motion.

Q17.2 We assume that a perfect vacuum surrounds the clock. The sound waves require a medium for them to travel to your ear. The hammer on the alarm will strike the bell, and the vibration will spread as sound waves through the body of the clock. If a bone of your skull were in contact with the clock, you would hear the bell. However, in the absence of a surrounding medium like air or water, no sound can be radiated away. A larger-scale example of the same effect: Colossal storms raging on the Sun are deathly still for us.

What happens to the sound energy within the clock? Here is the answer: As the sound wave travels through the steel and plastic, traversing joints and going around corners, its energy is converted into additional internal energy, raising the temperature of the materials. After the sound has died away, the clock will glow very slightly brighter in the infrared portion of the electromagnetic spectrum.

Q17.3 If an object is $\frac{1}{2}$ meter from the sonic ranger, then the sensor would have to measure how long it would take for a sound pulse to travel one meter. Since sound of any frequency moves at about 343 m/s, then the sonic ranger would have to be able to measure a time difference of under 0.003 seconds. This small time measurement is possible with modern electronics. But it would be more expensive to outfit sonic rangers with the more sensitive equipment than it is to print "do not use to measure distances less than $\frac{1}{2}$ meter" in the users' manual.

Q17.4 The speed of sound to two significant figures is 340 m/s. Let's assume that you can measure time to $\frac{1}{10}$ second by using a stopwatch. To get a speed to two significant figures, you need to measure a time of at least 1.0 seconds. Since $d = vt$, the minimum distance is 340 meters.

Q17.5 The frequency increases by a factor of 2 because the wave speed, which is dependent only on the medium through which the wave travels, remains constant.

Q17.6 When listening, you are approximately the same distance from all of the members of the group. If different frequencies traveled at different speeds, then you might hear the higher pitched frequencies before you heard the lower ones produced at the same time. Although it might be interesting to think that each listener heard his or her own personal performance depending on where they were seated, a time lag like this could make a Beethoven sonata sound as if it were written by Charles Ives.

Q17.7 Since air is a viscous fluid, some of the energy of sound vibration is turned into internal energy. At such great distances, the amplitude of the signal is so decreased by this effect you re unable to hear it.

Q17.8 We suppose that a point source has no structure, and radiates sound equally in all directions (isotropically). The sound wavefronts are expanding spheres, so the area over which the sound energy spreads increases according to $A = 4\pi r^2$. Thus, if the distance is tripled, the area increases by a factor of nine, and the new intensity will be one-ninth of the old intensity. This answer according to the inverse-square law applies if the medium is uniform and unbounded.

For contrast, suppose that the sound is confined to move in a horizontal layer. (Thermal stratification in an ocean can have this effect on sonar "pings.") Then the area over which the sound energy is dispersed will only increase according to the circumference of an expanding circle: $A = 2\pi rh$, and so three times the distance will result in one third the intensity.

In the case of an entirely enclosed speaking tube (such as a ship's telephone), the area perpendicular to the energy flow stays the same, and increasing the distance will not change the intensity appreciably.

Q17.9 He saw the first wave he encountered, light traveling at 3.00×10^8 m/s. At the same moment, infrared as well as visible light began warming his skin, but some time was required to raise the temperature of the outer skin layers before he noticed it. The meteor produced compressional waves in the air and in the ground. The wave in the ground, which can be called either sound or a seismic wave, traveled much faster than the wave in air, since the ground is much stiffer against compression. Our witness received it next and noticed it as a little earthquake. He was no doubt unable to distinguish the P and S waves. The first air-compression wave he received was a shock wave with an amplitude on the order of meters. It transported him off his doorstep. Then he could hear some additional direct sound, reflected sound, and perhaps the sound of the falling trees.

Q17.10 A microwave pulse is reflected from a moving object. The waves that are reflected back are Doppler shifted in frequency according to the speed of the target. The receiver in the radar gun detects the reflected wave and compares its frequency to that of the emitted pulse. Using the frequency shift, the speed can be calculated to high precision. Be forewarned: this technique works if you are either traveling toward or away from your local law enforcement agent!

Q17.11 As you move towards the canyon wall, the echo of your car horn would be shifted up in frequency; as you move away, the echo would be shifted down in frequency.

Q17.12 Normal conversation has an intensity level of about 60 dB.

Q17.13 A rock concert has an intensity level of about 120 dB.
A cheering crowd has an intensity level of about 90 dB.
Normal conversation has an intensity level of about 50–60 dB.
Turning a page in the textbook has an intensity level of about 10–20 dB.

Q17.14 One would expect the spectra of the light to be Doppler shifted up in frequency (blue shift) as the star approaches us. As the star recedes in its orbit, the frequency spectrum would be shifted down (red shift). While the star is moving perpendicular to our line of sight, there will be no frequency shift at all. Overall, the spectra would oscillate with a period equal to that of the orbiting stars.

Q17.15 For the sound from a source not to shift in frequency, the radial velocity of the source relative to the observer must be zero; that is, the source must not be moving toward or away from the observer. The source can be moving in a plane perpendicular to the line between it and the observer. Other possibilities: The source and observer might both have zero velocity. They might have equal velocities relative to the medium. The source might be moving around the observer on a sphere of constant radius. Even if the source speeds up on the sphere, slows down, or stops, the frequency heard will be equal to the frequency emitted by the source.

Q17.16 Wind can change a Doppler shift but cannot cause one. Both v_o and v_s in our equations must be interpreted as speeds of observer and source relative to the air. If source and observer are moving relative to each other, the observer will hear one shifted frequency in still air and a different shifted frequency if wind is blowing. If the distance between source and observer is constant, there will never be a Doppler shift.

Q17.17 If the object being tracked is moving away from the observer, then the sonic pulse would never reach the object, as the object is moving away faster than the wave speed. If the object being tracked is moving towards the observer, then the object itself would reach the detector before reflected pulse.

Q17.18 New-fallen snow is a wonderful acoustic absorber as it reflects very little of the sound that reaches it. It is full of tiny intricate air channels and does not spring back when it is distorted. It acts very much like acoustic tile in buildings. So where does the absorbed energy go? It turns into internal energy—albeit a very small amount.

Q17.19 As a sound wave moves away from the source, its intensity decreases. With an echo, the sound must move from the source to the reflector and then back to the observer, covering a significant distance.

Q17.20 The observer would most likely hear the sonic boom of the plane itself and then beep, baap, boop. Since the plane is supersonic, the loudspeaker would pull ahead of the leading "boop" wavefront before emitting the "baap", and so forth.

"How are you?" would be heard as "?uoy era woH"

Q17.21 This system would be seen as a star moving in an elliptical path. Just like the light from a star in a binary star system, described in the answer to question 14, the spectrum of light from the star would undergo a series of Doppler shifts depending on the star's speed and direction of motion relative to the observer. The repetition rate of the Doppler shift pattern is the period of the orbit. Information about the orbit size can be calculated from the size of the Doppler shifts.

SOLUTIONS TO PROBLEMS

Section 17.1 Speed of Sound Waves

P17.1 Since $v_{\text{light}} >> v_{\text{sound}}$: $d \approx (343\ \text{m/s})(16.2\ \text{s}) = \boxed{5.56\ \text{km}}$

P17.2 $v = \sqrt{\dfrac{B}{\rho}} = \sqrt{\dfrac{2.80 \times 10^{10}}{13.6 \times 10^3}} = \boxed{1.43\ \text{km/s}}$

P17.3 Sound takes this time to reach the man: $\frac{(20.0\text{ m} - 1.75\text{ m})}{343\text{ m/s}} = 5.32\times10^{-2}\text{ s}$

so the warning should be shouted no later than before the pot strikes. $0.300\text{ s} + 5.32\times10^{-2}\text{ s} = 0.353\text{ s}$

Since the whole time of fall is given by $y = \frac{1}{2}gt^2$: $18.25\text{ m} = \frac{1}{2}(9.80\text{ m/s}^2)t^2$

$t = 1.93\text{ s}$

the warning needs to come $1.93\text{ s} - 0.353\text{ s} = 1.58\text{ s}$

into the fall, when the pot has fallen $\frac{1}{2}(9.80\text{ m/s}^2)(1.58\text{ s})^2 = 12.2\text{ m}$

to be above the ground by $20.0\text{ m} - 12.2\text{ m} = \boxed{7.82\text{ m}}$

P17.4 (a) At 9 000 m, $\Delta T = \left(\frac{9\,000}{150}\right)(-1.00°\text{C}) = -60.0°\text{C}$ so $T = -30.0°\text{C}$.

Using the chain rule:

$$\frac{dv}{dt} = \frac{dv}{dT}\frac{dT}{dx}\frac{dx}{dt} = v\frac{dv}{dT}\frac{dT}{dx} = v(0.607)\left(\frac{1}{150}\right) = \frac{v}{247}, \text{ so } dt = (247\text{ s})\frac{dv}{v}$$

$$\int_0^t dt = (247\text{ s})\int_{v_i}^{v_f}\frac{dv}{v}$$

$$t = (247\text{ s})\ln\left(\frac{v_f}{v_i}\right) = (247\text{ s})\ln\left[\frac{331.5 + 0.607(30.0)}{331.5 + 0.607(-30.0)}\right]$$

$t = \boxed{27.2\text{ s}}$ for sound to reach ground.

(b) $t = \frac{h}{v} = \frac{9\,000}{[331.5 + 0.607(30.0)]} = \boxed{25.7\text{ s}}$

It takes longer when the air cools off than if it were at a uniform temperature.

***P17.5** Let x_1 represent the cowboy's distance from the nearer canyon wall and x_2 his distance from the farther cliff. The sound for the first echo travels distance $2x_1$. For the second, $2x_2$. For the third, $2x_1 + 2x_2$. For the fourth echo, $2x_1 + 2x_2 + 2x_1$. Then $\frac{2x_2 - 2x_1}{340\text{ m/s}} = 1.92\text{ s}$ and $\frac{2x_1 + 2x_2 - 2x_2}{340\text{ m/s}} = 1.47\text{ s}$.

Thus $x_1 = \frac{1}{2}340\text{ m/s } 1.47\text{ s} = 250\text{ m}$ and $\frac{2x_2}{340\text{ m/s}} = 1.92\text{ s} + 1.47\text{ s}$; $x_2 = 576\text{ m}$.

(a) So $x_1 + x_2 = \boxed{826\text{ m}}$

(b) $\frac{2x_1 + 2x_2 + 2x_1 - (2x_1 + 2x_2)}{340\text{ m/s}} = \boxed{1.47\text{ s}}$

P17.6 It is easiest to solve part (b) first:

(b) The distance the sound travels to the plane is $d_s = \sqrt{h^2 + \left(\frac{h}{2}\right)^2} = \frac{h\sqrt{5}}{2}$.

The sound travels this distance in 2.00 s, so

$$d_s = \frac{h\sqrt{5}}{2} = (343 \text{ m/s})(2.00 \text{ s}) = 686 \text{ m}$$

giving the altitude of the plane as $h = \frac{2(686 \text{ m})}{\sqrt{5}} = \boxed{614 \text{ m}}$.

(a) The distance the plane has traveled in 2.00 s is $v(2.00 \text{ s}) = \frac{h}{2} = 307$ m.

Thus, the speed of the plane is: $v = \frac{307 \text{ m}}{2.00 \text{ s}} = \boxed{153 \text{ m/s}}$.

Section 17.2 **Periodic Sound Waves**

P17.7 $\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{60.0 \times 10^3 \text{ s}^{-1}} = \boxed{5.67 \text{ mm}}$

***P17.8** The sound speed is $v = 331 \text{ m/s}\sqrt{1 + \frac{26°\text{C}}{273°\text{C}}} = 346 \text{ m/s}$

(a) Let t represent the time for the echo to return. Then

$$d = \frac{1}{2}vt = \frac{1}{2}346 \text{ m/s } 24 \times 10^{-3} \text{ s} = \boxed{4.16 \text{ m}}.$$

(b) Let Δt represent the duration of the pulse:

$$\Delta t = \frac{10\lambda}{v} = \frac{10\lambda}{f\lambda} = \frac{10}{f} = \frac{10}{22 \times 10^6 \text{ 1/s}} = \boxed{0.455 \ \mu\text{s}}.$$

(c) $L = 10\lambda = \frac{10v}{f} = \frac{10(346 \text{ m/s})}{22 \times 10^6 \text{ 1/s}} = \boxed{0.157 \text{ mm}}$

***P17.9** If $f = 1$ MHz, $\lambda = \frac{v}{f} = \frac{1\,500 \text{ m/s}}{10^6\text{/s}} = \boxed{1.50 \text{ mm}}$

If $f = 20$ MHz, $\lambda = \frac{1\,500 \text{ m/s}}{2 \times 10^7\text{/s}} = \boxed{75.0 \ \mu\text{m}}$

P17.10 $\Delta P_{\max} = \rho v \omega s_{\max}$

$$s_{\max} = \frac{\Delta P_{\max}}{\rho v \omega} = \frac{\left(4.00 \times 10^{-3} \text{ N/m}^2\right)}{\left(1.20 \text{ kg/m}^3\right)(343 \text{ m/s})(2\pi)\left(10.0 \times 10^3 \text{ s}^{-1}\right)} = \boxed{1.55 \times 10^{-10} \text{ m}}$$

P17.11 (a) $A = \boxed{2.00\ \mu\text{m}}$

$$\lambda = \frac{2\pi}{15.7} = 0.400\text{ m} = \boxed{40.0\text{ cm}}$$

$$v = \frac{\omega}{k} = \frac{858}{15.7} = \boxed{54.6\text{ m/s}}$$

(b) $s = 2.00\cos\left[(15.7)(0.050\,0) - (858)\left(3.00\times10^{-3}\right)\right] = \boxed{-0.433\ \mu\text{m}}$

(c) $v_{\max} = A\omega = (2.00\ \mu\text{m})\left(858\text{ s}^{-1}\right) = \boxed{1.72\text{ mm/s}}$

P17.12 (a) $\Delta P = (1.27\text{ Pa})\sin\left(\frac{\pi x}{\text{m}} - \frac{340\pi t}{\text{s}}\right)$ (SI units)

The pressure amplitude is: $\Delta P_{\max} = \boxed{1.27\text{ Pa}}$.

(b) $\omega = 2\pi f = 340\pi/\text{s}$, so $f = \boxed{170\text{ Hz}}$

(c) $k = \frac{2\pi}{\lambda} = \pi/\text{m}$, giving $\lambda = \boxed{2.00\text{ m}}$

(d) $v = \lambda f = (2.00\text{ m})(170\text{ Hz}) = \boxed{340\text{ m/s}}$

P17.13 $k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.100\text{ m})} = 62.8\text{ m}^{-1}$

$$\omega = \frac{2\pi v}{\lambda} = \frac{2\pi(343\text{ m/s})}{(0.100\text{ m})} = 2.16\times10^{4}\text{ s}^{-1}$$

Therefore, $\boxed{\Delta P = (0.200\text{ Pa})\sin\left[62.8\,x/\text{m} - 2.16\times10^{4}\,t/\text{s}\right]}$.

P17.14 $\omega = 2\pi f = \frac{2\pi v}{\lambda} = \frac{2\pi(343\text{ m/s})}{(0.100\text{ m})} = 2.16\times10^{4}\text{ rad/s}$

$$s_{\max} = \frac{\Delta P_{\max}}{\rho v\omega} = \frac{(0.200\text{ Pa})}{\left(1.20\text{ kg/m}^3\right)(343\text{ m/s})\left(2.16\times10^{4}\text{ s}^{-1}\right)} = 2.25\times10^{-8}\text{ m}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.100\text{ m})} = 62.8\text{ m}^{-1}$$

Therefore, $\boxed{s = s_{\max}\cos(kx - \omega t) = \left(2.25\times10^{-8}\text{ m}\right)\cos\left(62.8x/\text{m} - 2.16\times10^{4}\,t/\text{s}\right)}$.

P17.15 $\Delta P_{\max} = \rho v\omega s_{\max} = \rho v\left(\frac{2\pi v}{\lambda}\right)s_{\max}$

$$\lambda = \frac{2\pi\rho v^2 s_{\max}}{\Delta P_{\max}} = \frac{2\pi(1.20)(343)^2\left(5.50\times10^{-6}\right)}{0.840} = \boxed{5.81\text{ m}}$$

P17.16 (a) The sound "pressure" is extra tensile stress for one-half of each cycle. When it becomes $(0.500\%)(13.0\times10^{10}\text{ Pa})=6.50\times10^{8}\text{ Pa}$, the rod will break. Then, $\Delta P_{max}=\rho v\omega s_{max}$

$$s_{max}=\frac{\Delta P_{max}}{\rho v\omega}=\frac{6.50\times10^{8}\text{ N/m}^2}{(8.92\times10^{3}\text{ kg/m}^3)(5\,010\text{ m/s})(2\pi 500/\text{s})}=\boxed{4.63\text{ mm}}.$$

(b) From $s=s_{max}\cos(kx-\omega t)$

$$v=\frac{\partial s}{\partial t}=-\omega s_{max}\sin(kx-\omega t)$$
$$v_{max}=\omega s_{max}=(2\pi 500/\text{s})(4.63\text{ mm})=\boxed{14.5\text{ m/s}}$$

(c) $I=\frac{1}{2}\rho v(\omega s_{max})^2=\frac{1}{2}\rho v v_{max}^2=\frac{1}{2}(8.92\times10^{3}\text{ kg/m}^3)(5\,010\text{ m/s})(14.5\text{ m/s})^2$
$=\boxed{4.73\times10^{9}\text{ W/m}^2}$

***P17.17** Let $P(x)$ represent absolute pressure as a function of x. The net force to the right on the chunk of air is $+P(x)A-P(x+\Delta x)A$. Atmospheric pressure subtracts out, leaving $[-\Delta P(x+\Delta x)+\Delta P(x)]A=-\frac{\partial \Delta P}{\partial x}\Delta x A$.

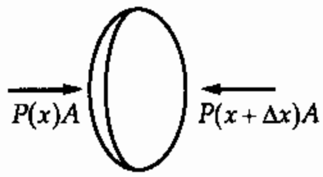


FIG. P17.17

The mass of the air is $\Delta m=\rho\Delta V=\rho A\Delta x$ and its acceleration is $\frac{\partial^2 s}{\partial t^2}$. So Newton's second law becomes

$$-\frac{\partial \Delta P}{\partial x}\Delta x A=\rho A\Delta x\frac{\partial^2 s}{\partial t^2}$$
$$-\frac{\partial}{\partial x}\left(-B\frac{\partial s}{\partial x}\right)=\rho\frac{\partial^2 s}{\partial t^2}$$
$$\frac{B}{\rho}\frac{\partial^2 s}{\partial x^2}=\frac{\partial^2 s}{\partial t^2}$$

Into this wave equation as a trial solution we substitute the wave function $s(x,t)=s_{max}\cos(kx-\omega t)$ we find

$$\frac{\partial s}{\partial x}=-ks_{max}\sin(kx-\omega t)$$
$$\frac{\partial^2 s}{\partial x^2}=-k^2 s_{max}\cos(kx-\omega t)$$
$$\frac{\partial s}{\partial t}=+\omega s_{max}\sin(kx-\omega t)$$
$$\frac{\partial^2 s}{\partial t^2}=-\omega^2 s_{max}\cos(kx-\omega t)$$

$\frac{B}{\rho}\frac{\partial^2 s}{\partial x^2}=\frac{\partial^2 s}{\partial t^2}$ becomes $-\frac{B}{\rho}k^2 s_{max}\cos(kx-\omega t)=-\omega^2 s_{max}\cos(kx-\omega t)$

This is true provided $\frac{B}{\rho}\frac{4\pi^2}{\lambda^2}=4\pi^2 f^2$.

The sound wave can propagate provided it has $\lambda^2 f^2=v^2=\frac{B}{\rho}$; that is, provided it propagates with speed $v=\sqrt{\frac{B}{\rho}}$.

Section 17.3 **Intensity of Periodic Sound Waves**

***P17.18** The sound power incident on the eardrum is $\wp = IA$ where I is the intensity of the sound and $A = 5.0\times10^{-5}\ \text{m}^2$ is the area of the eardrum.

(a) At the threshold of hearing, $I = 1.0\times10^{-12}\ \text{W/m}^2$, and

$$\wp = \left(1.0\times10^{-12}\ \text{W/m}^2\right)\left(5.0\times10^{-5}\ \text{m}^2\right) = \boxed{5.00\times10^{-17}\ \text{W}}.$$

(b) At the threshold of pain, $I = 1.0\ \text{W/m}^2$, and

$$\wp = \left(1.0\ \text{W/m}^2\right)\left(5.0\times10^{-5}\ \text{m}^2\right) = \boxed{5.00\times10^{-5}\ \text{W}}.$$

P17.19 $\beta = 10\log\left(\frac{I}{I_0}\right) = 10\log\left(\frac{4.00\times10^{-6}}{1.00\times10^{-12}}\right) = \boxed{66.0\ \text{dB}}$

P17.20 (a) $70.0\ \text{dB} = 10\log\left(\frac{I}{1.00\times10^{-12}\ \text{W/m}^2}\right)$

Therefore, $I = \left(1.00\times10^{-12}\ \text{W/m}^2\right)10^{(70.0/10)} = \boxed{1.00\times10^{-5}\ \text{W/m}^2}$.

(b) $I = \frac{\Delta P_{\text{max}}^2}{2\rho v}$, so

$$\Delta P_{\text{max}} = \sqrt{2\rho v I} = \sqrt{2\left(1.20\ \text{kg/m}^3\right)(343\ \text{m/s})\left(1.00\times10^{-5}\ \text{W/m}^2\right)}$$
$$\Delta P_{\text{max}} = \boxed{90.7\ \text{mPa}}$$

P17.21 $I = \frac{1}{2}\rho\omega^2 s_{\text{max}}^2 v$

(a) At $f = 2\,500\ \text{Hz}$, the frequency is increased by a factor of 2.50, so the intensity (at constant s_{max}) increases by $(2.50)^2 = 6.25$.

Therefore, $6.25(0.600) = \boxed{3.75\ \text{W/m}^2}$.

(b) $\boxed{0.600\ \text{W/m}^2}$

P17.22 The original intensity is $I_1 = \frac{1}{2}\rho\omega^2 s_{\text{max}}^2 v = 2\pi^2\rho v f^2 s_{\text{max}}^2$

(a) If the frequency is increased to f' while a constant displacement amplitude is maintained, the new intensity is

$$I_2 = 2\pi^2\rho v(f')^2 s_{\text{max}}^2 \text{ so } \frac{I_2}{I_1} = \frac{2\pi^2\rho v(f')s_{\text{max}}^2}{2\pi^2\rho v f^2 s_{\text{max}}^2} = \left(\frac{f'}{f}\right)^2 \text{ or } \boxed{I_2 = \left(\frac{f'}{f}\right)^2 I_1}.$$

continued on next page

(b) If the frequency is reduced to $f' = \frac{f}{2}$ while the displacement amplitude is doubled, the new intensity is

$$I_2 = 2\pi^2 \rho v \left(\frac{f}{2}\right)^2 (2s_{max})^2 = 2\pi^2 \rho v f^2 s_{max}^2 = I_1$$

or the $\boxed{\text{intensity is unchanged}}$.

***P17.23** (a) For the low note the wavelength is $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{146.8/\text{s}} = \boxed{2.34 \text{ m}}$.

For the high note $\lambda = \frac{343 \text{ m/s}}{880/\text{s}} = \boxed{0.390 \text{ m}}$.

We observe that the ratio of the frequencies of these two notes is $\frac{880 \text{ Hz}}{146.8 \text{ Hz}} = 5.99$ nearly equal to a small integer. This fact is associated with the consonance of the notes D and A.

(b) $\beta = 10 \text{ dB} \log\left(\frac{I}{10^{-12} \text{ W/m}^2}\right) = 75 \text{ dB}$ gives $I = 3.16 \times 10^{-5} \text{ W/m}^2$

$$I = \frac{\Delta P_{max}^2}{2\rho v}$$

$$\Delta P_{max} = \sqrt{3.16 \times 10^{-5} \text{ W/m}^2\, 2(1.20 \text{ kg/m}^3)(343 \text{ m/s})} = \boxed{0.161 \text{ Pa}}$$

for both low and high notes.

(c) $I = \frac{1}{2}\rho v(\omega s_{max})^2 = \frac{1}{2}\rho v 4\pi^2 f^2 s_{max}^2$

$$s_{max} = \sqrt{\frac{I}{2\pi^2 \rho v f^2}}$$

for the low note,

$$s_{max} = \sqrt{\frac{3.16 \times 10^{-5} \text{ W/m}^2}{2\pi^2 1.20 \text{ kg/m}^3\, 343 \text{ m/s}}}\, \frac{1}{146.8/\text{s}}$$

$$= \frac{6.24 \times 10^{-5}}{146.8} \text{ m} = \boxed{4.25 \times 10^{-7} \text{ m}}$$

for the high note,

$$s_{max} = \frac{6.24 \times 10^{-5}}{880} \text{ m} = \boxed{7.09 \times 10^{-8} \text{ m}}$$

(d) With both frequencies lower (numerically smaller) by the factor $\frac{146.8}{134.3} = \frac{880}{804.9} = 1.093$, the wavelengths and displacement amplitudes are made 1.093 times larger, and the pressure amplitudes are unchanged.

***P17.24** The power necessarily supplied to the speaker is the power carried away by the sound wave:

$$P = \frac{1}{2}\rho A v(\omega s_{max})^2 = 2\pi^2 \rho A v f^2 s_{max}^2$$

$$= 2\pi^2 (1.20 \text{ kg/m}^3)\pi\left(\frac{0.08 \text{ m}}{2}\right)^2 (343 \text{ m/s})(600 \text{ 1/s})^2 (0.12 \times 10^{-2} \text{ m})^2 = \boxed{21.2 \text{ W}}$$

P17.25 (a) $I_1 = (1.00\times10^{-12}\ \text{W/m}^2)10^{(\beta_1/10)} = (1.00\times10^{-12}\ \text{W/m}^2)10^{80.0/10}$

or $I_1 = 1.00\times10^{-4}\ \text{W/m}^2$

$I_2 = (1.00\times10^{-12}\ \text{W/m}^2)10^{(\beta_2/10)} = (1.00\times10^{-12}\ \text{W/m}^2)10^{75.0/10}$

or $I_2 = 1.00\times10^{-4.5}\ \text{W/m}^2 = 3.16\times10^{-5}\ \text{W/m}^2$

When both sounds are present, the total intensity is

$$I = I_1 + I_2 = 1.00\times10^{-4}\ \text{W/m}^2 + 3.16\times10^{-5}\ \text{W/m}^2 = \boxed{1.32\times10^{-4}\ \text{W/m}^2}.$$

(b) The decibel level for the combined sounds is

$$\beta = 10\log\left(\frac{1.32\times10^{-4}\ \text{W/m}^2}{1.00\times10^{-12}\ \text{W/m}^2}\right) = 10\log(1.32\times10^8) = \boxed{81.2\ \text{dB}}.$$

***P17.26** (a) We have $\lambda = \frac{v}{f}$ and f is the same for all three waves. Since the speed is smallest in air, λ is smallest in air. It is larger by $\frac{1\,493\ \text{m/s}}{331\ \text{m/s}} = \boxed{4.51\ \text{times}}$ in water and by $\frac{5\,950}{331} = \boxed{18.0\ \text{times in iron}}$.

(b) From $I = \frac{1}{2}\rho v\omega^2 s_{\max}^2$; $s_{\max} = \sqrt{\frac{2I_0}{\rho v\omega_0^2}}$, $s_{\max}$ is smallest in iron, larger in water by $\sqrt{\frac{\rho_{\text{iron}}v_{\text{iron}}}{\rho_{\text{water}}v_{\text{water}}}} = \sqrt{\frac{7\,860\cdot5\,950}{1\,000\cdot1\,493}} = \boxed{5.60\ \text{times}}$, and larger in air by $\sqrt{\frac{7\,860\cdot5\,950}{1.29\cdot331}} = \boxed{331\ \text{times}}$.

(c) From $I = \frac{\Delta P_{\max}^2}{2\rho v}$; $\Delta P_{\max} = \sqrt{2I\rho v}$, $\Delta P_{\max}$ is smallest in air, larger in water by $\sqrt{\frac{1\,000\cdot1\,493}{1.29\cdot331}} = \boxed{59.1\ \text{times}}$, and larger in iron by $\sqrt{\frac{7\,860\cdot5\,950}{1.29\cdot331}} = \boxed{331\ \text{times}}$.

(d) $\lambda = \frac{v}{f} = \frac{v2\pi}{\omega} = \frac{(331\ \text{m/s})2\pi}{2\,000\,\pi/\text{s}} = \boxed{0.331\ \text{m}}$ in air

$\lambda = \frac{1\,493\ \text{m/s}}{1\,000/\text{s}} = \boxed{1.49\ \text{m}}$ in water $\qquad \lambda = \frac{5\,950\ \text{m/s}}{1\,000/\text{s}} = \boxed{5.95\ \text{m}}$ in iron

$s_{\max} = \sqrt{\frac{2I_0}{\rho v\omega_0^2}} = \sqrt{\frac{2\times10^{-6}\ \text{W/m}^2}{(1.29\ \text{kg/m}^3)(331\ \text{m/s})(6\,283\ 1/\text{s})^2}} = \boxed{1.09\times10^{-8}\ \text{m}}$ in air

$s_{\max} = \sqrt{\frac{2\times10^{-6}}{1\,000(1\,493)}}\frac{1}{6\,283} = \boxed{1.84\times10^{-10}\ \text{m}}$ in water

$s_{\max} = \sqrt{\frac{2\times10^{-6}}{7\,860(5\,950)}}\frac{1}{6\,283} = \boxed{3.29\times10^{-11}\ \text{m}}$ in iron

$\Delta P_{\max} = \sqrt{2I\rho v} = \sqrt{2(10^{-6}\ \text{W/m}^2)(1.29\ \text{kg/m}^3)331\ \text{m/s}} = \boxed{0.029\,2\ \text{Pa}}$ in air

$\Delta P_{\max} = \sqrt{2\times10^{-6}(1\,000)1\,493} = \boxed{1.73\ \text{Pa}}$ in water

$\Delta P_{\max} = \sqrt{2\times10^{-6}(7\,860)(5\,950)} = \boxed{9.67\ \text{Pa}}$ in iron

P17.27 (a) $$120\text{ dB} = 10\text{ dB}\log\left[\frac{I}{10^{-12}\text{ W/m}^2}\right]$$

$$I = 1.00\text{ W/m}^2 = \frac{\wp}{4\pi r^2}$$

$$r = \sqrt{\frac{\wp}{4\pi I}} = \sqrt{\frac{6.00\text{ W}}{4\pi\left(1.00\text{ W/m}^2\right)}} = \boxed{0.691\text{ m}}$$

We have assumed the speaker is an isotropic point source.

(b) $$0\text{ dB} = 10\text{ dB}\log\left(\frac{I}{10^{-12}\text{ W/m}^2}\right)$$

$$I = 1.00\times10^{-12}\text{ W/m}^2$$

$$r = \sqrt{\frac{\wp}{4\pi I}} = \sqrt{\frac{6.00\text{ W}}{4\pi\left(1.00\times10^{-12}\text{ W/m}^2\right)}} = \boxed{691\text{ km}}$$

We have assumed a uniform medium that absorbs no energy.

P17.28 We begin with $\beta_2 = 10\log\left(\frac{I_2}{I_0}\right)$, and $\beta_1 = 10\log\left(\frac{I_1}{I_0}\right)$, so

$$\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right).$$

Also, $I_2 = \frac{\wp}{4\pi r_2^2}$, and $I_1 = \frac{\wp}{4\pi r_1^2}$, giving $\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$.

Then, $\beta_2 - \beta_1 = 10\log\left(\frac{r_1}{r_2}\right)^2 = \boxed{20\log\left(\frac{r_1}{r_2}\right)}$.

P17.29 Since intensity is inversely proportional to the square of the distance,

$$I_4 = \frac{1}{100}I_{0.4}\text{ and }I_{0.4} = \frac{\Delta P_{max}^2}{2\rho v} = \frac{(10.0)^2}{2(1.20)(343)} = 0.121\text{ W/m}^2.$$

The difference in sound intensity level is

$$\Delta\beta = 10\log\left(\frac{I_{4\text{ km}}}{I_{0.4\text{ km}}}\right) = 10(-2.00) = -20.0\text{ dB}.$$

At 0.400 km,

$$\beta_{0.4} = 10\log\left(\frac{0.121\text{ W/m}^2}{10^{-12}\text{ W/m}^2}\right) = 110.8\text{ dB}.$$

At 4.00 km,

$$\beta_4 = \beta_{0.4} + \Delta\beta = (110.8 - 20.0)\text{ dB} = 90.8\text{ dB}.$$

Allowing for absorption of the wave over the distance traveled,

$$\beta_4' = \beta_4 - (7.00\text{ dB/km})(3.60\text{ km}) = \boxed{65.6\text{ dB}}.$$

This is equivalent to the sound intensity level of heavy traffic.

P17.30 Let r_1 and r_2 be the distance from the speaker to the observer that hears 60.0 dB and 80.0 dB, respectively. Use the result of problem 28,

$\beta_2 - \beta_1 = 20\log\left(\frac{r_1}{r_2}\right)$, to obtain $80.0 - 60.0 = 20\log\left(\frac{r_1}{r_2}\right)$.

Thus, $\log\left(\frac{r_1}{r_2}\right) = 1$, so $r_1 = 10.0r_2$. Also: $r_1 + r_2 = 110\text{ m}$, so

$10.0r_2 + r_2 = 110\text{ m}$ giving $\boxed{r_2 = 10.0\text{ m}}$, and $\boxed{r_1 = 100\text{ m}}$.

P17.31 We presume the speakers broadcast equally in all directions.

(a)
$$r_{AC} = \sqrt{3.00^2 + 4.00^2}\text{ m} = 5.00\text{ m}$$
$$I = \frac{\wp}{4\pi r^2} = \frac{1.00\times10^{-3}\text{ W}}{4\pi(5.00\text{ m})^2} = 3.18\times10^{-6}\text{ W/m}^2$$
$$\beta = 10\text{ dB}\log\left(\frac{3.18\times10^{-6}\text{ W/m}^2}{10^{-12}\text{ W/m}^2}\right)$$
$$\beta = 10\text{ dB }6.50 = \boxed{65.0\text{ dB}}$$

(b)
$$r_{BC} = 4.47\text{ m}$$
$$I = \frac{1.50\times10^{-3}\text{ W}}{4\pi(4.47\text{ m})^2} = 5.97\times10^{-6}\text{ W/m}^2$$
$$\beta = 10\text{ dB}\log\left(\frac{5.97\times10^{-6}}{10^{-12}}\right)$$
$$\beta = \boxed{67.8\text{ dB}}$$

(c)
$$I = 3.18\ \mu\text{W/m}^2 + 5.97\ \mu\text{W/m}^2$$
$$\beta = 10\text{ dB}\log\left(\frac{9.15\times10^{-6}}{10^{-12}}\right) = \boxed{69.6\text{ dB}}$$

P17.32 In $I = \frac{\wp}{4\pi r^2}$, intensity I is proportional to $\frac{1}{r^2}$,

so between locations 1 and 2: $\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2}$.

In $I = \frac{1}{2}\rho v(\omega s_{\max})^2$, intensity is proportional to $s_{\max}^2$, so $\frac{I_2}{I_1} = \frac{s_2^2}{s_1^2}$.

Then, $\left(\frac{s_2}{s_1}\right)^2 = \left(\frac{r_1}{r_2}\right)^2$ or $\left(\frac{1}{2}\right)^2 = \left(\frac{r_1}{r_2}\right)^2$, giving $r_2 = 2r_1 = 2(50.0\text{ m}) = 100\text{ m}$.

But, $r_2 = \sqrt{(50.0\text{ m})^2 + d^2}$ yields $d = \boxed{86.6\text{ m}}$.

P17.33 $\beta = 10\log\left(\frac{I}{10^{-12}}\right) \qquad I = \left[10^{(\beta/10)}\right]\left(10^{-12}\right)\ \text{W/m}^2$

$I_{(120\text{ dB})} = 1.00\ \text{W/m}^2$; $I_{(100\text{ dB})} = 1.00\times10^{-2}\ \text{W/m}^2$; $I_{(10\text{ dB})} = 1.00\times10^{-11}\ \text{W/m}^2$

(a) $\wp = 4\pi r^2 I$ so that $r_1^2 I_1 = r_2^2 I_2$

$$r_2 = r_1\left(\frac{I_1}{I_2}\right)^{1/2} = (3.00\text{ m})\sqrt{\frac{1.00}{1.00\times10^{-2}}} = \boxed{30.0\text{ m}}$$

(b) $$r_2 = r_1\left(\frac{I_1}{I_2}\right)^{1/2} = (3.00\text{ m})\sqrt{\frac{1.00}{1.00\times10^{-11}}} = \boxed{9.49\times10^5\text{ m}}$$

P17.34 (a) $E = \wp t = 4\pi r^2 It = 4\pi(100\text{ m})^2\left(7.00\times10^{-2}\ \text{W/m}^2\right)(0.200\text{ s}) = \boxed{1.76\text{ kJ}}$

(b) $\beta = 10\log\left(\frac{7.00\times10^{-2}}{1.00\times10^{-12}}\right) = \boxed{108\text{ dB}}$

P17.35 (a) The sound intensity inside the church is given by

$$\beta = 10\ln\left(\frac{I}{I_0}\right)$$

$$101\text{ dB} = (10\text{ dB})\ln\left(\frac{I}{10^{-12}\ \text{W/m}^2}\right)$$

$$I = 10^{10.1}\left(10^{-12}\ \text{W/m}^2\right) = 10^{-1.90}\ \text{W/m}^2 = 0.0126\ \text{W/m}^2$$

We suppose that sound comes perpendicularly out through the windows and doors. Then, the radiated power is

$$\wp = IA = \left(0.0126\ \text{W/m}^2\right)\left(22.0\text{ m}^2\right) = 0.277\text{ W}.$$

Are you surprised by how small this is? The energy radiated in 20.0 minutes is

$$E = \wp t = (0.277\text{ J/s})(20.0\text{ min})\left(\frac{60.0\text{ s}}{1.00\text{ min}}\right) = \boxed{332\text{ J}}.$$

(b) If the ground reflects all sound energy headed downward, the sound power, $\wp = 0.277\text{ W}$, covers the area of a hemisphere. One kilometer away, this area is $A = 2\pi r^2 = 2\pi(1\,000\text{ m})^2 = 2\pi\times10^6\text{ m}^2$.
The intensity at this distance is

$$I = \frac{\wp}{A} = \frac{0.277\text{ W}}{2\pi\times10^6\text{ m}^2} = 4.41\times10^{-8}\ \text{W/m}^2$$

and the sound intensity level is

$$\beta = (10\text{ dB})\ln\left(\frac{4.41\times10^{-8}\ \text{W/m}^2}{1.00\times10^{-12}\ \text{W/m}^2}\right) = \boxed{46.4\text{ dB}}.$$

***P17.36** Assume you are 1 m away from your lawnmower and receiving 100 dB sound from it. The intensity of this sound is given by $100\text{ dB} = 10\text{ dB}\log\frac{I}{10^{-12}\text{ W/m}^2}$; $I = 10^{-2}\text{ W/m}^2$. If the lawnmower radiates as a point source, its sound power is given by $I = \frac{\wp}{4\pi r^2}$.

$$\wp = 4\pi(1\text{ m})^2 10^{-2}\text{ W/m}^2 = 0.126\text{ W}$$

Now let your neighbor have an identical lawnmower 20 m away. You receive from it sound with intensity $I = \frac{0.126\text{ W}}{4\pi(20\text{ m})^2} = 2.5\times10^{-5}\text{ W/m}^2$. The total sound intensity impinging on you is $10^{-2}\text{ W/m}^2 + 2.5\times10^{-5}\text{ W/m}^2 = 1.0025\times10^{-2}\text{ W/m}^2$. So its level is

$$10\text{ dB}\log\frac{1.0025\times10^{-2}}{10^{-12}} = 100.01\text{ dB}.$$

If the smallest noticeable difference is between 100 dB and 101 dB, this $\boxed{\text{cannot}}$ be heard as a change from 100 dB.

Section 17.4 The Doppler Effect

P17.37 $f' = f\frac{(v \pm v_O)}{(v \pm v_S)}$

(a) $f' = 320\frac{(343+40.0)}{(343+20.0)} = \boxed{338\text{ Hz}}$

(b) $f' = 510\frac{(343+20.0)}{(343+40.0)} = \boxed{483\text{ Hz}}$

P17.38 (a) $\omega = 2\pi f = 2\pi\left(\frac{115/\text{min}}{60.0\text{ s/min}}\right) = 12.0\text{ rad/s}$

$v_{max} = \omega A = (12.0\text{ rad/s})(1.80\times10^{-3}\text{ m}) = \boxed{0.0217\text{ m/s}}$

(b) The heart wall is a moving observer.

$f' = f\left(\frac{v+v_O}{v}\right) = (2\,000\,000\text{ Hz})\left(\frac{1\,500+0.0217}{1\,500}\right) = \boxed{2\,000\,028.9\text{ Hz}}$

(c) Now the heart wall is a moving source.

$f'' = f'\left(\frac{v}{v-v_s}\right) = (2\,000\,029\text{ Hz})\left(\frac{1\,500}{1\,500-0.0217}\right) = \boxed{2\,000\,057.8\text{ Hz}}$

P17.39 Approaching ambulance: $f' = \frac{f}{(1 - v_S/v)}$

Departing ambulance: $f'' = \frac{f}{(1 - (-v_S/v))}$

Since $f' = 560$ Hz and $f'' = 480$ Hz

$$560\left(1 - \frac{v_S}{v}\right) = 480\left(1 + \frac{v_S}{v}\right)$$

$$1\,040\frac{v_S}{v} = 80.0$$

$$v_S = \frac{80.0(343)}{1\,040}\text{ m/s} = \boxed{26.4\text{ m/s}}$$

P17.40 (a) The maximum speed of the speaker is described by

$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$$

$$v_{\max} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{20.0\text{ N/m}}{5.00\text{ kg}}}(0.500\text{ m}) = 1.00\text{ m/s}$$

The frequencies heard by the stationary observer range from

$$f'_{\min} = f\left(\frac{v}{v + v_{\max}}\right) \text{ to } f'_{\max} = f\left(\frac{v}{v - v_{\max}}\right)$$

where v is the speed of sound.

$$f'_{\min} = 440\text{ Hz}\left(\frac{343\text{ m/s}}{343\text{ m/s} + 1.00\text{ m/s}}\right) = \boxed{439\text{ Hz}}$$

$$f'_{\max} = 440\text{ Hz}\left(\frac{343\text{ m/s}}{343\text{ m/s} - 1.00\text{ m/s}}\right) = \boxed{441\text{ Hz}}$$

(b) $\beta = 10\text{ dB}\log\left(\frac{I}{I_0}\right) = 10\text{ dB}\log\left(\frac{\wp/4\pi r^2}{I_0}\right)$

The maximum intensity level (of 60.0 dB) occurs at $r = r_{\min} = 1.00$ m. The minimum intensity level occurs when the speaker is farthest from the listener (i.e., when $r = r_{\max} = r_{\min} + 2A = 2.00$ m).

Thus, $\beta_{\max} - \beta_{\min} = 10\text{ dB}\log\left(\frac{\wp}{4\pi I_0 r_{\min}^2}\right) - 10\text{ dB}\log\left(\frac{\wp}{4\pi I_0 r_{\max}^2}\right)$

or $\beta_{\max} - \beta_{\min} = 10\text{ dB}\log\left(\frac{\wp}{4\pi I_0 r_{\min}^2}\cdot\frac{4\pi I_0 r_{\max}^2}{\wp}\right) = 10\text{ dB}\log\left(\frac{r_{\max}^2}{r_{\min}^2}\right)$.

This gives: $60.0\text{ dB} - \beta_{\min} = 10\text{ dB}\log(4.00) = 6.02$ dB, and $\beta_{\min} = \boxed{54.0\text{ dB}}$.

P17.41 $f' = f\left(\frac{v}{v - v_s}\right)$ $$485 = 512\left(\frac{340}{340 - (-9.80t_{\text{fall}})}\right)$$

$$485(340) + (485)(9.80t_f) = (512)(340)$$

$$t_f = \left(\frac{512 - 485}{485}\right)\frac{340}{9.80} = 1.93\text{ s}$$

$d_1 = \frac{1}{2}gt_f^2 = 18.3\text{ m}:$ $$t_{\text{return}} = \frac{18.3}{340} = 0.053\,8\text{ s}$$

The fork continues to fall while the sound returns.

$$t_{\text{total fall}} = t_f + t_{\text{return}} = 1.93\text{ s} + 0.053\,8\text{ s} = 1.985\text{ s}$$

$$d_{\text{total}} = \frac{1}{2}gt_{\text{total fall}}^2 = \boxed{19.3\text{ m}}$$

P17.42 (a) $v = (331\text{ m/s}) + 0.6\frac{\text{m}}{\text{s}\cdot{}^\circ\text{C}}(-10^\circ\text{C}) = \boxed{325\text{ m/s}}$

(b) Approaching the bell, the athlete hears a frequency of $f' = f\left(\frac{v + v_O}{v}\right)$

After passing the bell, she hears a lower frequency of $f'' = f\left(\frac{v + (-v_O)}{v}\right)$

The ratio is $\frac{f''}{f'} = \frac{v - v_O}{v + v_O} = \frac{5}{6}$

which gives $6v - 6v_o = 5v + 5v_o$ or $v_O = \frac{v}{11} = \frac{325\text{ m/s}}{11} = \boxed{29.5\text{ m/s}}$

***P17.43** (a) Sound moves upwind with speed $(343 - 15)$ m/s. Crests pass a stationary upwind point at frequency 900 Hz.

Then $\lambda = \frac{v}{f} = \frac{328\text{ m/s}}{900/\text{s}} = \boxed{0.364\text{ m}}$

(b) By similar logic, $\lambda = \frac{v}{f} = \frac{(343 + 15)\text{ m/s}}{900/\text{s}} = \boxed{0.398\text{ m}}$

(c) The source is moving through the air at 15 m/s toward the observer. The observer is stationary relative to the air.

$$f' = f\left(\frac{v + v_o}{v - v_s}\right) = 900\text{ Hz}\left(\frac{343 + 0}{343 - 15}\right) = \boxed{941\text{ Hz}}$$

(d) The source is moving through the air at 15 m/s away from the downwind firefighter. Her speed relative to the air is 30 m/s toward the source.

$$f' = f\left(\frac{v + v_o}{v - v_s}\right) = 900\text{ Hz}\left(\frac{343 + 30}{343 - (-15)}\right) = 900\text{ Hz}\left(\frac{373}{358}\right) = \boxed{938\text{ Hz}}$$

***P17.44** The half-angle of the cone of the shock wave is θ where

$$\theta = \sin^{-1}\left(\frac{v_{\text{sound}}}{v_{\text{source}}}\right) = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8°.$$

As shown in the sketch, the angle between the direction of propagation of the shock wave and the direction of the plane's velocity is

$$\phi = 90° - \theta = 90° - 41.8° = \boxed{48.2°}.$$

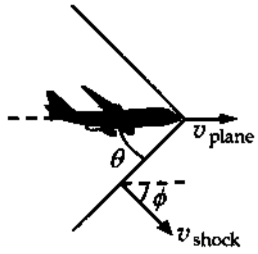


FIG. P17.44

P17.45 The *half angle* of the shock wave cone is given by $\sin\theta = \frac{v_{\text{light}}}{v_S}$.

$$v_S = \frac{v_{\text{light}}}{\sin\theta} = \frac{2.25 \times 10^8 \text{ m/s}}{\sin(53.0°)} = \boxed{2.82 \times 10^8 \text{ m/s}}$$

P17.46 $\theta = \sin^{-1}\frac{v}{v_S} = \sin^{-1}\frac{1}{1.38} = \boxed{46.4°}$

P17.47 (b) $\sin\theta = \frac{v}{v_S} = \frac{1}{3.00}$; $\theta = 19.5°$

$\tan\theta = \frac{h}{x}$; $x = \frac{h}{\tan\theta}$

$x = \frac{20\,000 \text{ m}}{\tan 19.5°} = 5.66 \times 10^4 \text{ m} = \boxed{56.6 \text{ km}}$

(a) It takes the plane $t = \frac{x}{v_S} = \frac{5.66 \times 10^4 \text{ m}}{3.00(335 \text{ m/s})} = \boxed{56.3 \text{ s}}$ to travel this distance.

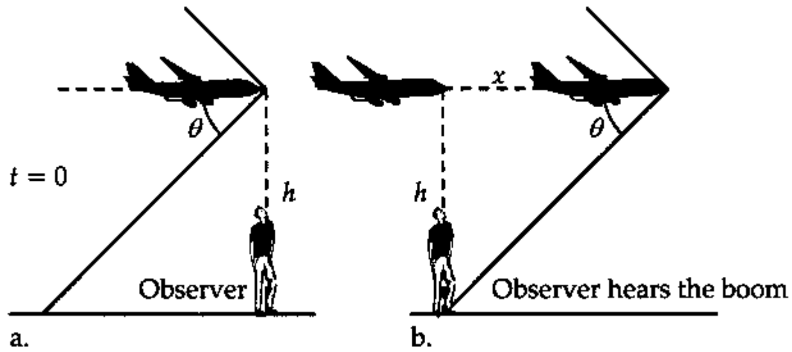


FIG. P17.47(a)

Section 17.5 Digital Sound Recording

Section 17.6 Motion Picture Sound

***P17.48** For a 40-dB sound,

$$40\ \text{dB} = 10\ \text{dB}\log\left[\frac{I}{10^{-12}\ \text{W/m}^2}\right]$$

$$I = 10^{-8}\ \text{W/m}^2 = \frac{\Delta P_{\max}^2}{2\rho v}$$

$$\Delta P_{\max} = \sqrt{2\rho v I} = \sqrt{2\left(1.20\ \text{kg/m}^2\right)(343\ \text{m/s})10^{-8}\ \text{W/m}^2} = 2.87\times10^{-3}\ \text{N/m}^2$$

(a) $$\text{code} = \frac{2.87\times10^{-3}\ \text{N/m}^2}{28.7\ \text{N/m}^2}65\,536 = \boxed{7}$$

(b) For sounds of 40 dB or softer, too few digital words are available to represent the wave form with good fidelity.

(c) In a sound wave ΔP is negative half of the time but this coding scheme has no words available for negative pressure variations.

***P17.49** If the source is to the left at angle θ from the direction you are facing, the sound must travel an extra distance $d\sin\theta$ to reach your right ear as shown, where d is the distance between your ears. The delay time is Δt in $v = \frac{d\sin\theta}{\Delta t}$. Then

$$\theta = \sin^{-1}\frac{v\Delta t}{d} = \sin^{-1}\frac{(343\ \text{m/s})210\times10^{-6}\ \text{s}}{0.19\ \text{m}} = \boxed{22.3°\ \text{left of center}}.$$

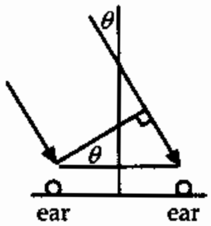


FIG. P17.49

***P17.50** $$103\ \text{dB} = 10\ \text{dB}\log\left[\frac{I}{10^{-12}\ \text{W/m}^2}\right]$$

(a) $$I = 2.00\times10^{-2}\ \text{W/m}^2 = \frac{\wp}{4\pi r^2} = \frac{\wp}{4\pi(1.6\ \text{m})^2}$$

$$\wp = \boxed{0.642\ \text{W}}$$

(b) $$\text{efficiency} = \frac{\text{sound output power}}{\text{total input power}} = \frac{0.642\ \text{W}}{150\ \text{W}} = \boxed{0.004\,28}$$

Additional Problems

P17.51 Model your loud, sharp sound impulse as a single narrow peak in a graph of air pressure versus time. It is a noise with no pitch, no frequency, wavelength, or period. It radiates away from you in all directions and some of it is incident on each one of the solid vertical risers of the bleachers. Suppose that, at the ambient temperature, sound moves at 340 m/s; and suppose that the horizontal width of each row of seats is 60 cm. Then there is a time delay of

$$\frac{0.6\ \text{m}}{(340\ \text{m/s})} = 0.002\ \text{s}$$

continued on next page

between your sound impulse reaching each riser and the next. Whatever its material, each will reflect much of the sound that reaches it. The reflected wave sounds very different from the sharp pop you made. If there are twenty rows of seats, you hear from the bleachers a tone with twenty crests, each separated from the next in time by

$$\frac{2(0.6\text{ m})}{(340\text{ m/s})}=0.004\text{ s}.$$

This is the extra time for it to cross the width of one seat twice, once as an incident pulse and once again after its reflection. Thus, you hear a sound of definite pitch, with period about 0.004 s, frequency

$$\frac{1}{0.003\,5\text{ s}}\boxed{\sim 300\text{ Hz}}$$

wavelength

$$\lambda=\frac{v}{f}=\frac{(340\text{ m/s})}{(300/\text{s})}=1.2\text{ m}\sim\boxed{10^{0}\text{ m}}$$

and duration

$$20(0.004\text{ s})\sim\boxed{10^{-1}\text{ s}}.$$

P17.52 (a) $\lambda=\frac{v}{f}=\frac{343\text{ m/s}}{1\,480\text{ s}^{-1}}=\boxed{0.232\text{ m}}$

(b) $\beta=81.0\text{ dB}=10\text{ dB}\log\left[\frac{I}{10^{-12}\text{ W/m}^2}\right]$

$$I=\left(10^{-12}\text{ W/m}^2\right)10^{8.10}=10^{-3.90}\text{ W/m}^2=1.26\times10^{-4}\text{ W/m}^2=\frac{1}{2}\rho v\omega^2 s_{\max}^2$$

$$s_{\max}=\sqrt{\frac{2I}{\rho v\omega^2}}=\sqrt{\frac{2\left(1.26\times10^{-4}\text{ W/m}^2\right)}{\left(1.20\text{ kg/m}^3\right)(343\text{ m/s})4\pi^2\left(1\,480\text{ s}^{-1}\right)^2}}=\boxed{8.41\times10^{-8}\text{ m}}$$

(c) $\lambda'=\frac{v}{f'}=\frac{343\text{ m/s}}{1\,397\text{ s}^{-1}}=0.246\text{ m}\quad\Delta\lambda=\lambda'-\lambda=\boxed{13.8\text{ mm}}$

P17.53 Since $\cos^2\theta+\sin^2\theta=1$, $\sin\theta=\pm\sqrt{1-\cos^2\theta}$ (each sign applying half the time)

$$\Delta P=\Delta P_{\max}\sin(kx-\omega t)=\pm\rho v\omega\, s_{\max}\sqrt{1-\cos^2(kx-\omega t)}$$

Therefore $\Delta P=\pm\rho v\omega\sqrt{s_{\max}^2-s_{\max}^2\cos^2(kx-\omega t)}=\pm\rho v\omega\sqrt{s_{\max}^2-s^2}$

P17.54 The trucks form a train analogous to a wave train of crests with speed $v=19.7\text{ m/s}$ and unshifted frequency $f=\frac{2}{3.00\text{ min}}=0.667\text{ min}^{-1}$.

(a) The cyclist as observer measures a lower Doppler-shifted frequency:

$$f'=f\left(\frac{v+v_o}{v}\right)=\left(0.667\text{ min}^{-1}\right)\left(\frac{19.7+(-4.47)}{19.7}\right)=\boxed{0.515/\text{min}}$$

(b) $f''=f\left(\frac{v+v_o'}{v}\right)=\left(0.667\text{ min}^{-1}\right)\left(\frac{19.7+(-1.56)}{19.7}\right)=\boxed{0.614/\text{min}}$

The cyclist's speed has decreased very significantly, but there is only a modest increase in the frequency of trucks passing him.

P17.55 $v = \frac{2d}{t}$: $d = \frac{vt}{2} = \frac{1}{2}(6.50\times10^3\text{ m/s})(1.85\text{ s}) = \boxed{6.01\text{ km}}$

P17.56 (a) The speed of a compression wave in a bar is

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{20.0\times10^{10}\text{ N/m}^2}{7\,860\text{ kg/m}^3}} = \boxed{5.04\times10^3\text{ m/s}}.$$

(b) The signal to stop passes between layers of atoms as a sound wave, reaching the back end of the bar in time

$$t = \frac{L}{v} = \frac{0.800\text{ m}}{5.04\times10^3\text{ m/s}} = \boxed{1.59\times10^{-4}\text{ s}}.$$

(c) As described by Newton's first law, the rearmost layer of steel has continued to move forward with its original speed v_i for this time, compressing the bar by

$$\Delta L = v_i t = (12.0\text{ m/s})(1.59\times10^{-4}\text{ s}) = 1.90\times10^{-3}\text{ m} = \boxed{1.90\text{ mm}}.$$

(d) The strain in the rod is: $\frac{\Delta L}{L} = \frac{1.90\times10^{-3}\text{ m}}{0.800\text{ m}} = \boxed{2.38\times10^{-3}}$.

(e) The stress in the rod is:

$$\sigma = Y\left(\frac{\Delta L}{L}\right) = (20.0\times10^{10}\text{ N/m}^2)(2.38\times10^{-3}) = \boxed{476\text{ MPa}}.$$

Since $\sigma > 400\text{ MPa}$, the rod will be permanently distorted.

(f) We go through the same steps as in parts (a) through (e), but use algebraic expressions rather than numbers:

The speed of sound in the rod is $v = \sqrt{\frac{Y}{\rho}}$.

The back end of the rod continues to move forward at speed v_i for a time of $t = \frac{L}{v} = L\sqrt{\frac{\rho}{Y}}$, traveling distance $\Delta L = v_i t$ after the front end hits the wall.

The strain in the rod is: $\frac{\Delta L}{L} = \frac{v_i t}{L} = v_i\sqrt{\frac{\rho}{Y}}$.

The stress is then: $\sigma = Y\left(\frac{\Delta L}{L}\right) = Yv_i\sqrt{\frac{\rho}{Y}} = v_i\sqrt{\rho Y}$.

For this to be less than the yield stress, σ_y, it is necessary that

$$v_i\sqrt{\rho Y} < \sigma_y \text{ or } \boxed{v_i < \frac{\sigma_y}{\sqrt{\rho Y}}}.$$

With the given numbers, this speed is 10.1 m/s. The fact that the length of the rod divides out means that the steel will start to bend right away at the front end of the rod. There it will yield enough so that eventually the remainder of the rod will experience only stress within the elastic range. You can see this effect when sledgehammer blows give a mushroom top to a rod used as a tent stake.

P17.57 (a) $f' = f\dfrac{v}{(v - v_{\text{diver}})}$

so $1 - \dfrac{v_{\text{diver}}}{v} = \dfrac{f}{f'}$

$\Rightarrow v_{\text{diver}} = v\left(1 - \dfrac{f}{f'}\right)$

with $v = 343$ m/s, $f' = 1\,800$ Hz and $f = 2\,150$ Hz

we find

$$v_{\text{diver}} = 343\left(1 - \frac{1\,800}{2\,150}\right) = \boxed{55.8 \text{ m/s}}.$$

(b) If the waves are reflected, and the skydiver is moving into them, we have

$$f'' = f'\frac{(v + v_{\text{diver}})}{v} \Rightarrow f'' = f\left[\frac{v}{(v - v_{\text{diver}})}\right]\frac{(v + v_{\text{diver}})}{v}$$

so $f'' = 1\,800\dfrac{(343 + 55.8)}{(343 - 55.8)} = \boxed{2\,500 \text{ Hz}}$.

P17.58 (a) $f' = \dfrac{fv}{v - u}$ $\quad f'' = \dfrac{fv}{v - (-u)}$ $\quad f' - f'' = fv\left(\dfrac{1}{v - u} - \dfrac{1}{v + u}\right)$

$$\Delta f = \frac{fv(v + u - v + u)}{v^2 - u^2} = \frac{2uvf}{v^2\left(1 - \left(u^2/v^2\right)\right)} = \boxed{\frac{2(u/v)}{1 - \left(u^2/v^2\right)}f}$$

(b) 130 km/h $= 36.1$ m/s $\quad \therefore \Delta f = \dfrac{2(36.1)(400)}{340\left[1 - (36.1)^2/340^2\right]} = \boxed{85.9 \text{ Hz}}$

P17.59 When observer is moving in front of and in the same direction as the source, $f' = f\dfrac{v - v_O}{v - v_S}$ where v_O and v_S are measured relative to the medium in which the sound is propagated. In this case the ocean current is opposite the direction of travel of the ships and

$$v_O = 45.0 \text{ km/h} - (-10.0 \text{ km/h}) = 55.0 \text{ km/h} = 15.3 \text{ m/s, and}$$
$$v_S = 64.0 \text{ km/h} - (-10.0 \text{ km/h}) = 74.0 \text{ km/h} = 20.55 \text{ m/s}$$

Therefore, $f' = (1\,200.0 \text{ Hz})\dfrac{1\,520 \text{ m/s} - 15.3 \text{ m/s}}{1\,520 \text{ m/s} - 20.55 \text{ m/s}} = \boxed{1\,204.2 \text{ Hz}}$.

P17.60 Use the Doppler formula, and remember that the bat is a moving source.

If the velocity of the insect is v_x,

$$40.4 = 40.0\frac{(340+5.00)(340-v_x)}{(340-5.00)(340+v_x)}.$$

Solving,

$$v_x = 3.31\ \text{m/s}.$$

Therefore, $\boxed{\text{the bat is gaining on its prey at } 1.69\ \text{m/s}}$.

P17.61 $\sin\beta = \dfrac{v}{v_S} = \dfrac{1}{N_M}$

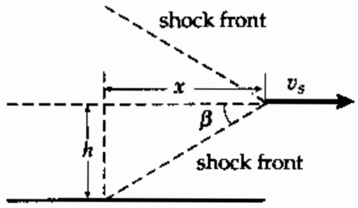


FIG. P17.61

$h = v(12.8\ \text{s})$

$x = v_S(10.0\ \text{s})$

$\tan\beta = \dfrac{h}{x} = 1.28\dfrac{v}{v_S} = \dfrac{1.28}{N_M}$

$\cos\beta = \dfrac{\sin\beta}{\tan\beta} = \dfrac{1}{1.28}$

$\beta = 38.6°$

$N_M = \dfrac{1}{\sin\beta} = \boxed{1.60}$

P17.62 (a)

FIG. P17.62(a)

(b) $\lambda = \dfrac{v}{f} = \dfrac{343\ \text{m/s}}{1\,000\ \text{s}^{-1}} = \boxed{0.343\ \text{m}}$

(c) $\lambda' = \dfrac{v}{f'} = \dfrac{v}{f}\left(\dfrac{v-v_S}{v}\right) = \dfrac{(343-40.0)\ \text{m/s}}{1\,000\ \text{s}^{-1}} = \boxed{0.303\ \text{m}}$

(d) $\lambda'' = \dfrac{v}{f''} = \dfrac{v}{f}\left(\dfrac{v+v_S}{v}\right) = \dfrac{(343+40.0)\ \text{m/s}}{1\,000\ \text{s}^{-1}} = \boxed{0.383\ \text{m}}$

(e) $f' = f\left(\dfrac{v-v_O}{v-v_S}\right) = (1\,000\ \text{Hz})\dfrac{(343-30.0)\ \text{m/s}}{(343-40.0)\ \text{m/s}} = \boxed{1.03\ \text{kHz}}$

P17.63 $\Delta t = L\left(\frac{1}{v_{air}} - \frac{1}{v_{cu}}\right) = L\frac{v_{cu} - v_{air}}{v_{air}v_{cu}}$

$$L = \frac{v_{air}v_{cu}}{v_{cu} - v_{air}}\Delta t = \frac{(331 \text{ m/s})(3.56 \times 10^3 \text{ m/s})}{(3\,560 - 331) \text{ m/s}}(6.40 \times 10^{-3} \text{ s})$$

$\boxed{L = 2.34 \text{ m}}$

P17.64 The shock wavefront connects all observers first hearing the plane, including our observer *O* and the plane *P*, so here it is vertical. The angle ϕ that the shock wavefront makes with the direction of the plane's line of travel is given by

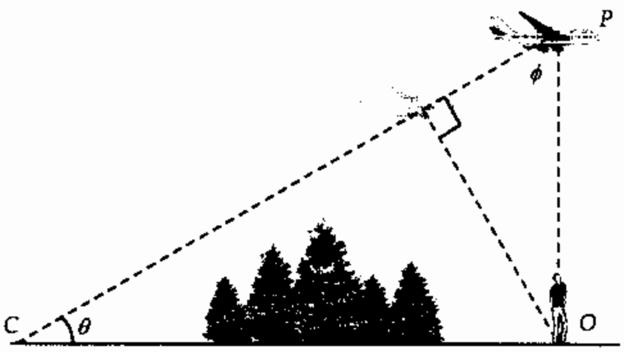


FIG. P17.64

$$\sin\phi = \frac{v}{v_S} = \frac{340 \text{ m/s}}{1\,963 \text{ m/s}} = 0.173$$

so $\phi = 9.97°$.

Using the right triangle *CPO*, the angle θ is seen to be

$$\theta = 90.0° - \phi = 90.0° - 9.97° = \boxed{80.0°}.$$

P17.65 (a) $\theta = \sin^{-1}\left(\frac{v_{sound}}{v_{obj}}\right) = \sin^{-1}\left(\frac{331}{20.0 \times 10^3}\right) = \boxed{0.948°}$

(b) $\theta' = \sin^{-1}\left(\frac{1\,533}{20.0 \times 10^3}\right) = \boxed{4.40°}$

P17.66 $\wp_2 = \frac{1}{20.0}\wp_1 \quad \beta_1 - \beta_2 = 10\log\frac{\wp_1}{\wp_2}$

$80.0 - \beta_2 = 10\log 20.0 = +13.0$

$\beta_2 = \boxed{67.0 \text{ dB}}$

P17.67 For the longitudinal wave $v_L = \left(\frac{Y}{\rho}\right)^{1/2}$.

For the transverse wave $v_T = \left(\frac{T}{\mu}\right)^{1/2}$.

If we require $\frac{v_L}{v_T} = 8.00$, we have $T = \frac{\mu Y}{64.0\rho}$ where $\mu = \frac{m}{L}$ and

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{\pi r^2 L}.$$

This gives $T = \frac{\pi r^2 Y}{64.0} = \frac{\pi(2.00 \times 10^{-3} \text{ m})^2(6.80 \times 10^{10} \text{ N/m}^2)}{64.0} = \boxed{1.34 \times 10^4 \text{ N}}$.

P17.68 The total output sound energy is $eE = \wp\Delta t$, where $\wp$ is the power radiated.

Thus, $\Delta t = \frac{eE}{\wp} = \frac{eE}{IA} = \frac{eE}{(4\pi r^2)I} = \frac{eE}{4\pi d^2 I}$.

But, $\beta = 10\log\left(\frac{I}{I_0}\right)$. Therefore, $I = I_0\left(10^{\beta/10}\right)$ and $\Delta t = \boxed{\frac{eE}{4\pi d^2 I_0 10^{\beta/10}}}$.

P17.69 (a) If the source and the observer are moving away from each other, we have: $\theta_S - \theta_0 = 180°$, and since $\cos 180° = -1$, we get Equation 17.12 with negative values for both v_O and v_S.

(b) If $v_O = 0$ m/s then $f' = \frac{v}{v - v_S\cos\theta_S} f$

Also, when the train is 40.0 m from the intersection, and the car is 30.0 m from the intersection,

$$\cos\theta_S = \frac{4}{5}$$

so $f' = \frac{343 \text{ m/s}}{343 \text{ m/s} - 0.800(25.0 \text{ m/s})}(500 \text{ Hz})$

or $f' = \boxed{531 \text{ Hz}}$.

Note that as the train approaches, passes, and departs from the intersection, θ_S varies from 0° to 180° and the frequency heard by the observer varies from:

$$f'_{\text{max}} = \frac{v}{v - v_S\cos 0°} f = \frac{343 \text{ m/s}}{343 \text{ m/s} - 25.0 \text{ m/s}}(500 \text{ Hz}) = 539 \text{ Hz}$$

$$f'_{\text{min}} = \frac{v}{v - v_S\cos 180°} f = \frac{343 \text{ m/s}}{343 \text{ m/s} + 25.0 \text{ m/s}}(500 \text{ Hz}) = 466 \text{ Hz}$$

P17.70 Let T represent the period of the source vibration, and E be the energy put into each wavefront. Then $\wp_{\text{av}} = \frac{E}{T}$. When the observer is at distance r in front of the source, he is receiving a spherical wavefront of radius vt, where t is the time since this energy was radiated, given by $vt - v_S t = r$. Then,

$$t = \frac{r}{v - v_S}.$$

The area of the sphere is $4\pi(vt)^2 = \frac{4\pi v^2 r^2}{(v - v_S)^2}$. The energy per unit area over the spherical wavefront is uniform with the value $\frac{E}{A} = \frac{\wp_{\text{av}}T(v - v_S)^2}{4\pi v^2 r^2}$.

The observer receives parcels of energy with the Doppler shifted frequency $f' = f\left(\frac{v}{v - v_S}\right) = \frac{v}{T(v - v_S)}$, so the observer receives a wave with intensity

$$I = \left(\frac{E}{A}\right)f' = \left(\frac{\wp_{\text{av}}T(v - v_S)^2}{4\pi v^2 r^2}\right)\left(\frac{v}{T(v - v_S)}\right) = \boxed{\frac{\wp_{\text{av}}}{4\pi r^2}\left(\frac{v - v_S}{v}\right)}.$$

P17.71 (a) The time required for a sound pulse to travel distance L at speed v is given by $t = \frac{L}{v} = \frac{L}{\sqrt{Y/\rho}}$. Using this expression we find

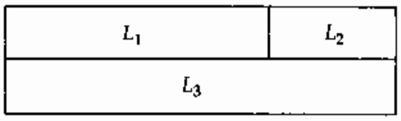


FIG. P17.71

$$t_1 = \frac{L_1}{\sqrt{Y_1/\rho_1}} = \frac{L_1}{\sqrt{\left(7.00\times10^{10}\ \text{N/m}^2\right)/\left(2\,700\ \text{kg/m}^3\right)}} = \left(1.96\times10^{-4}L_1\right)\text{ s}$$

$$t_2 = \frac{1.50\text{ m} - L_1}{\sqrt{Y_2/\rho_2}} = \frac{1.50\text{ m} - L_1}{\sqrt{\left(1.60\times10^{10}\ \text{N/m}^2\right)/\left(11.3\times10^3\ \text{kg/m}^3\right)}}$$

or $t_2 = \left(1.26\times10^{-3} - 8.40\times10^{-4}L_1\right)\text{ s}$

$$t_3 = \frac{1.50\text{ m}}{\sqrt{\left(11.0\times10^{10}\ \text{N/m}^3\right)/\left(8\,800\ \text{kg/m}^3\right)}}$$

$$t_3 = 4.24\times10^{-4}\text{ s}$$

We require $t_1 + t_2 = t_3$, or

$$1.96\times10^{-4}L_1 + 1.26\times10^{-3} - 8.40\times10^{-4}L_1 = 4.24\times10^{-4}.$$

This gives $L_1 = 1.30\text{ m}$ and $L_2 = 1.50 - 1.30 = 0.201\text{ m}$.

The ratio of lengths is then $\frac{L_1}{L_2} = \boxed{6.45}$.

(b) The ratio of lengths $\frac{L_1}{L_2}$ is adjusted in part (a) so that $t_1 + t_2 = t_3$. Sound travels the two paths in equal time and the phase difference, $\boxed{\Delta\phi = 0}$.

P17.72 To find the separation of adjacent molecules, use a model where each molecule occupies a sphere of radius r given by

$$\rho_{\text{air}} = \frac{\text{average mass per molecule}}{\frac{4}{3}\pi r^3}$$

or $1.20\ \text{kg/m}^3 = \dfrac{4.82\times10^{-26}\text{ kg}}{\frac{4}{3}\pi r^3}$, $r = \left[\dfrac{3\left(4.82\times10^{-26}\text{ kg}\right)}{4\pi\left(1.20\ \text{kg/m}^3\right)}\right]^{1/3} = 2.12\times10^{-9}\text{ m}$.

Intermolecular separation is $2r = 4.25\times10^{-9}\text{ m}$, so the highest possible frequency sound wave is

$$f_{\text{max}} = \frac{v}{\lambda_{\text{min}}} = \frac{v}{2r} = \frac{343\text{ m/s}}{4.25\times10^{-9}\text{ m}} = 8.03\times10^{10}\text{ Hz}\ \boxed{\sim10^{11}\text{ Hz}}.$$

P17.2 1.43 km/s

P17.4 (a) 27.2 s; (b) longer than 25.7 s, because the air is cooler

P17.6 (a) 153 m/s; (b) 614 m

P17.8 (a) 4.16 m; (b) 0.455 μs; (c) 0.157 mm

P17.10 1.55×10^{-10} m

P17.12 (a) 1.27 Pa; (b) 170 Hz; (c) 2.00 m; (d) 340 m/s

P17.14 $s = 22.5 \text{ nm} \cos(62.8x - 2.16 \times 10^4 t)$

P17.16 (a) 4.63 mm; (b) 14.5 m/s; (c) 4.73×10^9 W/m^2

P17.18 (a) 5.00×10^{-17} W; (b) 5.00×10^{-5} W

P17.20 (a) 1.00×10^{-5} W/m^2; (b) 90.7 mPa

P17.22 (a) $I_2 = \left(\frac{f'}{f}\right)^2 I_1$; (b) $I_2 = I_1$

P17.24 21.2 W

P17.26 (a) 4.51 times larger in water than in air and 18.0 times larger in iron; (b) 5.60 times larger in water than in iron and 331 times larger in air; (c) 59.1 times larger in water than in air and 331 times larger in iron; (d) 0.331 m; 1.49 m; 5.95 m; 10.9 nm; 184 pm; 32.9 pm; 29.2 mPa; 1.73 Pa; 9.67 Pa

P17.28 see the solution

P17.30 10.0 m; 100 m

P17.32 86.6 m

P17.34 (a) 1.76 kJ; (b) 108 dB

P17.36 no

P17.38 (a) 2.17 cm/s; (b) 2 000 028.9 Hz; (c) 2 000 057.8 Hz

P17.40 (a) 441 Hz; 439 Hz; (b) 54.0 dB

P17.42 (a) 325 m/s; (b) 29.5 m/s

P17.44 48.2°

P17.46 46.4°

P17.48 (a) 7; (b) and (c) see the solution

P17.50 (a) 0.642 W; (b) 0.004 28 = 0.428%

P17.52 (a) 0.232 m; (b) 84.1 nm; (c) 13.8 mm

P17.54 (a) 0.515/min; (b) 0.614/min

P17.56 (a) 5.04 km/s; (b) 159 μs; (c) 1.90 mm; (d) 0.002 38; (e) 476 MPa; (f) see the solution

P17.58 (a) see the solution; (b) 85.9 Hz

P17.60 The gap between bat and insect is closing at 1.69 m/s.

P17.62 (a) see the solution; (b) 0.343 m; (c) 0.303 m; (d) 0.383 m; (e) 1.03 kHz

P17.64 80.0°

P17.66 67.0 dB

P17.68 $\Delta t = \dfrac{eE}{4\pi d^2 I_0 10^{\beta/10}}$

P17.70 see the solution

P17.72 $\sim 10^{11}$ Hz

18

Superposition and Standing Waves

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q18.1 No. Waves with other waveforms are also trains of disturbance that add together when waves from different sources move through the same medium at the same time.

Q18.2 The energy has not disappeared, but is still carried by the wave pulses. Each particle of the string still has kinetic energy. This is similar to the motion of a simple pendulum. The pendulum does not stop at its equilibrium position during oscillation—likewise the particles of the string do not stop at the equilibrium position of the string when these two waves superimpose.

Q18.3 No. A wave is not a solid object, but a chain of disturbance. As described by the principle of superposition, the waves move through each other.

Q18.4 They can, wherever the two waves are nearly enough in phase that their displacements will add to create a total displacement greater than the amplitude of either of the two original waves.

When two one-dimensional sinusoidal waves of the same amplitude interfere, this condition is satisfied whenever the absolute value of the phase difference between the two waves is less than 120°.

Q18.5 When the two tubes together are not an efficient transmitter of sound from source to receiver, they are an efficient reflector. The incoming sound is reflected back to the source. The waves reflected by the two tubes separately at the junction interfere constructively.

Q18.6 No. The total energy of the pair of waves remains the same. Energy missing from zones of destructive interference appears in zones of constructive interference.

Q18.7 Each of these standing wave patterns is made of two superimposed waves of identical frequencies traveling, and hence transferring energy, in opposite directions. Since the energy transfer of the waves are equal, then no net transfer of energy occurs.

Q18.8 Damping, and non–linear effects in the vibration turn the energy of vibration into internal energy.

Q18.9 The air in the shower stall can vibrate in standing wave patterns to intensify those frequencies in your voice which correspond to its free vibrations. The hard walls of the bathroom reflect sound very well to make your voice more intense at all frequencies, giving the room a longer reverberation time. The reverberant sound may help you to stay on key.

Q18.10 The trombone slide and trumpet valves change the length of the air column inside the instrument, to change its resonant frequencies.

Q18.11 The vibration of the air must have zero amplitude at the closed end. For air in a pipe closed at one end, the diagrams show how resonance vibrations have NA distances that are odd integer submultiples of the NA distance in the fundamental vibration. If the pipe is open, resonance vibrations have NA distances that are all integer submultiples of the NA distance in the fundamental.

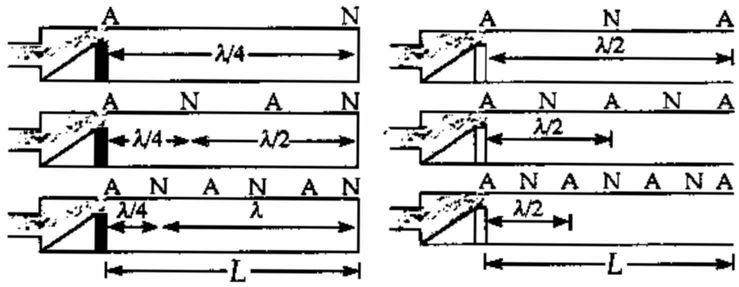


FIG. Q18.11

Q18.12 What is needed is a tuning fork—or other pure-tone generator—of the desired frequency. Strike the tuning fork and pluck the corresponding string on the piano at the same time. If they are precisely in tune, you will hear a single pitch with no amplitude modulation. If the two pitches are a bit off, you will hear beats. As they vibrate, retune the piano string until the beat frequency goes to zero.

Q18.13 Air blowing fast by a rim of the pipe creates a "shshshsh" sound called edgetone noise, a mixture of all frequencies, as the air turbulently switches between flowing on one side of the edge and the other. The air column inside the pipe finds one or more of its resonance frequencies in the noise. The air column starts vibrating with large amplitude in a standing wave vibration mode. It radiates sound into the surrounding air (and also locks the flapping airstream at the edge to its own frequency, making the noise disappear after just a few cycles).

Q18.14 A typical standing-wave vibration possibility for a bell is similar to that for the glass shown in Figure 18.17. Here six node-to-node distances fit around the circumference of the rim. The circumference is equal to three times the wavelength of the transverse wave of in-and-out bending of the material. In other states the circumference is two, four, five, or higher integers times the wavelengths of the higher-frequency vibrations. (The circumference being equal to the wavelength would describe the bell moving from side to side without bending, which it can do without producing any sound.) A tuned bell is cast and shaped so that some of these vibrations will have their frequencies constitute higher harmonics of a musical note, the strike tone. This tuning is lost if a crack develops in the bell. The sides of the crack vibrate as antinodes. Energy of vibration may be rapidly converted into internal energy at the end of the crack, so the bell may not ring for so long a time.

Q18.15 The bow string is pulled away from equilibrium and released, similar to the way that a guitar string is pulled and released when it is plucked. Thus, standing waves will be excited in the bow string. If the arrow leaves from the exact center of the string, then a series of odd harmonics will be excited. Even harmonies will not be excited because they have a node at the point where the string exhibits its maximum displacement.

Q18.16 Walking makes the person's hand vibrate a little. If the frequency of this motion is equal to the natural frequency of coffee sloshing from side to side in the cup, then a large–amplitude vibration of the coffee will build up in resonance. To get off resonance and back to the normal case of a small-amplitude disturbance producing a small–amplitude result, the person can walk faster, walk slower, or get a larger or smaller cup. Alternatively, even at resonance he can reduce the amplitude by adding damping, as by stirring high–fiber quick–cooking oatmeal into the hot coffee.

Q18.17 Beats. The propellers are rotating at slightly different frequencies.

Q18.18 Instead of just radiating sound very softly into the surrounding air, the tuning fork makes the chalkboard vibrate. With its large area this stiff sounding board radiates sound into the air with higher power. So it drains away the fork's energy of vibration faster and the fork stops vibrating sooner. This process exemplifies conservation of energy, as the energy of vibration of the fork is transferred through the blackboard into energy of vibration of the air.

Q18.19 The difference between static and kinetic friction makes your finger alternately slip and stick as it slides over the glass. Your finger produces a noisy vibration, a mixture of different frequencies, like new sneakers on a gymnasium floor. The glass finds one of its resonance frequencies in the noise. The thin stiff wall of the cup starts vibrating with large amplitude in a standing wave vibration mode. A typical possibility is shown in Figure 18.17. It radiates sound into the surrounding air, and also can lock your squeaking finger to its own frequency, making the noise disappear after just a few cycles. Get a lot of different thin–walled glasses of fine crystal and try them out. Each will generally produce a different note. You can tune them by adding wine.

Q18.20 Helium is less dense than air. It carries sound at higher speed. Each cavity in your vocal apparatus has a standing-wave resonance frequency, and each of these frequencies is shifted to a higher value. Your vocal chords can vibrate at the same fundamental frequency, but your vocal tract amplifies by resonance a different set of higher frequencies. Then your voice has a different quacky quality.

Warning: Inhaling any pressurized gas can cause a gas embolism which can lead to stroke or death, regardless of your age or health status. If you plan to try this demonstration in class, inhale your helium from a balloon, not directly from a pressurized tank.

Q18.21 Stick a bit of chewing gum to one tine of the second fork. If the beat frequency is then faster than 4 beats per second, the second has a lower frequency than the standard fork. If the beats have slowed down, the second fork has a higher frequency than the standard. Remove the gum, clean the fork, add or subtract 4 Hz according to what you found, and your answer will be the frequency of the second fork.

SOLUTIONS TO PROBLEMS

Section 18.1 Superposition and Interference

P18.1 $y = y_1 + y_2 = 3.00\cos(4.00x - 1.60t) + 4.00\sin(5.0x - 2.00t)$ evaluated at the given x values.

(a) $x = 1.00,\ t = 1.00$ $\quad y = 3.00\cos(2.40\text{ rad}) + 4.00\sin(+3.00\text{ rad}) = \boxed{-1.65\text{ cm}}$

(b) $x = 1.00,\ t = 0.500$ $\quad y = 3.00\cos(+3.20\text{ rad}) + 4.00\sin(+4.00\text{ rad}) = \boxed{-6.02\text{ cm}}$

(c) $x = 0.500,\ t = 0$ $\quad y = 3.00\cos(+2.00\text{ rad}) + 4.00\sin(+2.50\text{ rad}) = \boxed{1.15\text{ cm}}$

P18.2

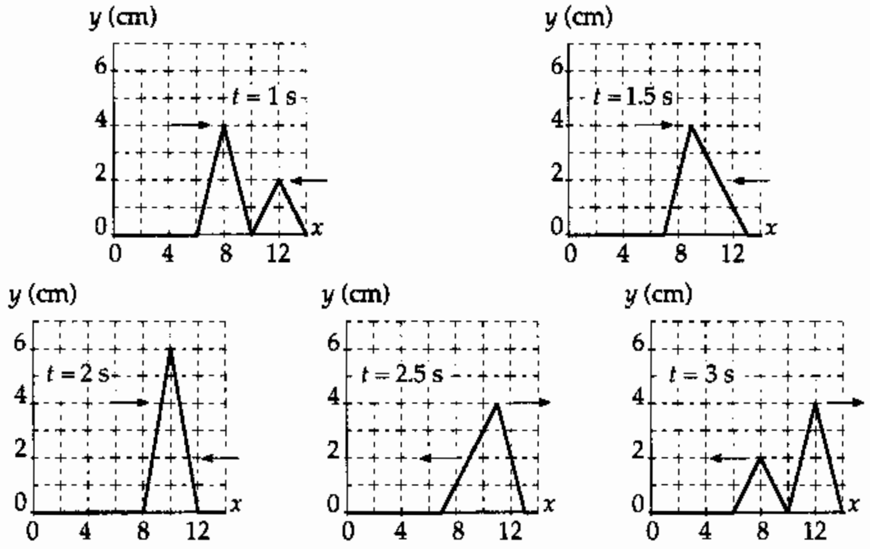


FIG. P18.2

P18.3 (a) $y_1 = f(x - vt)$, so wave 1 travels in the $\boxed{+x \text{ direction}}$

$y_2 = f(x + vt)$, so wave 2 travels in the $\boxed{-x \text{ direction}}$

(b) To cancel, $y_1 + y_2 = 0$:

$$\frac{5}{(3x-4t)^2+2} = \frac{+5}{(3x+4t-6)^2+2}$$

$$(3x-4t)^2 = (3x+4t-6)^2$$

$$3x - 4t = \pm(3x+4t-6)$$

for the positive root, $8t = 6$ $\boxed{t = 0.750 \text{ s}}$

(at $t = 0.750$ s, the waves cancel everywhere)

(c) for the negative root, $6x = 6$ $\boxed{x = 1.00 \text{ m}}$

(at $x = 1.00$ m, the waves cancel always)

P18.4 Suppose the waves are sinusoidal.

The sum is $(4.00 \text{ cm})\sin(kx - \omega t) + (4.00 \text{ cm})\sin(kx - \omega t + 90.0°)$

$$2(4.00 \text{ cm})\sin(kx - \omega t + 45.0°)\cos 45.0°$$

So the amplitude is $(8.00 \text{ cm})\cos 45.0° = \boxed{5.66 \text{ cm}}$.

P18.5 The resultant wave function has the form

$$y = 2A_0\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

(a) $A = 2A_0\cos\left(\frac{\phi}{2}\right) = 2(5.00)\cos\left[\frac{-\pi/4}{2}\right] = \boxed{9.24 \text{ m}}$

(b) $f = \frac{\omega}{2\pi} = \frac{1\,200\pi}{2\pi} = \boxed{600 \text{ Hz}}$

P18.6 $2A_0 \cos\left(\frac{\phi}{2}\right) = A_0$ so $\frac{\phi}{2} = \cos^{-1}\left(\frac{1}{2}\right) = 60.0° = \frac{\pi}{3}$

Thus, the phase difference is $\phi = 120° = \frac{2\pi}{3}$

This phase difference results if the time delay is $\frac{T}{3} = \frac{1}{3f} = \frac{\lambda}{3v}$

$$\text{Time delay} = \frac{3.00\text{ m}}{3(2.00\text{ m/s})} = \boxed{0.500\text{ s}}$$

P18.7 (a) If the end is fixed, there is inversion of the pulse upon reflection. Thus, when they meet, they cancel and the amplitude is $\boxed{\text{zero}}$.

(b) If the end is free, there is no inversion on reflection. When they meet, the amplitude is $2A = 2(0.150\text{ m}) = \boxed{0.300\text{ m}}$.

P18.8 (a) $\Delta x = \sqrt{9.00 + 4.00} - 3.00 = \sqrt{13} - 3.00 = 0.606\text{ m}$

The wavelength is $\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{300\text{ Hz}} = 1.14\text{ m}$

Thus, $\frac{\Delta x}{\lambda} = \frac{0.606}{1.14} = 0.530$ of a wave,

or $\Delta\phi = 2\pi(0.530) = \boxed{3.33\text{ rad}}$

(b) For destructive interference, we want $\frac{\Delta x}{\lambda} = 0.500 = f\frac{\Delta x}{v}$

where Δx is a constant in this set up. $f = \frac{v}{2\Delta x} = \frac{343}{2(0.606)} = \boxed{283\text{ Hz}}$

P18.9 We suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\sqrt{L^2 + d^2} - L$.

He hears a minimum when this is $\frac{(2n-1)\lambda}{2}$ with $n = 1, 2, 3, \ldots$

Then,

$$\sqrt{L^2 + d^2} - L = \frac{(n - 1/2)v}{f}$$

$$\sqrt{L^2 + d^2} = \frac{(n - 1/2)v}{f} + L$$

$$L^2 + d^2 = \frac{(n - 1/2)^2 v^2}{f^2} + L^2 + \frac{2(n - 1/2)vL}{f}$$

$$L = \frac{d^2 - (n - 1/2)^2 v^2/f^2}{2(n - 1/2)v/f} \qquad n = 1, 2, 3, \ldots$$

This will give us the answer to (b). The path difference starts from nearly zero when the man is very far away and increases to d when $L = 0$. The number of minima he hears is the greatest integer solution to $d \geq \frac{(n - 1/2)v}{f}$

$$n = \text{ greatest integer} \leq \frac{df}{v} + \frac{1}{2}.$$

continued on next page

(a) $$\frac{df}{v}+\frac{1}{2}=\frac{(4.00\text{ m})(200/\text{s})}{330\text{ m/s}}+\frac{1}{2}=2.92$$

He hears $\boxed{\text{two}}$ minima.

(b) With $n=1$,

$$L=\frac{d^2-(1/2)^2v^2/f^2}{2(1/2)v/f}=\frac{(4.00\text{ m})^2-(330\text{ m/s})^2/4(200/\text{s})^2}{(330\text{ m/s})/200/\text{s}}$$

$$L=\boxed{9.28\text{ m}}$$

with $n=2$

$$L=\frac{d^2-(3/2)^2v^2/f^2}{2(3/2)v/f}=\boxed{1.99\text{ m}}.$$

P18.10 Suppose the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\Delta r=\sqrt{L^2+d^2}-L$.

He hears a minimum when $\Delta r=(2n-1)\left(\frac{\lambda}{2}\right)$ with $n=1, 2, 3, \ldots$

Then, $$\sqrt{L^2+d^2}-L=\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)$$

$$\sqrt{L^2+d^2}=\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)+L$$

$$L^2+d^2=\left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2+2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)L+L^2$$

$$d^2-\left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2=2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)L \qquad (1)$$

Equation 1 gives the distances from the lower speaker at which the man will hear a minimum. The path difference Δr starts from nearly zero when the man is very far away and increases to d when $L=0$.

(a) The number of minima he hears is the greatest integer value for which $L\geq 0$. This is the same as the greatest integer solution to $d\geq\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)$, or

$$\boxed{\text{number of minima heard }=n_{\max}=\text{greatest integer }\leq d\left(\frac{f}{v}\right)+\frac{1}{2}}.$$

(b) From equation 1, the distances at which minima occur are given by

$$\boxed{L_n=\frac{d^2-(n-1/2)^2(v/f)^2}{2(n-1/2)(v/f)}\text{ where }n=1, 2, \ldots, n_{\max}}.$$

P18.11 (a) $\phi_1 = (20.0 \text{ rad/cm})(5.00 \text{ cm}) - (32.0 \text{ rad/s})(2.00 \text{ s}) = 36.0 \text{ rad}$

$\phi_1 = (25.0 \text{ rad/cm})(5.00 \text{ cm}) - (40.0 \text{ rad/s})(2.00 \text{ s}) = 45.0 \text{ rad}$

$\Delta\phi = 9.00 \text{ radians} = 516° = \boxed{156°}$

(b) $\Delta\phi = \left|20.0x - 32.0t - [25.0x - 40.0t]\right| = \left|-5.00x + 8.00t\right|$

At $t = 2.00$ s, the requirement is

$$\Delta\phi = \left|-5.00x + 8.00(2.00)\right| = (2n+1)\pi \text{ for any integer } n.$$

For $x < 3.20$, $-5.00x + 16.0$ is positive, so we have

$$-5.00x + 16.0 = (2n+1)\pi, \text{ or}$$

$$x = 3.20 - \frac{(2n+1)\pi}{5.00}$$

The smallest positive value of x occurs for $n = 2$ and is

$$x = 3.20 - \frac{(4+1)\pi}{5.00} = 3.20 - \pi = \boxed{0.058\,4 \text{ cm}}.$$

P18.12 (a) First we calculate the wavelength: $\lambda = \frac{v}{f} = \frac{344 \text{ m/s}}{21.5 \text{ Hz}} = 16.0 \text{ m}$

Then we note that the path difference equals $9.00 \text{ m} - 1.00 \text{ m} = \boxed{\frac{1}{2}\lambda}$

Therefore, the receiver will record a minimum in sound intensity.

(b) We choose the origin at the midpoint between the speakers. If the receiver is located at point (x, y), then we must solve:

$$\sqrt{(x+5.00)^2 + y^2} - \sqrt{(x-5.00)^2 + y^2} = \frac{1}{2}\lambda$$

Then, $\sqrt{(x+5.00)^2 + y^2} = \sqrt{(x-5.00)^2 + y^2} + \frac{1}{2}\lambda$

Square both sides and simplify to get: $20.0x - \frac{\lambda^2}{4} = \lambda\sqrt{(x-5.00)^2 + y^2}$

Upon squaring again, this reduces to: $400x^2 - 10.0\lambda^2 x + \frac{\lambda^4}{16.0} = \lambda^2(x-5.00)^2 + \lambda^2 y^2$

Substituting $\lambda = 16.0$ m, and reducing, $\boxed{9.00x^2 - 16.0y^2 = 144}$

or $\frac{x^2}{16.0} - \frac{y^2}{9.00} = 1$

(When plotted this yields a curve called a hyperbola.)

Section 18.2 **Standing Waves**

P18.13 $y = (1.50 \text{ m})\sin(0.400x)\cos(200t) = 2A_0 \sin kx \cos \omega t$

Therefore, $k = \frac{2\pi}{\lambda} = 0.400 \text{ rad/m}$ $\qquad \lambda = \frac{2\pi}{0.400 \text{ rad/m}} = \boxed{15.7 \text{ m}}$

and $\omega = 2\pi f$ so $\qquad f = \frac{\omega}{2\pi} = \frac{200 \text{ rad/s}}{2\pi \text{ rad}} = \boxed{31.8 \text{ Hz}}$

The speed of waves in the medium is $v = \lambda f = \frac{\lambda}{2\pi} 2\pi f = \frac{\omega}{k} = \frac{200 \text{ rad/s}}{0.400 \text{ rad/m}} = \boxed{500 \text{ m/s}}$

P18.14 $y = 0.030\,0 \text{ m} \cos\left(\frac{x}{2}\right)\cos(40t)$

(a) nodes occur where $y = 0$:

$$\frac{x}{2} = (2n+1)\frac{\pi}{2}$$

so $x = \boxed{(2n+1)\pi = \pi, 3\pi, 5\pi, \ldots}$.

(b) $y_{\max} = 0.030\,0 \text{ m} \cos\left(\frac{0.400}{2}\right) = \boxed{0.029\,4 \text{ m}}$

P18.15 The facing speakers produce a standing wave in the space between them, with the spacing between nodes being

$$d_{NN} = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343 \text{ m/s}}{2\left(800 \text{ s}^{-1}\right)} = 0.214 \text{ m}$$

If the speakers vibrate in phase, the point halfway between them is an antinode of pressure at a distance from either speaker of

$$\frac{1.25 \text{ m}}{2} = 0.625.$$

Then there isa node at $\quad 0.625 - \frac{0.214}{2} = \boxed{0.518 \text{ m}}$

a node at $\quad 0.518 \text{ m} - 0.214 \text{ m} = \boxed{0.303 \text{ m}}$

a node at $\quad 0.303 \text{ m} - 0.214 \text{ m} = \boxed{0.089\,1 \text{ m}}$

a node at $\quad 0.518 \text{ m} + 0.214 \text{ m} = \boxed{0.732 \text{ m}}$

a node at $\quad 0.732 \text{ m} + 0.214 \text{ m} = \boxed{0.947 \text{ m}}$

and a node at $\quad 0.947 \text{ m} + 0.214 \text{ m} = \boxed{1.16 \text{ m}}$ from either speaker.

P18.16 $y = 2A_0 \sin kx \cos \omega t$

$$\frac{\partial^2 y}{\partial x^2} = -2A_0 k^2 \sin kx \cos \omega t \qquad \frac{\partial^2 y}{\partial t^2} = -2A_0 \omega^2 \sin kx \cos \omega t$$

Substitution into the wave equation gives $-2A_0 k^2 \sin kx \cos \omega t = \left(\frac{1}{v^2}\right)\left(-2A_0 \omega^2 \sin kx \cos \omega t\right)$

This is satisfied, provided that $v = \frac{\omega}{k}$

P18.17 $y_1 = 3.00 \sin[\pi(x + 0.600t)]$ cm; $y_2 = 3.00 \sin[\pi(x - 0.600t)]$ cm

$y = y_1 + y_2 = [3.00 \sin(\pi x)\cos(0.600\pi t) + 3.00 \sin(\pi x)\cos(0.600\pi t)]$ cm

$y = (6.00 \text{ cm})\sin(\pi x)\cos(0.600\pi t)$

(a) We can take $\cos(0.600\pi t) = 1$ to get the maximum y.
At $x = 0.250$ cm, $y_{max} = (6.00 \text{ cm})\sin(0.250\pi) = \boxed{4.24 \text{ cm}}$

(b) At $x = 0.500$ cm, $y_{max} = (6.00 \text{ cm})\sin(0.500\pi) = \boxed{6.00 \text{ cm}}$

(c) Now take $\cos(0.600\pi t) = -1$ to get y_{max}:
At $x = 1.50$ cm, $y_{max} = (6.00 \text{ cm})\sin(1.50\pi)(-1) = \boxed{6.00 \text{ cm}}$

(d) The antinodes occur when $x = \frac{n\lambda}{4}$ $(n = 1, 3, 5, \ldots)$

But $k = \frac{2\pi}{\lambda} = \pi$, so $\lambda = 2.00$ cm

and $x_1 = \frac{\lambda}{4} = \boxed{0.500 \text{ cm}}$ as in (b)

$x_2 = \frac{3\lambda}{4} = \boxed{1.50 \text{ cm}}$ as in (c)

$x_3 = \frac{5\lambda}{4} = \boxed{2.50 \text{ cm}}$

P18.18 (a) The resultant wave is $y = 2A \sin\left(kx + \frac{\phi}{2}\right)\cos\left(\omega t - \frac{\phi}{2}\right)$

The nodes are located at $kx + \frac{\phi}{2} = n\pi$

so $x = \frac{n\pi}{k} - \frac{\phi}{2k}$

which means that each node is shifted $\frac{\phi}{2k}$ to the left.

(b) The separation of nodes is $\Delta x = \left[(n+1)\frac{\pi}{k} - \frac{\phi}{2k}\right] - \left[\frac{n\pi}{k} - \frac{\phi}{2k}\right]$ $\quad \Delta x = \frac{\pi}{k} = \frac{\lambda}{2}$

The nodes are still separated by half a wavelength.

Section 18.3 **Standing Waves in a String Fixed at Both Ends**

P18.19 $L = 30.0\text{ m}$; $\mu = 9.00\times10^{-3}\text{ kg/m}$; $T = 20.0\text{ N}$; $f_1 = \dfrac{v}{2L}$

where $$v = \left(\frac{T}{\mu}\right)^{1/2} = 47.1\text{ m/s}$$

so $$f_1 = \frac{47.1}{60.0} = \boxed{0.786\text{ Hz}} \qquad f_2 = 2f_1 = \boxed{1.57\text{ Hz}}$$

$$f_3 = 3f_1 = \boxed{2.36\text{ Hz}} \qquad f_4 = 4f_1 = \boxed{3.14\text{ Hz}}$$

***P18.20** The tension in the string is $T = (4\text{ kg})(9.8\text{ m/s}^2) = 39.2\text{ N}$

Its linear density is $\mu = \dfrac{m}{L} = \dfrac{8\times10^{-3}\text{ kg}}{5\text{ m}} = 1.6\times10^{-3}\text{ kg/m}$

and the wave speed on the string is $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{39.2\text{ N}}{1.6\times10^{-3}\text{ kg/m}}} = 156.5\text{ m/s}$

In its fundamental mode of vibration, we have $\lambda = 2L = 2(5\text{ m}) = 10\text{ m}$

Thus, $f = \dfrac{v}{\lambda} = \dfrac{156.5\text{ m/s}}{10\text{ m}} = \boxed{15.7\text{ Hz}}$

P18.21 (a) Let n be the number of nodes in the standing wave resulting from the 25.0-kg mass. Then $n+1$ is the number of nodes for the standing wave resulting from the 16.0-kg mass. For standing waves, $\lambda = \dfrac{2L}{n}$, and the frequency is $f = \dfrac{v}{\lambda}$.

Thus, $$f = \frac{n}{2L}\sqrt{\frac{T_n}{\mu}}$$

and also $$f = \frac{n+1}{2L}\sqrt{\frac{T_{n+1}}{\mu}}$$

Thus, $$\frac{n+1}{n} = \sqrt{\frac{T_n}{T_{n+1}}} = \sqrt{\frac{(25.0\text{ kg})g}{(16.0\text{ kg})g}} = \frac{5}{4}$$

Therefore, $4n + 4 = 5n$, or $n = 4$

Then, $$f = \frac{4}{2(2.00\text{ m})}\sqrt{\frac{(25.0\text{ kg})(9.80\text{ m/s}^2)}{0.002\,00\text{ kg/m}}} = \boxed{350\text{ Hz}}$$

(b) The largest mass will correspond to a standing wave of 1 loop

$(n = 1)$ so $$350\text{ Hz} = \frac{1}{2(2.00\text{ m})}\sqrt{\frac{m(9.80\text{ m/s}^2)}{0.002\,00\text{ kg/m}}}$$

yielding $m = \boxed{400\text{ kg}}$

***P18.22** The first string has linear density

$$\mu_1 = \frac{1.56\times10^{-3}\ \text{kg}}{0.658\ \text{m}} = 2.37\times10^{-3}\ \text{kg/m}.$$

The second, $\mu_2 = \frac{6.75\times10^{-3}\ \text{kg}}{0.950\ \text{m}} = 7.11\times10^{-3}\ \text{kg/m}$.

The tension in both is $T = 6.93\ \text{kg}\ 9.8\ \text{m/s}^2 = 67.9\ \text{N}$. The speed of waves in the first string is

$$v_1 = \sqrt{\frac{T}{\mu_1}} = \sqrt{\frac{67.9\ \text{N}}{2.37\times10^{-3}\ \text{kg/m}}} = 169\ \text{m/s}$$

and in the second $v_2 = \sqrt{\frac{T}{\mu_2}} = 97.8\ \text{m/s}$. The two strings vibrate at the same frequency, according to $\frac{n_1 v_1}{2L_1} = \frac{n_2 v_2}{2L_2}$

$$\frac{n_1 169\ \text{m/s}}{2(0.658\ \text{m})} = \frac{n_2 97.8\ \text{m/s}}{2(0.950\ \text{m})}$$

$\frac{n_2}{n_1} = 2.50 = \frac{5}{2}$. Thus $n_1 = 2$ and $n_2 = 5$ are the number of antinodes on each string in the lowest resonance with a node at the junction.

(b) The first string has $2+1=3$ nodes and the second string 5 more nodes, for a total of 8, or $\boxed{6}$ other than the vibrator and pulley.

FIG. P18.22(b)

(a) The frequency is $\frac{2(169\ \text{m/s})}{2(0.658\ \text{m})} = \boxed{257\ \text{Hz}}$.

***P18.23** For the E-string on a guitar vibrating as a whole, $v = f\lambda = 330\ \text{Hz}(2)64.0\ \text{cm}$. When it is stopped at the first fret we have $\sqrt[12]{2}\,330\ \text{Hz}(2)L_F = v = 330\ \text{Hz}(2)64.0\ \text{cm}$. So $L_F = \frac{64.0\ \text{cm}}{\sqrt[12]{2}}$. Similarly for the second fret, $2^{2/12}330\ \text{Hz}(2)L_{F\#} = v = 330\ \text{Hz}(2)64.0\ \text{cm}$. $L_{F\#} = \frac{64.0\ \text{cm}}{2^{2/12}}$. The spacing between the first and second frets is

$$64.0\ \text{cm}\left(\frac{1}{2^{1/12}} - \frac{1}{2^{2/12}}\right) = 64.0\ \text{cm}\left(\frac{1}{1.059\,5} - \frac{1}{1.059\,5^2}\right) = 3.39\ \text{cm}.$$

This is a more precise version of the answer to the example in the text.

Now the eighteenth fret is distant from the bridge by $L_{18} = \frac{64.0\ \text{cm}}{2^{18/12}}$. And the nineteenth lets this much string vibrate: $L_{19} = \frac{64.0\ \text{cm}}{2^{19/12}}$. The distance between them is

$$64.0\ \text{cm}\left(\frac{1}{2^{18/12}} - \frac{1}{2^{19/12}}\right) = 64.0\ \text{cm}\frac{1}{2^{1.5}}\left(1 - \frac{1}{2^{1/12}}\right) = \boxed{1.27\ \text{cm}}.$$

***P18.24** For the whole string vibrating, $d_{NN} = 0.64\text{ m} = \frac{\lambda}{2}$; $\lambda = 1.28\text{ m}$. The speed of a pulse on the string is $v = f\lambda = 330\frac{1}{s}1.28\text{ m} = 422\text{ m/s}$.

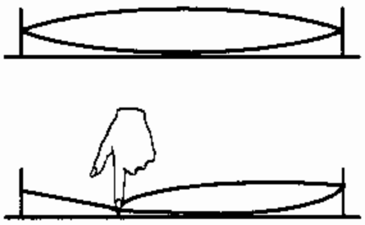


FIG. P18.24(a)

(a) When the string is stopped at the fret, $d_{NN} = \frac{2}{3}0.64\text{ m} = \frac{\lambda}{2}$; $\lambda = 0.853\text{ m}$

$$f = \frac{v}{\lambda} = \frac{422\text{ m/s}}{0.853\text{ m}} = \boxed{495\text{ Hz}}.$$

(b) The light touch at a point one third of the way along the string damps out vibration in the two lowest vibration states of the string as a whole. The whole string vibrates in its third resonance possibility: $3d_{NN} = 0.64\text{ m} = 3\frac{\lambda}{2}$; $\lambda = 0.427\text{ m}$

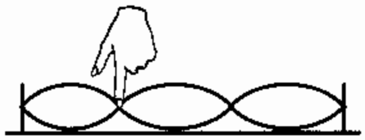


FIG. P18.24(b)

$$f = \frac{v}{\lambda} = \frac{422\text{ m/s}}{0.427\text{ m}} = \boxed{990\text{ Hz}}.$$

P18.25 $f_1 = \frac{v}{2L}$, where $v = \left(\frac{T}{\mu}\right)^{1/2}$

(a) If L is doubled, then $f_1 \propto L^{-1}$ will be reduced by a factor $\frac{1}{2}$.

(b) If μ is doubled, then $f_1 \propto \mu^{-1/2}$ will be reduced by a factor $\frac{1}{\sqrt{2}}$.

(c) If T is doubled, then $f_1 \propto \sqrt{T}$ will increase by a factor of $\sqrt{2}$.

P18.26 $L = 60.0\text{ cm} = 0.600\text{ m}$; $T = 50.0\text{ N}$; $\mu = 0.100\text{ g/cm} = 0.010\,0\text{ kg/m}$

$$f_n = \frac{nv}{2L}$$

where

$$v = \left(\frac{T}{\mu}\right)^{1/2} = 70.7\text{ m/s}$$

$$f_n = n\left(\frac{70.7}{1.20}\right) = 58.9n = 20\,000\text{ Hz}$$

Largest $n = 339 \Rightarrow f = \boxed{19.976\text{ kHz}}$.

P18.27 $d_{NN} = 0.700\text{ m}$

$\lambda = 1.40\text{ m}$

$$f\lambda = v = 308\text{ m/s} = \sqrt{\frac{T}{(1.20\times10^{-3})/(0.700)}}$$

(a) $T = \boxed{163\text{ N}}$

(b) $f_3 = \boxed{660\text{ Hz}}$

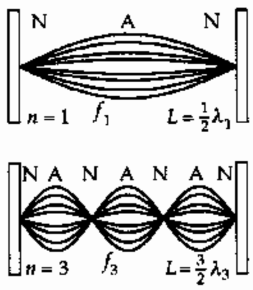


FIG. P18.27

P18.28 $\lambda_G = 2(0.350\text{ m}) = \dfrac{v}{f_G}$; $\lambda_A = 2L_A = \dfrac{v}{f_A}$

$$L_G - L_A = L_G - \left(\frac{f_G}{f_A}\right)L_G = L_G\left(1 - \frac{f_G}{f_A}\right) = (0.350\text{ m})\left(1 - \frac{392}{440}\right) = 0.038\,2\text{ m}$$

Thus, $L_A = L_G - 0.038\,2\text{ m} = 0.350\text{ m} - 0.038\,2\text{ m} = 0.312\text{ m}$,

or the finger should be placed $\boxed{\text{31.2 cm from the bridge}}$.

$$L_A = \frac{v}{2f_A} = \frac{1}{2f_A}\sqrt{\frac{T}{\mu}};\; dL_A = \frac{dT}{4f_A\sqrt{T\mu}};\; \frac{dL_A}{L_A} = \frac{1}{2}\frac{dT}{T}$$

$$\frac{dT}{T} = 2\frac{dL_A}{L_A} = 2\frac{0.600\text{ cm}}{(35.0 - 3.82)\text{ cm}} = \boxed{3.84\%}$$

P18.29 In the fundamental mode, the string above the rod has only two nodes, at A and B, with an anti-node halfway between A and B. Thus,

$$\frac{\lambda}{2} = \overline{AB} = \frac{L}{\cos\theta} \text{ or } \lambda = \frac{2L}{\cos\theta}.$$

Since the fundamental frequency is f, the wave speed in this segment of string is

$$v = \lambda f = \frac{2Lf}{\cos\theta}.$$

Also, $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{T}{m/\overline{AB}}} = \sqrt{\dfrac{TL}{m\cos\theta}}$

where T is the tension in this part of the string. Thus,

$$\frac{2Lf}{\cos\theta} = \sqrt{\frac{TL}{m\cos\theta}} \text{ or } \frac{4L^2f^2}{\cos^2\theta} = \frac{TL}{m\cos\theta}$$

and the mass of string above the rod is:

$$m = \frac{T\cos\theta}{4Lf^2} \qquad \text{[Equation 1]}$$

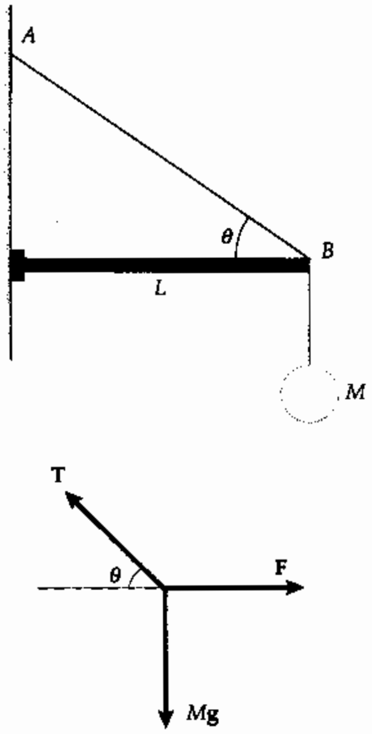


FIG. P18.29

Now, consider the tension in the string. The light rod would rotate about point P if the string exerted any vertical force on it. Therefore, recalling Newton's third law, the rod must exert only a horizontal force on the string. Consider a free-body diagram of the string segment in contact with the end of the rod.

$$\sum F_y = T\sin\theta - Mg = 0 \Rightarrow T = \frac{Mg}{\sin\theta}$$

Then, from Equation 1, the mass of string above the rod is

$$m = \left(\frac{Mg}{\sin\theta}\right)\frac{\cos\theta}{4Lf^2} = \boxed{\frac{Mg}{4Lf^2\tan\theta}}.$$

***P18.30** Let $m = \rho V$ represent the mass of the copper cylinder. The original tension in the wire is $T_1 = mg = \rho V g$. The water exerts a buoyant force $\rho_{\text{water}}\left(\frac{V}{2}\right)g$ on the cylinder, to reduce the tension to

$$T_2 = \rho V g - \rho_{\text{water}}\left(\frac{V}{2}\right)g = \left(\rho - \frac{\rho_{\text{water}}}{2}\right)Vg.$$

The speed of a wave on the string changes from $\sqrt{\frac{T_1}{\mu}}$ to $\sqrt{\frac{T_2}{\mu}}$. The frequency changes from

$$f_1 = \frac{v_1}{\lambda} = \sqrt{\frac{T_1}{\mu}}\frac{1}{\lambda} \text{ to } f_2 = \sqrt{\frac{T_2}{\mu}}\frac{1}{\lambda}$$

where we assume $\lambda = 2L$ is constant.

Then $\quad \dfrac{f_2}{f_1} = \sqrt{\dfrac{T_2}{T_1}} = \sqrt{\dfrac{\rho - \rho_{\text{water}}/2}{\rho}} = \sqrt{\dfrac{8.92 - 1.00/2}{8.92}}$

$$f_2 = 300 \text{ Hz}\sqrt{\frac{8.42}{8.92}} = \boxed{291 \text{ Hz}}$$

***P18.31** Comparing $\quad y = (0.002 \text{ m})\sin\left((\pi \text{ rad/m})x\right)\cos\left((100\pi \text{ rad/s})t\right)$

with $\quad y = 2A\sin kx\cos\omega t$

we find $\quad k = \frac{2\pi}{\lambda} = \pi \text{ m}^{-1}$, $\lambda = 2.00$ m, and $\omega = 2\pi f = 100\pi \text{ s}^{-1}$: $f = 50.0$ Hz

(a) Then the distance between adjacent nodes is $\quad d_{\text{NN}} = \frac{\lambda}{2} = 1.00$ m

and on the string are $\quad \dfrac{L}{d_{\text{NN}}} = \dfrac{3.00 \text{ m}}{1.00 \text{ m}} = \boxed{3 \text{ loops}}$

For the speed we have $\quad v = f\lambda = \left(50 \text{ s}^{-1}\right)2 \text{ m} = 100$ m/s

(b) In the simplest standing wave vibration, $d_{\text{NN}} = 3.00 \text{ m} = \frac{\lambda_b}{2}$, $\lambda_b = 6.00$ m

and $\quad f_b = \dfrac{v_a}{\lambda_b} = \dfrac{100 \text{ m/s}}{6.00 \text{ m}} = \boxed{16.7 \text{ Hz}}$

(c) In $v_0 = \sqrt{\frac{T_0}{\mu}}$, if the tension increases to $T_c = 9T_0$ and the string does not stretch, the speed increases to

$$v_c = \sqrt{\frac{9T_0}{\mu}} = 3\sqrt{\frac{T_0}{\mu}} = 3v_0 = 3(100 \text{ m/s}) = 300 \text{ m/s}$$

Then $\quad \lambda_c = \dfrac{v_c}{f_a} = \dfrac{300 \text{ m/s}}{50 \text{ s}^{-1}} = 6.00$ m $\qquad d_{\text{NN}} = \dfrac{\lambda_c}{2} = 3.00$ m

and $\boxed{\text{one}}$ loop fits onto the string.

Section 18.4 **Resonance**

P18.32 The natural frequency is

$$f=\frac{1}{T}=\frac{1}{2\pi}\sqrt{\frac{g}{L}}=\frac{1}{2\pi}\sqrt{\frac{9.80\ \text{m/s}^2}{2.00\ \text{m}}}=0.352\ \text{Hz}.$$

The big brother must push at this same frequency of $\boxed{0.352\ \text{Hz}}$ to produce resonance.

P18.33 (a) The wave speed is $v=\dfrac{9.15\ \text{m}}{2.50\ \text{s}}=\boxed{3.66\ \text{m/s}}$

(b) From the figure, there are antinodes at both ends of the pond, so the distance between adjacent antinodes

is $d_{AA}=\dfrac{\lambda}{2}=9.15\ \text{m}$,

and the wavelength is $\lambda=18.3\ \text{m}$

The frequency is then $f=\dfrac{v}{\lambda}=\dfrac{3.66\ \text{m/s}}{18.3\ \text{m}}=\boxed{0.200\ \text{Hz}}$

We have assumed the wave speed is the same for all wavelengths.

P18.34 The wave speed is $v=\sqrt{gd}=\sqrt{(9.80\ \text{m/s}^2)(36.1\ \text{m})}=18.8\ \text{m/s}$

The bay has one end open and one closed. Its simplest resonance is with a node of horizontal velocity, which is also an antinode of vertical displacement, at the head of the bay and an antinode of velocity, which is a node of displacement, at the mouth. The vibration of the water in the bay is like that in one half of the pond shown in Figure P18.33.

Then, $d_{NA}=210\times10^3\ \text{m}=\dfrac{\lambda}{4}$

and $\lambda=840\times10^3\ \text{m}$

Therefore, the period is $T=\dfrac{1}{f}=\dfrac{\lambda}{v}=\dfrac{840\times10^3\ \text{m}}{18.8\ \text{m/s}}=4.47\times10^4\ \text{s}=\boxed{12\ \text{h}\ 24\ \text{min}}$

$\boxed{\text{This agrees precisely with the period of the lunar excitation}}$, so we identify the extra-high tides as amplified by resonance.

P18.35 The distance between adjacent nodes is one-quarter of the circumference.

$$d_{NN}=d_{AA}=\frac{\lambda}{2}=\frac{20.0\ \text{cm}}{4}=5.00\ \text{cm}$$

so $\lambda=10.0\ \text{cm}$ and $f=\dfrac{v}{\lambda}=\dfrac{900\ \text{m/s}}{0.100\ \text{m}}=9\,000\ \text{Hz}=\boxed{9.00\ \text{kHz}}$.

The singer must match this frequency quite precisely for some interval of time to feed enough energy into the glass to crack it.

Section 18.5 **Standing Waves in Air Columns**

P18.36 $d_{AA} = 0.320\text{ m};\ \lambda = 0.640\text{ m}$

(a) $f = \dfrac{v}{\lambda} = \boxed{531\text{ Hz}}$

(b) $\lambda = 0.0850\text{ m};\ d_{AA} = \boxed{42.5\text{ mm}}$

P18.37 (a) For the fundamental mode in a closed pipe, $\lambda = 4L$, as in the diagram.

But $v = f\lambda$, therefore $L = \dfrac{v}{4f}$

So, $L = \dfrac{343\text{ m/s}}{4\left(240\text{ s}^{-1}\right)} = \boxed{0.357\text{ m}}$

(b) For an open pipe, $\lambda = 2L$, as in the diagram.

So, $L = \dfrac{v}{2f} = \dfrac{343\text{ m/s}}{2\left(240\text{ s}^{-1}\right)} = \boxed{0.715\text{ m}}$

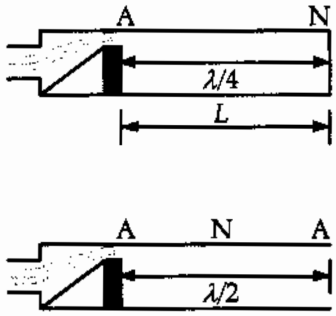


FIG. P18.37

P18.38 The wavelength is $\lambda = \dfrac{v}{f} = \dfrac{343\text{ m/s}}{261.6\text{/s}} = 1.31\text{ m}$

so the length of the open pipe vibrating in its simplest (A-N-A) mode is

$$d_{\text{A to A}} = \frac{1}{2}\lambda = \boxed{0.656\text{ m}}$$

A closed pipe has (N-A) for its simplest resonance,

(N-A-N-A) for the second,

and (N-A-N-A-N-A) for the third.

Here, the pipe length is $5d_{\text{N to A}} = \dfrac{5\lambda}{4} = \dfrac{5}{4}(1.31\text{ m}) = \boxed{1.64\text{ m}}$

***P18.39** Assuming an air temperature of $T = 37°\text{C} = 310\text{ K}$, the speed of sound inside the pipe is

$$v = (331\text{ m/s})\sqrt{\frac{310\text{ K}}{273\text{ K}}} = 353\text{ m/s}.$$

In the fundamental resonant mode, the wavelength of sound waves in a pipe closed at one end is $\lambda = 4L$. Thus, for the whooping crane

$$\lambda = 4(5.0\text{ ft}) = 2.0\times10^{1}\text{ ft and } f = \frac{v}{\lambda} = \frac{(353\text{ m/s})}{2.0\times10^{1}\text{ ft}}\left(\frac{3.281\text{ ft}}{1\text{ m}}\right) = \boxed{57.9\text{ Hz}}.$$

P18.40 The air in the auditory canal, about 3 cm long, can vibrate with a node at the closed end and antinode at the open end,

with $d_{\text{N to A}} = 3 \text{ cm} = \dfrac{\lambda}{4}$

so $\lambda = 0.12 \text{ m}$

and $f = \dfrac{v}{\lambda} = \dfrac{343 \text{ m/s}}{0.12 \text{ m}} \approx \boxed{3 \text{ kHz}}$

A small-amplitude external excitation at this frequency can, over time, feed energy into a larger-amplitude resonance vibration of the air in the canal, making it audible.

P18.41 For a closed box, the resonant frequencies will have nodes at both sides, so the permitted wavelengths will be $L = \dfrac{1}{2}n\lambda$, $(n = 1, 2, 3, \ldots)$.

i.e., $L = \dfrac{n\lambda}{2} = \dfrac{nv}{2f}$ and $f = \dfrac{nv}{2L}$.

Therefore, with $L = 0.860 \text{ m}$ and $L' = 2.10 \text{ m}$, the resonant frequencies are

$f_n = \boxed{n(206 \text{ Hz})}$ for $L = 0.860 \text{ m}$ for each n from 1 to 9

and $f_n' = \boxed{n(84.5 \text{ Hz})}$ for $L' = 2.10 \text{ m}$ for each n from 2 to 23.

P18.42 The wavelength of sound is $\lambda = \dfrac{v}{f}$

The distance between water levels at resonance is $d = \dfrac{v}{2f}$ $\quad \therefore Rt = \pi r^2 d = \dfrac{\pi r^2 v}{2f}$

and $t = \boxed{\dfrac{\pi r^2 v}{2Rf}}$.

P18.43 For both open and closed pipes, resonant frequencies are equally spaced as numbers. The set of resonant frequencies then must be 650 Hz, 550 Hz, 450 Hz, 350 Hz, 250 Hz, 150 Hz, 50 Hz. These are odd-integer multipliers of the fundamental frequency of $\boxed{50.0 \text{ Hz}}$. Then the pipe length is

$d_{NA} = \dfrac{\lambda}{4} = \dfrac{v}{4f} = \dfrac{340 \text{ m/s}}{4(50/\text{s})} = \boxed{1.70 \text{ m}}$.

P18.44 $\dfrac{\lambda}{2} = d_{\text{AA}} = \dfrac{L}{n}$ or $\quad L = \dfrac{n\lambda}{2} \quad$ for $n = 1, 2, 3, \ldots$

Since $\lambda = \dfrac{v}{f}$ $\quad L = n\left(\dfrac{v}{2f}\right) \quad$ for $n = 1, 2, 3, \ldots$

With $v = 343 \text{ m/s}$ and $\quad f = 680 \text{ Hz}$,

$$L = n\left(\frac{343 \text{ m/s}}{2(680 \text{ Hz})}\right) = n(0.252 \text{ m}) \quad \text{for } n = 1, 2, 3, \ldots$$

Possible lengths for resonance are: $L = \boxed{0.252 \text{ m}, 0.504 \text{ m}, 0.757 \text{ m}, \ldots, n(0.252) \text{ m}}$.

P18.45 For resonance in a narrow tube open at one end,

$$f = n\frac{v}{4L}\;(n = 1, 3, 5, \ldots).$$

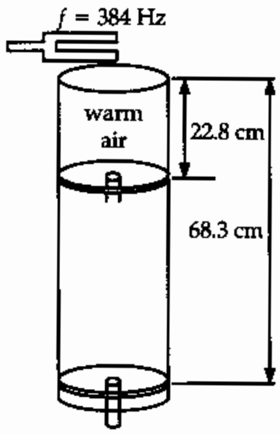


FIG. P18.45

(a) Assuming $n = 1$ and $n = 3$,

$$384 = \frac{v}{4(0.228)} \text{ and } 384 = \frac{3v}{4(0.683)}.$$

In either case, $v = \boxed{350 \text{ m/s}}$.

(b) For the next resonance $n = 5$, and $L = \frac{5v}{4f} = \frac{5(350 \text{ m/s})}{4(384 \text{ s}^{-1})} = \boxed{1.14 \text{ m}}$.

P18.46 The length corresponding to the fundamental satisfies $f = \frac{v}{4L}$: $L_1 = \frac{v}{4f} = \frac{34}{4(512)} = 0.167 \text{ m}$.

Since $L > 20.0$ cm, the *next* two modes will be observed, corresponding to $f = \frac{3v}{4L_2}$ and $f = \frac{5v}{4L_3}$.

or $L_2 = \frac{3v}{4f} = \boxed{0.502 \text{ m}}$ and $L_3 = \frac{5v}{4f} = \boxed{0.837 \text{ m}}$.

P18.47 We suppose these are the lowest resonances of the enclosed air columns.

For one, $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{256 \text{ s}^{-1}} = 1.34 \text{ m}$ length $= d_{AA} = \frac{\lambda}{2} = 0.670 \text{ m}$

For the other, $\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{440 \text{ s}^{-1}} = 0.780 \text{ m}$ length $= 0.390 \text{ m}$

So,

(b) original length $= \boxed{1.06 \text{ m}}$

$\lambda = 2d_{AA} = 2.12 \text{ m}$

(a) $f = \frac{343 \text{ m/s}}{2.12 \text{ m}} = \boxed{162 \text{ Hz}}$

P18.48 (a) For the fundamental mode of an open tube,

$$L = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343 \text{ m/s}}{2(880 \text{ s}^{-1})} = \boxed{0.195 \text{ m}}.$$

(b) $v = 331 \text{ m/s}\sqrt{1 + \frac{(-5.00)}{273}} = 328 \text{ m/s}$

We ignore the thermal expansion of the metal.

$$f = \frac{v}{\lambda} = \frac{v}{2L} = \frac{328 \text{ m/s}}{2(0.195 \text{ m})} = \boxed{841 \text{ Hz}}$$

The flute is flat by a semitone.

Section 18.6 Standing Waves in Rod and Plates

P18.49 (a) $f = \frac{v}{2L} = \frac{5\,100}{(2)(1.60)} = \boxed{1.59\text{ kHz}}$

(b) Since it is held in the center, there must be a node in the center as well as antinodes at the ends. The even harmonics have an antinode at the center so only $\boxed{\text{the odd harmonics}}$ are present.

(c) $f = \frac{v'}{2L} = \frac{3\,560}{(2)(1.60)} = \boxed{1.11\text{ kHz}}$

P18.50 When the rod is clamped at one-quarter of its length, the vibration pattern reads ANANA and the rod length is $L = 2d_{AA} = \lambda$.

Therefore, $L = \frac{v}{f} = \frac{5\,100\text{ m/s}}{4\,400\text{ Hz}} = \boxed{1.16\text{ m}}$

Section 18.7 Beats: Interference in Time

P18.51 $f \propto v \propto \sqrt{T}$ $\qquad f_{\text{new}} = 110\sqrt{\frac{540}{600}} = 104.4\text{ Hz}$

$\Delta f = \boxed{5.64\text{ beats/s}}$

P18.52 (a) The string could be tuned to either $\boxed{521\text{ Hz or }525\text{ Hz}}$ from this evidence.

(b) Tightening the string raises the wave speed and frequency. If the frequency were originally 521 Hz, the beats would slow down.

Instead, the frequency must have started at 525 Hz to become $\boxed{526\text{ Hz}}$.

(c) From $f = \frac{v}{\lambda} = \frac{\sqrt{T/\mu}}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \text{ and } T_2 = \left(\frac{f_2}{f_1}\right)^2 T_1 = \left(\frac{523\text{ Hz}}{526\text{ Hz}}\right)^2 T_1 = 0.989T_1.$$

The fractional change that should be made in the tension is then

$$\text{fractional change} = \frac{T_1 - T_2}{T_1} = 1 - 0.989 = 0.011\,4 = 1.14\%\text{ lower.}$$

The tension should be $\boxed{\text{reduced by }1.14\%}$.

P18.53 For an echo $f' = f\frac{(v+v_s)}{(v-v_s)}$ the beat frequency is $f_b = |f' - f|$.

Solving for f_b.

gives $f_b = f\frac{(2v_s)}{(v-v_s)}$ when approaching wall.

(a) $f_b = (256)\frac{2(1.33)}{(343-1.33)} = \boxed{1.99\text{ Hz}}$ beat frequency

(b) When he is moving away from the wall, v_s changes sign. Solving for v_s gives

$$v_s = \frac{f_b v}{2f - f_b} = \frac{(5)(343)}{(2)(256)-5} = \boxed{3.38\text{ m/s}}.$$

***P18.54** Using the $\boxed{4\text{ and }2\frac{2}{3}\text{-foot pipes}}$ produces actual frequencies of 131 Hz and 196 Hz and a combination tone at $(196-131)\text{Hz} = 65.4\text{ Hz}$, so this pair supplies the so-called missing fundamental. The 4 and 2-foot pipes produce a combination tone $(262-131)\text{Hz} = 131\text{ Hz}$, so this does not work.

The $\boxed{2\frac{2}{3}\text{ and 2-foot pipes}}$ produce a combination tone at $(262-196)\text{Hz} = 65.4\text{ Hz}$, so this works.

Also, $\boxed{4,\ 2\frac{2}{3},\text{ and 2-foot pipes}}$ all playing together produce the 65.4-Hz combination tone.

Section 18.8 **Non-Sinusoidal Wave Patterns**

P18.55 We list the frequencies of the harmonics of each note in Hz:

	Harmonic				
Note	1	2	3	4	5
A	440.00	880.00	1 320.0	1 760.0	2 200.0
C#	554.37	1 108.7	1 663.1	2 217.5	2 771.9
E	659.26	1 318.5	1 977.8	2 637.0	3 296.3

The second harmonic of E is close the the third harmonic of A, and the fourth harmonic of C# is close to the fifth harmonic of A.

P18.56 We evaluate

$$s = 100\sin\theta + 157\sin 2\theta + 62.9\sin 3\theta + 105\sin 4\theta$$
$$+51.9\sin 5\theta + 29.5\sin 6\theta + 25.3\sin 7\theta$$

where s represents particle displacement in nanometers and θ represents the phase of the wave in radians. As θ advances by 2π, time advances by (1/523) s. Here is the result:

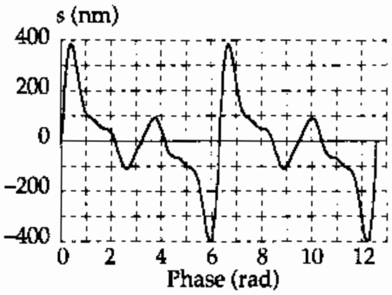


FIG. P18.56

Additional Problems

P18.57 $f = 87.0$ Hz

speed of sound in air: $v_a = 340$ m/s

(a) $\lambda_b = \ell$ $\quad v = f\lambda_b = (87.0\ \text{s}^{-1})(0.400\ \text{m})$

$$v = \boxed{34.8\ \text{m/s}}$$

(b) $\left.\begin{array}{l}\lambda_a = 4L \\ v_a = \lambda_a f\end{array}\right\}$ $\quad L = \dfrac{v_a}{4f} = \dfrac{340\ \text{m/s}}{4(87.0\ \text{s}^{-1})} = \boxed{0.977\ \text{m}}$

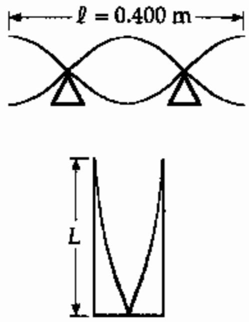


FIG. P18.57

***P18.58** (a) Use the Doppler formula

$$f' = f\frac{(v \pm v_0)}{(v \mp v_s)}.$$

With f_1' = frequency of the speaker in front of student and

f_2' = frequency of the speaker behind the student.

$$f_1' = (456\ \text{Hz})\frac{(343\ \text{m/s} + 1.50\ \text{m/s})}{(343\ \text{m/s} - 0)} = 458\ \text{Hz}$$

$$f_2' = (456\ \text{Hz})\frac{(343\ \text{m/s} - 1.50\ \text{m/s})}{(343\ \text{m/s} + 0)} = 454\ \text{Hz}$$

Therefore, $f_b = f_1' - f_2' = \boxed{3.99\ \text{Hz}}$.

(b) The waves broadcast by both speakers have $\lambda = \dfrac{v}{f} = \dfrac{343\ \text{m/s}}{456/\text{s}} = 0.752\ \text{m}$. The standing wave between them has $d_{AA} = \dfrac{\lambda}{2} = 0.376\ \text{m}$. The student walks from one maximum to the next in time $\Delta t = \dfrac{0.376\ \text{m}}{1.50\ \text{m/s}} = 0.251\ \text{s}$, so the frequency at which she hears maxima is $f = \dfrac{1}{T} = \boxed{3.99\ \text{Hz}}$.

P18.59 Moving away from station, frequency is depressed:

$f' = 180 - 2.00 = 178$ Hz: $\quad 178 = 180\dfrac{343}{343 - (-v)}$

Solving for v gives $\quad v = \dfrac{(2.00)(343)}{178}$

Therefore, $\quad v = \boxed{3.85\ \text{m/s away from station}}$

Moving toward the station, the frequency is enhanced:

$f' = 180 + 2.00 = 182$ Hz: $\quad 182 = 180\dfrac{343}{343 - v}$

Solving for v gives $\quad 4 = \dfrac{(2.00)(343)}{182}$

Therefore, $\quad v = \boxed{3.77\ \text{m/s toward the station}}$

P18.60 $v=\sqrt{\dfrac{(48.0)(2.00)}{4.80\times10^{-3}}}=141\ \text{m/s}$

$d_{NN}=1.00\ \text{m};\ \lambda=2.00\ \text{m};\ f=\dfrac{v}{\lambda}=70.7\ \text{Hz}$

$\lambda_a=\dfrac{v_a}{f}=\dfrac{343\ \text{m/s}}{70.7\ \text{Hz}}=\boxed{4.85\ \text{m}}$

P18.61 Call L the depth of the well and v the speed of sound.

Then for some integer n $\quad L=(2n-1)\dfrac{\lambda_1}{4}=(2n-1)\dfrac{v}{4f_1}=\dfrac{(2n-1)(343\ \text{m/s})}{4(51.5\ \text{s}^{-1})}$

and for the next resonance $\quad L=[2(n+1)-1]\dfrac{\lambda_2}{4}=(2n+1)\dfrac{v}{4f_2}=\dfrac{(2n+1)(343\ \text{m/s})}{4(60.0\ \text{s}^{-1})}$

Thus, $\quad \dfrac{(2n-1)(343\ \text{m/s})}{4(51.5\ \text{s}^{-1})}=\dfrac{(2n+1)(343\ \text{m/s})}{4(60.0\ \text{s}^{-1})}$

and we require an *integer* solution to $\quad \dfrac{2n+1}{60.0}=\dfrac{2n-1}{51.5}$

The equation gives $n=\dfrac{111.5}{17}=6.56$, so the best fitting integer is $n=7$.

Then $\quad L=\dfrac{[2(7)-1](343\ \text{m/s})}{4(51.5\ \text{s}^{-1})}=21.6\ \text{m}$

and $\quad L=\dfrac{[2(7)+1](343\ \text{m/s})}{4(60.0\ \text{s}^{-1})}=21.4\ \text{m}$

suggest the best value for the depth of the well is $\boxed{21.5\ \text{m}}$.

P18.62 The second standing wave mode of the air in the pipe reads ANAN, with $d_{NA}=\dfrac{\lambda}{4}=\dfrac{1.75\ \text{m}}{3}$

so $\quad \lambda=2.33\ \text{m}$

and $\quad f=\dfrac{v}{\lambda}=\dfrac{343\ \text{m/s}}{2.33\ \text{m}}=147\ \text{Hz}$

For the string, λ and v are different but f is the same.

$$\frac{\lambda}{2}=d_{NN}=\frac{0.400\ \text{m}}{2}$$

so $\quad \lambda=0.400\ \text{m}$

$v=\lambda f=(0.400\ \text{m})(147\ \text{Hz})=58.8\ \text{m/s}=\sqrt{\dfrac{T}{\mu}}$

$T=\mu v^2=(9.00\times10^{-3}\ \text{kg/m})(58.8\ \text{m/s})^2=\boxed{31.1\ \text{N}}$

P18.63 (a) Since the first node is at the weld, the wavelength in the thin wire is $2L$ or 80.0 cm. The frequency and tension are the same in both sections, so

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2(0.400)}\sqrt{\frac{4.60}{2.00\times10^{-3}}} = \boxed{59.9 \text{ Hz}}.$$

(b) As the thick wire is twice the diameter, the linear density is 4 times that of the thin wire.

$\mu' = 8.00$ g/m

so $L' = \frac{1}{2f}\sqrt{\frac{T}{\mu'}}$ $\qquad L' = \left[\frac{1}{(2)(59.9)}\right]\sqrt{\frac{4.60}{8.00\times10^{-3}}} = \boxed{20.0 \text{ cm}}$ half the length of the thin wire.

P18.64 (a) For the block:

$\sum F_x = T - Mg\sin 30.0° = 0$

so $T = Mg\sin 30.0° = \boxed{\frac{1}{2}Mg}$.

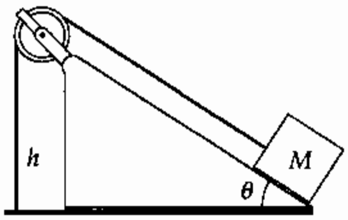


FIG. P18.64

(b) The length of the section of string parallel to the incline is $\frac{h}{\sin 30.0°} = 2h$. The total length of the string is then $\boxed{3h}$.

(c) The mass per unit length of the string is $\qquad \mu = \boxed{\frac{m}{3h}}$

(d) The speed of waves in the string is $\qquad v = \sqrt{\frac{T}{\mu}} = \sqrt{\left(\frac{Mg}{2}\right)\left(\frac{3h}{m}\right)} = \boxed{\sqrt{\frac{3Mgh}{2m}}}$

(e) In the fundamental mode, the segment of length h vibrates as one loop. The distance between adjacent nodes is then $d_{NN} = \frac{\lambda}{2} = h$, so the wavelength is $\lambda = 2h$.

The frequency is $\qquad f = \frac{v}{\lambda} = \frac{1}{2h}\sqrt{\frac{3Mgh}{2m}} = \boxed{\sqrt{\frac{3Mg}{8mh}}}$

(g) When the vertical segment of string vibrates with 2 loops (i.e., 3 nodes), then $h = 2\left(\frac{\lambda}{2}\right)$ and the wavelength is $\qquad \lambda = \boxed{h}$.

(f) The period of the standing wave of 3 nodes (or two loops) is

$$T = \frac{1}{f} = \frac{\lambda}{v} = h\sqrt{\frac{2m}{3Mgh}} = \boxed{\sqrt{\frac{2mh}{3Mg}}}$$

(h) $f_b = 1.02f - f = \left(2.00\times10^{-2}\right)f = \boxed{\left(2.00\times10^{-2}\right)\sqrt{\frac{3Mg}{8mh}}}$

P18.65 (a) $f=\frac{n}{2L}\sqrt{\frac{T}{\mu}}$

so $\frac{f'}{f}=\frac{L}{L'}=\frac{L}{2L}=\frac{1}{2}$

The $\boxed{\text{frequency should be halved}}$ to get the same number of antinodes for twice the length.

(b) $\frac{n'}{n}=\sqrt{\frac{T}{T'}}$ so $\frac{T'}{T}=\left(\frac{n}{n'}\right)^2=\left[\frac{n}{n+1}\right]^2$

The tension must be $T'=\boxed{\left[\frac{n}{n+1}\right]^2 T}$

(c) $\frac{f'}{f}=\frac{n'L}{nL'}\sqrt{\frac{T'}{T}}$ so $\frac{T'}{T}=\left(\frac{nfL'}{n'fL}\right)^2$

$\frac{T'}{T}=\left(\frac{3}{2\cdot 2}\right)^2$ $\boxed{\frac{T'}{T}=\frac{9}{16}}$ to get twice as many antinodes.

P18.66 For the wire, $\mu=\frac{0.010\,0\text{ kg}}{2.00\text{ m}}=5.00\times10^{-3}\text{ kg/m}$: $v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{(200\text{ kg}\cdot\text{m/s}^2)}{5.00\times10^{-3}\text{ kg/m}}}$

$v=200\text{ m/s}$

If it vibrates in its simplest state, $d_{NN}=2.00\text{ m}=\frac{\lambda}{2}$: $f=\frac{v}{\lambda}=\frac{(200\text{ m/s})}{4.00\text{ m}}=50.0\text{ Hz}$

(a) The tuning fork can have frequencies $\boxed{45.0\text{ Hz or }55.0\text{ Hz}}$.

(b) If $f=45.0\text{ Hz}$, $v=f\lambda=(45.0/\text{s})4.00\text{ m}=180\text{ m/s}$.

Then, $T=v^2\mu=(180\text{ m/s})^2(5.00\times10^{-3}\text{ kg/m})=\boxed{162\text{ N}}$

$\boxed{\text{or}}$ if $f=55.0\text{ Hz}$, $T=v^2\mu=f^2\lambda^2\mu=(55.0/\text{s})^2(4.00\text{ m})^2(5.00\times10^{-3}\text{ kg/m})=\boxed{242\text{ N}}$.

P18.67 We look for a solution of the form

$$5.00\sin(2.00x-10.0t)+10.0\cos(2.00x-10.0t)=A\sin(2.00x-10.0t+\phi)$$
$$=A\sin(2.00x-10.0t)\cos\phi+A\cos(2.00x-10.0t)\sin\phi$$

This will be true if both $5.00=A\cos\phi$ and $10.0=A\sin\phi$,

requiring $(5.00)^2+(10.0)^2=A^2$

$A=11.2$ and $\phi=63.4°$

The resultant wave $\boxed{11.2\sin(2.00x-10.0t+63.4°)}$ is sinusoidal.

P18.68 (a) With $k=\frac{2\pi}{\lambda}$ and $\omega=2\pi f=\frac{2\pi v}{\lambda}$: $y(x,t)=2A\sin kx\cos\omega t=\boxed{2A\sin\left(\frac{2\pi x}{\lambda}\right)\cos\left(\frac{2\pi vt}{\lambda}\right)}$

(b) For the fundamental vibration, $\lambda_1=2L$

so $y_1(x,t)=\boxed{2A\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi vt}{L}\right)}$

(c) For the second harmonic $\lambda_2=L$ and $y_2(x,t)=\boxed{2A\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi vt}{L}\right)}$

(d) In general, $\lambda_n=\frac{2L}{n}$ and $y_n(x,t)=\boxed{2A\sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi vt}{L}\right)}$

P18.69 (a) Let θ represent the angle each slanted rope makes with the vertical.

In the diagram, observe that:

$$\sin\theta=\frac{1.00\text{ m}}{1.50\text{ m}}=\frac{2}{3}$$

or $\theta=41.8°$.

Considering the mass,

$\sum F_y=0$: $2T\cos\theta=mg$

or $T=\frac{(12.0\text{ kg})(9.80\text{ m/s}^2)}{2\cos 41.8°}=\boxed{78.9\text{ N}}$

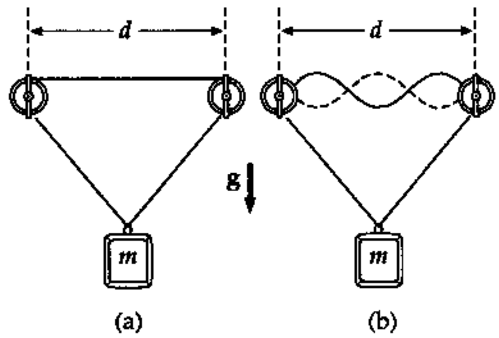


FIG. P18.69

(b) The speed of transverse waves in the string is $v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{78.9\text{ N}}{0.001\,00\text{ kg/m}}}=281\text{ m/s}$

For the standing wave pattern shown (3 loops), $d=\frac{3}{2}\lambda$

or $\lambda=\frac{2(2.00\text{ m})}{3}=1.33\text{ m}$

Thus, the required frequency is $f=\frac{v}{\lambda}=\frac{281\text{ m/s}}{1.33\text{ m}}=\boxed{211\text{ Hz}}$

***P18.70** $d_{AA}=\frac{\lambda}{2}=7.05\times10^{-3}$ m is the distance between antinodes.

Then $\lambda=14.1\times10^{-3}$ m

and $f=\frac{v}{\lambda}=\frac{3.70\times10^3\text{ m/s}}{14.1\times10^{-3}\text{ m}}=\boxed{2.62\times10^5\text{ Hz}}$.

The crystal can be tuned to vibrate at 2^{18} Hz, so that binary counters can derive from it a signal at precisely 1 Hz.

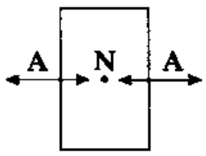


FIG. P18.70

ANSWERS TO EVEN PROBLEMS

P18.2 see the solution

P18.4 5.66 cm

P18.6 0.500 s

P18.8 (a) 3.33 rad; (b) 283 Hz

P18.10 (a) The number is the greatest integer $\le d\left(\frac{f}{v}\right)+\frac{1}{2}$;
(b) $L_n = \frac{d^2-(n-1/2)^2(v/f)^2}{2(n-1/2)(v/f)}$ where $n = 1, 2, \ldots, n_{\max}$

P18.12 (a) $\Delta x = \frac{\lambda}{2}$;
(b) along the hyperbola $9x^2 - 16y^2 = 144$

P18.14 (a) $(2n+1)\pi$ m for $n = 0, 1, 2, 3, \ldots$;
(b) 0.029 4 m

P18.16 see the solution

P18.18 see the solution

P18.20 15.7 Hz

P18.22 (a) 257 Hz; (b) 6

P18.24 (a) 495 Hz; (b) 990 Hz

P18.26 19.976 kHz

P18.28 3.84%

P18.30 291 Hz

P18.32 0.352 Hz

P18.34 see the solution

P18.36 (a) 531 Hz; (b) 42.5 mm

P18.38 0.656 m; 1.64 m

P18.40 3 kHz; see the solution

P18.42 $\Delta t = \frac{\pi r^2 v}{2Rf}$

P18.44 $L = 0.252$ m, 0.504 m, 0.757 m, …, $n(0.252)$ m for $n = 1, 2, 3, \ldots$

P18.46 0.502 m; 0.837 m

P18.48 (a) 0.195 m; (b) 841 m

P18.50 1.16 m

P18.52 (a) 521 Hz or 525 Hz; (b) 526 Hz;
(c) reduce by 1.14%

P18.54 4-foot and $2\frac{2}{3}$-foot; $2\frac{2}{3}$ and 2-foot; and all three together

P18.56 see the solution

P18.58 (a) and (b) 3.99 beats/s

P18.60 4.85 m

P18.62 31.1 N

P18.64 (a) $\frac{1}{2}Mg$; (b) $3h$; (c) $\frac{m}{3h}$; (d) $\sqrt{\frac{3Mgh}{2m}}$;
(e) $\sqrt{\frac{3Mg}{8mh}}$; (f) $\sqrt{\frac{2mh}{3Mg}}$; (g) h;
(h) $\left(2.00\times10^{-2}\right)\sqrt{\frac{3Mg}{8mh}}$

P18.66 (a) 45.0 Hz or 55.0 Hz; (b) 162 N or 242 N

P18.68 see the solution

P18.70 262 kHz

19

Temperature

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q19.1 Two objects in thermal equilibrium need not be in contact. Consider the two objects that are in thermal equilibrium in Figure 19.1(c). The act of separating them by a small distance does not affect how the molecules are moving inside either object, so they will still be in thermal equilibrium.

Q19.2 The copper's temperature drops and the water temperature rises until both temperatures are the same. Then the metal and the water are in thermal equilibrium.

Q19.3 The astronaut is referring to the temperature of the lunar surface, specifically a 400°F difference. A thermometer would register the temperature of the thermometer liquid. Since there is no atmosphere in the moon, the thermometer will not read a realistic temperature unless it is placed into the lunar soil.

Q19.4 Rubber contracts when it is warmed.

Q19.5 Thermal expansion of the glass bulb occurs first, since the wall of the bulb is in direct contact with the hot water. Then the mercury heats up, and it expands.

Q19.6 If the amalgam had a larger coefficient of expansion than your tooth, it would expand more than the cavity in your tooth when you take a sip of your ever-beloved coffee, resulting in a broken or cracked tooth! As you ice down your now excruciatingly painful broken tooth, the amalgam would contract more than the cavity in your tooth and fall out, leaving the nerve roots exposed. Isn't it nice that your dentist knows thermodynamics?

Q19.7 The measurements made with the heated steel tape will be too short—but only by a factor of 5×10^{-5} of the measured length.

Q19.8 (a) One mole of H_2 has a mass of 2.016 0 g.

(b) One mole of He has a mass of 4.002 6 g.

(c) One mole of CO has a mass of 28.010 g.

Q19.9 The ideal gas law, $PV = nRT$ predicts zero volume at absolute zero. This is incorrect because the ideal gas law cannot work all the way down to or below the temperature at which gas turns to liquid, or in the case of CO_2, a solid.

Q19.10 Call the process isobaric cooling or isobaric contraction. The rubber wall is easy to stretch. The air inside is nearly at atmospheric pressure originally and stays at atmospheric pressure as the wall moves in, just maintaining equality of pressure outside and inside. The air is nearly an ideal gas to start with, but $PV = nRT$ soon fails. Volume will drop by a larger factor than temperature as the water vapor liquefies and then freezes, as the carbon dioxide turns to snow, as the argon turns to slush, and as the oxygen liquefies. From the outside, you see contraction to a small fraction of the original volume.

Q19.11 Cylinder A must be at lower pressure. If the gas is thin, it will be at one-third the absolute pressure of B.

Q19.12 At high temperature and pressure, the steam inside exerts large forces on the pot and cover. Strong latches hold them together, but they would explode apart if you tried to open the hot cooker.

Q19.13 (a) The water level in the cave rises by a smaller distance than the water outside, as the trapped air is compressed. Air can escape from the cave if the rock is not completely airtight, and also by dissolving in the water.

(b) The ideal cave stays completely full of water at low tide. The water in the cave is supported by atmospheric pressure on the free water surface outside.

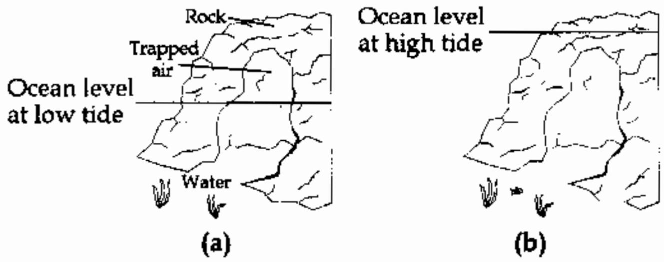


FIG. Q19.13

Q19.14 Absolute zero is a natural choice for the zero of a temperature scale. If an alien race had bodies that were mostly liquid water—or if they just liked its taste or its cleaning properties—it is conceivable that they might place one hundred degrees between its freezing and boiling points. It is very unlikely, on the other hand, that these would be our familiar "normal" ice and steam points, because atmospheric pressure would surely be different where the aliens come from.

Q19.15 As the temperature increases, the brass expands. This would effectively increase the distance, d, from the pivot point to the center of mass of the pendulum, and also increase the moment of inertia of the pendulum. Since the moment of inertia is proportional to d^2, and the period of a physical pendulum is $T = 2\pi\sqrt{\frac{I}{mgd}}$, the period would increase, and the clock would run slow.

Q19.16 As the water rises in temperature, it expands. The excess volume would spill out of the cooling system. Modern cooling systems have an overflow reservoir to take up excess volume when the coolant heats up and expands.

Q19.17 The coefficient of expansion of metal is larger than that of glass. When hot water is run over the jar, both the glass and the lid expand, but at different rates. Since *all* dimensions expand, there will be a certain temperature at which the inner diameter of the lid has expanded more than the top of the jar, and the lid will be easier to remove.

Q19.18 The sphere expands when heated, so that it no longer fits through the ring. With the sphere still hot, you can separate the sphere and ring by heating the ring. This more surprising result occurs because the thermal expansion of the ring is not like the inflation of a blood-pressure cuff. Rather, it is like a photographic enlargement; every linear dimension, including the hole diameter, increases by the same factor. The reason for this is that the atoms everywhere, including those around the inner circumference, push away from each other. The only way that the atoms can accommodate the greater distances is for the circumference—and corresponding diameter—to grow. This property was once used to fit metal rims to wooden wagon and horse-buggy wheels. If the ring is heated and the sphere left at room temperature, the sphere would pass through the ring with more space to spare.

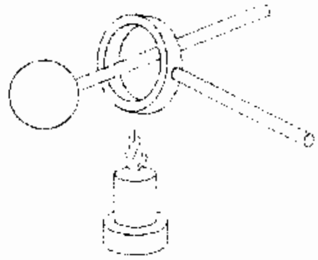


FIG. Q19.18

SOLUTIONS TO PROBLEMS

Section 19.1 Temperature and the Zeroth Law of Thermodynamics

No problems in this section

Section 19.2 Thermometers and the Celsius Temperature Scale

Section 19.3 The Constant-Volume Gas Thermometer and the Absolute Temperature Scale

P19.1 Since we have a linear graph, the pressure is related to the temperature as $P = A + BT$, where A and B are constants. To find A and B, we use the data

$$0.900 \text{ atm} = A + (-80.0°\text{C})B \quad (1)$$

$$1.635 \text{ atm} = A + (78.0°\text{C})B \quad (2)$$

Solving (1) and (2) simultaneously,

we find $A = 1.272$ atm

and $B = 4.652 \times 10^{-3}$ atm/°C

Therefore, $P = 1.272 \text{ atm} + \left(4.652 \times 10^{-3} \text{ atm/°C}\right)T$

(a) At absolute zero $P = 0 = 1.272 \text{ atm} + \left(4.652 \times 10^{-3} \text{ atm/°C}\right)T$

which gives $\boxed{T = -274°\text{C}}$.

(b) At the freezing point of water $P = 1.272 \text{ atm} + 0 = \boxed{1.27 \text{ atm}}$.

(c) And at the boiling point $P = 1.272 \text{ atm} + \left(4.652 \times 10^{-3} \text{ atm/°C}\right)(100°\text{C}) = \boxed{1.74 \text{ atm}}$.

P19.2 $P_1V = nRT_1$

and $P_2V = nRT_2$

imply that $\frac{P_2}{P_1} = \frac{T_2}{T_1}$

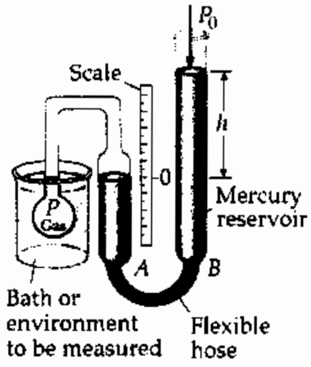


FIG. P19.2

(a) $P_2 = \frac{P_1T_2}{T_1} = \frac{(0.980 \text{ atm})(273 \text{ K} + 45.0 \text{ K})}{(273 + 20.0) \text{ K}} = \boxed{1.06 \text{ atm}}$

(b) $T_3 = \frac{T_1P_3}{P_1} = \frac{(293 \text{ K})(0.500 \text{ atm})}{0.980 \text{ atm}} = 149 \text{ K} = \boxed{-124°\text{C}}$

P19.3 (a) $T_F = \frac{9}{5}T_C + 32.0°\text{F} = \frac{9}{5}(-195.81) + 32.0 = \boxed{-320°\text{F}}$

(b) $T = T_C + 273.15 = -195.81 + 273.15 = \boxed{77.3 \text{ K}}$

P19.4 (a) To convert from Fahrenheit to Celsius, we use $T_C = \frac{5}{9}(T_F - 32.0) = \frac{5}{9}(98.6 - 32.0) = \boxed{37.0°\text{C}}$

and the Kelvin temperature is found as $T = T_C + 273 = \boxed{310 \text{ K}}$

(b) In a fashion identical to that used in (a), we find $T_C = \boxed{-20.6°\text{C}}$

and $T = \boxed{253 \text{ K}}$

P19.5 (a) $\Delta T = 450°\text{C} = 450°\text{C}\left(\frac{212°\text{F} - 32.0°\text{F}}{100°\text{C} - 0.00°\text{C}}\right) = \boxed{810°\text{F}}$

(b) $\Delta T = 450°\text{C} = \boxed{450 \text{ K}}$

P19.6 Require $0.00°\text{C} = a(-15.0°\text{S}) + b$

$100°\text{C} = a(60.0°\text{S}) + b$

Subtracting, $100°\text{C} = a(75.0°\text{S})$

$$a = 1.33 \text{ C°/S°}.$$

Then $0.00°\text{C} = 1.33(-15.0°\text{S})\text{C°} + b$

$$b = 20.0°\text{C}.$$

So the conversion is $\boxed{T_C = (1.33 \text{ C°/S°})T_S + 20.0°\text{C}}$.

P19.7 (a) $T = 1\,064 + 273 = \boxed{1\,337 \text{ K}}$ melting point

$T = 2\,660 + 273 = \boxed{2\,933 \text{ K}}$ boiling point

(b) $\Delta T = \boxed{1\,596°\text{C}} = \boxed{1\,596 \text{ K}}$. The differences are the same.

Section 19.4 Thermal Expansion of Solids and Liquids

P19.8 $\alpha = 1.10 \times 10^{-5}\,°\text{C}^{-1}$ for steel

$$\Delta L = 518\text{ m}\left(1.10 \times 10^{-5}\,°\text{C}^{-1}\right)\left[35.0°\text{C} - (-20.0°\text{C})\right] = \boxed{0.313\text{ m}}$$

P19.9 The wire is 35.0 m long when $T_C = -20.0°\text{C}$.

$$\Delta L = L_i\bar{\alpha}(T - T_i)$$

$$\bar{\alpha} = \alpha(20.0°\text{C}) = 1.70 \times 10^{-5}\ (\text{C}°)^{-1} \text{ for Cu.}$$

$$\Delta L = (35.0\text{ m})\left(1.70 \times 10^{-5}\ (\text{C}°)^{-1}\right)\left(35.0°\text{C} - (-20.0°\text{C})\right) = \boxed{+3.27\text{ cm}}$$

P19.10 $\Delta L = L_i\alpha\,\Delta T = (25.0\text{ m})\left(12.0 \times 10^{-6}/\text{C}°\right)(40.0°\text{C}) = \boxed{1.20\text{ cm}}$

P19.11 For the dimensions to increase, $\Delta L = \alpha L_i \Delta T$

$$1.00 \times 10^{-2}\text{ cm} = 1.30 \times 10^{-4}\,°\text{C}^{-1}(2.20\text{ cm})(T - 20.0°\text{C})$$

$$T = \boxed{55.0°\text{C}}$$

***P19.12** $\Delta L = \alpha L_i \Delta T = \left(22 \times 10^{-6}/\text{C}°\right)(2.40\text{ cm})(30°\text{C}) = \boxed{1.58 \times 10^{-3}\text{ cm}}$

P19.13 (a) $\Delta L = \alpha L_i \Delta T = 9.00 \times 10^{-6}\,°\text{C}^{-1}(30.0\text{ cm})(65.0°\text{C}) = \boxed{0.176\text{ mm}}$

(b) $\Delta L = \alpha L_i \Delta T = 9.00 \times 10^{-6}\,°\text{C}^{-1}(1.50\text{ cm})(65.0°\text{C}) = \boxed{8.78 \times 10^{-4}\text{ cm}}$

(c) $\Delta V = 3\alpha V_i \Delta T = 3\left(9.00 \times 10^{-6}\,°\text{C}^{-1}\right)\left(\dfrac{30.0(\pi)(1.50)^2}{4}\text{ cm}^3\right)(65.0°\text{C}) = \boxed{0.0930\text{ cm}^3}$

***P19.14** The horizontal section expands according to $\Delta L = \alpha L_i \Delta T$.

$$\Delta x = \left(17 \times 10^{-6}\,°\text{C}^{-1}\right)(28.0\text{ cm})(46.5°\text{C} - 18.0°\text{C}) = 1.36 \times 10^{-2}\text{ cm}$$

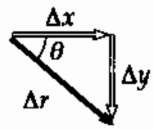


FIG. P19.14

The vertical section expands similarly by

$$\Delta y = \left(17 \times 10^{-6}\,°\text{C}^{-1}\right)(134\text{ cm})(28.5°\text{C}) = 6.49 \times 10^{-2}\text{ cm}.$$

The vector displacement of the pipe elbow has magnitude

$$\Delta r = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(0.136\text{ mm})^2 + (0.649\text{ mm})^2} = 0.663\text{ mm}$$

and is directed to the right below the horizontal at angle

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{0.649\text{ mm}}{0.136\text{ mm}}\right) = 78.2°$$

$$\boxed{\Delta r = 0.663\text{ mm to the right at } 78.2° \text{ below the horizontal}}$$

P19.15 (a) $L_{Al}(1+\alpha_{Al}\Delta T) = L_{Brass}(1+\alpha_{Brass}\Delta T)$

$$\Delta T = \frac{L_{Al} - L_{Brass}}{L_{Brass}\alpha_{Brass} - L_{Al}\alpha_{Al}}$$

$$\Delta T = \frac{(10.01 - 10.00)}{(10.00)(19.0\times10^{-6}) - (10.01)(24.0\times10^{-6})}$$

$\Delta T = -199°\text{C}$ so $T = \boxed{-179°\text{C. This is attainable.}}$

(b)
$$\Delta T = \frac{(10.02 - 10.00)}{(10.00)(19.0\times10^{-6}) - (10.02)(24.0\times10^{-6})}$$

$\Delta T = -396°\text{C}$ so $T = \boxed{-376°\text{C which is below 0 K so it cannot be reached.}}$

P19.16 (a) $\Delta A = 2\alpha A_i \Delta T$: $\Delta A = 2(17.0\times10^{-6}\,°\text{C}^{-1})(0.080\,0\text{ m})^2(50.0°\text{C})$

$\Delta A = 1.09\times10^{-5}\text{ m}^2 = \boxed{0.109\text{ cm}^2}$

(b) The length of each side of the hole has increased. Thus, this represents an $\boxed{\text{increase}}$ in the area of the hole.

P19.17 $\Delta V = (\beta - 3\alpha)V_i\Delta T = (5.81\times10^{-4} - 3(11.0\times10^{-6}))(50.0\text{ gal})(20.0) = \boxed{0.548\text{ gal}}$

P19.18 (a) $L = L_i(1+\alpha\Delta T)$: $5.050\text{ cm} = 5.000\text{ cm}[1 + 24.0\times10^{-6}\,°\text{C}^{-1}(T - 20.0°\text{C})]$

$T = \boxed{437°\text{C}}$

(b) We must get $L_{Al} = L_{Brass}$ for some ΔT, or

$L_{i,\,Al}(1+\alpha_{Al}\Delta T) = L_{i,\,Brass}(1+\alpha_{Brass}\Delta T)$

$5.000\text{ cm}[1+(24.0\times10^{-6}\,°\text{C}^{-1})\Delta T] = 5.050\text{ cm}[1+(19.0\times10^{-6}\,°\text{C}^{-1})\Delta T]$

Solving for ΔT, $\Delta T = 2\,080°\text{C}$,

so $\boxed{T = 3\,000°\text{C}}$

This will not work because $\boxed{\text{aluminum melts at }660°\text{C}}$.

P19.19 (a) $V_f = V_i(1+\beta\Delta T) = 100[1 + 1.50\times10^{-4}(-15.0)] = \boxed{99.8\text{ mL}}$

(b) $\Delta V_{acetone} = (\beta V_i \Delta T)_{acetone}$

$\Delta V_{flask} = (\beta V_i \Delta T)_{Pyrex} = (3\alpha V_i \Delta T)_{Pyrex}$

for same V_i, ΔT,

$$\frac{\Delta V_{acetone}}{\Delta V_{flask}} = \frac{\beta_{acetone}}{\beta_{flask}} = \frac{1.50\times10^{-4}}{3(3.20\times10^{-6})} = \frac{1}{6.40\times10^{-2}}$$

The volume change of flask is

$\boxed{\text{about 6\% of the change in the acetone's volume}}$.

P19.20 (a),(b) The material would expand by $\Delta L = \alpha L_i \Delta T$,

$$\frac{\Delta L}{L_i} = \alpha \Delta T \text{, but instead feels stress}$$

$$\frac{F}{A} = \frac{Y\Delta L}{L_i} = Y\alpha\Delta T = \left(7.00\times10^{9}\ \text{N/m}^2\right)12.0\times10^{-6}\left(\text{C}^\circ\right)^{-1}(30.0^\circ\text{C})$$

$$= \boxed{2.52\times10^{6}\ \text{N/m}^2}\text{. This will } \boxed{\text{not break}} \text{ concrete.}$$

P19.21 (a)

$$\Delta V = V_t\beta_t\Delta T - V_{\text{Al}}\beta_{\text{Al}}\Delta T = \left(\beta_t - 3\alpha_{\text{Al}}\right)V_i\Delta T$$

$$= \left(9.00\times10^{-4} - 0.720\times10^{-4}\right)^\circ\text{C}^{-1}\left(2\,000\ \text{cm}^3\right)(60.0^\circ\text{C})$$

$$\Delta V = \boxed{99.4\ \text{cm}^3} \text{ overflows.}$$

(b) The whole new volume of turpentine is

$$2\,000\ \text{cm}^3 + 9.00\times10^{-4}\ {}^\circ\text{C}^{-1}\left(2\,000\ \text{cm}^3\right)(60.0^\circ\text{C}) = 2\,108\ \text{cm}^3$$

so the fraction lost is $\dfrac{99.4\ \text{cm}^3}{2\,108\ \text{cm}^3} = 4.71\times10^{-2}$

and this fraction of the cylinder's depth will be empty upon cooling:

$$4.71\times10^{-2}(20.0\ \text{cm}) = \boxed{0.943\ \text{cm}}.$$

***P19.22** The volume of the sphere is

$$V_{\text{Pb}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(2\ \text{cm})^3 = 33.5\ \text{cm}^3.$$

The amount of mercury overflowing is

$$\text{overflow} = \Delta V_{\text{Hg}} + \Delta V_{\text{Pb}} - \Delta V_{\text{glass}} = \left(\beta_{\text{Hg}}V_{\text{Hg}} + \beta_{\text{Pb}}V_{\text{Pb}} - \beta_{\text{glass}}V_{\text{glass}}\right)\Delta T$$

where $V_{\text{glass}} = V_{\text{Hg}} + V_{\text{Pb}}$ is the initial volume. Then

$$\text{overflow} = \left[\left(\beta_{\text{Hg}} - \beta_{\text{glass}}\right)V_{\text{Hg}} + \left(\beta_{\text{Pb}} - \beta_{\text{glass}}\right)V_{\text{Pb}}\right]\Delta T = \left[\left(\beta_{\text{Hg}} - 3\alpha_{\text{glass}}\right)V_{\text{Hg}} + \left(3\alpha_{\text{Pb}} - 3\alpha_{\text{glass}}\right)V_{\text{Pb}}\right]\Delta T$$

$$= \left[(182-27)10^{-6}\frac{1}{\text{C}^\circ}118\ \text{cm}^3 + (87-27)10^{-6}\frac{1}{\text{C}^\circ}33.5\ \text{cm}^3\right]40^\circ\text{C} = \boxed{0.812\ \text{cm}^3}$$

P19.23 In $\dfrac{F}{A} = \dfrac{Y\Delta L}{L_i}$ require $\Delta L = \alpha L_i\Delta T$

$$\frac{F}{A} = Y\alpha\Delta T$$

$$\Delta T = \frac{F}{AY\alpha} = \frac{500\ \text{N}}{\left(2.00\times10^{-4}\ \text{m}^2\right)\left(20.0\times10^{10}\ \text{N/m}^2\right)\left(11.0\times10^{-6}/\text{C}^\circ\right)}$$

$$\Delta T = \boxed{1.14^\circ\text{C}}$$

***P19.24** Model the wire as contracting according to $\Delta L = \alpha L_i \Delta T$ and then stretching according to stress $= \frac{F}{A} = Y\frac{\Delta L}{L_i} = \frac{Y}{L_i}\alpha L_i \Delta T = Y\alpha\Delta T$.

(a) $F = YA\alpha\Delta T = \left(20\times 10^{10}\text{ N/m}^2\right)4\times 10^{-6}\text{ m}^2\ 11\times 10^{-6}\ \frac{1}{\text{C}°}45°\text{C} = \boxed{396\text{ N}}$

(b) $\Delta T = \frac{\text{stress}}{Y\alpha} = \frac{3\times 10^8\text{ N/m}^2}{\left(20\times 10^{10}\text{ N/m}^2\right)11\times 10^{-6}/\text{C}°} = 136°\text{C}$

To increase the stress the temperature must decrease to $35°\text{C} - 136°\text{C} = \boxed{-101°\text{C}}$.

(c) The original length divides out, so the answers would not change.

***P19.25** The area of the chip decreases according to $\Delta A = \gamma A_1 \Delta T = A_f - A_i$

$$A_f = A_i(1+\gamma\Delta T) = A_i(1+2\alpha\Delta T)$$

The star images are scattered uniformly, so the number N of stars that fit is proportional to the area. Then $N_f = N_i(1+2\alpha\Delta T) = 5\,342\left[1+2\left(4.68\times 10^{-6}\,°\text{C}^{-1}\right)(-100°\text{C}-20°\text{C})\right] = \boxed{5\,336\text{ star images}}$.

Section 19.5 Macroscopic Description of an Ideal Gas

P19.26 (a) $n = \frac{PV}{RT} = \frac{(9.00\text{ atm})\left(1.013\times 10^5\text{ Pa/atm}\right)\left(8.00\times 10^{-3}\text{ m}^3\right)}{(8.314\text{ N}\cdot\text{mol K})(293\text{ K})} = \boxed{2.99\text{ mol}}$

(b) $N = nN_A = (2.99\text{ mol})\left(6.02\times 10^{23}\text{ molecules/mol}\right) = \boxed{1.80\times 10^{24}\text{ molecules}}$

P19.27 (a) Initially, $P_iV_i = n_iRT_i$ $\quad(1.00\text{ atm})V_i = n_iR(10.0+273.15)\text{ K}$

Finally, $P_fV_f = n_fRT_f$ $\quad P_f(0.280V_i) = n_iR(40.0+273.15)\text{ K}$

Dividing these equations, $\quad \frac{0.280P_f}{1.00\text{ atm}} = \frac{313.15\text{ K}}{283.15\text{ K}}$

giving $\quad P_f = 3.95\text{ atm}$

or $\quad P_f = \boxed{4.00\times 10^5\text{ Pa(abs.)}}$.

(b) After being driven $\quad P_d(1.02)(0.280V_i) = n_iR(85.0+273.15)\text{ K}$

$$P_d = 1.121P_f = \boxed{4.49\times 10^5\text{ Pa}}$$

P19.28 $PV = NP'V' = \frac{4}{3}\pi r^3 NP'$: $\quad N = \frac{3PV}{4\pi r^3 P'} = \frac{3(150)(0.100)}{4\pi(0.150)^3(1.20)} = \boxed{884\text{ balloons}}$

If we have no special means for squeezing the last 100 L of helium out of the tank, the tank will be full of helium at 1.20 atm when the last balloon is inflated. The number of balloons is then reduced to to $884 - \frac{\left(0.100\text{ m}^3\right)3}{4\pi(0.15\text{ m})^3} = 877$.

P19.29 The equation of state of an ideal gas is $PV = nRT$ so we need to solve for the number of moles to find N.

$$n = \frac{PV}{RT} = \frac{\left(1.01\times10^5\ \text{N/m}^2\right)\left[(10.0\ \text{m})(20.0\ \text{m})(30.0\ \text{m})\right]}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = 2.49\times10^5\ \text{mol}$$

$$N = nN_A = 2.49\times10^5\ \text{mol}\left(6.022\times10^{23}\ \text{molecules/mol}\right) = \boxed{1.50\times10^{29}\ \text{molecules}}$$

***P19.30** (a) $$P_iV_i = n_iRT_i = \frac{m_i}{M}RT_i$$

$$m_i = \frac{MP_iV_i}{RT_i} = \frac{4.00\times10^{-3}\ \text{kg}}{\text{mole}}\frac{1.013\times10^5\ \text{N}}{\text{m}^2}\frac{4\pi\left(6.37\times10^6\ \text{m}\right)^3}{3}\frac{\text{mole}\cdot\text{K}}{8.314\ \text{Nm}\ 50\ \text{K}}$$

$$= \boxed{1.06\times10^{21}\ \text{kg}}$$

(b) $$\frac{P_fV_f}{P_iV_i} = \frac{n_fRT_f}{n_iRT_i}$$

$$2\cdot1 = \left(\frac{1.06\times10^{21}\ \text{kg} + 8.00\times10^{20}\ \text{kg}}{1.06\times10^{21}\ \text{kg}}\right)\frac{T_f}{50\ \text{K}}$$

$$T_f = 100\ \text{K}\left(\frac{1}{1.76}\right) = \boxed{56.9\ \text{K}}$$

P19.31 $$P = \frac{nRT}{V} = \left(\frac{9.00\ \text{g}}{18.0\ \text{g/mol}}\right)\left(\frac{8.314\ \text{J}}{\text{mol K}}\right)\left(\frac{773\ \text{K}}{2.00\times10^{-3}\ \text{m}^3}\right) = \boxed{1.61\ \text{MPa}} = 15.9\ \text{atm}$$

P19.32 (a) $$T_2 = T_1\frac{P_2}{P_1} = (300\ \text{K})(3) = \boxed{900\ \text{K}}$$

(b) $$T_2 = T_1\frac{P_2V_2}{P_1V_1} = 300(2)(2) = \boxed{1\,200\ \text{K}}$$

P19.33 $\sum F_y = 0$: $$\rho_{\text{out}}gV - \rho_{\text{in}}gV - (200\ \text{kg})g = 0$$

$$\left(\rho_{\text{out}} - \rho_{\text{in}}\right)\left(400\ \text{m}^3\right) = 200\ \text{kg}$$

The density of the air outside is $1.25\ \text{kg/m}^3$.

From $PV = nRT$, $\frac{n}{V} = \frac{P}{RT}$

The density is inversely proportional to the temperature, and the density of the hot air is

$$\rho_{\text{in}} = \left(1.25\ \text{kg/m}^3\right)\left(\frac{283\ \text{K}}{T_{\text{in}}}\right)$$

Then $$\left(1.25\ \text{kg/m}^3\right)\left(1 - \frac{283\ \text{K}}{T_{\text{in}}}\right)\left(400\ \text{m}^3\right) = 200\ \text{kg}$$

$$1 - \frac{283\ \text{K}}{T_{\text{in}}} = 0.400$$

$$0.600 = \frac{283\ \text{K}}{T_{\text{in}}} \qquad T_{\text{in}} = \boxed{472\ \text{K}}$$

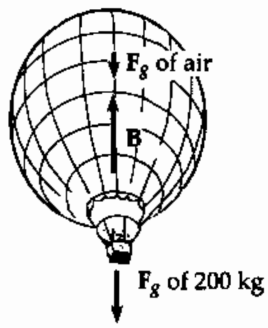


FIG. P19.33

***P19.34** Consider the air in the tank during one discharge process. We suppose that the process is slow enough that the temperature remains constant. Then as the pressure drops from 2.40 atm to 1.20 atm, the volume of the air doubles. During the first discharge, the air volume changes from 1 L to 2 L. Just 1 L of water is expelled and 3 L remains. In the second discharge, the air volume changes from 2 L to 4 L and 2 L of water is sprayed out. In the third discharge, only the last 1 L of water comes out. Were it not for male pattern dumbness, each person could more efficiently use his device by starting with the tank half full of water.

P19.35 (a) $PV = nRT$

$$n = \frac{PV}{RT} = \frac{(1.013\times10^5\ \text{Pa})(1.00\ \text{m}^3)}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = \boxed{41.6\ \text{mol}}$$

(b) $m = nM = (41.6\ \text{mol})(28.9\ \text{g/mol}) = \boxed{1.20\ \text{kg}}$, in agreement with the tabulated density of $1.20\ \text{kg/m}^3$ at 20.0°C.

***P19.36** The void volume is $0.765V_{\text{total}} = 0.765\pi r^2\ell = 0.765\pi\left(1.27\times10^{-2}\ \text{m}\right)^2 0.2\ \text{m} = 7.75\times10^{-5}\ \text{m}^3$. Now for the gas remaining $PV = nRT$

$$n = \frac{PV}{RT} = \frac{12.5(1.013\times10^5\ \text{N/m}^2)7.75\times10^{-5}\ \text{m}^3}{(8.314\ \text{Nm/mole K})(273+25)\ \text{K}} = \boxed{3.96\times10^{-2}\ \text{mol}}$$

P19.37 (a) $PV = nRT \qquad n = \frac{PV}{RT}$

$$m = nM = \frac{PVM}{RT} = \frac{1.013\times10^5\ \text{Pa}(0.100\ \text{m})^3(28.9\times10^{-3}\ \text{kg/mol})}{(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K})}$$

$$m = \boxed{1.17\times10^{-3}\ \text{kg}}$$

(b) $F_g = mg = 1.17\times10^{-3}\ \text{kg}(9.80\ \text{m/s}^2) = \boxed{11.5\ \text{mN}}$

(c) $F = PA = (1.013\times10^5\ \text{N/m}^2)(0.100\ \text{m})^2 = \boxed{1.01\ \text{kN}}$

(d) The $\boxed{\text{molecules must be moving very fast}}$ to hit the walls hard.

P19.38 At depth, $P = P_0 + \rho g h$ and $PV_i = nRT_i$

At the surface, $P_0V_f = nRT_f$: $\qquad \frac{P_0V_f}{(P_0+\rho g h)V_i} = \frac{T_f}{T_i}$

Therefore $V_f = V_i\left(\frac{T_f}{T_i}\right)\left(\frac{P_0+\rho g h}{P_0}\right)$

$$V_f = 1.00\ \text{cm}^3\left(\frac{293\ \text{K}}{278\ \text{K}}\right)\left(\frac{1.013\times10^5\ \text{Pa} + (1\,025\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(25.0\ \text{m})}{1.013\times10^5\ \text{Pa}}\right)$$

$$V_f = \boxed{3.67\ \text{cm}^3}$$

P19.39 $PV = nRT$: $\frac{m_f}{m_i} = \frac{n_f}{n_i} = \frac{P_f V_f}{RT_f}\frac{RT_i}{P_i V_i} = \frac{P_f}{P_i}$

so $m_f = m_i\left(\frac{P_f}{P_i}\right)$

$$|\Delta m| = m_i - m_f = m_i\left(\frac{P_i - P_f}{P_i}\right) = 12.0\text{ kg}\left(\frac{41.0\text{ atm} - 26.0\text{ atm}}{41.0\text{ atm}}\right) = \boxed{4.39\text{ kg}}$$

P19.40 My bedroom is 4 m long, 4 m wide, and 2.4 m high, enclosing air at 100 kPa and $20°C = 293\text{ K}$. Think of the air as 80.0% N_2 and 20.0% O_2.

Avogadro's number of molecules has mass

$$(0.800)(28.0\text{ g/mol}) + (0.200)(32.0\text{ g/mol}) = 0.028\,8\text{ kg/mol}$$

Then $PV = nRT = \left(\frac{m}{M}\right)RT$

gives $$m = \frac{PVM}{RT} = \frac{(1.00\times10^5\text{ N/m}^2)(38.4\text{ m}^3)(0.028\,8\text{ kg/mol})}{(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})} = 45.4\text{ kg}\boxed{\sim10^2\text{ kg}}$$

***P19.41** The CO_2 is far from liquefaction, so after it comes out of solution it behaves as an ideal gas. Its molar mass is $M = 12.0\text{ g/mol} + 2(16.0\text{ g/mol}) = 44.0\text{ g/mol}$. The quantity of gas in the cylinder is

$$n = \frac{m_{\text{sample}}}{M} = \frac{6.50\text{ g}}{44.0\text{ g/mol}} = 0.148\text{ mol}$$

Then $PV = nRT$

gives $$V = \frac{nRT}{P} = \frac{0.148\text{ mol}(8.314\text{ J/mol}\cdot\text{K})(273\text{ K} + 20\text{ K})}{1.013\times10^5\text{ N/m}^2}\left(\frac{1\text{ N}\cdot\text{m}}{1\text{ J}}\right)\left(\frac{10^3\text{ L}}{1\text{ m}^3}\right) = \boxed{3.55\text{ L}}$$

P19.42 $$N = \frac{PVN_A}{RT} = \frac{(10^{-9}\text{ Pa})(1.00\text{ m}^3)(6.02\times10^{23}\text{ molecules/mol})}{(8.314\text{ J/K}\cdot\text{mol})(300\text{ K})} = \boxed{2.41\times10^{11}\text{ molecules}}$$

P19.43 $P_0V = n_1RT_1 = \left(\frac{m_1}{M}\right)RT_1$

$P_0V = n_2RT_2 = \left(\frac{m_2}{M}\right)RT_2$

$$\boxed{m_1 - m_2 = \frac{P_0VM}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

P19.44 (a) Initially the air in the bell satisfies $P_0 V_{\text{bell}} = nRT_i$

or $P_0\left[(2.50\text{ m})A\right] = nRT_i$ (1)

When the bell is lowered, the air in the bell satisfies

$$P_{\text{bell}}(2.50\text{ m} - x)A = nRT_f \tag{2}$$

where x is the height the water rises in the bell. Also, the pressure in the bell, once it is lowered, is equal to the sea water pressure at the depth of the water level in the bell.

$$P_{\text{bell}} = P_0 + \rho g(82.3\text{ m} - x) \approx P_0 + \rho g(82.3\text{ m}) \tag{3}$$

The approximation is good, as $x < 2.50$ m. Substituting (3) into (2) and substituting nR from (1) into (2),

$$\left[P_0 + \rho g(82.3\text{ m})\right](2.50\text{ m} - x)A = P_0 V_{\text{bell}} \frac{T_f}{T_i}.$$

Using $P_0 = 1\text{ atm} = 1.013 \times 10^5\text{ Pa}$ and $\rho = 1.025 \times 10^3\text{ kg/m}^3$

$$x = (2.50\text{ m})\left[1 - \frac{T_f}{T_0}\left(1 + \frac{\rho g(82.3\text{ m})}{P_0}\right)^{-1}\right]$$

$$= (2.50\text{ m})\left[1 - \frac{277.15\text{ K}}{293.15\text{ K}}\left(1 + \frac{(1.025 \times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(82.3\text{ m})}{1.013 \times 10^5\text{ N/m}^2}\right)^{-1}\right]$$

$$x = \boxed{2.24\text{ m}}$$

(b) If the water in the bell is to be expelled, the air pressure in the bell must be raised to the water pressure at the bottom of the bell. That is,

$$P_{\text{bell}} = P_0 + \rho g(82.3\text{ m})$$

$$= 1.013 \times 10^5\text{ Pa} + (1.025 \times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(82.3\text{ m})$$

$$P_{\text{bell}} = 9.28 \times 10^5\text{ Pa} = \boxed{9.16\text{ atm}}$$

Additional Problems

P19.45 The excess expansion of the brass is $\Delta L_{\text{rod}} - \Delta L_{\text{tape}} = (\alpha_{\text{brass}} - \alpha_{\text{steel}})L_i \Delta T$

$$\Delta(\Delta L) = (19.0 - 11.0) \times 10^{-6}\ (°\text{C})^{-1}(0.950\text{ m})(35.0°\text{C})$$

$$\Delta(\Delta L) = 2.66 \times 10^{-4}\text{ m}$$

(a) The rod contracts more than tape to

a length reading $0.950\,0\text{ m} - 0.000\,266\text{ m} = \boxed{0.949\,7\text{ m}}$

(b) $0.950\,0\text{ m} + 0.000\,266\text{ m} = \boxed{0.950\,3\text{ m}}$

P19.46 At 0°C, 10.0 gallons of gasoline has mass,

from $\rho = \dfrac{m}{V}$

$$m = \rho V = \left(730\ \text{kg/m}^3\right)\left(10.0\ \text{gal}\right)\left(\frac{0.003\,80\ \text{m}^3}{1.00\ \text{gal}}\right) = 27.7\ \text{kg}$$

The gasoline will expand in volume by

$$\Delta V = \beta V_i \Delta T = 9.60\times10^{-4}\ {}^\circ\text{C}^{-1}(10.0\ \text{gal})(20.0^\circ\text{C} - 0.0^\circ\text{C}) = 0.192\ \text{gal}$$

At 20.0°C, $10.192\ \text{gal} = 27.7\ \text{kg}$

$$10.0\ \text{gal} = 27.7\ \text{kg}\left(\frac{10.0\ \text{gal}}{10.192\ \text{gal}}\right) = 27.2\ \text{kg}$$

The extra mass contained in 10.0 gallons at 0.0°C is

$$27.7\ \text{kg} - 27.2\ \text{kg} = \boxed{0.523\ \text{kg}}.$$

P19.47 Neglecting the expansion of the glass,

$$\Delta h = \frac{V}{A}\beta\Delta T$$

$$\Delta h = \frac{\frac{4}{3}\pi(0.250\ \text{cm}/2)^3}{\pi\left(2.00\times10^{-3}\ \text{cm}\right)^2}\left(1.82\times10^{-4}\ {}^\circ\text{C}^{-1}\right)(30.0^\circ\text{C}) = \boxed{3.55\ \text{cm}}$$

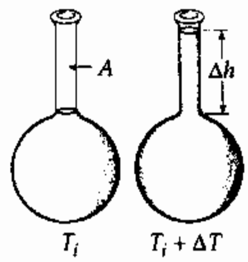


FIG. P19.47

P19.48 (a) The volume of the liquid increases as $\Delta V_\ell = V_i\beta\Delta T$. The volume of the flask increases as $\Delta V_g = 3\alpha V_i \Delta T$. Therefore, the overflow in the capillary is $V_c = V_i\Delta T(\beta - 3\alpha)$; and in the capillary $V_c = A\Delta h$.

Therefore, $\boxed{\Delta h = \dfrac{V_i}{A}(\beta - 3\alpha)\Delta T}$.

(b) For a mercury thermometer $\beta(\text{Hg}) = 1.82\times10^{-4}\ {}^\circ\text{C}^{-1}$

and for glass, $3\alpha = 3\times3.20\times10^{-6}\ {}^\circ\text{C}^{-1}$

Thus $\beta - 3\alpha \approx \beta$

or $\boxed{\alpha \ll \beta}$.

P19.49 The frequency played by the cold-walled flute is $f_i = \frac{v}{\lambda_i} = \frac{v}{2L_i}$.

When the instrument warms up

$$f_f = \frac{v}{\lambda_f} = \frac{v}{2L_f} = \frac{v}{2L_i(1+\alpha\Delta T)} = \frac{f_i}{1+\alpha\Delta T}.$$

The final frequency is lower. The change in frequency is

$$\Delta f = f_i - f_f = f_i\left(1 - \frac{1}{1+\alpha\Delta T}\right)$$

$$\Delta f = \frac{v}{2L_i}\left(\frac{\alpha\Delta T}{1+\alpha\Delta T}\right) \approx \frac{v}{2L_i}(\alpha\Delta T)$$

$$\Delta f \approx \frac{(343\text{ m/s})(24.0\times10^{-6}/\text{C}^\circ)(15.0^\circ\text{C})}{2(0.655\text{ m})} = \boxed{0.0943\text{ Hz}}$$

This change in frequency is imperceptibly small.

P19.50 (a) $\frac{P_0V}{T} = \frac{P'V'}{T'}$

$V' = V + Ah$

$P' = P_0 + \frac{kh}{A}$

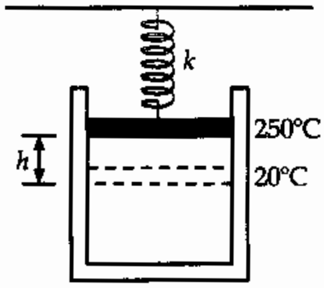


FIG. P19.50

$$\left(P_0 + \frac{kh}{A}\right)(V + Ah) = P_0V\left(\frac{T'}{T}\right)$$

$$\left(1.013\times10^5\text{ N/m}^2 + 2.00\times10^5\text{ N/m}^3\,h\right)\left(5.00\times10^{-3}\text{ m}^3 + (0.0100\text{ m}^2)h\right)$$

$$= \left(1.013\times10^5\text{ N/m}^2\right)\left(5.00\times10^{-3}\text{ m}^3\right)\left(\frac{523\text{ K}}{293\text{ K}}\right)$$

$$2000h^2 + 2013h - 397 = 0$$

$$h = \frac{-2013\pm2689}{4000} = \boxed{0.169\text{ m}}$$

(b) $$P' = P + \frac{kh}{A} = 1.013\times10^5\text{ Pa} + \frac{(2.00\times10^3\text{ N/m})(0.169)}{0.0100\text{ m}^2}$$

$$P' = \boxed{1.35\times10^5\text{ Pa}}$$

P19.51 (a) $\rho = \frac{m}{V}$ and $d\rho = -\frac{m}{V^2}dV$

For very small changes in V and ρ, this can be expressed as

$$\Delta\rho = -\frac{m}{V}\frac{\Delta V}{V} = -\rho\beta\Delta T.$$

The negative sign means that any increase in temperature causes the density to decrease and vice versa.

(b) For water we have $\beta = \left|\frac{\Delta\rho}{\rho\Delta T}\right| = \left|\frac{1.000\,0\ \text{g/cm}^3 - 0.999\,7\ \text{g/cm}^3}{(1.000\,0\ \text{g/cm}^3)(10.0°\text{C} - 4.0°\text{C})}\right| = \boxed{5\times10^{-5}\ °\text{C}^{-1}}$.

***P19.52** The astronauts exhale this much CO_2:

$$n = \frac{m_{\text{sample}}}{M} = \frac{1.09\ \text{kg}}{\text{astronaut}\cdot\text{day}}\left(\frac{1\,000\ \text{g}}{1\ \text{kg}}\right)(3\ \text{astronauts})(7\ \text{days})\left(\frac{1\ \text{mol}}{44.0\ \text{g}}\right) = 520\ \text{mol}.$$

Then 520 mol of methane is generated. It is far from liquefaction and behaves as an ideal gas.

$$P = \frac{nRT}{V} = \frac{520\ \text{mol}(8.314\ \text{J/mol}\cdot\text{K})(273\ \text{K} - 45\ \text{K})}{150\times10^{-3}\ \text{m}^3} = \boxed{6.57\times10^6\ \text{Pa}}$$

P19.53 (a) We assume that air at atmospheric pressure is above the piston.

In equilibrium $P_{\text{gas}} = \frac{mg}{A} + P_0$

Therefore, $\frac{nRT}{hA} = \frac{mg}{A} + P_0$

or $\boxed{h = \frac{nRT}{mg + P_0A}}$

where we have used $V = hA$ as the volume of the gas.

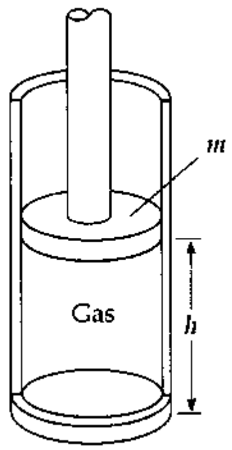


FIG. P19.53

(b) From the data given,

$$h = \frac{0.200\ \text{mol}(8.314\ \text{J/K}\cdot\text{mol})(400\ \text{K})}{20.0\ \text{kg}(9.80\ \text{m/s}^2) + (1.013\times10^5\ \text{N/m}^2)(0.008\,00\ \text{m}^2)}$$

$$= \boxed{0.661\ \text{m}}$$

P19.54 The angle of bending θ, between tangents to the two ends of the strip, is equal to the angle the strip subtends at its center of curvature. (The angles are equal because their sides are perpendicular, right side to the right side and left side to left side.)

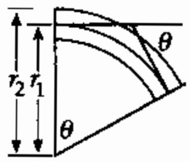


FIG. P19.54

(a) The definition of radian measure gives $L_i + \Delta L_1 = \theta r_1$

and $L_i + \Delta L_2 = \theta r_2$

By subtraction, $\Delta L_2 - \Delta L_1 = \theta(r_2 - r_1)$

$$\alpha_2 L_i \Delta T - \alpha_1 L_i \Delta T = \theta \Delta r$$

$$\boxed{\theta = \frac{(\alpha_2 - \alpha_1) L_i \Delta T}{\Delta r}}$$

(b) In the expression from part (a), θ is directly proportional to ΔT and also to $(\alpha_2 - \alpha_1)$. Therefore θ is zero when either of these quantities becomes zero.

(c) The material that expands more when heated contracts more when cooled, so the bimetallic strip bends the other way. It is fun to demonstrate this with liquid nitrogen.

(d)
$$\theta = \frac{2(\alpha_2 - \alpha_1) L_i \Delta T}{2\Delta r} = \frac{2\left(\left(19\times10^{-6} - 0.9\times10^{-6}\right)°\text{C}^{-1}\right)(200 \text{ mm})(1°\text{C})}{0.500 \text{ mm}}$$
$$= 1.45\times10^{-2} = 1.45\times10^{-2} \text{ rad}\left(\frac{180°}{\pi \text{ rad}}\right) = \boxed{0.830°}$$

P19.55 From the diagram we see that the change in area is

$$\Delta A = \ell \Delta w + w \Delta \ell + \Delta w \Delta \ell .$$

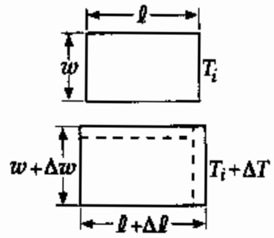


FIG. P19.55

Since $\Delta \ell$ and Δw are each small quantities, the product $\Delta w \Delta \ell$ will be very small. Therefore, we assume $\Delta w \Delta \ell \approx 0$.

Since $\Delta w = w\alpha\Delta T$ and $\Delta \ell = \ell \alpha \Delta T$,

we then have $\Delta A = \ell w \alpha \Delta T + \ell w \alpha \Delta T$

and since $A = \ell w$, $\boxed{\Delta A = 2\alpha A \Delta T}$.

The approximation assumes $\Delta w \Delta \ell \approx 0$, or $\alpha \Delta T \approx 0$. Another way of stating this is $\boxed{\alpha \Delta T << 1}$.

P19.56 (a) $T_i = 2\pi\sqrt{\dfrac{L_i}{g}}$ so $L_i = \dfrac{T_i^2 g}{4\pi^2} = \dfrac{(1.000\ \text{s})^2(9.80\ \text{m/s}^2)}{4\pi^2} = 0.248\,2\ \text{m}$

$$\Delta L = \alpha L_i \Delta T = 19.0\times10^{-6}\ {}^\circ\text{C}^{-1}(0.284\,2\ \text{m})(10.0^\circ\text{C}) = 4.72\times10^{-5}\ \text{m}$$

$$T_f = 2\pi\sqrt{\frac{L_i+\Delta L}{g}} = 2\pi\sqrt{\frac{0.248\,3\ \text{m}}{9.80\ \text{m/s}^2}} = 1.000\,095\,0\ \text{s}$$

$$\Delta T = \boxed{9.50\times10^{-5}\ \text{s}}$$

(b) In one week, the time lost is time lost $= 1$ week$\left(9.50\times10^{-5}\ \text{s lost per second}\right)$

$$\text{time lost} = (7.00\ \text{d/week})\left(\frac{86\,400\ \text{s}}{1.00\ \text{d}}\right)\left(9.50\times10^{-5}\ \frac{\text{s lost}}{\text{s}}\right)$$

$$\text{time lost} = \boxed{57.5\ \text{s lost}}$$

P19.57 $I = \int r^2 dm$ and since $r(T) = r(T_i)(1+\alpha\Delta T)$

for $\alpha\Delta T \ll 1$ we find $\dfrac{I(T)}{I(T_i)} = (1+\alpha\Delta T)^2$

thus $\dfrac{I(T)-I(T_i)}{I(T_i)} \approx 2\alpha\Delta T$

(a) With $\alpha = 17.0\times10^{-6}\ {}^\circ\text{C}^{-1}$ and $\Delta T = 100^\circ\text{C}$

we find for Cu: $\dfrac{\Delta I}{I} = 2\left(17.0\times10^{-6}\ {}^\circ\text{C}^{-1}\right)(100^\circ\text{C}) = \boxed{0.340\%}$

(b) With $\alpha = 24.0\times10^{-6}\ {}^\circ\text{C}^{-1}$

and $\Delta T = 100^\circ\text{C}$

we find for Al: $\dfrac{\Delta I}{I} = 2\left(24.0\times10^{-6}\ {}^\circ\text{C}^{-1}\right)(100^\circ\text{C}) = \boxed{0.480\%}$

P19.58 (a) $B = \rho g V'$ $\quad P' = P_0 + \rho g d$ $\quad P'V' = P_0 V_i$

$$B = \frac{\rho g P_0 V_i}{P'} = \boxed{\frac{\rho g P_0 V_i}{(P_0+\rho g d)}}$$

(b) Since d is in the denominator, B must $\boxed{\text{decrease}}$ as the depth increases. (The volume of the balloon becomes smaller with increasing pressure.)

(c) $$\frac{1}{2} = \frac{B(d)}{B(0)} = \frac{\rho g P_0 V_i/(P_0+\rho g d)}{\rho g P_0 V_i/P_0} = \frac{P_0}{P_0+\rho g d}$$

$$P_0 + \rho g d = 2P_0$$

$$d = \frac{P_0}{\rho g} = \frac{1.013\times10^5\ \text{N/m}^2}{\left(1.00\times10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)} = \boxed{10.3\ \text{m}}$$

***P19.59** The effective coefficient is defined by $\Delta L_{\text{total}} = \alpha_{\text{effective}} L_{\text{total}} \Delta T$ where $\Delta L_{\text{total}} = \Delta L_{\text{Cu}} + \Delta L_{\text{Pb}}$ and $L_{\text{total}} = L_{\text{Cu}} + L_{\text{Pb}} = xL_{\text{total}} + (1-x)L_{\text{total}}$. Then by substitution

$$\alpha_{\text{Cu}} L_{\text{Cu}} \Delta T + \alpha_{\text{Pb}} L_{\text{Pb}} \Delta T = \alpha_{\text{eff}} (L_{\text{Cu}} + L_{\text{Pb}}) \Delta T$$

$$\alpha_{\text{Cu}} x + \alpha_{\text{Pb}} (1-x) = \alpha_{\text{eff}}$$

$$(\alpha_{\text{Cu}} - \alpha_{\text{Pb}}) x = \alpha_{\text{eff}} - \alpha_{\text{Pb}}$$

$$x = \frac{20 \times 10^{-6}\ 1/\text{C}° - 29 \times 10^{-6}\ 1/\text{C}°}{17 \times 10^{-6}\ 1/\text{C}° - 29 \times 10^{-6}\ 1/\text{C}°} = \frac{9}{12} = \boxed{0.750}$$

***P19.60** (a) No torque acts on the disk so its angular momentum is constant. Its moment of inertia decreases as it contracts so its angular speed must $\boxed{\text{increase}}$.

(b) $$I_i \omega_i = I_f \omega_f = \frac{1}{2} M R_i^2 \omega_i = \frac{1}{2} M R_f^2 \omega_f = \frac{1}{2} M \left[R_i + R_i \alpha \Delta T \right]^2 \omega_f = \frac{1}{2} M R_i^2 \left[1 - \alpha |\Delta T| \right]^2 \omega_f$$

$$\omega_f = \omega_i \left[1 - \alpha |\Delta T| \right]^{-2} = \frac{25.0\ \text{rad/s}}{\left(1 - \left(17 \times 10^{-6}\ 1/\text{C}°\right) 830°\text{C}\right)^2} = \frac{25.0\ \text{rad/s}}{0.972} = \boxed{25.7\ \text{rad/s}}$$

P19.61 After expansion, the length of one of the spans is

$$L_f = L_i (1 + \alpha \Delta T) = 125\ \text{m} \left[1 + 12 \times 10^{-6}\ °\text{C}^{-1} (20.0°\text{C}) \right] = 125.03\ \text{m}.$$

L_f, y, and the original 125 m length of this span form a right triangle with y as the altitude. Using the Pythagorean theorem gives:

$$(125.03\ \text{m})^2 = y^2 + (125\ \text{m})^2$$

yielding $y = \boxed{2.74\ \text{m}}$.

P19.62 After expansion, the length of one of the spans is $L_f = L(1 + \alpha \Delta T)$. L_f, y, and the original length L of this span form a right triangle with y as the altitude. Using the Pythagorean theorem gives

$$L_f^2 = L^2 + y^2, \quad \text{or} \quad y = \sqrt{L_f^2 - L^2} = L\sqrt{(1 + \alpha \Delta T)^2 - 1} = L\sqrt{2\alpha \Delta T + (\alpha \Delta T)^2}$$

Since $\alpha \Delta T << 1$, $\quad y \approx \boxed{L\sqrt{2\alpha \Delta T}}$.

P19.63 (a) Let m represent the sample mass. The number of moles is $n = \frac{m}{M}$ and the density is $\rho = \frac{m}{V}$.

So $PV = nRT$ becomes $PV = \frac{m}{M} RT$ or $PM = \frac{m}{V} RT$.

Then, $\rho = \frac{m}{V} = \boxed{\frac{PM}{RT}}$.

(b) $$\rho = \frac{PM}{RT} = \frac{\left(1.013 \times 10^5\ \text{N/m}^2\right)(0.032\,0\ \text{kg/mol})}{(8.314\ \text{J/mol} \cdot \text{K})(293\ \text{K})} = \boxed{1.33\ \text{kg/m}^3}$$

P19.64 (a) From $PV = nRT$, the volume is: $V = \left(\frac{nR}{P}\right)T$

Therefore, when pressure is held constant, $\frac{dV}{dT} = \frac{nR}{P} = \frac{V}{T}$

Thus, $\beta \equiv \left(\frac{1}{V}\right)\frac{dV}{dT} = \left(\frac{1}{V}\right)\frac{V}{T}$, or $\boxed{\beta = \frac{1}{T}}$

(b) At $T = 0°C = 273$ K, this predicts $\beta = \frac{1}{273\text{ K}} = \boxed{3.66 \times 10^{-3}\text{ K}^{-1}}$

Experimental values are: $\beta_{He} = 3.665 \times 10^{-3}\text{ K}^{-1}$ and $\beta_{air} = 3.67 \times 10^{-3}\text{ K}^{-1}$

They agree within 0.06% and 0.2%, respectively.

P19.65 For each gas alone, $P_1 = \frac{N_1 kT}{V}$ and $P_2 = \frac{N_2 kT}{V}$ and $P_3 = \frac{N_3 kT}{V}$, etc.

For all gases

$$P_1V_1 + P_2V_2 + P_3V_3 \ldots = (N_1 + N_2 + N_3 \ldots)kT \text{ and}$$
$$(N_1 + N_2 + N_3 \ldots)kT = PV$$

Also, $V_1 = V_2 = V_3 = \ldots = V$, therefore $\boxed{P = P_1 + P_2 + P_3 \ldots}$.

P19.66 (a) Using the Periodic Table, we find the molecular masses of the air components to be

$$M(N_2) = 28.01\text{ u},\ M(O_2) = 32.00\text{ u},\ M(Ar) = 39.95\text{ u}$$

and $M(CO_2) = 44.01$ u.

Thus, the number of moles of each gas in the sample is

$$n(N_2) = \frac{75.52\text{ g}}{28.01\text{ g/mol}} = 2.696\text{ mol}$$
$$n(O_2) = \frac{23.15\text{ g}}{32.00\text{ g/mol}} = 0.723\,4\text{ mol}$$
$$n(Ar) = \frac{1.28\text{ g}}{39.95\text{ g/mol}} = 0.032\,0\text{ mol}$$
$$n(CO_2) = \frac{0.05\text{ g}}{44.01\text{ g/mol}} = 0.001\,1\text{ mol}$$

The total number of moles is $n_0 = \sum n_i = 3.453$ mol. Then, the partial pressure of N_2 is

$$P(N_2) = \frac{2.696\text{ mol}}{3.453\text{ mol}}\left(1.013 \times 10^5\text{ Pa}\right) = \boxed{79.1\text{ kPa}}.$$

Similarly,

$$P(O_2) = \boxed{21.2\text{ kPa}} \qquad P(Ar) = \boxed{940\text{ Pa}} \qquad P(CO_2) = \boxed{33.3\text{ Pa}}$$

continued on next page

(b) Solving the ideal gas law equation for V and using $T = 273.15 + 15.00 = 288.15$ K, we find

$$V = \frac{n_0 RT}{P} = \frac{(3.453\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(288.15\text{ K})}{1.013\times10^5\text{ Pa}} = \boxed{8.166\times10^{-2}\text{ m}^3}.$$

Then, $\rho = \frac{m}{V} = \frac{100\times10^{-3}\text{ kg}}{8.166\times10^{-2}\text{ m}^3} = \boxed{1.22\text{ kg/m}^3}$.

(c) The 100 g sample must have an appropriate molar mass to yield n_0 moles of gas: that is

$$M(\text{air}) = \frac{100\text{ g}}{3.453\text{ mol}} = \boxed{29.0\text{ g/mol}}.$$

***P19.67** Consider a spherical steel shell of inner radius r and much smaller thickness t, containing helium at pressure P. When it contains so much helium that it is on the point of bursting into two hemispheres, we have $P\pi r^2 = (5\times10^8\text{ N/m}^2)2\pi rt$. The mass of the steel is

$\rho_s V = \rho_s 4\pi r^2 t = \rho_s 4\pi r^2 \frac{Pr}{10^9\text{ Pa}}$. For the helium in the tank, $PV = nRT$ becomes

$$P\frac{4}{3}\pi r^3 = nRT = \frac{m_{\text{He}}}{M_{\text{He}}}RT = 1\text{ atm}V_{\text{balloon}}.$$

The buoyant force on the balloon is the weight of the air it displaces, which is described by $1\text{ atm}V_{\text{balloon}} = \frac{m_{\text{air}}}{M_{\text{air}}}RT = P\frac{4}{3}\pi r^3$. The net upward force on the balloon with the steel tank hanging from it is

$$+m_{\text{air}}g - m_{\text{He}}g - m_s g = \frac{M_{\text{air}}P4\pi r^3 g}{3RT} - \frac{M_{\text{He}}P4\pi r^3 g}{3RT} - \frac{\rho_s P4\pi r^3 g}{10^9\text{ Pa}}$$

The balloon will or will not lift the tank depending on whether this quantity is positive or negative, which depends on the sign of $\frac{(M_{\text{air}} - M_{\text{He}})}{3RT} - \frac{\rho_s}{10^9\text{ Pa}}$. At 20°C this quantity is

$$= \frac{(28.9 - 4.00)\times10^{-3}\text{ kg/mol}}{3(8.314\text{ J/mol}\cdot\text{K})293\text{ K}} - \frac{7\,860\text{ kg/m}^3}{10^9\text{ N/m}^2}$$
$$= 3.41\times10^{-6}\text{ s}^2/\text{m}^2 - 7.86\times10^{-6}\text{ s}^2/\text{m}^2$$

where we have used the density of iron. The net force on the balloon is downward so the helium balloon is $\boxed{\text{not able to lift}}$ its tank.

P19.68 With piston alone: $T = \text{constant}$, so $PV = P_0V_0$

or $P(Ah_i) = P_0(Ah_0)$

With $A = \text{constant}$, $P = P_0\left(\frac{h_0}{h_i}\right)$

But, $P = P_0 + \frac{m_p g}{A}$

where m_p is the mass of the piston.

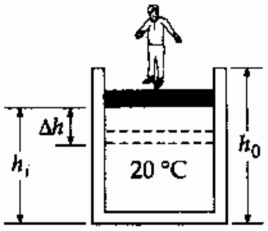


FIG. P19.68

Thus, $P_0 + \frac{m_p g}{A} = P_0\left(\frac{h_0}{h_i}\right)$

which reduces to $h_i = \frac{h_0}{1 + \frac{m_p g}{P_0 A}} = \frac{50.0 \text{ cm}}{1 + \frac{20.0 \text{ kg}(9.80 \text{ m/s}^2)}{1.013\times10^5 \text{ Pa}\left[\pi(0.400 \text{ m})^2\right]}} = 49.81 \text{ cm}$

With the man of mass M on the piston, a very similar calculation (replacing m_p by $m_p + M$) gives:

$$h' = \frac{h_0}{1 + \frac{(m_p+M)g}{P_0 A}} = \frac{50.0 \text{ cm}}{1 + \frac{95.0 \text{ kg}(9.80 \text{ m/s}^2)}{1.013\times10^5 \text{ Pa}\left[\pi(0.400 \text{ m})^2\right]}} = 49.10 \text{ cm}$$

Thus, when the man steps on the piston, it moves downward by

$$\Delta h = h_i - h' = 49.81 \text{ cm} - 49.10 \text{ cm} = 0.706 \text{ cm} = \boxed{7.06 \text{ mm}}.$$

(b) $P = \text{const}$, so $\frac{V}{T} = \frac{V'}{T_i}$ or $\frac{Ah_i}{T} = \frac{Ah'}{T_i}$

giving $T = T_i\left(\frac{h_i}{h'}\right) = 293 \text{ K}\left(\frac{49.81}{49.10}\right) = \boxed{297 \text{ K}}$ (or 24°C)

P19.69 (a) $\frac{dL}{L} = \alpha dT$: $\int_{T_i}^{T_f} \alpha dT = \int_{L_i}^{L_f} \frac{dL}{L} \Rightarrow \ln\left(\frac{L_f}{L_i}\right) = \alpha\Delta T \Rightarrow \boxed{L_f = L_i e^{\alpha\Delta T}}$

(b) $L_f = (1.00 \text{ m})e^{\left[2.00\times10^{-5} \text{ °C}^{-1}(100\text{°C})\right]} = 1.002\,002 \text{ m}$

$L'_f = 1.00 \text{ m}\left[1 + 2.00\times10^{-5} \text{ °C}^{-1}(100\text{°C})\right] = 1.002\,000 \text{ m}$: $\frac{L_f - L'_f}{L_f} = 2.00\times10^{-6} = \boxed{2.00\times10^{-4}\%}$

$L_f = (1.00 \text{ m})e^{\left[2.00\times10^{-2} \text{ °C}^{-1}(100\text{°C})\right]} = 7.389 \text{ m}$

$L'_f = 1.00 \text{ m}\left[1 + 0.020\,0\text{°C}^{-1}(100\text{°C})\right] = 3.000 \text{ m}$: $\frac{L_f - L'_f}{L_f} = \boxed{59.4\%}$

P19.70 At 20.0°C, the unstretched lengths of the steel and copper wires are

$$L_s(20.0°\text{C}) = (2.000\ \text{m})\left[1 + 11.0\times10^{-6}(\text{C}°)^{-1}(-20.0°\text{C})\right] = 1.999\,56\ \text{m}$$
$$L_c(20.0°\text{C}) = (2.000\ \text{m})\left[1 + 17.0\times10^{-6}(\text{C}°)^{-1}(-20.0°\text{C})\right] = 1.999\,32\ \text{m}$$

Under a tension F, the length of the steel and copper wires are

$$L_s' = L_s\left[1 + \frac{F}{YA}\right]_s \qquad L_c' = L_c\left[1 + \frac{F}{YA}\right]_c \qquad \text{where } L_s' + L_c' = 4.000\ \text{m}.$$

Since the tension, F, must be the same in each wire, solve for F:

$$F = \frac{(L_s' + L_c') - (L_s + L_c)}{\frac{L_s}{Y_s A_s} + \frac{L_c}{Y_c A_c}}.$$

When the wires are stretched, their areas become

$$A_s = \pi(1.000\times10^{-3}\ \text{m})^2\left[1 + (11.0\times10^{-6})(-20.0)\right]^2 = 3.140\times10^{-6}\ \text{m}^2$$
$$A_c = \pi(1.000\times10^{-3}\ \text{m})^2\left[1 + (17.0\times10^{-6})(-20.0)\right]^2 = 3.139\times10^{-6}\ \text{m}^2$$

Recall $Y_s = 20.0\times10^{10}$ Pa and $Y_c = 11.0\times10^{10}$ Pa. Substituting into the equation for F, we obtain

$$F = \frac{4.000\ \text{m} - (1.999\,56\ \text{m} + 1.999\,32\ \text{m})}{[1.999\,56\ \text{m}]/\left[(20.0\times10^{10}\ \text{Pa})(3.140\times10^{-6})\ \text{m}^2\right] + [1.999\,32\ \text{m}]/\left[(11.0\times10^{10}\ \text{Pa})(3.139\times10^{-6})\ \text{m}^2\right]}$$
$$F = \boxed{125\ \text{N}}$$

To find the x-coordinate of the junction,

$$L_s' = (1.999\,56\ \text{m})\left[1 + \frac{125\ \text{N}}{(20.0\times10^{10}\ \text{N/m}^2)(3.140\times10^{-6}\ \text{m}^2)}\right] = 1.999\,958\ \text{m}$$

Thus the x-coordinate is $-2.000 + 1.999\,958 = \boxed{-4.20\times10^{-5}\ \text{m}}$.

P19.71 (a) $\mu = \pi r^2 \rho = \pi\left(5.00\times10^{-4}\text{ m}\right)^2\left(7.86\times10^3\text{ kg/m}^3\right) = \boxed{6.17\times10^{-3}\text{ kg/m}}$

(b) $f_1 = \dfrac{v}{2L}$ and $v = \sqrt{\dfrac{T}{\mu}}$ so $f_1 = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$

Therefore, $T = \mu(2Lf_1)^2 = \left(6.17\times10^{-3}\right)(2\times0.800\times200)^2 = \boxed{632\text{ N}}$

(c) First find the unstressed length of the string at 0°C:

$$L = L_{\text{natural}}\left(1+\frac{T}{AY}\right) \text{ so } L_{\text{natural}} = \frac{L}{1+T/AY}$$

$$A = \pi\left(5.00\times10^{-4}\text{ m}\right)^2 = 7.854\times10^{-7}\text{ m}^2 \text{ and } Y = 20.0\times10^{10}\text{ Pa}$$

Therefore, $\dfrac{T}{AY} = \dfrac{632}{\left(7.854\times10^{-7}\right)\left(20.0\times10^{10}\right)} = 4.02\times10^{-3}$, and

$$L_{\text{natural}} = \frac{(0.800\text{ m})}{\left(1+4.02\times10^{-3}\right)} = 0.796\,8\text{ m}.$$

The unstressed length at 30.0°C is $L_{30^\circ\text{C}} = L_{\text{natural}}\left[1+\alpha(30.0^\circ\text{C}-0.0^\circ\text{C})\right]$,

or $L_{30^\circ\text{C}} = (0.796\,8\text{ m})\left[1+\left(11.0\times10^{-6}\right)(30.0)\right] = 0.797\,06\text{ m}$.

Since $L = L_{30^\circ\text{C}}\left[1+\dfrac{T'}{AY}\right]$, where T' is the tension in the string at 30.0°C,

$$T' = AY\left[\frac{L}{L_{30^\circ\text{C}}}-1\right] = \left(7.854\times10^{-7}\right)\left(20.0\times10^{10}\right)\left[\frac{0.800}{0.797\,06}-1\right] = 580\text{ N}.$$

To find the frequency at 30.0°C, realize that

$$\frac{f_1'}{f_1} = \sqrt{\frac{T'}{T}} \text{ so } f_1' = (200\text{ Hz})\sqrt{\frac{580\text{ N}}{632\text{ N}}} = \boxed{192\text{ Hz}}.$$

***P19.72** Some gas will pass through the porous plug from the reaction chamber 1 to the reservoir 2 as the reaction chamber is heated, but the net quantity of gas stays constant according to

$$n_{i1}+n_{i2} = n_{f1}+n_{f2}.$$

Assuming the gas is ideal, we apply $n = \dfrac{PV}{RT}$ to each term:

$$\frac{P_iV_0}{(300\text{ K})R}+\frac{P_i(4V_0)}{(300\text{ K})R} = \frac{P_fV_0}{(673\text{ K})R}+\frac{P_f(4V_0)}{(300\text{ K})R}$$

$$1\text{ atm}\left(\frac{5}{300\text{ K}}\right) = P_f\left(\frac{1}{673\text{ K}}+\frac{4}{300\text{ K}}\right) \qquad \boxed{P_f = 1.12\text{ atm}}$$

P19.73 Let 2θ represent the angle the curved rail subtends. We have

$$L_i + \Delta L = 2\theta R = L_i(1+\alpha\Delta T)$$

and $$\sin\theta = \frac{\frac{L_i}{2}}{R} = \frac{L_i}{2R}$$

Thus, $$\theta = \frac{L_i}{2R}(1+\alpha\Delta T) = (1+\alpha\Delta T)\sin\theta$$

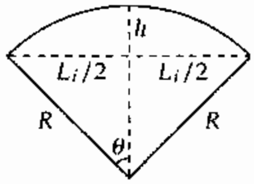


FIG. P19.73

and we must solve the transcendental equation $\theta = (1+\alpha\Delta T)\sin\theta = (1.000\,005\,5)\sin\theta$

Homing in on the non-zero solution gives, to four digits, $\theta = 0.018\,16 \text{ rad} = 1.040\,5°$

Now, $$h = R - R\cos\theta = \frac{L_i(1-\cos\theta)}{2\sin\theta}$$

This yields $\boxed{h = 4.54 \text{ m}}$, a remarkably large value compared to $\Delta L = 5.50$ cm.

***P19.74** (a) Let xL represent the distance of the stationary line below the top edge of the plate. The normal force on the lower part of the plate is $mg(1-x)\cos\theta$ and the force of kinetic friction on it is $\mu_k mg(1-x)\cos\theta$ up the roof. Again, $\mu_k mgx\cos\theta$ acts down the roof on the upper part of the plate. The near-equilibrium of the plate requires $\sum F_x = 0$

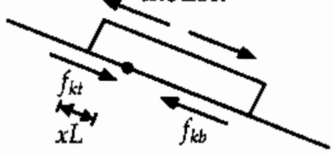


FIG. P19.74(a)

$$-\mu_k mgx\cos\theta + \mu_k mg(1-x)\cos\theta - mg\sin\theta = 0$$
$$-2\mu_k mgx\cos\theta = mg\sin\theta - \mu_k mg\cos\theta$$
$$2\mu_k x = \mu_k - \tan\theta$$
$$x = \frac{1}{2} - \frac{\tan\theta}{2\mu_k}$$

and the stationary line is indeed below the top edge by $xL = \frac{L}{2}\left(1 - \frac{\tan\theta}{\mu_k}\right)$.

(b) With the temperature falling, the plate contracts faster than the roof. The upper part slides down and feels an upward frictional force $\mu_k mg(1-x)\cos\theta$. The lower part slides up and feels downward frictional force $\mu_k mgx\cos\theta$. The equation $\sum F_x = 0$ is then the same as in part (a) and the stationary line is above the bottom edge by $xL = \frac{L}{2}\left(1 - \frac{\tan\theta}{\mu_k}\right)$.

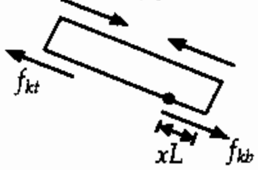


FIG. P19.74(b)

(c) Start thinking about the plate at dawn, as the temperature starts to rise. As in part (a), a line at distance xL below the top edge of the plate stays stationary relative to the roof as long as the temperature rises. The point P on the plate at distance xL above the bottom edge is destined to become the fixed point when the temperature starts falling. As the temperature rises, this point moves down the roof because of the expansion of the central part of the plate. Its displacement for the day is

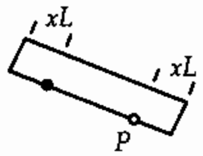


FIG. P19.74(c)

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$$\Delta L = (\alpha_2 - \alpha_1)(L - xL - xL)\Delta T$$
$$= (\alpha_2 - \alpha_1)\left[L - 2\frac{L}{2}\left(1 - \frac{\tan\theta}{\mu_k}\right)\right](T_h - T_c)$$
$$= (\alpha_2 - \alpha_1)\left(\frac{L\tan\theta}{\mu_k}\right)(T_h - T_c).$$

At dawn the next day the point P is farther down the roof by the distance ΔL. It represents the displacement of every other point on the plate.

(d) $(\alpha_2 - \alpha_1)\left(\frac{L\tan\theta}{\mu_k}\right)(T_h - T_c) = \left(24\times10^{-6}\,\frac{1}{\text{C}^\circ} - 15\times10^{-6}\,\frac{1}{\text{C}^\circ}\right)\frac{1.20\text{ m}\tan 18.5^\circ}{0.42}32^\circ\text{C} = \boxed{0.275\text{ mm}}$

(e) If $\alpha_2 < \alpha_1$, the diagram in part (a) applies to temperature falling and the diagram in part (b) applies to temperature rising. The weight of the plate still pulls it step by step down the roof. The same expression describes how far it moves each day.

ANSWERS TO EVEN PROBLEMS

P19.2 (a) 1.06 atm; (b) $-124°\text{C}$

P19.4 (a) $37.0°\text{C} = 310\text{ K}$; (b) $-20.6°\text{C} = 253\text{ K}$

P19.6 $T_C = (1.33\text{ C}°/\text{S}°)T_S + 20.0°\text{C}$

P19.8 0.313 m

P19.10 1.20 cm

P19.12 $15.8\,\mu\text{m}$

P19.14 0.663 mm to the right at 78.2° below the horizontal

P19.16 (a) 0.109 cm^2; (b) increase

P19.18 (a) $437°\text{C}$; (b) $3\,000°\text{C}$; no

P19.20 (a) $2.52\times10^6\text{ N/m}^2$; (b) no

P19.22 0.812 cm^3

P19.24 (a) 396 N; (b) $-101°\text{C}$; (c) no change

P19.26 (a) 2.99 mol; (b) 1.80×10^{24} molecules

P19.28 884 balloons

P19.30 (a) 1.06×10^{21} kg; (b) 56.9 K

P19.32 (a) 900 K; (b) 1 200 K

P19.34 see the solution

P19.36 3.96×10^{-2} mol

P19.38 3.67 cm^3

P19.40 between 10^1 kg and 10^2 kg

P19.42 2.41×10^{11} molecules

P19.44 (a) 2.24 m; (b) 9.28×10^5 Pa

P19.46 0.523 kg

P19.48 (a) see the solution; (b) $\alpha \ll \beta$

P19.50 (a) 0.169 m; (b) 1.35×10^5 Pa

P19.52 6.57 MPa

P19.54 (a) $\theta = \frac{(\alpha_2 - \alpha_1)L_i\Delta T}{\Delta r}$; (b) see the solution; (c) it bends the other way; (d) 0.830°

P19.56 (a) increase by $95.0\,\mu\text{s}$; (b) loses 57.5 s

P19.58 (a) $B = \rho g P_0 V_i (P_0 + \rho g d)^{-1}$ up; (b) decrease; (c) 10.3 m

P19.60 (a) yes; see the solution; (b) $25.7\ \text{rad/s}$

P19.62 $y \approx L(2\alpha\Delta T)^{1/2}$

P19.64 (a) see the solution;
(b) $3.66 \times 10^{-3}\ \text{K}^{-1}$, within 0.06% and 0.2% of the experimental values

P19.66 (a) 79.1 kPa for N_2; 21.2 kPa for O_2;
940 Pa for Ar; 33.3 Pa for CO_2;
(b) 81.7 L; $1.22\ \text{kg/m}^3$; (c) $29.0\ \text{g/mol}$

P19.68 (a) 7.06 mm; (b) 297 K

P19.70 125 N; $-42.0\ \mu\text{m}$

P19.72 1.12 atm

P19.74 (a), (b), (c) see the solution; (d) 0.275 mm;
(e) see the solution

Heat and the First Law of Thermodynamics

ANSWERS TO QUESTIONS

Q20.1 Temperature is a measure of molecular motion. Heat is energy in the process of being transferred between objects by random molecular collisions. Internal energy is an object's energy of random molecular motion and molecular interaction.

Q20.2 The ΔT is twice as great in the ethyl alcohol.

Q20.3 The final equilibrium temperature will show no significant increase over the initial temperature of the water.

Q20.4 Some water may boil away. You would have to very precisely measure how much, and very quickly measure the temperature of the steam; it is not necessarily $100°C$.

Q20.5 The fingers are wetted to create a layer of steam between the fingers and the molten lead. The steam acts as an insulator and can prevent or delay serious burns. The molten lead demonstration is dangerous, and we do not recommend it.

Q20.6 Heat is energy being transferred, not energy contained in an object. Further, a large-mass object, or an object made of a material with high specific heat, can contain more internal energy than a higher-temperature object.

Q20.7 There are three properties to consider here: thermal conductivity, specific heat, and mass. With dry aluminum, the thermal conductivity of aluminum is much greater than that of (dry) skin. This means that the internal energy in the aluminum can more readily be transferred to the atmosphere than to your fingers. In essence, your skin acts as a thermal insulator to some degree (pun intended). If the aluminum is wet, it can wet the outer layer of your skin to make it into a good conductor of heat; then more internal energy from the aluminum can get into you. Further, the water itself, with additional mass and with a relatively large specific heat compared to aluminum, can be a significant source of extra energy to burn you. In practical terms, when you let go of a hot, dry piece of aluminum foil, the heat transfer immediately ends. When you let go of a hot *and* wet piece of aluminum foil, the hot water sticks to your skin, continuing the heat transfer, and resulting in more energy transfer to you!

Q20.8 Write $1\,000\text{ kg}(4\,186\text{ J/kg}\cdot°\text{C})(1°\text{C}) = V(1.3\text{ kg/m}^3)(1\,000\text{ J/kg}\cdot°\text{C})(1°\text{C})$ to find $V = 3.2\times10^3\text{ m}^3$.

Q20.9 The large amount of energy stored in concrete during the day as the sun falls on it is released at night, resulting in an higher average evening temperature than the countryside. The cool air in the surrounding countryside exerts a buoyant force on the warmer air in the city, pushing it upward and moving into the city in the process. Thus, evening breezes tend to blow from country to city.

Q20.10 If the system is isolated, no energy enters or leaves the system by heat, work, or other transfer processes. Within the system energy can change from one form to another, but since energy is conserved these transformations cannot affect the total amount of energy. The total energy is constant.

Q20.11 (a) and (b) both increase by minuscule amounts.

Q20.12 The steam locomotive engine is a perfect example of turning internal energy into mechanical energy. Liquid water is heated past the point of vaporization. Through a controlled mechanical process, the expanding water vapor is allowed to push a piston. The translational kinetic energy of the piston is usually turned into rotational kinetic energy of the drive wheel.

Q20.13 Yes. If you know the different specific heats of zinc and copper, you can determine the fraction of each by heating a known mass of pennies to a specific initial temperature, say $100°C$, and dumping them into a known quantity of water, at say $20°C$. The final temperature T will reveal the metal content:

$$m_{\text{pennies}}\left[xc_{\text{Cu}}+(1-x)c_{\text{Zn}}\right](100°\text{C}-T)=m_{\text{H}_2\text{O}}c_{\text{H}_2\text{O}}(T-20°\text{C}).$$

Since all quantities are known, except x, the fraction of the penny that is copper will be found by putting in the experimental numbers m_{pennies}, $m_{\text{H}_2\text{O}}$, $T(\text{final})$, c_{Zn}, and c_{Cu}.

Q20.14 The materials used to make the support structure of the roof have a higher thermal conductivity than the insulated spaces in between. The heat from the barn conducts through the rafters and melts the snow.

Q20.15 The tile is a better thermal conductor than carpet. Thus, energy is conducted away from your feet more rapidly by the tile than by the carpeted floor.

Q20.16 The question refers to baking in a conventional oven, not to microwaving. The metal has much higher thermal conductivity than the potato. The metal quickly conducts energy from the hot oven into the center of potato.

Q20.17 Copper has a higher thermal conductivity than the wood. Heat from the flame is conducted through the copper away from the paper, so that the paper need not reach its kindling temperature. The wood does not conduct the heat away from the paper as readily as the copper, so the energy in the paper can increase enough to make it ignite.

Q20.18 In winter the interior of the house is warmer than the air outside. On a summer day we want the interior to stay cooler than the exterior. Heavy draperies over the windows can slow down energy transfer by conduction, by convection, and by radiation, to make it easier to maintain the desired difference in temperature.

Q20.19 You must allow time for the flow of energy into the center of the piece of meat. To avoid burning the outside, the meat should be relatively far from the flame. If the outer layer does char, the carbon will slow subsequent energy flow to the interior.

Q20.20 At night, the Styrofoam beads would decrease the overall thermal conductivity of the windows, and thus decrease the amount of heat conducted from inside to outside. The air pockets in the Styrofoam are an efficient insulator. During the winter day, the influx of sunlight coming through the window warms the living space.

An interesting aside—the majority of the energy that goes into warming a home from sunlight through a window is *not* the infrared light given off by the sun. Glass is a relatively good insulator of infrared. If not, the window on your cooking oven might as well be just an open hole! Glass is opaque to a large portion of the ultraviolet range. The glass molecules absorb ultraviolet light from the sun and re-emit the energy in the infrared region. It is this re-emitted infrared radiation that contributes to warming your home, along with visible light.

Q20.21 In winter the produce is protected from freezing. The heat capacity of the earth is so high that soil freezes only to a depth of a few decimeters in temperate regions. Throughout the year the temperature will stay nearly constant all day and night. Factors to be considered are the insulating properties of soil, the absence of a path for energy to be radiated away from or to the vegetables, and the hindrance to the formation of convection currents in the small, enclosed space.

Q20.22 The high mass and specific heat of the barrel of water and its high heat of fusion mean that a large amount of energy would have to leak out of the cellar before the water and the produce froze solid. Evaporation of the water keeps the relative humidity high to protect foodstuffs from drying out.

Q20.23 The sunlight hitting the peaks warms the air immediately around them. This air, which is slightly warmer and less dense than the surrounding air, rises, as it is buoyed up by cooler air from the valley below. The air from the valley flows up toward the sunny peaks, creating the morning breeze.

Q20.24 Sunlight hits the earth and warms the air immediately above it. This warm, less-dense air rises, creating an up-draft. Many raptors, like eagles, hawks and falcons use updrafts to aid in hunting. These birds can often be seen flying without flapping their wings—just sitting in an updraft with wings extended.

Q20.25 The bit of water immediately over the flame warms up and expands. It is buoyed up and rises through the rest of the water. Colder, more dense water flows in to take its place. Convection currents are set up. This effectively warms the bulk of the water all at once, much more rapidly than it would be by heat being conducted through the water from the flame.

Q20.26 The porcelain of the teacup is a thermal insulator. That is, it is a thermal conductor of relatively low conductivity. When you wrap your hands around a cup of hot tea, you make A large and L small in the equation $\mathcal{P} = kA\frac{T_h - T_c}{L}$ for the rate of energy transfer by heat from tea into you. When you hold the cup by the handle, you make the rate of energy transfer much smaller by reducing A and increasing L. The air around the cup handle will also reduce the temperature where you are touching it. A paper cup can be fitted into a tubular jacket of corrugated cardboard, with the channels running vertically, for remarkably effective insulation, according to the same principles.

Q20.27 As described in the answer to question 20.25, convection currents in the water serve to bring more of the heat into the water from the paper cup than the specific heats and thermal conductivities of paper and water would suggest. Since the boiling point of water is far lower than the kindling temperature of the cup, the extra energy goes into boiling the water.

Q20.28 Keep them dry. The air pockets in the pad conduct energy by heat, but only slowly. Wet pads would absorb some energy in warming up themselves, but the pot would still be hot and the water would quickly conduct and convect a lot of energy right into you.

Q20.29 The person should add the cream immediately when the coffee is poured. Then the smaller temperature difference between coffee and environment will reduce the rate of energy loss during the several minutes.

Q20.30 The cup without the spoon will be warmer. Heat is conducted from the coffee up through the metal. The energy then radiates and convects into the atmosphere.

Q20.31 Convection. The bridge deck loses energy rapidly to the air both above it and below it.

Q20.32 The marshmallow has very small mass compared to the saliva in the teacher's mouth and the surrounding tissues. Mostly air and sugar, the marshmallow also has a low specific heat compared to living matter. Then the marshmallow can zoom up through a large temperature change while causing only a small temperature drop of the teacher's mouth. The marshmallow is a foam with closed cells and it carries very little liquid nitrogen into the mouth. The liquid nitrogen still on the marshmallow comes in contact with the much hotter saliva and immediately boils into cold gaseous nitrogen. This nitrogen gas has very low thermal conductivity. It creates an insulating thermal barrier between the marshmallow and the teacher's mouth (the Leydenfrost effect). A similar effect can be seen when water droplets are put on a hot skillet. Each one dances around as it slowly shrinks, because it is levitated on a thin film of steam. The most extreme demonstration of this effect is pouring liquid nitrogen into one's mouth and blowing out a plume of nitrogen gas. We strongly recommended that you read of Jearl Walker's adventures with this demonstration rather than trying it.

Q20.33

(a) Warm a pot of coffee on a hot stove.

(b) Place an ice cube at $0°C$ in warm water—the ice will absorb energy while melting, but not increase in temperature.

(c) Let a high-pressure gas at room temperature slowly expand by pushing on a piston. Work comes out of the gas in a constant-temperature expansion as the same quantity of heat flows in from the surroundings.

(d) Warm your hands by rubbing them together. Heat your tepid coffee in a microwave oven. Energy input by work, by electromagnetic radiation, or by other means, can all alike produce a temperature increase.

(e) Davy's experiment is an example of this process.

(f) This is not necessarily true. Consider some supercooled liquid water, unstable but with temperature below $0°C$. Drop in a snowflake or a grain of dust to trigger its freezing into ice, and the loss of internal energy measured by its latent heat of fusion can actually push its temperature up.

Q20.34 Heat is conducted from the warm oil to the pipe that carries it. That heat is then conducted to the cooling fins and up through the solid material of the fins. The energy then radiates off in all directions and is efficiently carried away by convection into the air. The ground below is left frozen.

SOLUTIONS TO PROBLEMS

Section 20.1 Heat and Internal Energy

P20.1 Taking $m = 1.00$ kg, we have

$$\Delta U_g = mgh = (1.00\text{ kg})(9.80\text{ m/s}^2)(50.0\text{ m}) = 490\text{ J}.$$

But $\Delta U_g = Q = mc\Delta T = (1.00\text{ kg})(4\,186\text{ J/kg}\cdot{}^\circ\text{C})\Delta T = 490\text{ J}$ so $\Delta T = 0.117^\circ\text{C}$

$$T_f = T_i + \Delta T = \boxed{(10.0 + 0.117)^\circ\text{C}}$$

P20.2 The container is thermally insulated, so no energy flows by heat:

$$Q = 0$$

and $\Delta E_{\text{int}} = Q + W_{\text{input}} = 0 + W_{\text{input}} = 2mgh$

The work on the falling weights is equal to the work done on the water in the container by the rotating blades. This work results in an increase in internal energy of the water:

$$2mgh = \Delta E_{\text{int}} = m_{\text{water}} c\Delta T$$

$$\Delta T = \frac{2mgh}{m_{\text{water}}c} = \frac{2 \times 1.50\text{ kg}(9.80\text{ m/s}^2)(3.00\text{ m})}{0.200\text{ kg}(4\,186\text{ J/kg}\cdot{}^\circ\text{C})} = \frac{88.2\text{ J}}{837\text{ J/}^\circ\text{C}}$$

$$= \boxed{0.105^\circ\text{C}}$$

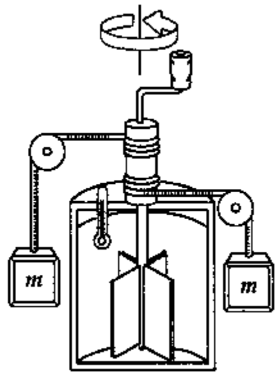


FIG. P20.2

Section 20.2 Specific Heat and Calorimetry

P20.3 $\Delta Q = mc_{\text{silver}}\Delta T$

$$1.23\text{ kJ} = (0.525\text{ kg})c_{\text{silver}}(10.0^\circ\text{C})$$

$$c_{\text{silver}} = \boxed{0.234\text{ kJ/kg}\cdot{}^\circ\text{C}}$$

P20.4 From $Q = mc\Delta T$

we find $\Delta T = \dfrac{Q}{mc} = \dfrac{1\,200\text{ J}}{0.050\,0\text{ kg}(387\text{ J/kg}\cdot{}^\circ\text{C})} = 62.0^\circ\text{C}$

Thus, the final temperature is $\boxed{87.0^\circ\text{C}}$.

***P20.5** We imagine the stone energy reservoir has a large area in contact with air and is always at nearly the same temperature as the air. Its overnight loss of energy is described by

$$\mathcal{P} = \frac{Q}{\Delta t} = \frac{mc\Delta T}{\Delta t}$$

$$m = \frac{\mathcal{P}\Delta t}{c\Delta T} = \frac{(-6\,000\text{ J/s})(14\text{ h})(3\,600\text{ s/h})}{(850\text{ J/kg}\cdot{}^\circ\text{C})(18^\circ\text{C} - 38^\circ\text{C})} = \frac{3.02 \times 10^8\text{ J}\cdot\text{kg}\cdot{}^\circ\text{C}}{850\text{ J}(20^\circ\text{C})} = \boxed{1.78 \times 10^4\text{ kg}}$$

***P20.6** The laser energy output:

$$\mathscr{P}\Delta t = \left(1.60\times10^{13}\ \text{J/s}\right)2.50\times10^{-9}\ \text{s} = 4.00\times10^{4}\ \text{J}.$$

The teakettle input:

$$Q = mc\Delta T = 0.800\ \text{kg}(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})80^\circ\text{C} = 2.68\times10^{5}\ \text{J}.$$

This is larger by 6.70 times.

P20.7 $Q_{\text{cold}} = -Q_{\text{hot}}$

$(mc\Delta T)_{\text{water}} = -(mc\Delta T)_{\text{iron}}$

$20.0\ \text{kg}(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(T_f - 25.0^\circ\text{C}) = -(1.50\ \text{kg})(448\ \text{J/kg}\cdot{}^\circ\text{C})(T_f - 600^\circ\text{C})$

$T_f = \boxed{29.6^\circ\text{C}}$

P20.8 Let us find the energy transferred in one minute.

$$Q = \left[m_{\text{cup}}c_{\text{cup}} + m_{\text{water}}c_{\text{water}}\right]\Delta T$$

$$Q = \left[(0.200\ \text{kg})(900\ \text{J/kg}\cdot{}^\circ\text{C}) + (0.800\ \text{kg})(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})\right](-1.50^\circ\text{C}) = -5\,290\ \text{J}$$

If this much energy is removed from the system each minute, the rate of removal is

$$\mathscr{P} = \frac{|Q|}{\Delta t} = \frac{5\,290\ \text{J}}{60.0\ \text{s}} = 88.2\ \text{J/s} = \boxed{88.2\ \text{W}}.$$

P20.9 (a) $Q_{\text{cold}} = -Q_{\text{hot}}$

$(m_w c_w + m_c c_c)(T_f - T_c) = -m_{\text{Cu}}c_{\text{Cu}}(T_f - T_{\text{Cu}}) - m_{unk}c_{unk}(T_f - T_{unk})$

where *w* is for water, *c* the calorimeter, Cu the copper sample, and *unk* the unknown.

$$\left[250\ \text{g}(1.00\ \text{cal/g}\cdot{}^\circ\text{C}) + 100\ \text{g}(0.215\ \text{cal/g}\cdot{}^\circ\text{C})\right](20.0 - 10.0)^\circ\text{C}$$
$$= -(50.0\ \text{g})(0.092\,4\ \text{cal/g}\cdot{}^\circ\text{C})(20.0 - 80.0)^\circ\text{C} - (70.0\ \text{g})c_{unk}(20.0 - 100)^\circ\text{C}$$
$$2.44\times10^{3}\ \text{cal} = \left(5.60\times10^{3}\ \text{g}\cdot{}^\circ\text{C}\right)c_{unk}$$

or $c_{unk} = \boxed{0.435\ \text{cal/g}\cdot{}^\circ\text{C}}$.

(b) The material of the sample is $\boxed{\text{beryllium}}$.

P20.10 (a) $(f)(mgh) = mc\Delta T$

$$\frac{(0.600)(3.00\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)(50.0\text{ m})}{4.186\text{ J/cal}} = (3.00\text{ g})(0.092\,4\text{ cal/g}\cdot{}^\circ\text{C})(\Delta T)$$

$$\Delta T = 0.760^\circ\text{C};\ \boxed{T = 25.8^\circ\text{C}}$$

(b) $\boxed{\text{No}}$. Both the change in potential energy and the heat absorbed are proportional to the mass; hence, the mass cancels in the energy relation.

***P20.11** We do not know whether the aluminum will rise or drop in temperature. The energy the water can absorb in rising to 26°C is $mc\Delta T = 0.25\text{ kg }4\,186\frac{\text{J}}{\text{kg °C}}6^\circ\text{C} = 6\,279\text{ J}$. The energy the copper can put out in dropping to 26°C is $mc\Delta T = 0.1\text{ kg }387\frac{\text{J}}{\text{kg °C}}74^\circ\text{C} = 2\,864\text{ J}$. Since $6\,279\text{ J} > 2\,864\text{ J}$, the final temperature is less than 26°C. We can write $Q_h = -Q_c$ as

$$Q_{\text{water}} + Q_{\text{Al}} + Q_{\text{Cu}} = 0$$

$$0.25\text{ kg }4\,186\frac{\text{J}}{\text{kg °C}}\left(T_f - 20^\circ\text{C}\right) + 0.4\text{ kg }900\frac{\text{J}}{\text{kg °C}}\left(T_f - 26^\circ\text{C}\right)$$

$$+0.1\text{ kg }387\frac{\text{J}}{\text{kg °C}}\left(T_f - 100^\circ\text{C}\right) = 0$$

$$1\,046.5T_f - 20\,930^\circ\text{C} + 360T_f - 9\,360^\circ\text{C} + 38.7T_f - 3\,870^\circ\text{C} = 0$$

$$1\,445.2T_f = 34\,160^\circ\text{C}$$

$$T_f = \boxed{23.6^\circ\text{C}}$$

P20.12 $Q_{\text{cold}} = -Q_{\text{hot}}$

$$m_{\text{Al}}c_{\text{Al}}\left(T_f - T_c\right) + m_c c_w\left(T_f - T_c\right) = -m_h c_w\left(T_f - T_h\right)$$

$$\left(m_{\text{Al}}c_{\text{Al}} + m_c c_w\right)T_f - \left(m_{\text{Al}}c_{\text{Al}} + m_c c_w\right)T_c = -m_h c_w T_f + m_h c_w T_h$$

$$\left(m_{\text{Al}}c_{\text{Al}} + m_c c_w + m_h c_w\right)T_f = \left(m_{\text{Al}}c_{\text{Al}} + m_c c_w\right)T_c + m_h c_w T_h$$

$$T_f = \boxed{\frac{\left(m_{\text{Al}}c_{\text{Al}} + m_c c_w\right)T_c + m_h c_w T_h}{m_{\text{Al}}c_{\text{Al}} + m_c c_w + m_h c_w}}$$

P20.13 The rate of collection of energy is $\mathscr{P} = 550\text{ W/m}^2\left(6.00\text{ m}^2\right) = 3\,300\text{ W}$. The amount of energy required to raise the temperature of 1 000 kg of water by 40.0°C is:

$$Q = mc\Delta T = 1\,000\text{ kg}(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(40.0^\circ\text{C}) = 1.67\times10^8\text{ J}$$

Thus, $\mathscr{P}\Delta t = 1.67\times10^8\text{ J}$

or $\Delta t = \dfrac{1.67\times10^8\text{ J}}{3\,300\text{ W}} = \boxed{50.7\text{ ks}} = 14.1\text{ h}$.

***P20.14** Vessel one contains oxygen according to $PV = nRT$:

$$n_c = \frac{PV}{RT} = \frac{1.75(1.013\times10^5 \text{ Pa})16.8\times10^{-3} \text{ m}^3}{8.314 \text{ Nm/mol}\cdot\text{K } 300 \text{ K}} = 1.194 \text{ mol}.$$

Vessel two contains this much oxygen:

$$n_h = \frac{2.25(1.013\times10^5)22.4\times10^{-3}}{8.314(450)} \text{ mol} = 1.365 \text{ mol}.$$

(a) The gas comes to an equilibrium temperature according to

$$(mc\Delta T)_{\text{cold}} = -(mc\Delta T)_{\text{hot}}$$
$$n_c Mc(T_f - 300 \text{ K}) + n_h Mc(T_f - 450 \text{ K}) = 0$$

The molar mass M and specific heat divide out:

$$1.194T_f - 358.2 \text{ K} + 1.365T_f - 614.1 \text{ K} = 0$$
$$T_f = \frac{972.3 \text{ K}}{2.559} = \boxed{380 \text{ K}}$$

(b) The pressure of the whole sample in its final state is

$$P = \frac{nRT}{V} = \frac{2.559 \text{ mol } 8.314 \text{ J } 380 \text{ K}}{\text{mol K}(22.4+16.8)\times10^{-3} \text{ m}^3} = \boxed{2.06\times10^5 \text{ Pa}} = 2.04 \text{ atm}.$$

Section 20.3 Latent Heat

P20.15 The heat needed is the sum of the following terms:

$$Q_{\text{needed}} = (\text{heat to reach melting point}) + (\text{heat to melt})$$
$$+ (\text{heat to reach melting point}) + (\text{heat to vaporize}) + (\text{heat to reach } 110°\text{C})$$

Thus, we have

$$Q_{\text{needed}} = 0.0400 \text{ kg}\Big[(2\,090 \text{ J/kg}\cdot°\text{C})(10.0°\text{C}) + (3.33\times10^5 \text{ J/kg})$$
$$+ (4\,186 \text{ J/kg}\cdot°\text{C})(100°\text{C}) + (2.26\times10^6 \text{ J/kg}) + (2\,010 \text{ J/kg}\cdot°\text{C})(10.0°\text{C})\Big]$$
$$Q_{\text{needed}} = \boxed{1.22\times10^5 \text{ J}}$$

P20.16 $Q_{\text{cold}} = -Q_{\text{hot}}$

$$(m_w c_w + m_c c_c)(T_f - T_i) = -m_s\left[-L_v + c_w(T_f - 100)\right]$$
$$\left[0.250 \text{ kg}(4\,186 \text{ J/kg}\cdot°\text{C}) + 0.0500 \text{ kg}(387 \text{ J/kg}\cdot°\text{C})\right](50.0°\text{C} - 20.0°\text{C})$$
$$= -m_s\left[-2.26\times10^6 \text{ J/kg} + (4\,186 \text{ J/kg}\cdot°\text{C})(50.0°\text{C} - 100°\text{C})\right]$$
$$m_s = \frac{3.20\times10^4 \text{ J}}{2.47\times10^6 \text{ J/kg}} = 0.0129 \text{ kg} = \boxed{12.9 \text{ g steam}}$$

P20.17 The bullet will not melt all the ice, so its final temperature is 0°C.

Then $\left(\frac{1}{2}mv^2 + mc|\Delta T|\right)_{\text{bullet}} = m_w L_f$

where m_w is the melt water mass

$$m_w = \frac{0.500\left(3.00\times10^{-3}\text{ kg}\right)(240\text{ m/s})^2 + 3.00\times10^{-3}\text{ kg}(128\text{ J/kg}\cdot°\text{C})(30.0°\text{C})}{3.33\times10^5\text{ J/kg}}$$

$$m_w = \frac{86.4\text{ J} + 11.5\text{ J}}{333\,000\text{ J/kg}} = \boxed{0.294\text{ g}}$$

P20.18 (a) $Q_1 =$ heat to melt all the ice $= \left(50.0\times10^{-3}\text{ kg}\right)\left(3.33\times10^5\text{ J/kg}\right) = 1.67\times10^4\text{ J}$

$Q_2 =$ (heat to raise temp of ice to 100°C)

$= \left(50.0\times10^{-3}\text{ kg}\right)(4\,186\text{ J/kg}\cdot°\text{C})(100°\text{C}) = 2.09\times10^4\text{ J}$

Thus, the total heat to melt ice and raise temp to 100°C $= 3.76\times10^4$ J

$Q_3 = \frac{\text{heat available}}{\text{as steam condenses}} = \left(10.0\times10^{-3}\text{ kg}\right)\left(2.26\times10^6\text{ J/kg}\right) = 2.26\times10^4\text{ J}$

Thus, we see that $Q_3 > Q_1$, but $Q_3 < Q_1 + Q_2$.

Therefore, all the ice melts but $T_f < 100°\text{C}$. Let us now find T_f

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

$$\left(50.0\times10^{-3}\text{ kg}\right)\left(3.33\times10^5\text{ J/kg}\right) + \left(50.0\times10^{-3}\text{ kg}\right)(4\,186\text{ J/kg}\cdot°\text{C})\left(T_f - 0°\text{C}\right)$$

$$= -\left(10.0\times10^{-3}\text{ kg}\right)\left(-2.26\times10^6\text{ J/kg}\right) - \left(10.0\times10^{-3}\text{ kg}\right)(4\,186\text{ J/kg}\cdot°\text{C})\left(T_f - 100°\text{C}\right)$$

From which, $\boxed{T_f = 40.4°\text{C}}$.

(b) $Q_1 =$ heat to melt all ice $= 1.67\times10^4$ J [See part (a)]

$Q_2 = \frac{\text{heat given up}}{\text{as steam condenses}} = \left(10^{-3}\text{ kg}\right)\left(2.26\times10^6\text{ J/kg}\right) = 2.26\times10^3\text{ J}$

$Q_3 = \frac{\text{heat given up as condensed}}{\text{steam cools to }0°\text{C}} = \left(10^{-3}\text{ kg}\right)(4\,186\text{ J/kg}\cdot°\text{C})(100°\text{C}) = 419\text{ J}$

Note that $Q_2 + Q_3 < Q_1$. Therefore, the final temperature will be 0°C with some ice remaining. Let us find the mass of ice which must melt to condense the steam and cool the condensate to 0°C.

$$mL_f = Q_2 + Q_3 = 2.68\times10^3\text{ J}$$

Thus, $m = \frac{2.68\times10^3\text{ J}}{3.33\times10^5\text{ J/kg}} = 8.04\times10^{-3}\text{ kg} = 8.04\text{ g}$.

Therefore, there is $\boxed{\text{42.0 g of ice left over}}$.

P20.19 $Q = m_{Cu}c_{Cu}\Delta T = m_{N_2}\left(L_{vap}\right)_{N_2}$

$1.00\text{ kg}(0.092\,0\text{ cal/g}\cdot°\text{C})(293-77.3)°\text{C} = m(48.0\text{ cal/g})$

$m = \boxed{0.414\text{ kg}}$

***P20.20** The original gravitational energy of the hailstone-Earth system changes entirely into additional internal energy in the hailstone, to produce its phase change. No temperature change occurs, either in the hailstone, in the air, or in sidewalk. Then

$$mgy = mL$$
$$y = \frac{L}{g} = \frac{3.33\times10^5\text{ J/kg}}{9.8\text{ m/s}^2}\left(\frac{1\text{ kg}\cdot\text{m}^2/\text{s}^2}{1\text{ J}}\right) = \boxed{3.40\times10^4\text{ m}}$$

P20.21 (a) Since the heat required to melt 250 g of ice at 0°C *exceeds* the heat required to cool 600 g of water from 18°C to 0°C, the final temperature of the system (water + ice) must be $\boxed{0°\text{C}}$.

(b) Let m represent the mass of ice that melts before the system reaches equilibrium at 0°C.

$$Q_{cold} = -Q_{hot}$$
$$mL_f = -m_w c_w(0°\text{C}-T_i)$$
$$m\left(3.33\times10^5\text{ J/kg}\right) = -(0.600\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C})(0°\text{C}-18.0°\text{C})$$
$$m = 136\text{ g, so the ice remaining} = 250\text{ g} - 136\text{ g} = \boxed{114\text{ g}}$$

P20.22 The original kinetic energy all becomes thermal energy:

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = 2\left(\frac{1}{2}\right)\left(5.00\times10^{-3}\text{ kg}\right)(500\text{ m/s})^2 = 1.25\text{ kJ}.$$

Raising the temperature to the melting point requires

$$Q = mc\Delta T = 10.0\times10^{-3}\text{ kg}(128\text{ J/kg}\cdot°\text{C})(327°\text{C}-20.0°\text{C}) = 393\text{ J}.$$

Since $1\,250\text{ J} > 393\text{ J}$, the lead starts to melt. Melting it all requires

$$Q = mL = \left(10.0\times10^{-3}\text{ kg}\right)\left(2.45\times10^4\text{ J/kg}\right) = 245\text{ J}.$$

Since $1\,250\text{ J} > 393 + 245\text{ J}$, it all melts. If we assume $\boxed{\text{liquid lead}}$ has the same specific heat as solid lead, the final temperature is given by

$$1.25\times10^3\text{ J} = 393\text{ J} + 245\text{ J} + 10.0\times10^{-3}\text{ kg}(128\text{ J/kg}\cdot°\text{C})\left(T_f - 327°\text{C}\right)$$
$$\boxed{T_f = 805°\text{C}}$$

Section 20.4 Work and Heat in Thermodynamic Processes

P20.23 $W_{if} = -\int_i^f P\,dV$

The work done on the gas is the negative of the area under the curve $P = \alpha V^2$ between V_i and V_f.

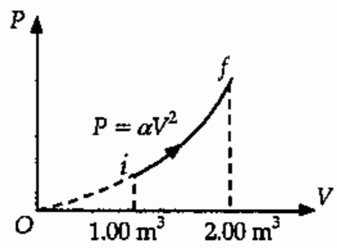


FIG. P20.23

$$W_{if} = -\int_i^f \alpha V^2 dV = -\frac{1}{3}\alpha\left(V_f^3 - V_i^3\right)$$

$$V_f = 2V_i = 2\left(1.00\text{ m}^3\right) = 2.00\text{ m}^3$$

$$W_{if} = -\frac{1}{3}\left[\left(5.00\text{ atm/m}^6\right)\left(1.013\times10^5\text{ Pa/atm}\right)\right]\left[\left(2.00\text{ m}^3\right)^3 + \left(1.00\text{ m}^3\right)^3\right] = \boxed{-1.18\text{ MJ}}$$

P20.24 (a) $W = -\int P\,dV$

$$W = -\left(6.00\times10^6\text{ Pa}\right)(2.00-1.00)\text{ m}^3 + -\left(4.00\times10^6\text{ Pa}\right)(3.00-2.00)\text{ m}^3 + -\left(2.00\times10^6\text{ Pa}\right)(4.00-3.00)\text{ m}^3$$

$$W_{i\to f} = \boxed{-12.0\text{ MJ}}$$

(b) $W_{f\to i} = \boxed{+12.0\text{ MJ}}$

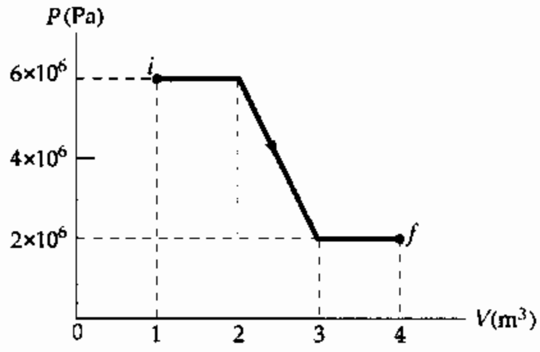


FIG. P20.24

P20.25 $W = -P\Delta V = -P\left(\frac{nR}{P}\right)\left(T_f - T_i\right) = -nR\Delta T = -(0.200)(8.314)(280) = \boxed{-466\text{ J}}$

P20.26 $W = -\int_i^f P\,dV = -P\int_i^f dV = -P\Delta V = -nR\Delta T = \boxed{-nR\left(T_2 - T_1\right)}$

P20.27 During the heating process $P = \left(\frac{P_i}{V_i}\right)V$.

(a) $W = -\int_i^f P\,dV = -\int_{V_i}^{3V_i}\left(\frac{P_i}{V_i}\right)V\,dV$

$$W = -\left(\frac{P_i}{V_i}\right)\frac{V^2}{2}\Bigg|_{V_i}^{3V_i} = -\frac{P_i}{2V_i}\left(9V_i^2 - V_i^2\right) = \boxed{-4P_iV_i}$$

(b) $PV = nRT$

$$\left[\left(\frac{P_i}{V_i}\right)V\right]V = nRT$$

$$\boxed{T = \left(\frac{P_i}{nRV_i}\right)V^2}$$

Temperature must be proportional to the square of volume, rising to nine times its original value.

Section 20.5 The First Law of Thermodynamics

P20.28 (a) $W = -P\Delta V = -(0.800 \text{ atm})(-7.00 \text{ L})(1.013\times10^5 \text{ Pa/atm})(10^{-3} \text{ m}^3/\text{L}) = \boxed{+567 \text{ J}}$

(b) $\Delta E_{\text{int}} = Q + W = -400 \text{ J} + 567 \text{ J} = \boxed{167 \text{ J}}$

P20.29 $\Delta E_{\text{int}} = Q + W$

$Q = \Delta E_{\text{int}} - W = -500 \text{ J} - 220 \text{ J} = \boxed{-720 \text{ J}}$

The negative sign indicates that positive energy is transferred *from* the system by heat.

P20.30 (a) $Q = -W = \text{Area of triangle}$

$Q = \frac{1}{2}(4.00 \text{ m}^3)(6.00 \text{ kPa}) = \boxed{12.0 \text{ kJ}}$

(b) $Q = -W = \boxed{-12.0 \text{ kJ}}$

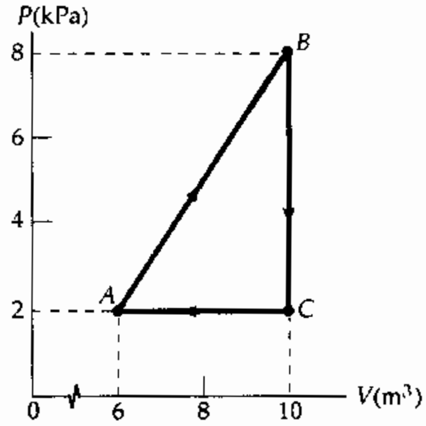


FIG. P20.30

P20.31

	Q	W	ΔE_{int}	
BC	–	0	–	($Q = \Delta E_{\text{int}}$ since $W_{BC} = 0$)
CA	–	+	–	($\Delta E_{\text{int}} < 0$ and $W > 0$, so $Q < 0$)
AB	+	–	+	($W < 0$, $\Delta E_{\text{int}} > 0$ since $\Delta E_{\text{int}} < 0$ for $B \to C \to A$; so $Q > 0$)

P20.32 $W_{BC} = -P_B(V_C - V_B) = -3.00 \text{ atm}(0.400 - 0.0900) \text{ m}^3$

$= -94.2 \text{ kJ}$

$\Delta E_{\text{int}} = Q + W$

$E_{\text{int, C}} - E_{\text{int, B}} = (100 - 94.2) \text{ kJ}$

$E_{\text{int, C}} - E_{\text{int, B}} = 5.79 \text{ kJ}$

Since T is constant,

$E_{\text{int, D}} - E_{\text{int, C}} = 0$

$W_{DA} = -P_D(V_A - V_D) = -1.00 \text{ atm}(0.200 - 1.20) \text{ m}^3$

$= +101 \text{ kJ}$

$E_{\text{int, A}} - E_{\text{int, D}} = -150 \text{ kJ} + (+101 \text{ kJ}) = -48.7 \text{ kJ}$

Now, $E_{\text{int, B}} - E_{\text{int, A}} = -\left[(E_{\text{int, C}} - E_{\text{int, B}}) + (E_{\text{int, D}} - E_{\text{int, C}}) + (E_{\text{int, A}} - E_{\text{int, D}})\right]$

$E_{\text{int, B}} - E_{\text{int, A}} = -[5.79 \text{ kJ} + 0 - 48.7 \text{ kJ}] = \boxed{42.9 \text{ kJ}}$

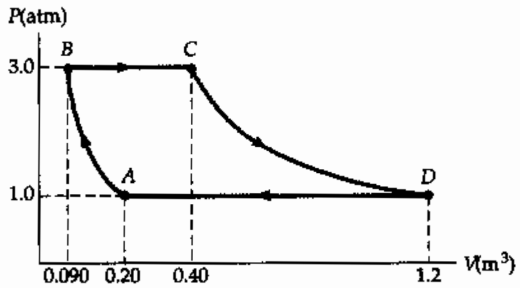


FIG. P20.32

***P20.33** The area of a true semicircle is $\frac{1}{2}\pi r^2$. The arrow in Figure P20.33 looks like a semicircle when the scale makes 1.2 L fill the same space as 100 kPa. Its area is

$$\frac{1}{2}\pi(2.4\text{ L})(200\text{ kPa})=\frac{1}{2}\pi\left(2.4\times10^{-3}\text{ m}^3\right)\left(2\times10^5\text{ N/m}^2\right).$$

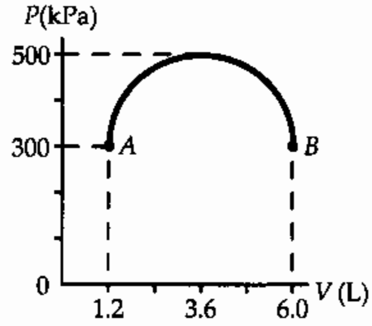


FIG. P20.33

The work on the gas is

$$W=-\int_A^B PdV=-\text{area under the arch shown in the graph}$$
$$=-\left(\frac{1}{2}\pi 2.4(200)\text{ J}+3\times10^5\text{ N/m}^2\ 4.8\times10^{-3}\text{ m}^3\right)$$
$$=-(754\text{ J}+1\,440\text{ J})=-2\,190\text{ J}$$
$$\Delta E_{\text{int}}=Q+W=5\,790\text{ J}-2\,190\text{ J}=\boxed{3.60\text{ kJ}}$$

Section 20.6 Some Applications of the First Law of Thermodynamics

P20.34 (a) $W=-nRT\ln\left(\frac{V_f}{V_i}\right)=-P_fV_f\ln\left(\frac{V_f}{V_i}\right)$

so $V_i=V_f\exp\left(+\frac{W}{P_fV_f}\right)=(0.025\,0)\exp\left[\frac{-3\,000}{0.025\,0\left(1.013\times10^5\right)}\right]=\boxed{0.007\,65\text{ m}^3}$

(b) $T_f=\frac{P_fV_f}{nR}=\frac{1.013\times10^5\text{ Pa}\left(0.025\,0\text{ m}^3\right)}{1.00\text{ mol}(8.314\text{ J/K}\cdot\text{mol})}=\boxed{305\text{ K}}$

P20.35 (a) $\Delta E_{\text{int}}=Q-P\Delta V=12.5\text{ kJ}-2.50\text{ kPa}(3.00-1.00)\text{ m}^3=\boxed{7.50\text{ kJ}}$

(b) $\frac{V_1}{T_1}=\frac{V_2}{T_2}$

$T_2=\frac{V_2}{V_1}T_1=\frac{3.00}{1.00}(300\text{ K})=\boxed{900\text{ K}}$

P20.36 (a) $W=-P\Delta V=-P[3\alpha V\Delta T]$

$$=-\left(1.013\times10^5\text{ N/m}^2\right)\left[3\left(24.0\times10^{-6}\,{}^\circ\text{C}^{-1}\right)\left(\frac{1.00\text{ kg}}{2.70\times10^3\text{ kg/m}^3}\right)(18.0^\circ\text{C})\right]$$

$W=\boxed{-48.6\text{ mJ}}$

(b) $Q=cm\Delta T=(900\text{ J/kg}\cdot{}^\circ\text{C})(1.00\text{ kg})(18.0^\circ\text{C})=\boxed{16.2\text{ kJ}}$

(c) $\Delta E_{\text{int}}=Q+W=16.2\text{ kJ}-48.6\text{ mJ}=\boxed{16.2\text{ kJ}}$

P20.37 $W = -P\Delta V = -P(V_s - V_w) = -\frac{P(nRT)}{P} + P\left[\frac{18.0\text{ g}}{(1.00\text{ g/cm}^3)(10^6\text{ cm}^3/\text{m}^3)}\right]$

$W = -(1.00\text{ mol})(8.314\text{ J/K}\cdot\text{mol})(373\text{ K}) + (1.013\times10^5\text{ N/m}^2)\left(\frac{18.0\text{ g}}{10^6\text{ g/m}^3}\right) = \boxed{-3.10\text{ kJ}}$

$Q = mL_v = 0.018\,0\text{ kg}(2.26\times10^6\text{ J/kg}) = 40.7\text{ kJ}$

$\Delta E_{\text{int}} = Q + W = \boxed{37.6\text{ kJ}}$

P20.38 (a) The work done during each step of the cycle equals the negative of the area under that segment of the PV curve.

$W = W_{DA} + W_{AB} + W_{BC} + W_{CD}$

$W = -P_i(V_i - 3V_i) + 0 - 3P_i(3V_i - V_i) + 0 = \boxed{-4P_iV_i}$

(b) The initial and final values of T for the system are equal. Therefore, $\Delta E_{\text{int}} = 0$ and $Q = -W = \boxed{4P_iV_i}$.

(c) $W = -4P_iV_i = -4nRT_i = -4(1.00)(8.314)(273) = \boxed{-9.08\text{ kJ}}$

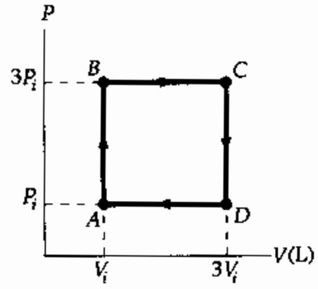


FIG. P20.38

P20.39 (a) $P_iV_i = P_fV_f = nRT = 2.00\text{ mol}(8.314\text{ J/K}\cdot\text{mol})(300\text{ K}) = 4.99\times10^3\text{ J}$

$V_i = \frac{nRT}{P_i} = \frac{4.99\times10^3\text{ J}}{0.400\text{ atm}}$

$V_f = \frac{nRT}{P_f} = \frac{4.99\times10^3\text{ J}}{1.20\text{ atm}} = \frac{1}{3}V_i = \boxed{0.041\,0\text{ m}^3}$

(b) $W = -\int P\,dV = -nRT\ln\left(\frac{V_f}{V_i}\right) = -(4.99\times10^3)\ln\left(\frac{1}{3}\right) = \boxed{+5.48\text{ kJ}}$

(c) $\Delta E_{\text{int}} = 0 = Q + W$

$Q = \boxed{-5.48\text{ kJ}}$

P20.40 $\Delta E_{\text{int},ABC} = \Delta E_{\text{int},AC}$ (conservation of energy)

(a) $\Delta E_{\text{int},ABC} = Q_{ABC} + W_{ABC}$ (First Law)

$Q_{ABC} = 800\text{ J} + 500\text{ J} = \boxed{1\,300\text{ J}}$

(b) $W_{CD} = -P_C\Delta V_{CD}$, $\Delta V_{AB} = -\Delta V_{CD}$, and $P_A = 5P_C$

Then, $W_{CD} = \frac{1}{5}P_A\Delta V_{AB} = -\frac{1}{5}W_{AB} = \boxed{100\text{ J}}$

(+ means that work is done on the system)

(c) $W_{CDA} = W_{CD}$ so that $Q_{CA} = \Delta E_{\text{int},CA} - W_{CDA} = -800\text{ J} - 100\text{ J} = \boxed{-900\text{ J}}$

(– means that energy must be removed from the system by heat)

(d) $\Delta E_{\text{int},CD} = \Delta E_{\text{int},CDA} - \Delta E_{\text{int},DA} = -800\text{ J} - 500\text{ J} = -1\,300\text{ J}$

and $Q_{CD} = \Delta E_{\text{int},CD} - W_{CD} = -1\,300\text{ J} - 100\text{ J} = \boxed{-1\,400\text{ J}}$

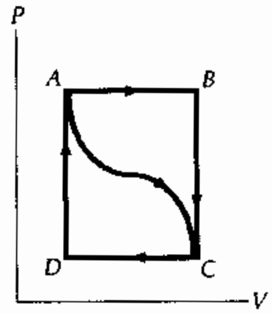


FIG. P20.40

Section 20.7 **Energy Transfer Mechanisms**

P20.41 $\mathcal{P} = kA\dfrac{\Delta T}{L}$

$$k = \frac{\mathcal{P}L}{A\Delta T} = \frac{10.0\text{ W}(0.040\,0\text{ m})}{1.20\text{ m}^2(15.0°\text{C})} = \boxed{2.22\times10^{-2}\text{ W/m}\cdot°\text{C}}$$

P20.42 $$\mathcal{P} = \frac{kA\Delta T}{L} = \frac{(0.800\text{ W/m}\cdot°\text{C})(3.00\text{ m}^2)(25.0°\text{C})}{6.00\times10^{-3}\text{ m}} = 1.00\times10^4\text{ W} = \boxed{10.0\text{ kW}}$$

P20.43 In the steady state condition, $\mathcal{P}_{\text{Au}} = \mathcal{P}_{\text{Ag}}$

so that $k_{\text{Au}}A_{\text{Au}}\left(\dfrac{\Delta T}{\Delta x}\right)_{\text{Au}} = k_{\text{Ag}}A_{\text{Ag}}\left(\dfrac{\Delta T}{\Delta x}\right)_{\text{Ag}}$

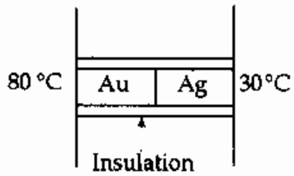


FIG. P20.43

In this case $A_{\text{Au}} = A_{\text{Ag}}$

$\Delta x_{\text{Au}} = \Delta x_{\text{Ag}}$

$\Delta T_{\text{Au}} = (80.0 - T)$

and $\Delta T_{\text{Ag}} = (T - 30.0)$

where T is the temperature of the junction.

Therefore, $k_{\text{Au}}(80.0 - T) = k_{\text{Ag}}(T - 30.0)$

And $\boxed{T = 51.2°\text{C}}$

P20.44 $$\mathcal{P} = \frac{A\Delta T}{\sum_i \frac{L_i}{k_i}} = \frac{(6.00\text{ m}^2)(50.0°\text{C})}{\left[2(4.00\times10^{-3}\text{ m})\right]/\left[0.800\text{ W/m}\cdot°\text{C}\right] + \left[5.00\times10^{-3}\text{ m}\right]/\left[0.023\,4\text{ W/m}\cdot°\text{C}\right]} = \boxed{1.34\text{ kW}}$$

***P20.45** We suppose that the area of the transistor is so small that energy flow by heat from the transistor directly to the air is negligible compared to energy conduction through the mica.

$$\mathcal{P} = kA\frac{(T_h - T_c)}{L}$$

$$T_h = T_c + \frac{\mathcal{P}L}{kA} = 35.0°\text{C} + \frac{1.50\text{ W}(0.085\,2\times10^{-3}\text{ m})}{(0.075\,3\text{ W/m}\cdot°\text{C})(8.25\times6.25)10^{-6}\text{ m}^2} = \boxed{67.9°\text{C}}$$

P20.46 From Table 20.4,

(a) $R = \boxed{0.890\text{ ft}^2\cdot°\text{F}\cdot\text{h/Btu}}$

(b) The insulating glass in the table must have sheets of glass less than $\dfrac{1}{8}$ inch thick. So we estimate the R-value of a 0.250-inch air space as $\dfrac{0.250}{3.50}$ times that of the thicker air space. Then for the double glazing

$$R_b = \left[0.890 + \left(\frac{0.250}{3.50}\right)1.01 + 0.890\right]\frac{\text{ft}^2\cdot°\text{F}\cdot\text{h}}{\text{Btu}} = \boxed{1.85\frac{\text{ft}^2\cdot°\text{F}\cdot\text{h}}{\text{Btu}}}.$$

(c) Since A and $(T_2 - T_1)$ are constants, heat flow is reduced by a factor of $\dfrac{1.85}{0.890} = \boxed{2.08}$.

P20.47 $\mathcal{P} = \sigma AeT^4 = \left(5.669\,6\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left[4\pi\left(6.96\times10^{8}\ \text{m}\right)^2\right](0.965)(5\,800\ \text{K})^4$

$\mathcal{P} = \boxed{3.77\times10^{26}\ \text{W}}$

P20.48 Suppose the pizza is 70 cm in diameter and $\ell = 2.0$ cm thick, sizzling at 100°C. It cannot lose heat by conduction or convection. It radiates according to $\mathcal{P} = \sigma AeT^4$. Here, A is its surface area,

$$A = 2\pi r^2 + 2\pi r\ell = 2\pi(0.35\ \text{m})^2 + 2\pi(0.35\ \text{m})(0.02\ \text{m}) = 0.81\ \text{m}^2.$$

Suppose it is dark in the infrared, with emissivity about 0.8. Then

$$\mathcal{P} = \left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(0.81\ \text{m}^2\right)(0.80)(373\ \text{K})^4 = 710\ \text{W}\boxed{\sim10^3\ \text{W}}.$$

If the density of the pizza is half that of water, its mass is

$$m = \rho V = \rho\pi r^2\ell = \left(500\ \text{kg/m}^3\right)\pi(0.35\ \text{m})^2(0.02\ \text{m}) = 4\ \text{kg}.$$

Suppose its specific heat is $c = 0.6\ \text{cal/g}\cdot{}^\circ\text{C}$. The drop in temperature of the pizza is described by:

$$Q = mc\left(T_f - T_i\right)$$

$$\mathcal{P} = \frac{dQ}{dt} = mc\frac{dT_f}{dt} - 0$$

$$\frac{dT_f}{dt} = \frac{\mathcal{P}}{mc} = \frac{710\ \text{J/s}}{(4\ \text{kg})(0.6\cdot4\,186\ \text{J/kg}\cdot{}^\circ\text{C})} = 0.07\ {}^\circ\text{C/s}\boxed{\sim10^{-1}\ \text{K/s}}$$

P20.49 $\mathcal{P} = \sigma AeT^4$

$2.00\ \text{W} = \left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(0.250\times10^{-6}\ \text{m}^2\right)(0.950)T^4$

$T = \left(1.49\times10^{14}\ \text{K}^4\right)^{1/4} = \boxed{3.49\times10^3\ \text{K}}$

P20.50 We suppose the earth below is an insulator. The square meter must radiate in the infrared as much energy as it absorbs, $\mathcal{P} = \sigma AeT^4$. Assuming that $e = 1.00$ for blackbody blacktop:

$1\,000\ \text{W} = \left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(1.00\ \text{m}^2\right)(1.00)T^4$

$T = \left(1.76\times10^{10}\ \text{K}^4\right)^{1/4} = \boxed{364\ \text{K}}$ (You can cook an egg on it.)

P20.51 The sphere of radius R absorbs sunlight over the area of its day hemisphere, projected as a flat circle perpendicular to the light: πR^2. It radiates in all directions, over area $4\pi R^2$. Then, in steady state,

$$\mathcal{P}_{\text{in}} = \mathcal{P}_{\text{out}}$$

$$e\left(1\,340\ \text{W/m}^2\right)\pi R^2 = e\sigma\left(4\pi R^2\right)T^4$$

The emissivity e, the radius R, and π all cancel.

Therefore, $T = \left[\dfrac{1\,340\ \text{W/m}^2}{4\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4} = \boxed{277\ \text{K}} = 4°\text{C}.$

Additional Problems

P20.52 $77.3\text{ K} = -195.8°\text{C}$ is the boiling point of nitrogen. It gains no heat to warm as a liquid, but gains heat to vaporize:

$$Q = mL_v = (0.100\text{ kg})(2.01\times10^5\text{ J/kg}) = 2.01\times10^4\text{ J}.$$

The water first loses heat by cooling. Before it starts to freeze, it can lose

$$Q = mc\Delta T = (0.200\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C})(5.00°\text{C}) = 4.19\times10^3\text{ J}.$$

The remaining $(2.01\times10^4 - 4.19\times10^3)\text{ J} = 1.59\times10^4\text{ J}$ that is removed from the water can freeze a mass x of water:

$$Q = mL_f$$

$$1.59\times10^4\text{ J} = x(3.33\times10^5\text{ J/kg})$$

$$x = 0.047\,7\text{ kg} = \boxed{47.7\text{ g}}\text{ of water can be frozen}$$

P20.53 The increase in internal energy required to melt 1.00 kg of snow is

$$\Delta E_{\text{int}} = (1.00\text{ kg})(3.33\times10^5\text{ J/kg}) = 3.33\times10^5\text{ J}$$

The force of friction is $f = \mu n = \mu mg = 0.200(75.0\text{ kg})(9.80\text{ m/s}^2) = 147\text{ N}$

According to the problem statement, the loss of mechanical energy of the skier is assumed to be equal to the increase in internal energy of the snow. This increase in internal energy is

$$\Delta E_{\text{int}} = f\Delta r = (147\text{ N})\Delta r = 3.33\times10^5\text{ J}$$

and $\Delta r = \boxed{2.27\times10^3\text{ m}}$.

P20.54 (a) The energy thus far gained by the copper equals the energy loss by the silver. Your down parka is an excellent insulator.

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

or $m_{\text{Cu}}c_{\text{Cu}}(T_f - T_i)_{\text{Cu}} = -m_{\text{Ag}}c_{\text{Ag}}(T_f - T_i)_{\text{Ag}}$

$$(9.00\text{ g})(387\text{ J/kg}\cdot°\text{C})(16.0°\text{C}) = -(14.0\text{ g})(234\text{ J/kg}\cdot°\text{C})(T_f - 30.0°\text{C})_{\text{Ag}}$$

$$(T_f - 30.0°\text{C})_{\text{Ag}} = -17.0°\text{C}$$

so $T_{f,\text{Ag}} = \boxed{13.0°\text{C}}$.

(b) Differentiating the energy gain-and-loss equation gives: $m_{\text{Ag}}c_{\text{Ag}}\left(\frac{dT}{dt}\right)_{\text{Ag}} = -m_{\text{Cu}}c_{\text{Cu}}\left(\frac{dT}{dt}\right)_{\text{Cu}}$

$$\left(\frac{dT}{dt}\right)_{\text{Ag}} = -\frac{m_{\text{Cu}}c_{\text{Cu}}}{m_{\text{Ag}}c_{\text{Ag}}}\left(\frac{dT}{dt}\right)_{\text{Cu}} = -\frac{9.00\text{ g}(387\text{ J/kg}\cdot°\text{C})}{14.0\text{ g}(234\text{ J/kg}\cdot°\text{C})}(+0.500°\text{C/s})$$

$$\left(\frac{dT}{dt}\right)_{\text{Ag}} = \boxed{-0.532°\text{C/s}}\text{ (negative sign }\Rightarrow\text{ decreasing temperature)}$$

P20.55 (a) Before conduction has time to become important, the energy lost by the rod equals the energy gained by the helium. Therefore,

$$(mL_v)_{He} = (mc|\Delta T|)_{Al}$$

or $$(\rho V L_v)_{He} = (\rho V c|\Delta T|)_{Al}$$

so $$V_{He} = \frac{(\rho V c|\Delta T|)_{Al}}{(\rho L_v)_{He}}$$

$$V_{He} = \frac{(2.70\ \text{g/cm}^3)(62.5\ \text{cm}^3)(0.210\ \text{cal/g}\cdot{}^\circ\text{C})(295.8^\circ\text{C})}{(0.125\ \text{g/cm}^3)(2.09\times10^4\ \text{J/kg})(1.00\ \text{cal/4.186 J})(1.00\ \text{kg/1 000 g})}$$

$$V_{He} = 1.68\times10^4\ \text{cm}^3 = \boxed{16.8\ \text{liters}}$$

(b) The rate at which energy is supplied to the rod in order to maintain constant temperatures is given by

$$\mathcal{P} = kA\left(\frac{dT}{dx}\right) = (31.0\ \text{J/s}\cdot\text{cm}\cdot\text{K})(2.50\ \text{cm}^2)\left(\frac{295.8\ \text{K}}{25.0\ \text{cm}}\right) = 917\ \text{W}$$

This power supplied to the helium will produce a "boil-off" rate of

$$\frac{\mathcal{P}}{\rho L_v} = \frac{(917\ \text{W})(10^3\ \text{g/kg})}{(0.125\ \text{g/cm}^3)(2.09\times10^4\ \text{J/kg})} = 351\ \text{cm}^3/\text{s} = \boxed{0.351\ \text{L/s}}$$

***P20.56** At the equilibrium temperature T_{eq} the diameters of the sphere and ring are equal:

$$d_s + d_s\alpha_{Al}(T_{eq} - T_i) = d_r + d_r\alpha_{Cu}(T_{eq} - 15^\circ\text{C})$$

$$5.01\ \text{cm} + 5.01\ \text{cm}(2.40\times10^{-5}\ 1/^\circ\text{C})(T_{eq} - T_i) = 5.00\ \text{cm} + 5.00\ \text{cm}(1.70\times10^{-5}\ 1/^\circ\text{C})(T_{eq} - 15^\circ\text{C})$$

$$0.01^\circ\text{C} + 1.202\,4\times10^{-4}T_{eq} - 1.202\,4\times10^{-4}T_i = 8.5\times10^{-5}T_{eq} - 1.275\times10^{-3\,\circ}\text{C}$$

$$1.127\,5\times10^{-2\,\circ}\text{C} + 3.524\times10^{-5}T_{eq} = 1.202\,4\times10^{-4}T_i$$

$$319.95^\circ\text{C} + T_{eq} = 3.412\,0T_i$$

At the equilibrium temperature, the energy lost is equal to the energy gained:

$$m_s c_{Al}(T_{eq} - T_i) = -m_r c_{Cu}(T_{eq} - 15^\circ\text{C})$$

$$10.9\ \text{g}\ 0.215\ \text{cal/g}\cdot{}^\circ\text{C}(T_{eq} - T_i) = -25\ \text{g}\ 0.092\,4\ \text{cal/g}\cdot{}^\circ\text{C}(T_{eq} - 15^\circ\text{C})$$

$$2.343\,5T_{eq} - 2.343\,5T_i = 34.65^\circ\text{C} - 2.31T_{eq}$$

$$4.653\,5T_{eq} = 34.65^\circ\text{C} + 2.343\,5T_i$$

Solving by substitution,

$$4.653\,5(3.412\,0T_i - 319.95^\circ\text{C}) = 34.65^\circ\text{C} + 2.343\,5T_i$$

$$15.877\,7T_i - 1\,488.89^\circ\text{C} = 34.65^\circ\text{C} + 2.343\,5T_i$$

(b) $$T_i = \frac{1\,523.54^\circ\text{C}}{13.534} = \boxed{113^\circ\text{C}}$$

(a) $$T_{eq} = -319.95 + 3.412\,0(112.57) = \boxed{64.1^\circ\text{C}}$$

P20.57 $Q = mc\Delta T = (\rho V)c\Delta T$ so that when a constant temperature difference ΔT is maintained,

the rate of adding energy to the liquid is $\mathcal{P} = \frac{dQ}{dt} = \rho\left(\frac{dV}{dt}\right)c\Delta T = \rho Rc\Delta T$

and the specific heat of the liquid is $c = \boxed{\frac{\mathcal{P}}{\rho R \Delta T}}$.

P20.58 (a) Work done by the gas is the negative of the area under the PV curve

$$W = -P_i\left(\frac{V_i}{2} - V_i\right) = \boxed{+\frac{P_i V_i}{2}}.$$

(b) In this case the area under the curve is $W = -\int P dV$. Since the process is isothermal,

$$PV = P_i V_i = 4P_i\left(\frac{V_i}{4}\right) = nRT_i$$

and $$W = -\int_{V_i}^{V_i/4}\left(\frac{dV}{V}\right)(P_i V_i) = -P_i V_i \ln\left(\frac{V_i/4}{V_i}\right) = P_i V_i \ln 4$$

$$= \boxed{+1.39 P_i V_i}$$

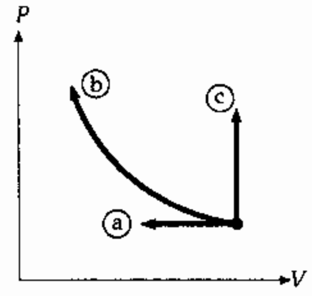


FIG. P20.58

(c) The area under the curve is 0 and $\boxed{W = 0}$.

P20.59 Call the initial pressure P_1. In the constant volume process $1 \to 2$ the work is zero.

$$P_1 V_1 = nRT_1$$
$$P_2 V_2 = nRT_2$$

so $$\frac{P_2 V_2}{P_1 V_1} = \frac{T_2}{T_1};\ T_2 = 300\ \text{K}\left(\frac{1}{4}\right)(1) = 75.0\ \text{K}$$

Now in $2 \to 3$

$$W = -\int_2^3 P dV = -P_2(V_3 - V_2) = -P_3 V_3 + P_2 V_2$$
$$W = -nRT_3 + nRT_2 = -(1.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K} - 75.0\ \text{K})$$
$$W = \boxed{-1.87\ \text{kJ}}$$

***P20.60** The initial moment of inertia of the disk is

$$\frac{1}{2}MR^2 = \frac{1}{2}\rho VR^2 = \frac{1}{2}\rho\pi R^2 tR^2 = \frac{1}{2}\left(8\,920\ \text{kg/m}^3\right)\pi(28\ \text{m})^4 1.2\ \text{m} = 1.033\times10^{10}\ \text{kg}\cdot\text{m}^2$$

The rotation speeds up as the disk cools off, according to

$$I_i\omega_i = I_f\omega_f$$
$$\frac{1}{2}MR_i^2\omega_i = \frac{1}{2}MR_f^2\omega_f = \frac{1}{2}MR_i^2\left(1-\alpha|\Delta T|\right)^2\omega_f$$
$$\omega_f = \omega_i\frac{1}{\left(1-\alpha|\Delta T|\right)^2} = 25\ \text{rad/s}\frac{1}{\left[1-\left(17\times10^{-6}\ 1/°\text{C}\right)830°\text{C}\right]^2} = 25.720\,7\ \text{rad/s}$$

(a) The kinetic energy increases by

$$\frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}I_i\omega_i\omega_f - \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}I_i\omega_i\left(\omega_f-\omega_i\right)$$
$$= \frac{1}{2}1.033\times10^{10}\ \text{kg}\cdot\text{m}^2(25\ \text{rad/s})0.720\,7\ \text{rad/s} = \boxed{9.31\times10^{10}\ \text{J}}$$

(b) $\Delta E_{\text{int}} = mc\Delta T = 2.64\times10^7\ \text{kg}(387\ \text{J/kg}\cdot°\text{C})(20°\text{C}-850°\text{C}) = \boxed{-8.47\times10^{12}\ \text{J}}$

(c) As 8.47×10^{12} J leaves the fund of internal energy, 9.31×10^{10} J changes into extra kinetic energy, and the rest, $\boxed{8.38\times10^{12}\ \text{J}}$ is radiated.

***P20.61** The loss of mechanical energy is

$$\frac{1}{2}mv_i^2 + \frac{GM_Em}{R_E} = \frac{1}{2}670\ \text{kg}\left(1.4\times10^4\ \text{m/s}\right)^2 + \frac{6.67\times10^{-11}\ \text{Nm}^2\ 5.98\times10^{24}\ \text{kg}\ 670\ \text{kg}}{\text{kg}^2\ 6.37\times10^6\ \text{m}}$$
$$= 6.57\times10^{10}\ \text{J} + 4.20\times10^{10}\ \text{J} = 1.08\times10^{11}\ \text{J}$$

One half becomes extra internal energy in the aluminum: $\Delta E_{\text{int}} = 5.38\times10^{10}$ J. To raise its temperature to the melting point requires energy

$$mc\Delta T = 670\ \text{kg}\ 900\frac{\text{J}}{\text{kg}°\text{C}}\left(660-(-15°\text{C})\right) = 4.07\times10^8\ \text{J}.$$

To melt it, $mL = 670\ \text{kg}\ 3.97\times10^5\ \text{J/kg} = 2.66\times10^8\ \text{J}$. To raise it to the boiling point, $mc\Delta T = 670(1\,170)(2\,450-600)\text{J} = 1.40\times10^9\ \text{J}$. To boil it, $mL = 670\ \text{kg}\ 1.14\times10^7\ \text{J/kg} = 7.64\times10^9\ \text{J}$. Then

$$5.38\times10^{10}\ \text{J} = 9.71\times10^9\ \text{J} + 670(1\,170)\left(T_f - 2\,450°\text{C}\right)\text{J/°C}$$
$$T_f = \boxed{5.87\times10^4\ °\text{C}}$$

P20.62 (a) $Fv = (50.0\text{ N})(40.0\text{ m/s}) = \boxed{2\,000\text{ W}}$

(b) Energy received by each object is $(1\,000\text{ W})(10\text{ s}) = 10^4\text{ J} = 2\,389\text{ cal}$. The specific heat of iron is $0.107\text{ cal/g}\cdot{}^\circ\text{C}$, so the heat capacity of each object is $5.00\times10^3\times0.107 = 535.0\text{ cal/}^\circ\text{C}$.

$$\Delta T = \frac{2\,389\text{ cal}}{535.0\text{ cal/}^\circ\text{C}} = \boxed{4.47^\circ\text{C}}$$

P20.63 The power incident on the solar collector is

$$\mathcal{P}_i = IA = \left(600\text{ W/m}^2\right)\left[\pi(0.300\text{ m})^2\right] = 170\text{ W}.$$

For a 40.0% reflector, the collected power is $\mathcal{P}_c = 67.9\text{ W}$. The total energy required to increase the temperature of the water to the boiling point and to evaporate it is $Q = cm\Delta T + mL_V$:

$$Q = 0.500\text{ kg}\left[(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(80.0^\circ\text{C}) + 2.26\times10^6\text{ J/kg}\right] = 1.30\times10^6\text{ J}.$$

The time interval required is $\Delta t = \frac{Q}{\mathcal{P}_c} = \frac{1.30\times10^6\text{ J}}{67.9\text{ W}} = \boxed{5.31\text{ h}}$.

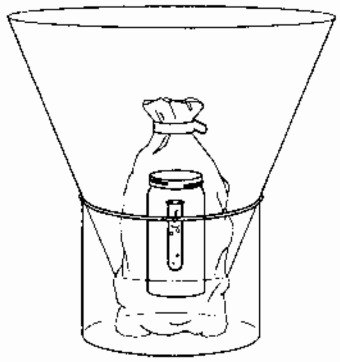


FIG. P20.63

P20.64 From $Q = mL_V$ the rate of boiling is described by

$$\mathcal{P} = \frac{Q}{\Delta t} = \frac{L_V m}{\Delta t} \qquad \therefore \frac{m}{\Delta t} = \frac{\mathcal{P}}{L_V}$$

Model the water vapor as an ideal gas

$$P_0V_0 = nRT = \left(\frac{m}{M}\right)RT$$

$$\frac{P_0V}{\Delta t} = \frac{m}{\Delta t}\left(\frac{RT}{M}\right)$$

$$P_0Av = \frac{\mathcal{P}}{L_V}\left(\frac{RT}{M}\right)$$

$$v = \frac{\mathcal{P}RT}{ML_VP_0A} = \frac{1\,000\text{ W}(8.314\text{ J/mol}\cdot\text{K})(373\text{ K})}{(0.018\,0\text{ kg/mol})\left(2.26\times10^6\text{ J/kg}\right)\left(1.013\times10^5\text{ N/m}^2\right)\left(2.00\times10^{-4}\text{ m}^2\right)}$$

$$v = \boxed{3.76\text{ m/s}}$$

P20.65 Energy goes in at a constant rate $\wp$. For the period from

$$50.0\text{ min to }60.0\text{ min, }Q = mc\Delta T$$

$$\wp(10.0\text{ min}) = (10\text{ kg} + m_i)(4\,186\text{ J/kg}\cdot°\text{C})(2.00°\text{C} - 0°\text{C})$$

$$\wp(10.0\text{ min}) = 83.7\text{ kJ} + (8.37\text{ kJ/kg})m_i \qquad (1)$$

For the period from 0 to 50.0 min, $Q = m_i L_f$

$$\wp(50.0\text{ min}) = m_i\left(3.33\times10^5\text{ J/kg}\right)$$

Substitute $\wp = \dfrac{m_i\left(3.33\times10^5\text{ J/kg}\right)}{50.0\text{ min}}$ into Equation (1) to find

$$\frac{m_i\left(3.33\times10^5\text{ J/kg}\right)}{5.00} = 83.7\text{ kJ} + (8.37\text{ kJ/kg})m_i$$

$$m_i = \frac{83.7\text{ kJ}}{(66.6-8.37)\text{ kJ/kg}} = \boxed{1.44\text{ kg}}$$

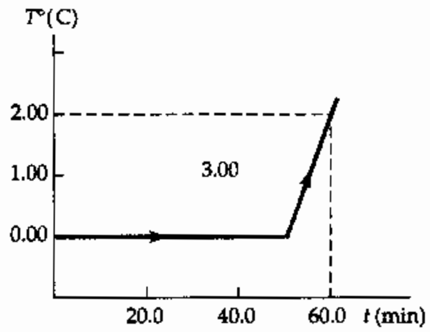


FIG. P20.65

P20.66 (a) The block starts with $K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(1.60\text{ kg})(2.50\text{ m/s})^2 = 5.00\text{ J}$

All this becomes extra internal energy in ice, melting some according to "Q" $= m_{\text{ice}}L_f$. Thus, the mass of ice that melts is

$$m_{\text{ice}} = \frac{"Q"}{L_f} = \frac{K_i}{L_f} = \frac{5.00\text{ J}}{3.33\times10^5\text{ J/kg}} = 1.50\times10^{-5}\text{ kg} = \boxed{15.0\text{ mg}}.$$

For the block: $Q = 0$ (no energy flows by heat since there is no temperature difference)

$W = -5.00\text{ J}$

$\Delta E_{\text{int}} = 0$ (no temperature change)

and $\Delta K = -5.00\text{ J}$

For the ice, $Q = 0$

$W = +5.00\text{ J}$

$\Delta E_{\text{int}} = +5.00\text{ J}$

and $\Delta K = 0$

(b) Again, $K_i = 5.00\text{ J}$ and $m_{\text{ice}} = \boxed{15.0\text{ mg}}$

For the block of ice: $Q = 0$; $\Delta E_{\text{int}} = +5.00\text{ J}$; $\Delta K = -5.00\text{ J}$

so $W = 0$.

For the copper, nothing happens: $Q = \Delta E_{\text{int}} = \Delta K = W = 0$.

continued on next page

(c) Again, $K_i = 5.00$ J. Both blocks must rise equally in temperature.

$$\text{“}Q\text{”} = mc\Delta T: \qquad \Delta T = \frac{\text{“}Q\text{”}}{mc} = \frac{5.00\text{ J}}{2(1.60\text{ kg})(387\text{ J/kg}\cdot{}^\circ\text{C})} = \boxed{4.04\times10^{-3}\,{}^\circ\text{C}}$$

At any instant, the two blocks are at the same temperature, so for both $Q = 0$.

For the moving block: $\Delta K = -5.00$ J

and $\Delta E_{\text{int}} = +2.50$ J

so $W = -2.50$ J

For the stationary block: $\Delta K = 0$

and $\Delta E_{\text{int}} = +2.50$ J

so $W = +2.50$ J

For each object in each situation, the general continuity equation for energy, in the form $\Delta K + \Delta E_{\text{int}} = W + Q$, correctly describes the relationship between energy transfers and changes in the object's energy content.

P20.67 $A = A_{\text{end walls}} + A_{\text{ends of attic}} + A_{\text{side walls}} + A_{\text{roof}}$

$$A = 2(8.00\text{ m}\times5.00\text{ m}) + 2\left[2\times\frac{1}{2}\times4.00\text{ m}\times(4.00\text{ m})\tan37.0^\circ\right]$$

$$+2(10.0\text{ m}\times5.00\text{ m}) + 2(10.0\text{ m})\left(\frac{4.00\text{ m}}{\cos37.0^\circ}\right)$$

$A = 304\text{ m}^2$

$$\wp = \frac{kA\Delta T}{L} = \frac{(4.80\times10^{-4}\text{ kW/m}\cdot{}^\circ\text{C})(304\text{ m}^2)(25.0^\circ\text{C})}{0.210\text{ m}} = 17.4\text{ kW} = 4.15\text{ kcal/s}$$

Thus, the energy lost per day by heat is $(4.15\text{ kcal/s})(86\,400\text{ s}) = 3.59\times10^5\text{ kcal/day}$.

The gas needed to replace this loss is $\dfrac{3.59\times10^5\text{ kcal/day}}{9\,300\text{ kcal/m}^3} = \boxed{38.6\text{ m}^3\text{/day}}$.

P20.68 $\dfrac{L\rho A dx}{dt} = kA\left(\dfrac{\Delta T}{x}\right)$

$$L\rho\int_{4.00}^{8.00} x\,dx = k\Delta T\int_0^{\Delta t} dt$$

$$L\rho\left.\frac{x^2}{2}\right|_{4.00}^{8.00} = k\Delta T\Delta t$$

$$(3.33\times10^5\text{ J/kg})(917\text{ kg/m}^3)\left(\frac{(0.080\,0\text{ m})^2 - (0.040\,0\text{ m})^2}{2}\right) = (2.00\text{ W/m}\cdot{}^\circ\text{C})(10.0^\circ\text{C})\Delta t$$

$\Delta t = 3.66\times10^4\text{ s} = \boxed{10.2\text{ h}}$

P20.69

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$W = -\int_A^B PdV - \int_B^C PdV - \int_C^D PdV - \int_D^A PdV$$

$$W = -nRT_1 \int_A^B \frac{dV}{V} - P_2 \int_B^C dV - nRT_2 \int_C^D \frac{dV}{V} - P_1 \int_D^A dV$$

$$W = -nRT_1 \ln\left(\frac{V_B}{V_1}\right) - P_2(V_C - V_B) - nRT_2 \ln\left(\frac{V_2}{V_C}\right) - P_1(V_A - V_D)$$

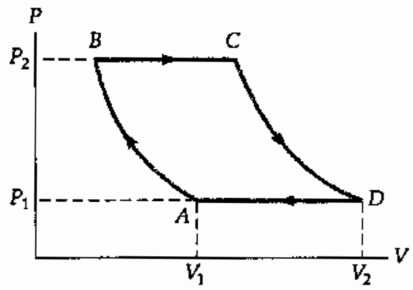


FIG. P20.69

Now $P_1V_A = P_2V_B$ and $P_2V_C = P_1V_D$, so only the logarithmic terms do not cancel out.

Also, $\frac{V_B}{V_1} = \frac{P_1}{P_2}$ and $\frac{V_2}{V_C} = \frac{P_2}{P_1}$

$$\sum W = -nRT_1 \ln\left(\frac{P_1}{P_2}\right) - nRT_2 \ln\left(\frac{P_2}{P_1}\right) = +nRT_1 \ln\left(\frac{P_2}{P_1}\right) - nRT_2 \ln\left(\frac{P_2}{P_1}\right) = -nR(T_2 - T_1)\ln\left(\frac{P_2}{P_1}\right)$$

Moreover $P_1V_2 = nRT_2$ and $P_1V_1 = nRT_1$

$$\sum W = \boxed{-P_1(V_2 - V_1)\ln\left(\frac{P_2}{P_1}\right)}$$

P20.70 For a cylindrical shell of radius *r*, height *L*, and thickness *dr*, the equation for thermal conduction,

$$\frac{dQ}{dt} = -kA\frac{dT}{dx} \quad \text{becomes} \quad \frac{dQ}{dt} = -k(2\pi rL)\frac{dT}{dr}$$

Under equilibrium conditions, $\frac{dQ}{dt}$ is constant; therefore,

$$dT = -\frac{dQ}{dt}\left(\frac{1}{2\pi kL}\right)\left(\frac{dr}{r}\right) \quad \text{and} \quad T_b - T_a = -\frac{dQ}{dt}\left(\frac{1}{2\pi kL}\right)\ln\left(\frac{b}{a}\right)$$

But $T_a > T_b$, so

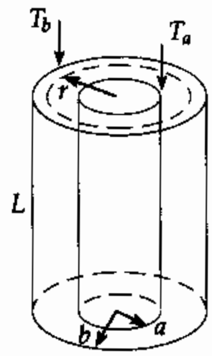
$$\boxed{\frac{dQ}{dt} = \frac{2\pi kL(T_a - T_b)}{\ln(b/a)}}$$

P20.71 From problem 70, the rate of energy flow through the wall is

$$\frac{dQ}{dt} = \frac{2\pi kL(T_a - T_b)}{\ln(b/a)}$$

$$\frac{dQ}{dt} = \frac{2\pi(4.00 \times 10^{-5}\ \text{cal/s}\cdot\text{cm}\cdot{}^\circ\text{C})(3\,500\ \text{cm})(60.0^\circ\text{C})}{\ln(256\ \text{cm}/250\ \text{cm})}$$

$$\frac{dQ}{dt} = 2.23 \times 10^3\ \text{cal/s} = \boxed{9.32\ \text{kW}}$$



This is the rate of energy loss from the plane by heat, and consequently is the rate at which energy must be supplied in order to maintain a constant temperature.

FIG. P20.71

P20.72 $Q_{\text{cold}} = -Q_{\text{hot}}$

or $Q_{\text{Al}} = -(Q_{\text{water}} + Q_{\text{calo}})$

$$m_{\text{Al}}c_{\text{Al}}(T_f - T_i)_{\text{Al}} = -(m_w c_w + m_c c_c)(T_f - T_i)_w$$

$$(0.200\text{ kg})c_{\text{Al}}(+39.3°\text{C}) = -[0.400\text{ kg}(4\,186\text{ J/kg}\cdot°\text{C}) + 0.040\,0\text{ kg}(630\text{ J/kg}\cdot°\text{C})](-3.70°\text{C})$$

$$c_{\text{Al}} = \frac{6.29\times10^3\text{ J}}{7.86\text{ kg}\cdot°\text{C}} = \boxed{800\text{ J/kg}\cdot°\text{C}}$$

***P20.73** (a) $\mathscr{P} = \sigma A e T^4 = (5.67\times10^{-8}\text{ W/m}^2\text{K}^4)5.1\times10^{14}\text{ m}^2\,(0.965)(5\,800\text{ K})^4 = \boxed{3.16\times10^{22}\text{ W}}$

(b) $T_{\text{avg}} = 0.1(4\,800\text{ K}) + 0.9(5\,890\text{ K}) = \boxed{5.78\times10^3\text{ K}}$

This is cooler than 5 800 K by $\dfrac{5\,800 - 5\,781}{5\,800} = 0.327\%$.

(c) $\mathscr{P} = (5.67\times10^{-8}\text{ W/m}^2\text{K}^4)0.1(5.1\times10^{14}\text{ m}^2)0.965(4\,800\text{ K})^4$

$+5.67\times10^{-8}\text{ W}\,0.9(5.1\times10^{14})0.965(5\,890)^4 = \boxed{3.17\times10^{22}\text{ W}}$

This is larger than 3.158×10^{22} W by $\dfrac{1.29\times10^{20}\text{ W}}{3.16\times10^{22}\text{ W}} = 0.408\%$.

ANSWERS TO EVEN PROBLEMS

P20.2 0.105°C

P20.4 87.0°C

P20.6 The energy input to the water is 6.70 times larger than the laser output of 40.0 kJ.

P20.8 88.2 W

P20.10 (a) 25.8°C; (b) no

P20.12 $T_f = \dfrac{(m_{\text{Al}}c_{\text{Al}} + m_c c_w)T_c + m_h c_w T_h}{m_{\text{Al}}c_{\text{Al}} + m_c c_w + m_h c_w}$

P20.14 (a) 380 K; (b) 206 kPa

P20.16 12.9 g

P20.18 (a) all the ice melts; 40.4°C; (b) 8.04 g melts; 0°C

P20.20 34.0 km

P20.22 liquid lead at 805°C

P20.24 (a) −12.0 MJ; (b) +12.0 MJ

P20.26 $-nR(T_2 - T_1)$

P20.28 (a) 567 J; (b) 167 J

P20.30 (a) 12.0 kJ; (b) −12.0 kJ

P20.32 42.9 kJ

P20.34 (a) 7.65 L; (b) 305 K

P20.36 (a) −48.6 mJ; (b) 16.2 kJ; (c) 16.2 kJ

P20.38 (a) $-4P_iV_i$; (b) $+4P_iV_i$; (c) −9.08 kJ

P20.40 (a) 1 300 J; (b) 100 J; (c) −900 J; (d) −1 400 J

P20.42 10.0 kW

P20.44 1.34 kW

P20.46 (a) $0.890\ \text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h/Btu}$; (b) $1.85\,\dfrac{\text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h}}{\text{Btu}}$; (c) 2.08

P20.48 (a) $\sim 10^3$ W; (b) $\sim -10^{-1}$ K/s

P20.50 364 K

P20.52 47.7 g

P20.54 (a) 13.0°C; (b) −0.532 °C/s

P20.56 (a) 64.1°C; (b) 113°C

P20.58 see the solution (a) $\frac{1}{2}P_iV_i$; (b) $1.39P_iV_i$; (c) 0

P20.60 (a) 9.31×10^{10} J; (b) -8.47×10^{12} J; (c) 8.38×10^{12} J

P20.62 (a) 2 000 W; (b) 4.47°C

P20.64 3.76 m/s

P20.66 (a) 15.0 mg; block: $Q=0$; $W=-5.00$ J; $\Delta E_{\text{int}}=0$; $\Delta K=-5.00$ J;
ice: $Q=0$; $W=5.00$ J; $\Delta E_{\text{int}}=5.00$ J; $\Delta K=0$
(b) 15.0 mg; block: $Q=0$; $W=0$; $\Delta E_{\text{int}}=5.00$ J; $\Delta K=-5.00$ J;
metal: $Q=0$; $W=0$; $\Delta E_{\text{int}}=0$; $\Delta K=0$
(c) 0.004 04°C; moving block: $Q=0$; $W=-2.50$ J; $\Delta E_{\text{int}}=2.50$ J; $\Delta K=-5.00$ J;
stationary block: $Q=0$; $W=2.50$ J; $\Delta E_{\text{int}}=2.50$ J; $\Delta K=0$

P20.68 10.2 h

P20.70 see the solution

P20.72 800 J/kg·°C

21

The Kinetic Theory of Gases

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q21.1 The molecules of all different kinds collide with the walls of the container, so molecules of all different kinds exert partial pressures that contribute to the total pressure. The molecules can be so small that they collide with one another relatively rarely and each kind exerts partial pressure as if the other kinds of molecules were absent. If the molecules collide with one another often, the collisions exactly conserve momentum and so do not affect the net force on the walls.

Q21.2 The helium must have the higher rms speed. According to Equation 21.4, the gas with the smaller mass per atom must have the higher average speed-squared and thus the higher rms speed.

Q21.3 Yes. As soon as the gases are mixed, they come to thermal equilibrium. Equation 21.4 predicts that the lighter helium atoms will on average have a greater speed than the heavier nitrogen molecules. Collisions between the different kinds of molecules gives each kind the same average kinetic energy of translation.

Q21.4 If the average velocity were non-zero, then the bulk sample of gas would be moving in the direction of the average velocity. In a closed tank, this motion would result in a pressure difference within the tank that could not be sustained.

Q21.5 The alcohol evaporates, absorbing energy from the skin to lower the skin temperature.

Q21.6 Partially evacuating the container is equivalent to letting the remaining gas expand. This means that the gas does work, making its internal energy and hence its temperature decrease. The liquid in the container will eventually reach thermal equilibrium with the low pressure gas. This effect of an expanding gas decreasing in temperature is a key process in your refrigerator or air conditioner.

Q21.7 Since the volume is fixed, the density of the cooled gas cannot change, so the mean free path does not change. The collision frequency decreases since each molecule of the gas has a lower average speed.

Q21.8 The mean free path decreases as the density of the gas increases.

Q21.9 The volume of the balloon will decrease. The pressure inside the balloon is nearly equal to the constant exterior atmospheric pressure. Then from $PV = nRT$, volume must decrease in proportion to the absolute temperature. Call the process isobaric contraction.

Q21.10 The dry air is more dense. Since the air and the water vapor are at the same temperature, they have the same kinetic energy per molecule. For a controlled experiment, the humid and dry air are at the same pressure, so the number of molecules per unit volume must be the same for both. The water molecule has a smaller molecular mass (18.0 u) than any of the gases that make up the air, so the humid air must have the smaller mass per unit volume.

Q21.11 Suppose the balloon rises into air uniform in temperature. The air cannot be uniform in pressure because the lower layers support the weight of all the air above them. The rubber in a typical balloon is easy to stretch and stretches or contracts until interior and exterior pressures are nearly equal. So as the balloon rises it expands. This is an isothermal expansion, with P decreasing as V increases by the same factor in $PV = nRT$. If the rubber wall is very strong it will eventually contain the helium at higher pressure than the air outside but at the same density, so that the balloon will stop rising. More likely, the rubber will stretch and break, releasing the helium to keep rising and "boil out" of the Earth's atmosphere.

Q21.12 A diatomic gas has more degrees of freedom—those of vibration and rotation—than a monatomic gas. The energy content per mole is proportional to the number of degrees of freedom.

Q21.13 (a) Average molecular kinetic energy increases by a factor of 3.

(b) The rms speed increases by a factor of $\sqrt{3}$.

(c) Average momentum change increases by $\sqrt{3}$.

(d) Rate of collisions increases by a factor of $\sqrt{3}$ since the mean free path remains unchanged.

(e) Pressure increases by a factor of 3.

Q21.14 They can, as this possibility is not contradicted by any of our descriptions of the motion of gases. If the vessel contains more than a few molecules, it is highly improbable that all will have the same speed. Collisions will make their speeds scatter according to the Boltzmann distribution law.

Q21.15 Collisions between molecules are mediated by electrical interactions among their electrons. On an atomic level, collisions of billiard balls work the same way. Collisions between gas molecules are perfectly elastic. Collisions between macroscopic spheres can be very nearly elastic. So the hard-sphere model is very good. On the other hand, an atom is not 'solid,' but has small-mass electrons moving through empty space as they orbit the nucleus.

Q21.16 As a parcel of air is pushed upward, it moves into a region of lower pressure, so it expands and does work on its surroundings. Its fund of internal energy drops, and so does its temperature. As mentioned in the question, the low thermal conductivity of air means that very little heat will be conducted into the now-cool parcel from the denser but warmer air below it.

Q21.17 A more massive diatomic or polyatomic molecule will generally have a lower frequency of vibration. At room temperature, vibration has a higher probability of being excited than in a less massive molecule. The absorption of energy into vibration shows up in higher specific heats.

SOLUTIONS TO PROBLEMS

Section 21.1 Molecular Model of an Ideal Gas

P21.1
$$\overline{F} = Nm\frac{\Delta v}{\Delta t} = 500\left(5.00\times10^{-3}\text{ kg}\right)\frac{\left[8.00\sin 45.0°-(-8.00\sin 45.0°)\right]\text{ m/s}}{30.0\text{ s}} = \boxed{0.943\text{ N}}$$

$$P = \frac{\overline{F}}{A} = 1.57\text{ N/m}^2 = \boxed{1.57\text{ Pa}}$$

P21.2 $\overline{F}=\frac{(5.00\times10^{23})[2(4.68\times10^{-26}\text{ kg})(300\text{ m/s})]}{1.00\text{ s}}=14.0\text{ N}$

and $P=\frac{\overline{F}}{A}=\frac{14.0\text{ N}}{8.00\times10^{-4}\text{ m}^2}=\boxed{17.6\text{ kPa}}$.

P21.3 We first find the pressure exerted by the gas on the wall of the container.

$$P=\frac{NkT}{V}=\frac{3N_Ak_BT}{V}=\frac{3RT}{V}=\frac{3(8.314\text{ N}\cdot\text{m/mol}\cdot\text{K})(293\text{ K})}{8.00\times10^{-3}\text{ m}^3}=9.13\times10^5\text{ Pa}$$

Thus, the force on one of the walls of the cubical container is

$$F=PA=(9.13\times10^5\text{ Pa})(4.00\times10^{-2}\text{ m}^2)=\boxed{3.65\times10^4\text{ N}}.$$

P21.4 Use Equation 21.2, $P=\frac{2N}{3V}\left(\frac{mv^2}{2}\right)$, so that

$$K_{av}=\frac{mv^2}{2}=\frac{3PV}{2N}\quad\text{where } N=nN_A=2N_A$$

$$K_{av}=\frac{3PV}{2(2N_A)}=\frac{3(8.00\text{ atm})(1.013\times10^5\text{ Pa/atm})(5.00\times10^{-3}\text{ m}^3)}{2(2\text{ mol})(6.02\times10^{23}\text{ molecules/mol})}$$

$$K_{av}=\boxed{5.05\times10^{-21}\text{ J/molecule}}$$

P21.5 $P=\frac{2}{3}\frac{N}{V}\left(\overline{KE}\right)$ Equation 21.2

$$N=\frac{3}{2}\frac{PV}{\left(\overline{KE}\right)}=\frac{3}{2}\frac{(1.20\times10^5)(4.00\times10^{-3})}{(3.60\times10^{-22})}=2.00\times10^{24}\text{ molecules}$$

$$n=\frac{N}{N_A}=\frac{2.00\times10^{24}\text{ molecules}}{6.02\times10^{23}\text{ molecules/mol}}=\boxed{3.32\text{ mol}}$$

P21.6 One mole of helium contains Avogadro's number of molecules and has a mass of 4.00 g. Let us call m the mass of one atom, and we have

$$N_Am=4.00\text{ g/mol}$$

or $$m=\frac{4.00\text{ g/mol}}{6.02\times10^{23}\text{ molecules/mol}}=6.64\times10^{-24}\text{ g/molecule}$$

$$m=\boxed{6.64\times10^{-27}\text{ kg}}$$

P21.7 (a) $PV=Nk_BT$: $N=\frac{PV}{k_BT}=\frac{1.013\times10^5\text{ Pa}\left[\frac{4}{3}\pi(0.150\text{ m})^3\right]}{(1.38\times10^{-23}\text{ J/K})(293\text{ K})}=\boxed{3.54\times10^{23}\text{ atoms}}$

(b) $\overline{K}=\frac{3}{2}k_BT=\frac{3}{2}(1.38\times10^{-23})(293)\text{ J}=\boxed{6.07\times10^{-21}\text{ J}}$

(c) For helium, the atomic mass is $m=\frac{4.00\text{ g/mol}}{6.02\times10^{23}\text{ molecules/mol}}=6.64\times10^{-24}\text{ g/molecule}$

$$m=6.64\times10^{-27}\text{ kg/molecule}$$

$\frac{1}{2}m\overline{v^2}=\frac{3}{2}k_BT$: $\quad\therefore v_{rms}=\sqrt{\frac{3k_BT}{m}}=\boxed{1.35\text{ km/s}}$

P21.8 $v=\sqrt{\dfrac{3k_BT}{m}}$

$$\frac{v_{\rm O}}{v_{\rm He}}=\sqrt{\frac{M_{\rm He}}{M_{\rm O}}}=\sqrt{\frac{4.00}{32.0}}=\sqrt{\frac{1}{8.00}}$$

$$v_{\rm O}=\frac{1\,350\text{ m/s}}{\sqrt{8.00}}=\boxed{477\text{ m/s}}$$

P21.9 (a) $\overline{K}=\dfrac{3}{2}k_BT=\dfrac{3}{2}\left(1.38\times10^{-23}\text{ J/K}\right)(423\text{ K})=\boxed{8.76\times10^{-21}\text{ J}}$

(b) $\overline{K}=\dfrac{1}{2}mv_{rms}^2=8.76\times10^{-21}\text{ J}$

so $\qquad v_{rms}=\sqrt{\dfrac{1.75\times10^{-20}\text{ J}}{m}}\qquad$ (1)

For helium, $\qquad m=\dfrac{4.00\text{ g/mol}}{6.02\times10^{23}\text{ molecules/mol}}=6.64\times10^{-24}\text{ g/molecule}$

$m=6.64\times10^{-27}\text{ kg/molecule}$

Similarly for argon, $\qquad m=\dfrac{39.9\text{ g/mol}}{6.02\times10^{23}\text{ molecules/mol}}=6.63\times10^{-23}\text{ g/molecule}$

$m=6.63\times10^{-26}\text{ kg/molecule}$

Substituting in (1) above,
we find for helium, $\qquad \boxed{v_{rms}=1.62\text{ km/s}}$

and for argon, $\qquad \boxed{v_{rms}=514\text{ m/s}}$

P21.10 (a) $PV=nRT=\dfrac{Nmv^2}{3}$

The total translational kinetic energy is $\dfrac{Nmv^2}{2}=E_{\rm trans}$:

$E_{\rm trans}=\dfrac{3}{2}PV=\dfrac{3}{2}\left(3.00\times1.013\times10^5\right)\left(5.00\times10^{-3}\right)=\boxed{2.28\text{ kJ}}$

(b) $\dfrac{mv^2}{2}=\dfrac{3k_BT}{2}=\dfrac{3RT}{2N_A}=\dfrac{3(8.314)(300)}{2\left(6.02\times10^{23}\right)}=\boxed{6.21\times10^{-21}\text{ J}}$

P21.11 (a) $1\text{ Pa}=(1\text{ Pa})\left(\dfrac{1\text{ N/m}^2}{1\text{ Pa}}\right)\left(\dfrac{1\text{ J}}{1\text{ N}\cdot\text{m}}\right)=\boxed{1\text{ J/m}^3}$

(b) For a monatomic ideal gas, $E_{\rm int}=\dfrac{3}{2}nRT$

For any ideal gas, the energy of molecular translation is the same,

$$E_{\rm trans}=\frac{3}{2}nRT=\frac{3}{2}PV.$$

Thus, the energy per volume is $\dfrac{E_{\rm trans}}{V}=\boxed{\dfrac{3}{2}P}$.

Section 21.2 Molar Specific Heat of an Ideal Gas

P21.12 $E_{int} = \frac{3}{2}nRT$

$$\Delta E_{int} = \frac{3}{2}nR\Delta T = \frac{3}{2}(3.00 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(2.00 \text{ K}) = \boxed{74.8 \text{ J}}$$

P21.13 We us the tabulated values for C_P and C_V

(a) $Q = nC_P\Delta T = 1.00 \text{ mol}(28.8 \text{ J/mol}\cdot\text{K})(420-300) \text{ K} = \boxed{3.46 \text{ kJ}}$

(b) $\Delta E_{int} = nC_V\Delta T = 1.00 \text{ mol}(20.4 \text{ J/mol}\cdot\text{K})(120 \text{ K}) = \boxed{2.45 \text{ kJ}}$

(c) $W = -Q + \Delta E_{int} = -3.46 \text{ kJ} + 2.45 \text{ kJ} = \boxed{-1.01 \text{ kJ}}$

P21.14 The piston moves to keep pressure constant. Since $V = \frac{nRT}{P}$, then

$$\Delta V = \frac{nR\Delta T}{P} \text{ for a constant pressure process.}$$

$$Q = nC_P\Delta T = n(C_V + R)\Delta T \text{ so } \Delta T = \frac{Q}{n(C_V+R)} = \frac{Q}{n(5R/2+R)} = \frac{2Q}{7nR}$$

and $$\Delta V = \frac{nR}{P}\left(\frac{2Q}{7nR}\right) = \frac{2Q}{7P} = \frac{2}{7}\frac{QV}{nRT}$$

$$\Delta V = \frac{2}{7}\frac{(4.40\times10^3 \text{ J})(5.00 \text{ L})}{(1.00 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(300 \text{ K})} = 2.52 \text{ L}$$

Thus, $V_f = V_i + \Delta V = 5.00 \text{ L} + 2.52 \text{ L} = \boxed{7.52 \text{ L}}$

P21.15 $n = 1.00 \text{ mol}$, $T_i = 300 \text{ K}$

(b) Since $V = $ constant, $W = \boxed{0}$

(a) $\Delta E_{int} = Q + W = 209 \text{ J} + 0 = \boxed{209 \text{ J}}$

(c) $\Delta E_{int} = nC_V\Delta T = n\left(\frac{3}{2}R\right)\Delta T$

so $$\Delta T = \frac{2\Delta E_{int}}{3nR} = \frac{2(209 \text{ J})}{3(1.00 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})} = 16.8 \text{ K}$$

$$T = T_i + \Delta T = 300 \text{ K} + 16.8 \text{ K} = \boxed{317 \text{ K}}$$

P21.16 (a) Consider heating it at constant pressure. Oxygen and nitrogen are diatomic, so $C_P = \frac{7R}{2}$

$$Q = nC_P\Delta T = \frac{7}{2}nR\Delta T = \frac{7}{2}\left(\frac{PV}{T}\right)\Delta T$$

$$Q = \frac{7}{2}\frac{\left(1.013\times10^5\ \text{N/m}^2\right)\left(100\ \text{m}^3\right)}{300\ \text{K}}(1.00\ \text{K}) = \boxed{118\ \text{kJ}}$$

(b) $U_g = mgy$

$$m = \frac{U_g}{gy} = \frac{1.18\times10^5\ \text{J}}{\left(9.80\ \text{m/s}^2\right)2.00\ \text{m}} = \boxed{6.03\times10^3\ \text{kg}}$$

***P21.17** (a) We assume that the bulb does not expand. Then this is a constant-volume heating process. The quantity of the gas is $n = \frac{P_iV}{RT_i}$. The energy input is $Q = \wp\Delta t = nC_V\Delta T$ so

$$\Delta T = \frac{\wp\Delta t}{nC_V} = \frac{\wp\Delta tRT_i}{P_iVC_V}.$$

The final temperature is $T_f = T_i + \Delta T = T_i\left(1 + \frac{\wp\Delta tR}{P_iVC_V}\right)$.

The final pressure is $P_f = P_i\frac{T_f}{T_i} = P_i\left(1 + \frac{\wp\Delta tR}{P_iVC_V}\right)$.

(b) $$P_f = 1\ \text{atm}\left(1 + \frac{3.60\ \text{J}\ 4\ \text{s}\ 8.314\ \text{J}\cdot\text{m}^2\ 3\ \text{mol}\cdot\text{K}}{\text{s}\cdot\text{mol}\cdot\text{K}\ 1.013\times10^5\ \text{N}\ 4\pi(0.05\ \text{m})^3 12.5\ \text{J}}\right) = \boxed{1.18\ \text{atm}}$$

P21.18 (a) $$C_V = \frac{5}{2}R = \frac{5}{2}(8.314\ \text{J/mol}\cdot\text{K})\left(\frac{1.00\ \text{mol}}{0.028\,9\ \text{kg}}\right) = 719\ \text{J/kg}\cdot\text{K} = \boxed{0.719\ \text{kJ/kg}\cdot\text{K}}$$

(b) $m = Mn = M\left(\frac{PV}{RT}\right)$

$$m = (0.028\,9\ \text{kg/mol})\left(\frac{200\times10^3\ \text{Pa}\left(0.350\ \text{m}^3\right)}{(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K})}\right) = \boxed{0.811\ \text{kg}}$$

(c) We consider a constant volume process where no work is done.

$$Q = mC_V\Delta T = 0.811\ \text{kg}(0.719\ \text{kJ/kg}\cdot\text{K})(700\ \text{K} - 300\ \text{K}) = \boxed{233\ \text{kJ}}$$

(d) We now consider a constant pressure process where the internal energy of the gas is increased and work is done.

$$Q = mC_P\Delta T = m(C_V + R)\Delta T = m\left(\frac{7R}{2}\right)\Delta T = m\left(\frac{7C_V}{5}\right)\Delta T$$

$$Q = 0.811\ \text{kg}\left[\frac{7}{5}(0.719\ \text{kJ/kg}\cdot\text{K})\right](400\ \text{K}) = \boxed{327\ \text{kJ}}$$

P21.19 Consider 800 cm^3 of (flavored) water at 90.0 °C mixing with 200 cm^3 of diatomic ideal gas at 20.0°C:

$$Q_{cold} = -Q_{hot}$$

or $$m_{air}c_{P,\,air}\left(T_f - T_{i,\,air}\right) = -m_w c_w (\Delta T)_w$$

$$(\Delta T)_w = \frac{-m_{air}c_{P,\,air}\left(T_f - T_{i,\,air}\right)}{m_w c_w} = \frac{-(\rho V)_{air}\,c_{P,\,air}(90.0°\text{C} - 20.0°\text{C})}{(\rho_w V_w)c_w}$$

where we have anticipated that the final temperature of the mixture will be close to 90.0°C.

The molar specific heat of air is $C_{P,\,air} = \frac{7}{2}R$

So the specific heat per gram is $c_{P,\,air} = \frac{7}{2}\left(\frac{R}{M}\right) = \frac{7}{2}(8.314\ \text{J/mol}\cdot\text{K})\left(\frac{1.00\ \text{mol}}{28.9\ \text{g}}\right) = 1.01\ \text{J/g}\cdot°\text{C}$

$$(\Delta T)_w = -\frac{\left[\left(1.20\times10^{-3}\ \text{g/cm}^3\right)\left(200\ \text{cm}^3\right)\right](1.01\ \text{J/g}\cdot°\text{C})(70.0°\text{C})}{\left[\left(1.00\ \text{g/cm}^3\right)\left(800\ \text{cm}^3\right)\right](4.186\ \text{J/kg}\cdot°\text{C})}$$

or $$(\Delta T)_w \approx -5.05\times10^{-3}\,°\text{C}$$

The change of temperature for the water is $\boxed{\text{between } 10^{-3}\,°\text{C and } 10^{-2}\,°\text{C}}$.

P21.20 $Q = (nC_P\Delta T)_{isobaric} + (nC_V\Delta T)_{isovolumetric}$

In the isobaric process, V doubles so T must double, to $2T_i$.

In the isovolumetric process, P triples so T changes from $2T_i$ to $6T_i$.

$$Q = n\left(\frac{7}{2}R\right)(2T_i - T_i) + n\left(\frac{5}{2}R\right)(6T_i - 2T_i) = 13.5nRT_i = \boxed{13.5PV}$$

P21.21 In the isovolumetric process $A \to B$, $W = 0$ and $Q = nC_V\Delta T = 500\ \text{J}$

$$500\ \text{J} = n\left(\frac{3R}{2}\right)(T_B - T_A) \text{ or } T_B = T_A + \frac{2(500\ \text{J})}{3nR}$$

$$T_B = 300\ \text{K} + \frac{2(500\ \text{J})}{3(1.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})} = 340\ \text{K}$$

In the isobaric process $B \to C$,

$$Q = nC_P\Delta T = \frac{5nR}{2}(T_C - T_B) = -500\ \text{J}.$$

Thus,

(a) $$T_C = T_B - \frac{2(500\ \text{J})}{5nR} = 340\ \text{K} - \frac{1\,000\ \text{J}}{5(1.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})} = \boxed{316\ \text{K}}$$

(b) The work done on the gas during the isobaric process is

$$W_{BC} = -P_B\Delta V = -nR(T_C - T_B) = -(1.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(316\ \text{K} - 340\ \text{J})$$

or $$W_{BC} = +200\ \text{J}$$

The work done on the gas in the isovolumetric process is zero, so in total

$$W_{on\ gas} = \boxed{+200\ \text{J}}.$$

***P21.22** (a) At any point in the heating process, $P_i = kV_i$ and $P = kV = \frac{P_i}{V_i}V = \frac{nRT_i}{V_i^2}V$. At the end,

$$P_f = \frac{nRT_i}{V_i^2}2V_i = 2P_i \text{ and } T_f = \frac{P_fV_f}{nR} = \frac{2P_i 2V_i}{nR} = \boxed{4T_i}.$$

(b) The work input is $W = -\int_i^f PdV = -\int_{V_i}^{2V_i}\frac{nRT_i}{V_i^2}VdV = -\frac{nRT_i}{V_i^2}\frac{V^2}{2}\Big|_{V_i}^{2V_i} = -\frac{nRT_i}{2V_i^2}\left(4V_i^2 - V_i^2\right) = -\frac{3}{2}nRT_i$.

The change in internal energy, is $\Delta E_{int} = nC_V\Delta T = n\frac{5}{2}R(4T_i - T_i) = +\frac{15}{2}nRT_i$. The heat input is $Q = \Delta E_{int} - W = \frac{18}{2}nRT_i = \boxed{9(1\text{ mol})RT_i}$.

P21.23 (a) The heat required to produce a temperature change is

$$Q = n_1C_1\Delta T + n_2C_2\Delta T$$

The number of molecules is $N_1 + N_2$, so the number of "moles of the mixture" is $n_1 + n_2$ and $Q = (n_1 + n_2)C\Delta T$,

so $$C = \boxed{\frac{n_1C_1 + n_2C_2}{n_1 + n_2}}.$$

(b) $$Q = \sum_{i=1}^{m} n_iC_i\Delta T = \left(\sum_{i=1}^{m} n_i\right)C\Delta T$$

$$C = \boxed{\frac{\sum_{i=1}^{m} n_iC_i}{\sum_{i=1}^{m} n_i}}$$

Section 21.3 Adiabatic Processes for an Ideal Gas

P21.24 (a) $P_iV_i^\gamma = P_fV_f^\gamma$ so $\frac{V_f}{V_i} = \left(\frac{P_i}{P_f}\right)^{1/\gamma} = \left(\frac{1.00}{20.0}\right)^{5/7} = \boxed{0.118}$

(b) $\frac{T_f}{T_i} = \frac{P_fV_f}{P_iV_i} = \left(\frac{P_f}{P_i}\right)\left(\frac{V_f}{V_i}\right) = (20.0)(0.118)$ $\frac{T_f}{T_i} = \boxed{2.35}$

(c) Since the process is adiabatic, $\boxed{Q = 0}$

Since $\gamma = 1.40 = \frac{C_P}{C_V} = \frac{R + C_V}{C_V}$, $C_V = \frac{5}{2}R$ and $\Delta T = 2.35T_i - T_i = 1.35T_i$

$$\Delta E_{int} = nC_V\Delta T = (0.016\,0\text{ mol})\left(\frac{5}{2}\right)(8.314\text{ J/mol}\cdot\text{K})[1.35(300\text{ K})] = \boxed{135\text{ J}}$$

and $W = -Q + \Delta E_{int} = 0 + 135\text{ J} = \boxed{+135\text{ J}}$.

P21.25 (a) $P_iV_i^\gamma = P_fV_f^\gamma$

$$P_f = P_i\left(\frac{V_i}{V_f}\right)^\gamma = 5.00\text{ atm}\left(\frac{12.0}{30.0}\right)^{1.40} = \boxed{1.39\text{ atm}}$$

(b)
$$T_i = \frac{P_iV_i}{nR} = \frac{5.00\left(1.013\times10^5\text{ Pa}\right)\left(12.0\times10^{-3}\text{ m}^3\right)}{2.00\text{ mol}(8.314\text{ J/mol}\cdot\text{K})} = \boxed{365\text{ K}}$$

$$T_f = \frac{P_fV_f}{nR} = \frac{1.39\left(1.013\times10^5\text{ Pa}\right)\left(30.0\times10^{-3}\text{ m}^3\right)}{2.00\text{ mol}(8.314\text{ J/mol}\cdot\text{K})} = \boxed{253\text{ K}}$$

(c) The process is adiabatic: $\boxed{Q = 0}$

$$\gamma = 1.40 = \frac{C_P}{C_V} = \frac{R + C_V}{C_V},\ C_V = \frac{5}{2}R$$

$$\Delta E_{\text{int}} = nC_V\Delta T = 2.00\text{ mol}\left(\frac{5}{2}(8.314\text{ J/mol}\cdot\text{K})\right)(253\text{ K} - 365\text{ K}) = \boxed{-4.66\text{ kJ}}$$

$$W = \Delta E_{\text{int}} - Q = -4.66\text{ kJ} - 0 = \boxed{-4.66\text{ kJ}}$$

P21.26
$$V_i = \pi\left(\frac{2.50\times10^{-2}\text{ m}}{2}\right)^2 0.500\text{ m} = 2.45\times10^{-4}\text{ m}^3$$

The quantity of air we find from $P_iV_i = nRT_i$

$$n = \frac{P_iV_i}{RT_i} = \frac{\left(1.013\times10^5\text{ Pa}\right)\left(2.45\times10^{-4}\text{ m}^3\right)}{(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})}$$

$$n = 9.97\times10^{-3}\text{ mol}$$

Adiabatic compression: $P_f = 101.3\text{ kPa} + 800\text{ kPa} = 901.3\text{ kPa}$

(a) $P_iV_i^\gamma = P_fV_f^\gamma$

$$V_f = V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma} = 2.45\times10^{-4}\text{ m}^3\left(\frac{101.3}{901.3}\right)^{5/7}$$

$$V_f = \boxed{5.15\times10^{-5}\text{ m}^3}$$

(b) $P_fV_f = nRT_f$

$$T_f = T_i\frac{P_fV_f}{P_iV_i} = T_i\frac{P_f}{P_i}\left(\frac{P_i}{P_f}\right)^{1/\gamma} = T_i\left(\frac{P_i}{P_f}\right)^{(1/\gamma - 1)}$$

$$T_f = 300\text{ K}\left(\frac{101.3}{901.3}\right)^{(5/7-1)} = \boxed{560\text{ K}}$$

(c) The work put into the gas in compressing it is $\Delta E_{\text{int}} = nC_V\Delta T$

$$W = \left(9.97\times10^{-3}\text{ mol}\right)\frac{5}{2}(8.314\text{ J/mol}\cdot\text{K})(560 - 300)\text{ K}$$

$$W = 53.9\text{ J}$$

continued on next page

Now imagine this energy being shared with the inner wall as the gas is held at constant volume. The pump wall has outer diameter 25.0 mm + 2.00 mm + 2.00 mm = 29.0 mm, and volume

$$\left[\pi\left(14.5\times10^{-3}\text{ m}\right)^2-\pi\left(12.5\times10^{-3}\text{ m}\right)^2\right]4.00\times10^{-2}\text{ m}=6.79\times10^{-6}\text{ m}^3$$

and mass $\rho V=\left(7.86\times10^3\text{ kg/m}^3\right)\left(6.79\times10^{-6}\text{ m}^3\right)=53.3\text{ g}$

The overall warming process is described by

$$53.9\text{ J}=nC_V\Delta T+mc\Delta T$$

$$53.9\text{ J}=\left(9.97\times10^{-3}\text{ mol}\right)\frac{5}{2}(8.314\text{ J/mol}\cdot\text{K})\left(T_{ff}-300\text{ K}\right)$$
$$+\left(53.3\times10^{-3}\text{ kg}\right)(448\text{ J/kg}\cdot\text{K})\left(T_{ff}-300\text{ K}\right)$$

$$53.9\text{ J}=(0.207\text{ J/K}+23.9\text{ J/K})\left(T_{ff}-300\text{ K}\right)$$

$$T_{ff}-300\text{ K}=\boxed{2.24\text{ K}}$$

P21.27 $\dfrac{T_f}{T_i}=\left(\dfrac{V_i}{V_f}\right)^{\gamma-1}=\left(\dfrac{1}{2}\right)^{0.400}$

If $T_i=300\text{ K}$, then $T_f=\boxed{227\text{ K}}$.

***P21.28** (a) In $P_iV_i^\gamma=P_fV_f^\gamma$ we have $P_f=P_i\left(\dfrac{V_i}{V_f}\right)^\gamma$

$$P_f=P_i\left(\frac{0.720\text{ m}^3}{0.240\text{ m}^3}\right)^{1.40}=4.66P_i$$

Then $\dfrac{P_iV_i}{T_i}=\dfrac{P_fV_f}{T_f}$ $\qquad T_f=T_i\dfrac{P_fV_f}{P_iV_i}=T_i(4.66)\dfrac{1}{3}=1.55$

The factor of increase in temperature is the same as the factor of increase in internal energy, according to $E_{\text{int}}=nC_VT$. Then $\dfrac{E_{\text{int},f}}{E_{\text{int},i}}=\boxed{1.55}$.

(b) In $\dfrac{T_f}{T_i}=\dfrac{P_fV_f}{P_iV_i}=\left(\dfrac{V_i}{V_f}\right)^\gamma\dfrac{V_f}{V_i}=\left(\dfrac{V_i}{V_f}\right)^{\gamma-1}$ we have

$$2=\left(\frac{0.720\text{ m}^3}{V_f}\right)^{0.40}$$

$$\frac{0.720\text{ m}^3}{V_f}=2^{1/0.4}=2^{2.5}=5.66$$

$$V_f=\frac{0.720\text{ m}^3}{5.66}=\boxed{0.127\text{ m}^3}$$

P21.29 (a) See the diagram at the right.

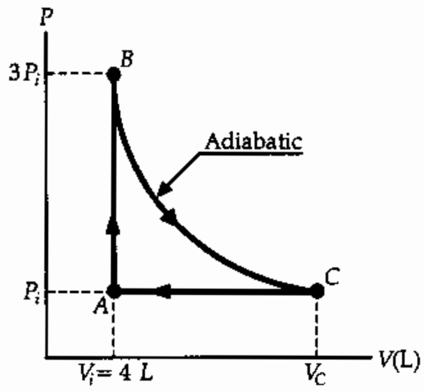


FIG. P21.29

(b) $P_B V_B^\gamma = P_C V_C^\gamma$

$3P_i V_i^\gamma = P_i V_C^\gamma$

$V_C = \left(3^{1/\gamma}\right)V_i = \left(3^{5/7}\right)V_i = 2.19V_i$

$V_C = 2.19(4.00\text{ L}) = \boxed{8.77\text{ L}}$

(c) $P_B V_B = nRT_B = 3P_i V_i = 3nRT_i$

$T_B = 3T_i = 3(300\text{ K}) = \boxed{900\text{ K}}$

(d) After one whole cycle, $T_A = T_i = \boxed{300\text{ K}}$.

(e) In AB, $Q_{AB} = nC_V\Delta V = n\left(\frac{5}{2}R\right)(3T_i - T_i) = (5.00)nRT_i$

$Q_{BC} = 0$ as this process is adiabatic

$P_C V_C = nRT_C = P_i(2.19V_i) = (2.19)nRT_i$

so $T_C = 2.19T_i$

$Q_{CA} = nC_P\Delta T = n\left(\frac{7}{2}R\right)(T_i - 2.19T_i) = (-4.17)nRT_i$

For the whole cycle,

$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17)nRT_i = (0.829)nRT_i$

$(\Delta E_{\text{int}})_{ABCA} = 0 = Q_{ABCA} + W_{ABCA}$

$W_{ABCA} = -Q_{ABCA} = -(0.829)nRT_i = -(0.829)P_iV_i$

$W_{ABCA} = -(0.829)\left(1.013\times10^5\text{ Pa}\right)\left(4.00\times10^{-3}\text{ m}^3\right) = \boxed{-336\text{ J}}$

P21.30 (a) See the diagram at the right.

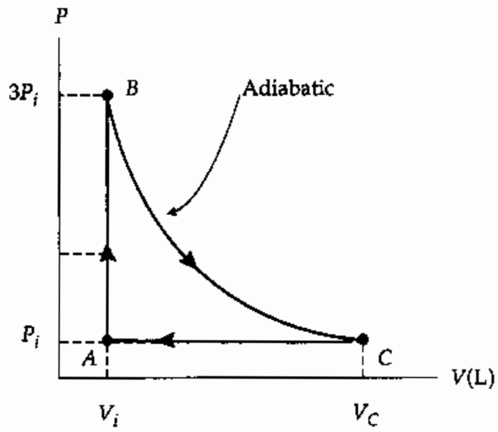


FIG. P21.30

(b) $P_B V_B^\gamma = P_C V_C^\gamma$

$3P_i V_i^\gamma = P_i V_C^\gamma$

$V_C = 3^{1/\gamma}V_i = 3^{5/7}V_i = \boxed{2.19V_i}$

(c) $P_B V_B = nRT_B = 3P_i V_i = 3nRT_i$

$T_B = \boxed{3T_i}$

(d) After one whole cycle, $T_A = \boxed{T_i}$

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(e) In AB, $Q_{AB} = nC_V\Delta T = n\left(\frac{5}{2}R\right)(3T_i - T_i) = (5.00)nRT_i$

$$Q_{BC} = 0 \text{ as this process is abiabatic}$$
$$P_C V_C = nRT_C = P_i(2.19V_i) = 2.19nRT_i \text{ so } T_C = 2.19T_i$$
$$Q_{CA} = nC_P\Delta T = n\left(\frac{7}{2}R\right)(T_i - 2.19T_i) = -4.17nRT_i$$

For the whole cycle,

$$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17)nRT_i = 0.830nRT_i$$
$$(\Delta E_{\text{int}})_{ABCA} = 0 = Q_{ABCA} + W_{ABCA}$$
$$W_{ABCA} = -Q_{ABCA} = -0.830nRT_i = \boxed{-0.830P_iV_i}$$

P21.31 (a) The work done *on* the gas is

$$W_{ab} = -\int_{V_a}^{V_b} PdV.$$

For the isothermal process,

$$W_{ab'} = -nRT_a \int_{V_a}^{V_{b'}} \left(\frac{1}{V}\right)dV$$
$$W_{ab'} = -nRT_a \ln\left(\frac{V_{b'}}{V_a}\right) = nRT\ln\left(\frac{V_a}{V_{b'}}\right).$$

Thus, $W_{ab'} = 5.00 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})(293 \text{ K})\ln(10.0)$

$$W_{ab'} = \boxed{28.0 \text{ kJ}}.$$

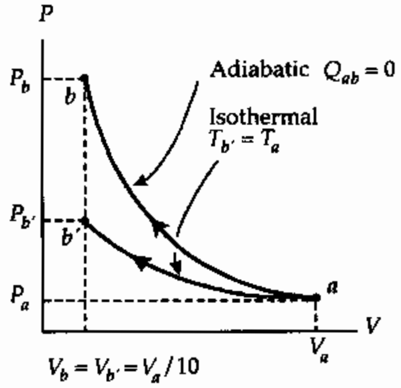

FIG. P21.31

(b) For the adiabatic process, we must first find the final temperature, T_b. Since air consists primarily of diatomic molecules, we shall use

$$\gamma_{\text{air}} = 1.40 \text{ and } C_{V,\text{air}} = \frac{5R}{2} = \frac{5(8.314)}{2} = 20.8 \text{ J/mol}\cdot\text{K}.$$

Then, for the adiabatic preocess

$$T_b = T_a\left(\frac{V_a}{V_b}\right)^{\gamma-1} = 293 \text{ K}(10.0)^{0.400} = 736 \text{ K}.$$

Thus, the work done on the gas during the adiabatic process is

$$W_{ab}(-Q + \Delta E_{\text{int}})_{ab} = (-0 + nC_V\Delta T)_{ab} = nC_V(T_b - T_a)$$

or $W_{ab} = 5.00 \text{ mol}(20.8 \text{ J/mol}\cdot\text{K})(736 - 293) \text{ K} = \boxed{46.0 \text{ kJ}}$.

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(c) For the isothermal process, we have

$$P_{b'}V_{b'} = P_aV_a.$$

Thus, $P_{b'} = P_a\left(\frac{V_a}{V_{b'}}\right) = 1.00\text{ atm}(10.0) = \boxed{10.0\text{ atm}}$.

For the adiabatic process, we have $P_bV_b^\gamma = P_aV_a^\gamma$.

Thus, $P_b = P_a\left(\frac{V_a}{V_b}\right)^\gamma = 1.00\text{ atm}(10.0)^{1.40} = \boxed{25.1\text{ atm}}$.

P21.32 We suppose the air plus burnt gasoline behaves like a diatomic ideal gas. We find its final absolute pressure:

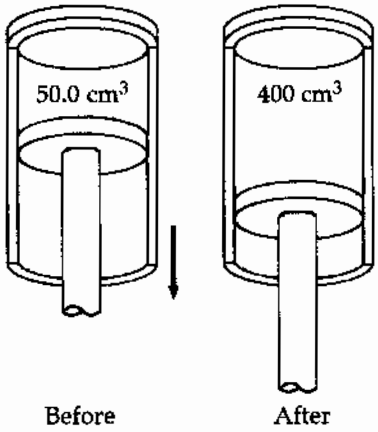

FIG. P21.32

$$21.0\text{ atm}\left(50.0\text{ cm}^3\right)^{7/5} = P_f\left(400\text{ cm}^3\right)^{7/5}$$

$$P_f = 21.0\text{ atm}\left(\frac{1}{8}\right)^{7/5} = 1.14\text{ atm}$$

Now $Q = 0$

and $W = \Delta E_{\text{int}} = nC_V\left(T_f - T_i\right)$

$$\therefore W = \frac{5}{2}nRT_f - \frac{5}{2}nRT_i = \frac{5}{2}\left(P_fV_f - P_iV_i\right)$$

$$W = \frac{5}{2}\left[1.14\text{ atm}\left(400\text{ cm}^3\right) - 21.0\text{ atm}\left(50.0\text{ cm}^3\right)\right]\left(\frac{1.013\times10^5\text{ N/m}^2}{1\text{ atm}}\right)\left(10^{-6}\text{ m}^3/\text{cm}^3\right)$$

$$W = -150\text{ J}$$

The output work is $-W = +150$ J

The time for this stroke is $\frac{1}{4}\left(\frac{1\text{ min}}{2\,500}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 6.00\times10^{-3}\text{ s}$

$$\mathcal{P} = \frac{-W}{\Delta t} = \frac{150\text{ J}}{6.00\times10^{-3}\text{ s}} = \boxed{25.0\text{ kW}}$$

Section 21.4 The Equipartition of Energy

P21.33 The heat capacity at constant volume is nC_V. An ideal gas of diatomic molecules has three degrees of freedom for translation in the x, y, and z directions. If we take the y axis along the axis of a molecule, then outside forces cannot excite rotation about this axis, since they have no lever arms. Collisions will set the molecule spinning only about the x and z axes.

(a) If the molecules do not vibrate, they have five degrees of freedom. Random collisions put equal amounts of energy $\frac{1}{2}k_BT$ into all five kinds of motion. The average energy of one molecule is $\frac{5}{2}k_BT$. The internal energy of the two-mole sample is

$$N\left(\frac{5}{2}k_BT\right)=nN_A\left(\frac{5}{2}k_BT\right)=n\left(\frac{5}{2}R\right)T=nC_VT.$$

The molar heat capacity is $C_V=\frac{5}{2}R$ and the sample's heat capacity is

$$nC_V=n\left(\frac{5}{2}R\right)=2\text{ mol}\left(\frac{5}{2}(8.314\text{ J/mol}\cdot\text{K})\right)$$

$$\boxed{nC_V=41.6\text{ J/K}}$$

For the heat capacity at constant pressure we have

$$nC_P=n(C_V+R)=n\left(\frac{5}{2}R+R\right)=\frac{7}{2}nR=2\text{ mol}\left(\frac{7}{2}(8.314\text{ J/mol}\cdot\text{K})\right)$$

$$\boxed{nC_P=58.2\text{ J/K}}$$

(b) In vibration with the center of mass fixed, both atoms are always moving in opposite directions with equal speeds. Vibration adds two more degrees of freedom for two more terms in the molecular energy, for kinetic and for elastic potential energy. We have

$$nC_V=n\left(\frac{7}{2}R\right)=\boxed{58.2\text{ J/K}}$$

and $$nC_P=n\left(\frac{9}{2}R\right)=\boxed{74.8\text{ J/K}}$$

P21.34 (1) $$E_{\text{int}}=Nf\left(\frac{k_BT}{2}\right)=f\left(\frac{nRT}{2}\right)$$

(2) $$C_V=\frac{1}{n}\left(\frac{dE_{\text{int}}}{dT}\right)=\frac{1}{2}fR$$

(3) $$C_P=C_V+R=\frac{1}{2}(f+2)R$$

(4) $$\gamma=\frac{C_P}{C_V}=\frac{f+2}{f}$$

P21.35 Rotational Kinetic Energy $= \frac{1}{2}I\omega^2$

$I = 2mr^2$, $m = 35.0 \times 1.67 \times 10^{-27}$ kg, $r = 10^{-10}$ m

$I = 1.17 \times 10^{-45}$ kg $\cdot$ m^2 $\qquad \omega = 2.00 \times 10^{12}$ s^{-1}

$\therefore K_{rot} = \frac{1}{2}I\omega^2 = \boxed{2.33 \times 10^{-21} \text{ J}}$

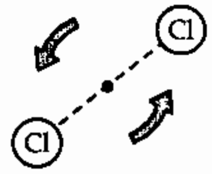

FIG. P21.35

Section 21.5 The Boltzmann Distribution Law

Section 21.6 Distribution of Molecular Speeds

P21.36 (a) The ratio of the number at higher energy to the number at lower energy is $e^{-\Delta E/k_BT}$ where ΔE is the energy difference. Here,

$$\Delta E = (10.2 \text{ eV})\left(\frac{1.60 \times 10^{-19} \text{ J}}{1 \text{ eV}}\right) = 1.63 \times 10^{-18} \text{ J}$$

and at 0°C,

$$k_BT = \left(1.38 \times 10^{-23} \text{ J/K}\right)(273 \text{ K}) = 3.77 \times 10^{-21} \text{ J}.$$

Since this is much less than the excitation energy, nearly all the atoms will be in the ground state and the number excited is

$$\left(2.70 \times 10^{25}\right)\exp\left(\frac{-1.63 \times 10^{-18} \text{ J}}{3.77 \times 10^{-21} \text{ J}}\right) = \left(2.70 \times 10^{25}\right)e^{-433}.$$

This number is much less than one, so almost all of the time $\boxed{\text{no atom is excited}}$.

(b) At 10 000°C,

$$k_BT = \left(1.38 \times 10^{-23} \text{ J/K}\right)10\,273 \text{ K} = 1.42 \times 10^{-19} \text{ J}.$$

The number excited is

$$\left(2.70 \times 10^{25}\right)\exp\left(\frac{-1.63 \times 10^{-18} \text{ J}}{1.42 \times 10^{-19} \text{ J}}\right) = \left(2.70 \times 10^{25}\right)e^{-11.5} = \boxed{2.70 \times 10^{20}}.$$

P21.37 (a) $v_{av} = \frac{\sum n_i v_i}{N} = \frac{1}{15}[1(2)+2(3)+3(5)+4(7)+3(9)+2(12)] = \boxed{6.80 \text{ m/s}}$

(b) $\left(v^2\right)_{av} = \frac{\sum n_i v_i^2}{N} = 54.9 \text{ m}^2/\text{s}^2$

so $v_{rms} = \sqrt{\left(v^2\right)_{av}} = \sqrt{54.9} = \boxed{7.41 \text{ m/s}}$

(c) $v_{mp} = \boxed{7.00 \text{ m/s}}$

P21.38 (a) $\frac{V_{rms,\,35}}{V_{rms,\,37}} = \frac{\sqrt{\frac{3RT}{M_{35}}}}{\sqrt{\frac{3RT}{M_{37}}}} = \left(\frac{37.0 \text{ g/mol}}{35.0 \text{ g/mol}}\right)^{1/2} = \boxed{1.03}$

(b) The lighter atom, $\boxed{^{35}\text{Cl}}$, moves faster.

P21.39 In the Maxwell Boltzmann speed distribution function take $\frac{dN_v}{dv} = 0$ to find

$$4\pi N\left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left(-\frac{mv^2}{2k_B T}\right)\left(2v - \frac{2mv^3}{2k_B T}\right) = 0$$

and solve for v to find the most probable speed.

Reject as solutions $v = 0$ and $v = \infty$

Retain only $2 - \frac{mv^2}{k_B T} = 0$

Then $v_{mp} = \sqrt{\frac{2k_B T}{m}}$

P21.40 The most probable speed is $v_{mp} = \sqrt{\frac{2k_B T}{m}} = \sqrt{\frac{2\left(1.38\times10^{-23} \text{ J/K}\right)(4.20 \text{ K})}{6.64\times10^{-27} \text{ kg}}} = \boxed{132 \text{ m/s}}$.

P21.41 (a) From $v_{av} = \sqrt{\frac{8k_B T}{\pi m}}$

we find the temperature as $T = \frac{\pi\left(6.64\times10^{-27} \text{ kg}\right)\left(1.12\times10^4 \text{ m/s}\right)^2}{8\left(1.38\times10^{-23} \text{ J/mol}\cdot\text{K}\right)} = \boxed{2.37\times10^4 \text{ K}}$

(b) $T = \frac{\pi\left(6.64\times10^{-27} \text{ kg}\right)\left(2.37\times10^3 \text{ m/s}\right)^2}{8\left(1.38\times10^{-23} \text{ J/mol}\cdot\text{K}\right)} = \boxed{1.06\times10^3 \text{ K}}$

P21.42 At 0°C, $\frac{1}{2}mv_{rms0}^2 = \frac{3}{2}k_B T_0$

At the higher temperature, $\frac{1}{2}m(2v_{rms0})^2 = \frac{3}{2}k_B T$

$$T = 4T_0 = 4(273 \text{ K}) = 1\,092 \text{ K} = \boxed{819°\text{C}}.$$

***P21.43** (a) From the Boltzmann distribution law, the number density of molecules with gravitational energy mgy is $n_0 e^{-mgy/k_BT}$. These are the molecules with height y, so this is the number per volume at height y as a function of y.

(b) $$\frac{n(y)}{n_0} = e^{-mgy/k_BT} = e^{-Mgy/N_Ak_BT} = e^{-Mgy/RT}$$

$$= e^{-(28.9\times10^{-3}\ \text{kg/mol})(9.8\ \text{m/s}^2)(11\times10^3\ \text{m})/(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})}$$

$$= e^{-1.279} = \boxed{0.278}$$

***P21.44** (a) We calculate

$$\int_0^\infty e^{-mgy/k_BT}dy = \int_{y=0}^\infty e^{-mgy/k_BT}\left(-\frac{mgdy}{k_BT}\right)\left(-\frac{k_BT}{mg}\right)$$

$$= -\frac{k_BT}{mg}e^{-mgy/k_BT}\Big|_0^\infty = -\frac{k_BT}{mg}(0-1) = \frac{k_BT}{mg}$$

Using Table B.6 in the appendix

$$\int_0^\infty ye^{-mgy/k_BT}dy = \frac{1!}{(mg/k_BT)^2} = \left(\frac{k_BT}{mg}\right)^2.$$

Then $\bar{y} = \dfrac{\int_0^\infty ye^{-mgy/k_BT}dy}{\int_0^\infty e^{-mgy/k_BT}dy} = \dfrac{(k_BT/mg)^2}{k_BT/mg} = \dfrac{k_BT}{mg}$.

(b) $$\bar{y} = \frac{k_BT}{(M/N_A)g} = \frac{RT}{Mg} = \frac{8.314\ \text{J}\ 283\ \text{K s}^2}{\text{mol}\cdot\text{K}\ 28.9\times10^{-3}\ \text{kg}\ 9.8\ \text{m}} = \boxed{8.31\times10^3\ \text{m}}$$

Section 21.7 Mean Free Path

P21.45 (a) $PV = \left(\dfrac{N}{N_A}\right)RT$ and $N = \dfrac{PVN_A}{RT}$ so that

$$N = \frac{(1.00\times10^{-10})(133)(1.00)(6.02\times10^{23})}{(8.314)(300)} = \boxed{3.21\times10^{12}\ \text{molecules}}$$

(b) $$\ell = \frac{1}{n_V\pi d^2 2^{1/2}} = \frac{V}{N\pi d^2 2^{1/2}} = \frac{1.00\ \text{m}^3}{(3.21\times10^{12}\ \text{molecules})\pi(3.00\times10^{-10}\ \text{m})^2(2)^{1/2}}$$

$$\ell = \boxed{779\ \text{km}}$$

(c) $$f = \frac{v}{\ell} = \boxed{6.42\times10^{-4}\ \text{s}^{-1}}$$

P21.46 The average molecular speed is

$$v=\sqrt{\frac{8k_BT}{\pi m}}=\sqrt{\frac{8k_BN_AT}{\pi N_Am}}$$

$$v=\sqrt{\frac{8RT}{\pi M}}$$

$$v=\sqrt{\frac{8(8.314\ \text{J/mol}\cdot\text{K})3.00\ \text{K}}{\pi\left(2.016\times10^{-3}\ \text{kg/mol}\right)}}$$

$$v=178\ \text{m/s}$$

(a) The mean free path is

$$\ell=\frac{1}{\sqrt{2}\pi d^2n_V}=\frac{1}{\sqrt{2}\pi\left(0.200\times10^{-9}\ \text{m}\right)^2\ 1/\text{m}^3}$$

$$\ell=\boxed{5.63\times10^{18}\ \text{m}}$$

The mean free time is

$$\frac{\ell}{v}=\frac{5.63\times10^{18}\ \text{m}}{178\ \text{m/s}}=3.17\times10^{16}\ \text{s}=\boxed{1.00\times10^{9}\ \text{yr}}.$$

(b) Now n_V is 10^6 times larger, to make ℓ smaller by 10^6 times:

$$\ell=\boxed{5.63\times10^{12}\ \text{m}}.$$

Thus, $\frac{\ell}{v}=3.17\times10^{10}\ \text{s}=\boxed{1.00\times10^{3}\ \text{yr}}$.

P21.47 From Equation 21.30, $\ell=\frac{1}{\sqrt{2}\pi d^2n_V}$

For an ideal gas, $n_V=\frac{N}{V}=\frac{P}{k_BT}$

Therefore, $\ell=\frac{k_BT}{\sqrt{2}\pi d^2P}$, as required.

P21.48 $\ell=\left[\sqrt{2}\pi d^2n_V\right]^{-1}$ $\qquad n_V=\frac{P}{k_BT}$

$d=3.60\times10^{-10}\ \text{m}$ $\qquad n_V=\frac{1.013\times10^5}{\left(1.38\times10^{-23}\right)(293)}=2.51\times10^{25}/\text{m}^3$

$\therefore\ \ell=6.93\times10^{-8}$ m, or about $\boxed{\text{193 molecular diameters}}$.

P21.49 Using $P = n_V k_B T$, Equation 21.30 becomes $\ell = \dfrac{k_B T}{\sqrt{2}\pi P d^2}$ (1)

(a) $$\ell = \frac{(1.38\times10^{-23}\ \text{J/K})(293\ \text{K})}{\sqrt{2}\pi(1.013\times10^{5}\ \text{Pa})(3.10\times10^{-10}\ \text{m})^2} = \boxed{9.36\times10^{-8}\ \text{m}}$$

(b) Equation (1) shows that $P_1\ell_1 = P_2\ell_2$. Taking $P_1\ell_1$ from (a) and with $\ell_2 = 1.00$ m, we find

$$P_2 = \frac{(1.00\ \text{atm})(9.36\times10^{-8}\ \text{m})}{1.00\ \text{m}} = \boxed{9.36\times10^{-8}\ \text{atm}}.$$

(c) For $\ell_3 = 3.10\times10^{-10}$ m, we have

$$P_3 = \frac{(1.00\ \text{atm})(9.36\times10^{-8}\ \text{m})}{3.10\times10^{-10}\ \text{m}} = \boxed{302\ \text{atm}}.$$

Additional Problems

P21.50 (a) $$n = \frac{PV}{RT} = \frac{(1.013\times10^{5}\ \text{Pa})(4.20\ \text{m}\times3.00\ \text{m}\times2.50\ \text{m})}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = 1.31\times10^{3}\ \text{mol}$$

$$N = nN_A = (1.31\times10^{3}\ \text{mol})(6.02\times10^{23}\ \text{molecules/mol})$$

$$N = \boxed{7.89\times10^{26}\ \text{molecules}}$$

(b) $$m = nM = (1.31\times10^{3}\ \text{mol})(0.028\,9\ \text{kg/mol}) = \boxed{37.9\ \text{kg}}$$

(c) $$\frac{1}{2}m_0v^2 = \frac{3}{2}k_BT = \frac{3}{2}(1.38\times10^{-23}\ \text{J/k})(293\ \text{K}) = \boxed{6.07\times10^{-21}\ \text{J/molecule}}$$

(d) For one molecule,

$$m_0 = \frac{M}{N_A} = \frac{0.028\,9\ \text{kg/mol}}{6.02\times10^{23}\ \text{molecules/mol}} = 4.80\times10^{-26}\ \text{kg/molecule}$$

$$v_{\text{rms}} = \sqrt{\frac{2(6.07\times10^{-21}\ \text{J/molecule})}{4.80\times10^{-26}\ \text{kg/molecule}}} = \boxed{503\ \text{m/s}}$$

(e),(f) $$E_{\text{int}} = nC_VT = n\left(\frac{5}{2}R\right)T = \frac{5}{2}PV$$

$$E_{\text{int}} = \frac{5}{2}(1.013\times10^{5}\ \text{Pa})(31.5\ \text{m}^3) = \boxed{7.98\ \text{MJ}}$$

P21.51 (a) $P_f = \boxed{100\text{ kPa}}$ $T_f = \boxed{400\text{ K}}$

$$V_f = \frac{nRT_f}{P_f} = \frac{2.00\text{ mol}(8.314\text{ J/mol}\cdot\text{K})(400\text{ K})}{100\times 10^3\text{ Pa}} = 0.066\,5\text{ m}^3 = \boxed{66.5\text{ L}}$$

$$\Delta E_{\text{int}} = (3.50)nR\Delta T = 3.50(2.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(100\text{ K}) = \boxed{5.82\text{ kJ}}$$

$$W = -P\Delta V = -nR\Delta T = -(2.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(100\text{ K}) = \boxed{-1.66\text{ kJ}}$$

$$Q = \Delta E_{\text{int}} - W = 5.82\text{ kJ} + 1.66\text{ kJ} = \boxed{7.48\text{ kJ}}$$

(b) $T_f = \boxed{400\text{ K}}$ $V_f = V_i = \dfrac{nRT_i}{P_i} = \dfrac{2.00\text{ mol}(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})}{100\times 10^3\text{ Pa}} = 0.049\,9\text{ m}^3 = \boxed{49.9\text{ L}}$

$P_f = P_i\left(\dfrac{T_f}{T_i}\right) = 100\text{ kPa}\left(\dfrac{400\text{ K}}{300\text{ K}}\right) = \boxed{133\text{ kPa}}$ $W = -\int P\,dV = \boxed{0}$ since $V = \text{constant}$

$\Delta E_{\text{int}} = \boxed{5.82\text{ kJ}}$ as in part (a) $Q = \Delta E_{\text{int}} - W = 5.82\text{ kJ} - 0 = \boxed{5.82\text{ kJ}}$

(c) $P_f = \boxed{120\text{ kPa}}$ $T_f = \boxed{300\text{ K}}$

$V_f = V_i\left(\dfrac{P_i}{P_f}\right) = 49.9\text{ L}\left(\dfrac{100\text{ kPa}}{120\text{ kPa}}\right) = \boxed{41.6\text{ L}}$ $\Delta E_{\text{int}} = (3.50)nR\Delta T = \boxed{0}$ since $T = \text{constant}$

$$W = -\int P\,dV = -nRT_i\int_{V_i}^{V_f}\frac{dV}{V} = -nRT_i\ln\left(\frac{V_f}{V_i}\right) = -nRT_i\ln\left(\frac{P_i}{P_f}\right)$$

$$W = -(2.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})\ln\left(\frac{100\text{ kPa}}{120\text{ kPa}}\right) = \boxed{+909\text{ J}}$$

$$Q = \Delta E_{\text{int}} - W = 0 - 910\text{ J} = \boxed{-909\text{ J}}$$

(d) $P_f = \boxed{120\text{ kPa}}$ $\gamma = \dfrac{C_P}{C_V} = \dfrac{C_V + R}{C_V} = \dfrac{3.50R + R}{3.50R} = \dfrac{4.50}{3.50} = \dfrac{9}{7}$

$P_f V_f^\gamma = P_i V_i^\gamma$: so $V_f = V_i\left(\dfrac{P_i}{P_f}\right)^{1/\gamma} = 49.9\text{ L}\left(\dfrac{100\text{ kPa}}{120\text{ kPa}}\right)^{7/9} = \boxed{43.3\text{ L}}$

$$T_f = T_i\left(\frac{P_f V_f}{P_i V_i}\right) = 300\text{ K}\left(\frac{120\text{ kPa}}{100\text{ kPa}}\right)\left(\frac{43.3\text{ L}}{49.9\text{ L}}\right) = \boxed{312\text{ K}}$$

$$\Delta E_{\text{int}} = (3.50)nR\Delta T = 3.50(2.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(12.4\text{ K}) = \boxed{722\text{ J}}$$

$Q = \boxed{0}$ (adiabatic process)

$$W = -Q + \Delta E_{\text{int}} = 0 + 722\text{ J} = \boxed{+722\text{ J}}$$

P21.52 (a) The average speed v_{av} is just the weighted average of all the speeds.

$$v_{av} = \frac{[2(v)+3(2v)+5(3v)+4(4v)+3(5v)+2(6v)+1(7v)]}{(2+3+5+4+3+2+1)} = \boxed{3.65v}$$

(b) First find the average of the square of the speeds,

$$v_{av}^2 = \frac{\left[2(v)^2+3(2v)^2+5(3v)^2+4(4v)^2+3(5v)^2+2(6v)^2+1(7v)^2\right]}{2+3+5+4+3+2+1} = 15.95v^2.$$

The root-mean square speed is then $v_{rms} = \sqrt{v_{av}^2} = \boxed{3.99v}$.

(c) The most probable speed is the one that most of the particles have;
i.e., five particles have speed $\boxed{3.00v}$.

(d) $PV = \frac{1}{3}Nmv_{av}^2$

Therefore, $P = \frac{20}{3}\frac{\left[m(15.95)v^2\right]}{V} = \boxed{106\left(\frac{mv^2}{V}\right)}$.

(e) The average kinetic energy for each particle is
$\overline{K} = \frac{1}{2}mv_{av}^2 = \frac{1}{2}m\left(15.95v^2\right) = \boxed{7.98mv^2}$.

P21.53 (a) $PV^\gamma = k$. So, $W = -\int_i^f PdV = -k\int_i^f \frac{dV}{V^\gamma} = \frac{P_fV_f - P_iV_i}{\gamma - 1}$

(b) $dE_{int} = dQ + dW$ and $dQ = 0$ for an adiabatic process.
Therefore, $W = +\Delta E_{int} = nC_V\left(T_f - T_i\right)$.

To show consistency between these 2 equations, consider that $\gamma = \frac{C_P}{C_V}$ and $C_P - C_V = R$.

Therefore, $\frac{1}{\gamma - 1} = \frac{C_V}{R}$.

Using this, the result found in part (a) becomes

$$W = \left(P_fV_f - P_iV_i\right)\frac{C_V}{R}.$$

Also, for an ideal gas $\frac{PV}{R} = nT$ so that $W = nC_V\left(T_f - T_i\right)$.

***P21.54** (a) $W = nC_V(T_f - T_i)$

$$-2\,500\text{ J} = 1\text{ mol}\frac{3}{2}8.314\text{ J/mol}\cdot\text{K}(T_f - 500\text{ K})$$

$$T_f = \boxed{300\text{ K}}$$

(b) $P_iV_i^\gamma = P_fV_f^\gamma$

$$P_i\left(\frac{nRT_i}{P_i}\right)^\gamma = P_f\left(\frac{nRT_f}{P_f}\right)^\gamma \qquad T_i^\gamma P_i^{1-\gamma} = T_f^\gamma P_f^{1-\gamma}$$

$$\frac{T_i^{\gamma/(\gamma-1)}}{P_i} = \frac{T_f^{\gamma/(\gamma-1)}}{P_f} \qquad P_f = P_i\left(\frac{T_f}{T_i}\right)^{\gamma/(\gamma-1)}$$

$$P_f = P_i\left(\frac{T_f}{T_i}\right)^{(5/3)(3/2)} = 3.60\text{ atm}\left(\frac{300}{500}\right)^{5/2} = \boxed{1.00\text{ atm}}$$

***P21.55** Let the subscripts '1' and '2' refer to the hot and cold compartments, respectively. The pressure is higher in the hot compartment, therefore the hot compartment expands and the cold compartment contracts. The work done by the adiabatically expanding gas is equal and opposite to the work done by the adiabatically compressed gas.

$$\frac{nR}{\gamma-1}(T_{1i} - T_{1f}) = -\frac{nR}{\gamma-1}(T_{2i} - T_{2f})$$

$$\therefore T_{1f} + T_{2f} = T_{1i} + T_{2i} = 800\text{ K} \qquad (1)$$

Consider the adiabatic changes of the gases.

$P_{1i}V_{1i}^\gamma = P_{1f}V_{1f}^\gamma$ and $P_{2i}V_{2i}^\gamma = P_{2f}V_{2f}^\gamma$

$$\therefore \frac{P_{1i}V_{1i}^\gamma}{P_{2i}V_{2i}^\gamma} = \frac{P_{1f}V_{1f}^\gamma}{P_{2f}V_{2f}^\gamma}$$

$$\therefore \frac{P_{1i}}{P_{2i}} = \left(\frac{V_{1f}}{V_{2f}}\right)^\gamma, \text{ since } V_{1i} = V_{2i} \text{ and } P_{1f} = P_{2f}$$

$$\therefore \frac{nRT_{1i}/V_{1i}}{nRT_{2i}/V_{2i}} = \left(\frac{nRT_{1f}/P_{1f}}{nRT_{2f}/P_{2f}}\right)^\gamma, \text{ using the ideal gas law}$$

$$\therefore \frac{T_{1i}}{T_{2i}} = \left(\frac{T_{1f}}{T_{2f}}\right)^\gamma, \text{ since } V_{1i} = V_{2i} \text{ and } P_{1f} = P_{2f}$$

$$\therefore \frac{T_{1f}}{T_{2f}} = \left(\frac{T_{1i}}{T_{2i}}\right)^{1/\gamma} = \left(\frac{550\text{ K}}{250\text{ K}}\right)^{1/1.4} = 1.756 \quad (2)$$

Solving equations (1) and (2) simultaneously gives

$$\boxed{T_{1f} = 510\text{ K},\ T_{2f} = 290\text{ K}}.$$

***P21.56** The work done by the gas on the bullet becomes its kinetic energy:

$$\frac{1}{2}mv^2 = \frac{1}{2}1.1\times10^{-3}\text{ kg}(120\text{ m/s})^2 = 7.92\text{ J}.$$

The work on the gas is

$$\frac{1}{\gamma-1}\left(P_fV_f - P_iV_i\right) = -7.92\text{ J}.$$

Also $P_fV_f^\gamma = P_iV_i^\gamma$ $\quad P_f = P_i\left(\frac{V_i}{V_f}\right)^\gamma$.

So $-7.92\text{ J} = \frac{1}{0.40}P_i\left[V_f\left(\frac{V_i}{V_f}\right)^\gamma - V_i\right]$.

And $V_f = 12\text{ cm}^3 + 50\text{ cm }0.03\text{ cm}^2 = 13.5\text{ cm}^3$.

Then $P_i = \frac{-7.92\text{ J}(0.40)10^6\text{ cm}^3/\text{m}^3}{\left[13.5\text{ cm}^3\left(\frac{12}{13.5}\right)^{1.40} - 12\text{ cm}^3\right]} = \boxed{5.74\times10^6\text{ Pa}} = 56.6\text{ atm}$.

P21.57 The pressure of the gas in the lungs of the diver must be the same as the absolute pressure of the water at this depth of 50.0 meters. This is:

$$P = P_0 + \rho gh = 1.00\text{ atm} + \left(1.03\times10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)(50.0\text{ m})$$

or $\quad P = 1.00\text{ atm} + 5.05\times10^5\text{ Pa}\left(\frac{1.00\text{ atm}}{1.013\times10^5\text{ Pa}}\right) = 5.98\text{ atm}$

If the partial pressure due to the oxygen in the gas mixture is to be 1.00 atmosphere (or the fraction $\frac{1}{5.98}$ of the total pressure) oxygen molecules should make up only $\frac{1}{5.98}$ of the total number of molecules. This will be true if 1.00 mole of oxygen is used for every 4.98 mole of helium. The ratio by weight is then

$$\frac{(4.98\text{ mol He})(4.003\text{ g/mol He})g}{(1.00\text{ mol O}_2)(2\times15.999\text{ g/mol O}_2)g} = \boxed{0.623}.$$

P21.58 (a) Maxwell's speed distribution function is

$$N_v = 4\pi N\left(\frac{m}{2\pi k_BT}\right)^{3/2} v^2e^{-mv^2/2k_BT}$$

With $\quad N = 1.00\times10^4$,

$m = \frac{M}{N_A} = \frac{0.032\text{ kg}}{6.02\times10^{23}} = 5.32\times10^{-26}\text{ kg}$

$T = 500\text{ K}$

and $\quad k_B = 1.38\times10^{-23}\text{ J/molecule}\cdot\text{K}$

this becomes $\quad N_v = \left(1.71\times10^{-4}\right)v^2e^{-\left(3.85\times10^{-6}\right)v^2}$

To the right is a plot of this function for the range $0 \le v \le 1500$ m/s.

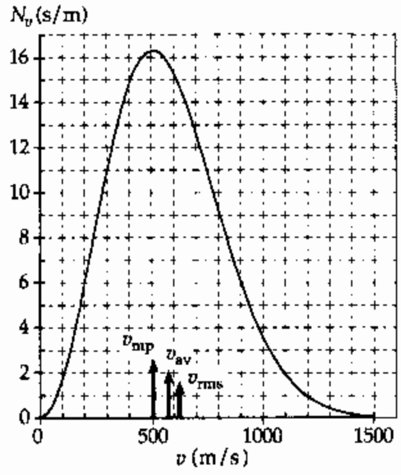

FIG. P21.58(a)

continued on next page

(b) The most probable speed occurs where N_v is a maximum.

From the graph, $\boxed{v_{mp} \approx 510 \text{ m/s}}$

(c) $$v_{av} = \sqrt{\frac{8k_BT}{\pi m}} = \sqrt{\frac{8(1.38\times10^{-23})(500)}{\pi(5.32\times10^{-26})}} = \boxed{575 \text{ m/s}}$$

Also,

$$v_{rms} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3(1.38\times10^{-23})(500)}{5.32\times10^{-26}}} = \boxed{624 \text{ m/s}}$$

(d) The fraction of particles in the range $300 \text{ m/s} \le v \le 600 \text{ m/s}$

is $$\frac{\int_{300}^{600} N_v dv}{N}$$

where $N = 10^4$

and the integral of N_v is read from the graph as the area under the curve.

This is approximately 4 400 and the fraction is 0.44 or $\boxed{44\%}$.

P21.59 (a) Since $\boxed{\text{pressure increases as volume decreases}}$ (and vice versa),

$$\frac{dV}{dP} < 0 \text{ and } -\frac{1}{V}\left[\frac{dV}{dP}\right] > 0.$$

(b) For an ideal gas, $V = \frac{nRT}{P}$ and $\kappa_1 = -\frac{1}{V}\frac{d}{dP}\left(\frac{nRT}{P}\right)$.

If the compression is isothermal, T is constant and

$$\kappa_1 = -\frac{nRT}{V}\left(-\frac{1}{P^2}\right) = \frac{1}{P}.$$

(c) For an adiabatic compression, $PV^\gamma = C$ (where C is a constant) and

$$\kappa_2 = -\frac{1}{V}\frac{d}{dP}\left(\frac{C}{P}\right)^{1/\gamma} = \frac{1}{V}\left(\frac{1}{\gamma}\right)\frac{C^{1/\gamma}}{P^{(1/\gamma)+1}} = \frac{P^{1/\gamma}}{\gamma P^{1/\gamma+1}} = \frac{1}{\gamma P}.$$

(d) $$\kappa_1 = \frac{1}{P} = \frac{1}{(2.00 \text{ atm})} = \boxed{0.500 \text{ atm}^{-1}}$$

$\gamma = \frac{C_P}{C_V}$ and for a monatomic ideal gas, $\gamma = \frac{5}{3}$, so that

$$\kappa_2 = \frac{1}{\gamma P} = \frac{1}{\frac{5}{3}(2.00 \text{ atm})} = \boxed{0.300 \text{ atm}^{-1}}$$

P21.60 (a) The speed of sound is $v=\sqrt{\frac{B}{\rho}}$ where $B=-V\frac{dP}{dV}$.

According to Problem 59, in an adiabatic process, this is $B=\frac{1}{\kappa_2}=\gamma P$.

Also, $\rho=\frac{m_s}{V}=\frac{nM}{V}=\frac{(nRT)M}{V(RT)}=\frac{PM}{RT}$ where m_s is the sample mass. Then, the speed of sound in the ideal gas is $v=\sqrt{\frac{B}{\rho}}=\sqrt{\gamma P\left(\frac{RT}{PM}\right)}=\boxed{\sqrt{\frac{\gamma RT}{M}}}$.

(b) $v=\sqrt{\frac{1.40(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})}{0.028\,9\ \text{kg/mol}}}=\boxed{344\ \text{m/s}}$

This nearly agrees with the 343 m/s listed in Table 17.1.

(c) We use $k_B=\frac{R}{N_A}$ and $M=mN_A$: $v=\sqrt{\frac{\gamma RT}{M}}=\sqrt{\frac{\gamma k_B N_A T}{mN_A}}=\sqrt{\frac{\gamma k_B T}{m}}$.

The most probable molecular speed is $\sqrt{\frac{2k_BT}{m}}$,

the average speed is $\sqrt{\frac{8k_BT}{\pi m}}$, and the rms speed is $\sqrt{\frac{3k_BT}{m}}$.

$\boxed{\text{All are somewhat larger}}$ than the speed of sound.

P21.61 $n=\frac{m}{M}=\frac{1.20\ \text{kg}}{0.028\,9\ \text{kg/mol}}=41.5\ \text{mol}$

(a) $V_i=\frac{nRT_i}{P_i}=\frac{(41.5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(298\ \text{K})}{200\times10^3\ \text{Pa}}=\boxed{0.514\ \text{m}^3}$

(b) $\frac{P_f}{P_i}=\frac{\sqrt{V_f}}{\sqrt{V_i}}$ so $V_f=V_i\left(\frac{P_f}{P_i}\right)^2=\left(0.514\ \text{m}^3\right)\left(\frac{400}{200}\right)^2=\boxed{2.06\ \text{m}^3}$

(c) $T_f=\frac{P_fV_f}{nR}=\frac{\left(400\times10^3\ \text{Pa}\right)\left(2.06\ \text{m}^3\right)}{(41.5\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})}=\boxed{2.38\times10^3\ \text{K}}$

(d) $W=-\int_{V_i}^{V_f}PdV=-C\int_{V_i}^{V_f}V^{1/2}dV=-\left(\frac{P_i}{V_i^{1/2}}\right)\frac{2V^{3/2}}{3}\Bigg|_{V_i}^{V_f}=-\frac{2}{3}\left(\frac{P_i}{V_i^{1/2}}\right)\left(V_f^{3/2}-V_i^{3/2}\right)$

$W=-\frac{2}{3}\left(\frac{200\times10^3\ \text{Pa}}{\sqrt{0.514\ \text{m}}}\right)\left[\left(2.06\ \text{m}^3\right)^{3/2}-(0.514\ \text{m})^{3/2}\right]=\boxed{-4.80\times10^5\ \text{J}}$

(e) $\Delta E_{\text{int}}=nC_V\Delta T=(41.5\ \text{mol})\left[\frac{5}{2}(8.314\ \text{J/mol}\cdot\text{K})\right]\left(2.38\times10^3-298\right)\text{K}$

$\Delta E_{\text{int}}=1.80\times10^6\ \text{J}$

$Q=\Delta E_{\text{int}}-W=1.80\times10^6\ \text{J}+4.80\times10^5\ \text{J}=2.28\times10^6\ \text{J}=\boxed{2.28\ \text{MJ}}$

P21.62 The ball loses energy $\frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2 = \frac{1}{2}(0.142\text{ kg})\left[(47.2)^2 - (42.5)^2\right]\text{ m}^2/\text{s}^2 = 29.9\text{ J}$

The air volume is $V = \pi(0.037\,0\text{ m})^2(19.4\text{ m}) = 0.083\,4\text{ m}^3$

and its quantity is $n = \frac{PV}{RT} = \frac{1.013\times10^5\text{ Pa}(0.083\,4\text{ m}^3)}{(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})} = 3.47\text{ mol}$

The air absorbs energy according to

$$Q = nC_P\Delta T$$

So $\Delta T = \frac{Q}{nC_P} = \frac{29.9\text{ J}}{3.47\text{ mol}\left(\frac{7}{2}\right)(8.314\text{ J/mol}\cdot\text{K})} = \boxed{0.296°\text{C}}$

P21.63 $N_v(v) = 4\pi N\left(\frac{m}{2\pi k_BT}\right)^{3/2} v^2\exp\left(\frac{-mv^2}{2k_BT}\right)$

Note that $v_{\text{mp}} = \left(\frac{2k_BT}{m}\right)^{1/2}$

Thus, $N_v(v) = 4\pi N\left(\frac{m}{2\pi k_BT}\right)^{3/2} v^2 e^{\left(-v^2/v_{\text{mp}}^2\right)}$

And $\frac{N_v(v)}{N_v(v_{\text{mp}})} = \left(\frac{v}{v_{\text{mp}}}\right)^2 e^{\left(1-v^2/v_{\text{mp}}^2\right)}$

For $v = \frac{v_{\text{mp}}}{50}$

$$\frac{N_v(v)}{N_v(v_{\text{mp}})} = \left(\frac{1}{50}\right)^2 e^{\left[1-(1/50)^2\right]} = 1.09\times10^{-3}$$

The other values are computed similarly, with the following results:

$\frac{v}{v_{\text{mp}}}$	$\frac{N_v(v)}{N_v(v_{\text{mp}})}$
$\frac{1}{50}$	1.09×10^{-3}
$\frac{1}{10}$	2.69×10^{-2}
$\frac{1}{2}$	0.529
1	1.00
2	0.199
10	1.01×10^{-41}
50	$1.25\times10^{-1\,082}$

To find the last value, note:

$$(50)^2 e^{1-2\,500} = 2\,500e^{-2\,499}$$

$$10^{\log 2\,500}e^{(\ln 10)(-2\,499/\ln 10)} = 10^{\log 2\,500}10^{-2\,499/\ln 10} = 10^{\log 2\,500 - 2\,499/\ln 10} = 10^{-1\,081.904}$$

P21.64 (a) The effect of high angular speed is like the effect of a very high gravitational field on an atmosphere. The result is:

The larger-mass molecules settle to the outside while the region at smaller r has a higher concentration of low-mass molecules.

(b) Consider a single kind of molecules, all of mass m. To cause the centripetal acceleration of the molecules between r and $r+dr$, the pressure must increase outward according to $\sum F_r = ma_r$. Thus,

$$PA-(P+dP)A = -(nmA\,dr)(r\omega^2)$$

where n is the number of molecules per unit volume and A is the area of any cylindrical surface. This reduces to $dP = nm\omega^2 r dr$.

But also $P = nk_BT$, so $dP = k_BTdn$. Therefore, the equation becomes

$$\frac{dn}{n} = \frac{m\omega^2}{k_BT}rdr \text{ giving } \int_{n_0}^{n}\frac{dn}{n} = \frac{m\omega^2}{k_BT}\int_0^r rdr \text{ or}$$

$$\ln(n)\Big|_{n_0}^{n} = \frac{m\omega^2}{k_BT}\left(\frac{r^2}{2}\right)\Bigg|_0^r$$

$$\ln\left(\frac{n}{n_0}\right) = \frac{m\omega^2}{2k_BT}r^2 \text{ and solving for } n\text{: } \boxed{n = n_0 e^{mr^2\omega^2/2k_BT}}.$$

P21.65 First find v_{av}^2 as $v_{av}^2 = \frac{1}{N}\int_0^\infty v^2 N_v dv$. Let $a = \frac{m}{2k_BT}$.

Then, $v_{av}^2 = \frac{\left[4N\pi^{-1/2}a^{3/2}\right]}{N}\int_0^\infty v^4 e^{-av^2 dv} = \left[4a^{3/2}\pi^{-1/2}\right]\frac{3}{8a^2}\sqrt{\frac{\pi}{a}} = \frac{3k_BT}{m}$

The root-mean square speed is then $v_{rms} = \sqrt{v_{av}^2} = \boxed{\sqrt{\frac{3k_BT}{m}}}$.

To find the average speed, we have

$$v_{av} = \frac{1}{N}\int_0^\infty vN_v dv = \frac{\left(4Na^{3/2}\pi^{-1/2}\right)}{N}\int_0^\infty v^3 e^{-av^2}dv = \frac{4a^{3/2}\pi^{-1/2}}{2a^2} = \boxed{\sqrt{\frac{8k_BT}{\pi m}}}.$$

***P21.66** We want to evaluate $\frac{dP}{dV}$ for the function implied by $PV = nRT = \text{constant}$, and also for the different function implied by $PV^\gamma = \text{constant}$. We can use implicit differentiation:

From $PV = \text{constant}$ $\quad P\frac{dV}{dV}+V\frac{dP}{dV} = 0 \quad \left(\frac{dP}{dV}\right)_{\text{isotherm}} = -\frac{P}{V}$

From $PV^\gamma = \text{constant}$ $\quad P\gamma V^{\gamma-1}+V^\gamma\frac{dP}{dV} = 0 \quad \left(\frac{dP}{dV}\right)_{\text{adiabat}} = -\frac{\gamma P}{V}$

Therefore, $\quad \left(\frac{dP}{dV}\right)_{\text{adiabat}} = \gamma\left(\frac{dP}{dV}\right)_{\text{isotherm}}$

The theorem is proved.

P21.67 (a) $n = \frac{PV}{RT} = \frac{(1.013 \times 10^5 \text{ Pa})(5.00 \times 10^{-3} \text{ m}^3)}{(8.314 \text{ J/mol}\cdot\text{K})(300 \text{ K})} = \boxed{0.203 \text{ mol}}$

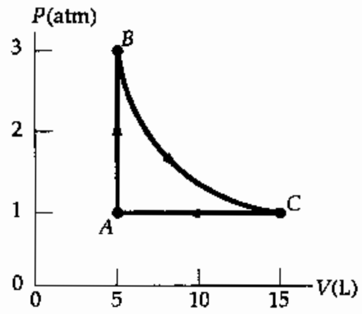

FIG. P21.67

(b) $T_B = T_A\left(\frac{P_B}{P_A}\right) = 300 \text{ K}\left(\frac{3.00}{1.00}\right) = \boxed{900 \text{ K}}$

$T_C = T_B = \boxed{900 \text{ K}}$

$V_C = V_A\left(\frac{T_C}{T_A}\right) = 5.00 \text{ L}\left(\frac{900}{300}\right) = \boxed{15.0 \text{ L}}$

(c) $E_{\text{int},A} = \frac{3}{2}nRT_A = \frac{3}{2}(0.203 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(300 \text{ K}) = \boxed{760 \text{ J}}$

$E_{\text{int},B} = E_{\text{int},C} = \frac{3}{2}nRT_B = \frac{3}{2}(0.203 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(900 \text{ K}) = \boxed{2.28 \text{ kJ}}$

(d)

	P (atm)	V(L)	T(K)	E_{int} (kJ)
A	1.00	5.00	300	0.760
B	3.00	5.00	900	2.28
C	1.00	15.00	900	2.28

(e) For the process *AB*, lock the piston in place and put the cylinder into an oven at 900 K. For *BC*, keep the sample in the oven while gradually letting the gas expand to lift a load on the piston as far as it can. For *CA*, carry the cylinder back into the room at 300 K and let the gas cool without touching the piston.

(f) For *AB*: $W = \boxed{0}$ $\quad \Delta E_{\text{int}} = E_{\text{int},B} - E_{\text{int},A} = (2.28 - 0.760) \text{ kJ} = \boxed{1.52 \text{ kJ}}$

$Q = \Delta E_{\text{int}} - W = \boxed{1.52 \text{ kJ}}$

For *BC*: $\Delta E_{\text{int}} = \boxed{0}$, $W = -nRT_B \ln\left(\frac{V_C}{V_B}\right)$

$W = -(0.203 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(900 \text{ K})\ln(3.00) = \boxed{-1.67 \text{ kJ}}$

$Q = \Delta E_{\text{int}} - W = \boxed{1.67 \text{ kJ}}$

For *CA*: $\Delta E_{\text{int}} = E_{\text{int},A} - E_{\text{int},C} = (0.760 - 2.28) \text{ kJ} = \boxed{-1.52 \text{ kJ}}$

$W = -P\Delta V = -nR\Delta T = -(0.203 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(-600 \text{ K}) = \boxed{1.01 \text{ kJ}}$

$Q = \Delta E_{\text{int}} - W = -1.52 \text{ kJ} - 1.01 \text{ kJ} = \boxed{-2.53 \text{ kJ}}$

(g) We add the amounts of energy for each process to find them for the whole cycle.

$Q_{ABCA} = +1.52 \text{ kJ} + 1.67 \text{ kJ} - 2.53 \text{ kJ} = \boxed{0.656 \text{ kJ}}$

$W_{ABCA} = 0 - 1.67 \text{ kJ} + 1.01 \text{ kJ} = \boxed{-0.656 \text{ kJ}}$

$(\Delta E_{\text{int}})_{ABCA} = +1.52 \text{ kJ} + 0 - 1.52 \text{ kJ} = \boxed{0}$

P21.68 (a) $(10\,000\text{ g})\left(\dfrac{1.00\text{ mol}}{18.0\text{ g}}\right)\left(\dfrac{6.02\times10^{23}\text{ molecules}}{1.00\text{ mol}}\right)=\boxed{3.34\times10^{26}\text{ molecules}}$

(b) After one day, 10^{-1} of the original molecules would remain. After two days, the fraction would be 10^{-2}, and so on. After 26 days, only 3 of the original molecules would likely remain, and after $\boxed{27\text{ days}}$, likely none.

(c) The soup is this fraction of the hydrosphere: $\left(\dfrac{10.0\text{ kg}}{1.32\times10^{21}\text{ kg}}\right)$.

Therefore, today's soup likely contains this fraction of the original molecules. The number of original molecules likely in the pot again today is:

$$\left(\frac{10.0\text{ kg}}{1.32\times10^{21}\text{ kg}}\right)(3.34\times10^{26}\text{ molecules})=\boxed{2.53\times10^{6}\text{ molecules}}.$$

P21.69 (a) For escape, $\frac{1}{2}mv^2=\frac{GmM}{R_E}$. Since the free-fall acceleration at the surface is $g=\frac{GM}{R_E^2}$, this can also be written as: $\frac{1}{2}mv^2=\frac{GmM}{R_E}=\boxed{mgR_E}$.

(b) For O_2, the mass of one molecule is

$$m=\frac{0.032\,0\text{ kg/mol}}{6.02\times10^{23}\text{ molecules/mol}}=5.32\times10^{-26}\text{ kg/molecule}.$$

Then, if $mgR_E=10\left(\frac{3k_BT}{2}\right)$, the temperature is

$$T=\frac{mgR_E}{15k_B}=\frac{(5.32\times10^{-26}\text{ kg})(9.80\text{ m/s}^2)(6.37\times10^{6}\text{ m})}{15(1.38\times10^{-23}\text{ J/mol}\cdot\text{K})}=\boxed{1.60\times10^{4}\text{ K}}.$$

P21.70 (a) For sodium atoms (with a molar mass $M=32.0$ g/mol)

$$\frac{1}{2}mv^2=\frac{3}{2}k_BT$$

$$\frac{1}{2}\left(\frac{M}{N_A}\right)v^2=\frac{3}{2}k_BT$$

$$v_{rms}=\sqrt{\frac{3RT}{M}}=\sqrt{\frac{3(8.314\text{ J/mol}\cdot\text{K})(2.40\times10^{-4}\text{ K})}{23.0\times10^{-3}\text{ kg}}}=\boxed{0.510\text{ m/s}}$$

(b) $t=\dfrac{d}{v_{rms}}=\dfrac{0.010\text{ m}}{0.510\text{ m/s}}=\boxed{20\text{ ms}}$

ANSWERS TO EVEN PROBLEMS

P21.2 17.6 kPa

P21.4 5.05×10^{-21} J/molecule

P21.6 6.64×10^{-27} kg

P21.8 477 m/s

P21.10 (a) 2.28 kJ; (b) 6.21×10^{-21} J

P21.12 74.8 J

P21.14 7.52 L

P21.16 (a) 118 kJ; (b) 6.03×10^{3} kg

P21.18 (a) 719 J/kg·K; (b) 0.811 kg; (c) 233 kJ; (d) 327 kJ

P21.20 $13.5PV$

P21.22 (a) $4T_i$; (b) $9(1\text{ mol})RT_i$

P21.24 (a) 0.118; (b) 2.35; (c) 0; 135 J; 135 J

P21.26 (a) 5.15×10^{-5} m^3; (b) 560 K; (c) 2.24 K

P21.28 (a) 1.55; (b) 0.127 m^3

P21.30 (a) see the solution; (b) $2.19V_i$; (c) $3T_i$; (d) T_i; (e) $-0.830P_iV_i$

P21.32 25.0 kW

P21.34 see the solution

P21.36 (a) No atom, almost all the time; (b) 2.70×10^{20}

P21.38 (a) 1.03; (b) ^{35}Cl

P21.40 132 m/s

P21.42 819°C

P21.44 (a) see the solution; (b) 8.31 km

P21.46 (a) 5.63×10^{18} m; 1.00×10^{9} yr; (b) 5.63×10^{12} m; 1.00×10^{3} yr

P21.48 193 molecular diameters

P21.50 (a) 7.89×10^{26} molecules; (b) 37.9 kg; (c) 6.07×10^{-21} J/molecule; (d) 503 m/s; (e) 7.98 MJ; (f) 7.98 MJ

P21.52 (a) $3.65v$; (b) $3.99v$; (c) $3.00v$; (d) $106\left(\frac{mv^2}{V}\right)$; (e) $7.98mv^2$

P21.54 (a) 300 K; (b) 1.00 atm

P21.56 5.74×10^{6} Pa

P21.58 (a) see the solution; (b) 5.1×10^{2} m/s; (c) $v_{av} = 575$ m/s; $v_{rms} = 624$ m/s; (d) 44%

P21.60 (a) see the solution; (b) 344 m/s nearly agreeing with the tabulated value; (c) see the solution; somewhat smaller than each

P21.62 0.296°C

P21.64 see the solution

P21.66 see the solution

P21.68 (a) 3.34×10^{26} molecules; (b) during the 27th day; (c) 2.53×10^{6} molecules

P21.70 (a) 0.510 m/s; (b) 20 ms

22

Heat Engines, Entropy, and the Second Law of Thermodynamics

CHAPTER OUTLINE

ANSWERS TO QUESTIONS

Q22.1 First, the efficiency of the automobile engine cannot exceed the Carnot efficiency: it is limited by the temperature of burning fuel and the temperature of the environment into which the exhaust is dumped. Second, the engine block cannot be allowed to go over a certain temperature. Third, any practical engine has friction, incomplete burning of fuel, and limits set by timing and energy transfer by heat.

Q22.2 It is easier to control the temperature of a hot reservoir. If it cools down, then heat can be added through some external means, like an exothermic reaction. If it gets too hot, then heat can be allowed to "escape" into the atmosphere. To maintain the temperature of a cold reservoir, one must remove heat if the reservoir gets too hot. Doing this requires either an "even colder" reservoir, which you also must maintain, or an endothermic process.

Q22.3 A higher steam temperature means that more energy can be extracted from the steam. For a constant temperature heat sink at T_c, and steam at T_h, the efficiency of the power plant goes as $\frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h}$ and is maximized for a high T_h.

Q22.4 No. Any heat engine takes in energy by heat and must also put out energy by heat. The energy that is dumped as exhaust into the low-temperature sink will always be thermal pollution in the outside environment. So-called 'steady growth' in human energy use cannot continue.

Q22.5 No. The first law of thermodynamics is a statement about energy conservation, while the second is a statement about stable thermal equilibrium. They are by no means mutually exclusive. For the particular case of a cycling heat engine, the first law implies $|Q_h| = W_{eng} + |Q_c|$, and the second law implies $|Q_c| > 0$.

Q22.6 Take an automobile as an example. According to the first law or the idea of energy conservation, it must take in all the energy it puts out. Its energy source is chemical energy in gasoline. During the combustion process, some of that energy goes into moving the pistons and eventually into the mechanical motion of the car. Clearly much of the energy goes into heat, which, through the cooling system, is dissipated into the atmosphere. Moreover, there are numerous places where friction, both mechanical and fluid, turns mechanical energy into heat. In even the most efficient internal combustion engine cars, less than 30% of the energy from the fuel actually goes into moving the car. The rest ends up as useless heat in the atmosphere.

Q22.7 Suppose the ambient temperature is 20°C. A gas can be heated to the temperature of the bottom of the pond, and allowed to cool as it blows through a turbine. The Carnot efficiency of such an engine is about $e_c = \frac{\Delta T}{T_h} = \frac{80}{373} = 22\%$.

Q22.8 No, because the work done to run the heat pump represents energy transferred into the house by heat.

Q22.9 A slice of hot pizza cools off. Road friction brings a skidding car to a stop. A cup falls to the floor and shatters. Your cat dies. Any process is irreversible if it looks funny or frightening when shown in a videotape running backwards. The free flight of a projectile is nearly reversible.

Q22.10 Below the frost line, the winter temperature is much higher than the air or surface temperature. The earth is a huge reservoir of internal energy, but digging a lot of deep trenches is much more expensive than setting a heat-exchanger out on a concrete pad. A heat pump can have a much higher coefficient of performance when it is transferring energy by heat between reservoirs at close to the same temperature.

Q22.11 (a) When the two sides of the semiconductor are at different temperatures, an electric potential (voltage) is generated across the material, which can drive electric current through an external circuit. The two cups at 50°C contain the same amount of internal energy as the pair of hot and cold cups. But no energy flows by heat through the converter bridging between them and no voltage is generated across the semiconductors.

(b) A heat engine must put out exhaust energy by heat. The cold cup provides a sink to absorb output or wasted energy by heat, which has nowhere to go between two cups of equally warm water.

Q22.12 Energy flows by heat from a hot bowl of chili into the cooler surrounding air. Heat lost by the hot stuff is equal to heat gained by the cold stuff, but the entropy decrease of the hot stuff is less than the entropy increase of the cold stuff. As you inflate a soft car tire at a service station, air from a tank at high pressure expands to fill a larger volume. That air increases in entropy and the surrounding atmosphere undergoes no significant entropy change. The brakes of your car get warm as you come to a stop. The shoes and drums increase in entropy and nothing loses energy by heat, so nothing decreases in entropy.

Q22.13 (a) For an expanding ideal gas at constant temperature, $\Delta S = \frac{\Delta Q}{T} = nR\ln\left(\frac{V_2}{V_1}\right)$.

(b) For a reversible adiabatic expansion $\Delta Q = 0$, and $\Delta S = 0$. An ideal gas undergoing an irreversible adiabatic expansion can have any positive value for ΔS up to the value given in part (a).

Q22.14 The rest of the Universe must have an entropy change of +8.0 J/K, or more.

Q22.15 Even at essentially constant temperature, energy must flow by heat out of the solidifying sugar into the surroundings, to raise the entropy of the environment. The water molecules become less ordered as they leave the liquid in the container to mix into the whole atmosphere and hydrosphere. Thus the entropy of the surroundings increases, and the second law describes the situation correctly.

Q22.16 To increase its entropy, raise its temperature. To decrease its entropy, lower its temperature. "Remove energy from it by heat" is not such a good answer, for if you hammer on it or rub it with a blunt file and at the same time remove energy from it by heat into a constant temperature bath, its entropy can stay constant.

Q22.17 An analogy used by Carnot is instructive: A waterfall continuously converts mechanical energy into internal energy. It continuously creates entropy as the organized motion of the falling water turns into disorganized molecular motion. We humans put turbines into the waterfall, diverting some of the energy stream to our use. Water flows spontaneously from high to low elevation and energy spontaneously flows by heat from high to low temperature. Into the great flow of solar radiation from Sun to Earth, living things put themselves. They live on energy flow, more than just on energy. A basking snake diverts energy from a high-temperature source (the Sun) through itself temporarily, before the energy inevitably is radiated from the body of the snake to a low-temperature sink (outer space). A tree builds organized cellulose molecules and we build libraries and babies who look like their grandmothers, all out of a thin diverted stream in the universal flow of energy crashing down to disorder. We do not violate the second law, for we build local reductions in the entropy of one thing within the inexorable increase in the total entropy of the Universe. Your roommate's exercise puts energy into the room by heat.

Q22.18 (a) Entropy increases as the yeast dies and as energy is transferred from the hot oven into the originally cooler dough and then from the hot bread into the surrounding air.

(b) Entropy increases some more as you metabolize the starches, converting chemical energy into internal energy.

Q22.19 Either statement can be considered an instructive analogy. We choose to take the first view. All processes require energy, either as energy content or as energy input. The kinetic energy which it possessed at its formation continues to make the Earth go around. Energy released by nuclear reactions in the core of the Sun drives weather on the Earth and essentially all processes in the biosphere. The energy intensity of sunlight controls how lush a forest or jungle can be and how warm a planet is. Continuous energy input is not required for the motion of the planet. Continuous energy input is required for life because energy tends to be continuously degraded, as heat flows into lower-temperature sinks. The continuously increasing entropy of the Universe is the index to energy-transfers completed.

Q22.20 The statement is not true. Although the probability is not exactly zero that this will happen, the probability of the concentration of air in one corner of the room is very nearly zero. If some billions of molecules are heading toward that corner just now, other billions are heading away from the corner in their random motion. Spontaneous compression of the air would violate the second law of thermodynamics. It would be a spontaneous departure from thermal and mechanical equilibrium.

Q22.21 Shaking opens up spaces between jellybeans. The smaller ones more often can fall down into spaces below them. The accumulation of larger candies on top and smaller ones on the bottom implies a small increase in order, a small decrease in one contribution to the total entropy, but the second law is not violated. The total entropy increases as the system warms up, its increase in internal energy coming from the work put into shaking the box and also from a bit of gravitational energy loss as the beans settle compactly together.

SOLUTIONS TO PROBLEMS

Section 22.1 Heat Engines and the Second Law of Thermodynamics

P22.1 (a) $e=\dfrac{W_{\text{eng}}}{|Q_h|}=\dfrac{25.0\text{ J}}{360\text{ J}}=\boxed{0.069\,4}$ or $\boxed{6.94\%}$

(b) $|Q_c|=|Q_h|-W_{\text{eng}}=360\text{ J}-25.0\text{ J}=\boxed{335\text{ J}}$

P22.2 $W_{\text{eng}}=|Q_h|-|Q_c|=200\text{ J}$ (1)

$e=\dfrac{W_{\text{eng}}}{|Q_h|}=1-\dfrac{|Q_c|}{|Q_h|}=0.300$ (2)

From (2), $|Q_c|=0.700|Q_h|$ (3)

Solving (3) and (1) simultaneously,

we have

(a) $\boxed{|Q_h|=667\text{ J}}$ and

(b) $\boxed{|Q_c|=467\text{ J}}$.

P22.3 (a) We have $e=\dfrac{W_{\text{eng}}}{|Q_h|}=\dfrac{|Q_h|-|Q_c|}{|Q_h|}=1-\dfrac{|Q_c|}{|Q_h|}=0.250$

with $|Q_c|=8\,000\text{ J}$, we have $|Q_h|=\boxed{10.7\text{ kJ}}$

(b) $W_{\text{eng}}=|Q_h|-|Q_c|=2\,667\text{ J}$

and from $\mathcal{P}=\dfrac{W_{\text{eng}}}{\Delta t}$, we have $\Delta t=\dfrac{W_{\text{eng}}}{\mathcal{P}}=\dfrac{2\,667\text{ J}}{5\,000\text{ J/s}}=\boxed{0.533\text{ s}}$.

***P22.4** We have $Q_{hx}=4Q_{hy}$, $W_{\text{eng}\,x}=2W_{\text{eng}\,y}$ and $Q_{cx}=7Q_{cy}$. As well as $Q_{hx}=W_{\text{eng}\,x}+Q_{cx}$ and $Q_{hy}=W_{\text{eng}\,y}+Q_{cy}$. Substituting, $4Q_{hy}=2W_{\text{eng}\,y}+7Q_{cy}$

$4Q_{hy}=2W_{\text{eng}\,y}+7Q_{hy}-7W_{\text{eng}\,y}$
$5W_{\text{eng}\,y}=3Q_{hy}$

(b) $e_y=\dfrac{W_{\text{eng}\,y}}{Q_{hy}}=\dfrac{3}{5}=\boxed{60.0\%}$

(a) $e_x=\dfrac{W_{\text{eng}\,x}}{Q_{hx}}=\dfrac{2W_{\text{eng}\,y}}{4Q_{hy}}=\dfrac{2}{4}(0.600)=0.300=\boxed{30.0\%}$

***P22.5** (a) The input energy each hour is

$$\left(7.89\times10^{3}\ \text{J/revolution}\right)\left(2\,500\ \text{rev/min}\right)\frac{60\ \text{min}}{1\ \text{h}}=1.18\times10^{9}\ \text{J/h}$$

implying fuel input $\left(1.18\times10^{9}\ \text{J/h}\right)\left(\frac{1\ \text{L}}{4.03\times10^{7}\ \text{J}}\right)=\boxed{29.4\ \text{L/h}}$

(b) $Q_h=W_{\text{eng}}+|Q_c|$. For a continuous-transfer process we may divide by time to have

$$\frac{Q_h}{\Delta t}=\frac{W_{\text{eng}}}{\Delta t}+\frac{|Q_c|}{\Delta t}$$

$$\text{Useful power output}=\frac{W_{\text{eng}}}{\Delta t}=\frac{Q_h}{\Delta t}-\frac{|Q_c|}{\Delta t}$$

$$=\left(\frac{7.89\times10^{3}\ \text{J}}{\text{revolution}}-\frac{4.58\times10^{3}\ \text{J}}{\text{revolution}}\right)\frac{2\,500\ \text{rev}}{1\ \text{min}}\frac{1\ \text{min}}{60\ \text{s}}=1.38\times10^{5}\ \text{W}$$

$$\mathcal{P}_{\text{eng}}=1.38\times10^{5}\ \text{W}\left(\frac{1\ \text{hp}}{746\ \text{W}}\right)=\boxed{185\ \text{hp}}$$

(c) $\mathcal{P}_{\text{eng}}=\tau\omega\Rightarrow\tau=\frac{\mathcal{P}_{\text{eng}}}{\omega}=\frac{1.38\times10^{5}\ \text{J/s}}{(2\,500\ \text{rev/60 s})}\left(\frac{1\ \text{rev}}{2\pi\,\text{rad}}\right)=\boxed{527\ \text{N}\cdot\text{m}}$

(d) $\frac{|Q_c|}{\Delta t}=\frac{4.58\times10^{3}\ \text{J}}{\text{revolution}}\left(\frac{2\,500\ \text{rev}}{60\ \text{s}}\right)=\boxed{1.91\times10^{5}\ \text{W}}$

P22.6 The heat to melt 15.0 g of Hg is $|Q_c|=mL_f=\left(15\times10^{-3}\ \text{kg}\right)\left(1.18\times10^{4}\ \text{J/kg}\right)=177\ \text{J}$

The energy absorbed to freeze 1.00 g of aluminum is

$$|Q_h|=mL_f=\left(10^{-3}\ \text{kg}\right)\left(3.97\times10^{5}\ \text{J/kg}\right)=397\ \text{J}$$

and the work output is $W_{\text{eng}}=|Q_h|-|Q_c|=220\ \text{J}$

$$e=\frac{W_{\text{eng}}}{|Q_h|}=\frac{220\ \text{J}}{397\ \text{J}}=0.554,\ \text{or}\ \boxed{55.4\%}$$

The theoretical (Carnot) efficiency is $\frac{T_h-T_c}{T_h}=\frac{933\ \text{K}-243.1\ \text{K}}{933\ \text{K}}=0.749=74.9\%$

Section 22.2 Heat Pumps and Refrigerators

P22.7 $\text{COP(refrigerator)}=\frac{Q_c}{W}$

(a) If $Q_c=120\ \text{J}$ and $\text{COP}=5.00$, then $\boxed{W=24.0\ \text{J}}$

(b) Heat expelled = Heat removed + Work done.

$$Q_h=Q_c+W=120\ \text{J}+24\ \text{J}=\boxed{144\ \text{J}}$$

P22.8 $\text{COP} = 3.00 = \frac{Q_c}{W}$. Therefore, $W = \frac{Q_c}{3.00}$.

The heat removed each minute is

$$\frac{Q_C}{t} = (0.030\,0 \text{ kg})(4\,186 \text{ J/kg}\,°\text{C})(22.0°\text{C}) + (0.030\,0 \text{ kg})\left(3.33\times10^5 \text{ J/kg}\right)$$
$$+(0.030\,0 \text{ kg})(2\,090 \text{ J/kg}\,°\text{C})(20.0°\text{C}) = 1.40\times10^4 \text{ J/min}$$

or, $\frac{Q_c}{t} = 233 \text{ J/s}$.

Thus, the work done per sec $= \mathscr{P} = \frac{233 \text{ J/s}}{3.00} = \boxed{77.8 \text{ W}}$.

P22.9 (a) $\left(10.0 \frac{\text{Btu}}{\text{h}\cdot\text{W}}\right)\left(\frac{1055 \text{ J}}{1 \text{ Btu}}\right)\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right)\left(\frac{1 \text{ W}}{1 \text{ J/s}}\right) = \boxed{2.93}$

(b) Coefficient of performance for a refrigerator: $\boxed{(\text{COP})_{\text{refrigerator}}}$

(c) With EER 5, $5 \frac{\text{Btu}}{\text{h}\cdot\text{W}} = \frac{10\,000 \text{ Btu/h}}{\mathscr{P}}$: $\mathscr{P} = \frac{10\,000 \text{ Btu/h}}{\frac{5 \text{ Btu}}{\text{h}\cdot\text{W}}} = 2\,000 \text{ W} = 2.00 \text{ kW}$

Energy purchased is $\mathscr{P}\Delta t = (2.00 \text{ kW})(1\,500 \text{ h}) = 3.00\times10^3 \text{ kWh}$

$\text{Cost} = \left(3.00\times10^3 \text{ kWh}\right)(0.100 \text{ \$/kWh}) = \$300$

With EER 10, $10 \frac{\text{Btu}}{\text{h}\cdot\text{W}} = \frac{10\,000 \text{ Btu/h}}{\mathscr{P}}$: $\mathscr{P} = \frac{10\,000 \text{ Btu/h}}{\frac{10 \text{ Btu}}{\text{h}\cdot\text{W}}} = 1\,000 \text{ W} = 1.00 \text{ kW}$

Energy purchased is $\mathscr{P}\Delta t = (1.00 \text{ kW})(1\,500 \text{ h}) = 1.50\times10^3 \text{ kWh}$

$\text{Cost} = \left(1.50\times10^3 \text{ kWh}\right)(0.100 \text{ \$/kWh}) = \$150$

Thus, the cost for air conditioning is $\boxed{\text{half as much with EER 10}}$

Section 22.3 Reversible and Irreversible Processes

No problems in this section

Section 22.4 The Carnot Engine

P22.10 When $e = e_c$, $1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|}$ and $\frac{\frac{W_{\text{eng}}}{\Delta t}}{\frac{|Q_h|}{\Delta t}} = 1 - \frac{T_c}{T_h}$

(a) $$|Q_h| = \frac{\left(\frac{W_{\text{eng}}}{\Delta t}\right)\Delta t}{1 - \frac{T_c}{T_h}} = \frac{\left(1.50\times10^5 \text{ W}\right)(3\,600 \text{ s})}{1 - \frac{293}{773}}$$

$|Q_h| = 8.69\times10^8 \text{ J} = \boxed{869 \text{ MJ}}$

(b) $|Q_c| = |Q_h| - \left(\frac{W_{\text{eng}}}{\Delta t}\right)\Delta t = 8.69\times10^8 - \left(1.50\times10^5\right)(3\,600) = 3.30\times10^8 \text{ J} = \boxed{330 \text{ MJ}}$

P22.11 $T_c = 703\text{ K} \qquad T_h = 2\,143\text{ K}$

(a) $$e_c = \frac{\Delta T}{T_h} = \frac{1\,440}{2\,143} = \boxed{67.2\%}$$

(b) $|Q_h| = 1.40 \times 10^5\text{ J}$, $W_{\text{eng}} = 0.420|Q_h|$

$$\mathcal{P} = \frac{W_{\text{eng}}}{\Delta t} = \frac{5.88 \times 10^4\text{ J}}{1\text{ s}} = \boxed{58.8\text{ kW}}$$

P22.12 The Carnot efficiency of the engine is $e_c = \dfrac{\Delta T}{T_h} = \dfrac{120\text{ K}}{473\text{ K}} = 0.253$

At 20.0% of this maximum efficiency, $e = 0.200(0.253) = 0.050\,6$

From the definition of efficiency $W_{\text{eng}} = |Q_h|e$

and $|Q_h| = \dfrac{W_{\text{eng}}}{e} = \dfrac{10.0\text{ kJ}}{0.050\,6} = \boxed{197\text{ kJ}}$

P22.13 Isothermal expansion at $T_h = 523\text{ K}$

Isothermal compression at $T_c = 323\text{ K}$

Gas absorbs 1 200 J during expansion.

(a) $$|Q_c| = |Q_h|\left(\frac{T_c}{T_h}\right) = 1\,200\text{ J}\left(\frac{323}{523}\right) = \boxed{741\text{ J}}$$

(b) $$W_{\text{eng}} = |Q_h| - |Q_c| = (1\,200 - 741)\text{ J} = \boxed{459\text{ J}}$$

P22.14 We use $e_c = 1 - \dfrac{T_c}{T_h}$

as $0.300 = 1 - \dfrac{573\text{ K}}{T_h}$

From which, $T_h = 819\text{ K} = \boxed{546°\text{C}}$

***P22.15** The efficiency is $e_c = 1 - \dfrac{T_c}{T_h} = 1 - \dfrac{|Q_c|}{|Q_h|}$

Then $$\frac{T_c}{T_h} = \frac{\frac{|Q_c|}{\Delta t}}{\frac{|Q_h|}{\Delta t}}$$

$$\frac{|Q_h|}{\Delta t} = \frac{|Q_c|}{\Delta t}\frac{T_h}{T_c} = 15.4\text{ W}\frac{(273+100)\text{ K}}{(273+20)\text{ K}} = 19.6\text{ W}$$

(a) $|Q_h| = W_{\text{eng}} + |Q_c|$

The useful power output is $\dfrac{W_{\text{eng}}}{\Delta t} = \dfrac{|Q_h|}{\Delta t} - \dfrac{|Q_c|}{\Delta t} = 19.6\text{ W} - 15.4\text{ W} = \boxed{4.20\text{ W}}$

(b) $|Q_h| = \left(\dfrac{|Q_h|}{\Delta t}\right)\Delta t = mL_V \qquad m = \dfrac{|Q_h|}{\Delta t}\dfrac{\Delta t}{L_V} = (19.6\text{ J/s})\left(\dfrac{3\,600\text{ s}}{2.26 \times 10^6\text{ J/kg}}\right) = \boxed{3.12 \times 10^{-2}\text{ kg}}$

P22.16 The Carnot summer efficiency is $e_{c,s} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273+20)\text{ K}}{(273+350)\text{ K}} = 0.530$

And in winter, $e_{c,w} = 1 - \frac{283}{623} = 0.546$

Then the actual winter efficiency is $0.320\left(\frac{0.546}{0.530}\right) = \boxed{0.330}$ or $\boxed{33.0\%}$

P22.17 (a) In an adiabatic process, $P_f V_f^\gamma = P_i V_i^\gamma$. Also, $\left(\frac{P_f V_f}{T_f}\right)^\gamma = \left(\frac{P_i V_i}{T_i}\right)^\gamma$.

Dividing the second equation by the first yields $T_f = T_i\left(\frac{P_f}{P_i}\right)^{(\gamma-1)/\gamma}$.

Since $\gamma = \frac{5}{3}$ for Argon, $\frac{\gamma-1}{\gamma} = \frac{2}{5} = 0.400$ and we have

$$T_f = (1\,073\text{ K})\left(\frac{300 \times 10^3\text{ Pa}}{1.50 \times 10^6\text{ Pa}}\right)^{0.400} = \boxed{564\text{ K}}.$$

(b) $\Delta E_{\text{int}} = nC_V\Delta T = Q - W_{\text{eng}} = 0 - W_{\text{eng}}$, so $W_{\text{eng}} = -nC_V\Delta T$,

and the power output is

$$\mathcal{P} = \frac{W_{\text{eng}}}{t} = \frac{-nC_V\Delta T}{t} \text{ or}$$

$$= \frac{(-80.0\text{ kg})\left(\frac{1.00\text{ mol}}{0.0399\text{ kg}}\right)\left(\frac{3}{2}\right)(8.314\text{ J/mol}\cdot\text{K})(564 - 1\,073)\text{K}}{60.0\text{ s}}$$

$$\mathcal{P} = 2.12 \times 10^5\text{ W} = \boxed{212\text{ kW}}$$

(c) $e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{564\text{ K}}{1\,073\text{ K}} = 0.475$ or $\boxed{47.5\%}$

P22.18 (a) $e_{\max} = 1 - \frac{T_c}{T_h} = 1 - \frac{278}{293} = 5.12 \times 10^{-2} = \boxed{5.12\%}$

(b) $\mathcal{P} = \frac{W_{\text{eng}}}{\Delta t} = 75.0 \times 10^6\text{ J/s}$

Therefore, $W_{\text{eng}} = (75.0 \times 10^6\text{ J/s})(3\,600\text{ s/h}) = 2.70 \times 10^{11}\text{ J/h}$

From $e = \frac{W_{\text{eng}}}{|Q_h|}$ we find $|Q_h| = \frac{W_{\text{eng}}}{e} = \frac{2.70 \times 10^{11}\text{ J/h}}{5.12 \times 10^{-2}} = 5.27 \times 10^{12}\text{ J/h} = \boxed{5.27\text{ TJ/h}}$

(c) As fossil-fuel prices rise, this way to use solar energy will become a good buy.

***P22.19** (a) $e=\frac{W_{\text{eng1}}+W_{\text{eng2}}}{Q_{h1}}=\frac{e_1Q_{1h}+e_2Q_{2h}}{Q_{h1}}$

Now $Q_{2h}=Q_{1c}=Q_{1h}-W_{\text{eng1}}=Q_{h1}-e_1Q_{1h}$.

So $e=\frac{e_1Q_{1h}+e_2(Q_{1h}-e_1Q_{1h})}{Q_{1h}}=\boxed{e_1+e_2-e_1e_2}$.

(b) $$e=e_1+e_2-e_1e_2=1-\frac{T_i}{T_h}+1-\frac{T_c}{T_i}-\left(1-\frac{T_i}{T_h}\right)\left(1-\frac{T_c}{T_i}\right)=2-\frac{T_i}{T_h}-\frac{T_c}{T_i}-1+\frac{T_i}{T_h}+\frac{T_c}{T_i}-\frac{T_c}{T_h}=\boxed{1-\frac{T_c}{T_h}}$$

The combination of reversible engines is itself a reversible engine so it has the Carnot efficiency.

(c) With $W_{\text{eng2}}=W_{\text{eng1}}$, $e=\frac{W_{\text{eng1}}+W_{\text{eng2}}}{Q_{1h}}=\frac{2W_{\text{eng1}}}{Q_{1h}}=2e_1$

$$1-\frac{T_c}{T_h}=2\left(1-\frac{T_i}{T_h}\right)$$

$$0-\frac{T_c}{T_h}=1-\frac{2T_i}{T_h}$$

$$2T_i=T_h+T_c$$

$$\boxed{T_i=\frac{1}{2}(T_h+T_c)}$$

(d) $e_1=e_2=1-\frac{T_i}{T_h}=1-\frac{T_c}{T_i}$

$$T_i^2=T_cT_h$$

$$\boxed{T_i=(T_hT_c)^{1/2}}$$

P22.20 The work output is $W_{\text{eng}}=\frac{1}{2}m_{\text{train}}(5.00\text{ m/s})^2$.

We are told $e=\frac{W_{\text{eng}}}{Q_h}$

$$0.200=\frac{1}{2}m_t\frac{(5.00\text{ m/s})^2}{Q_h}$$

and $e_C=1-\frac{300\text{ K}}{T_h}=\frac{1}{2}m_t\frac{(6.50\text{ m/s})^2}{Q_h}$.

Substitute $Q_h=\frac{1}{2}m_t\frac{(5.00\text{ m/s})^2}{0.200}$.

Then, $1-\frac{300\text{ K}}{T_h}=0.200\left(\frac{\frac{1}{2}m_t(6.50\text{ m/s})^2}{\frac{1}{2}m_t(5.00\text{ m/s})^2}\right)$

$$1-\frac{300\text{ K}}{T_h}=0.338$$

$$T_h=\frac{300\text{ K}}{0.662}=\boxed{453\text{ K}}$$

P22.21 For the Carnot engine, $e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{300\text{ K}}{750\text{ K}} = 0.600$.

Also, $e_c = \frac{W_{\text{eng}}}{|Q_h|}$.

so $|Q_h| = \frac{W_{\text{eng}}}{e_c} = \frac{150\text{ J}}{0.600} = 250\text{ J}$.

and $|Q_c| = |Q_h| - W_{\text{eng}} = 250\text{ J} - 150\text{ J} = 100\text{ J}$.

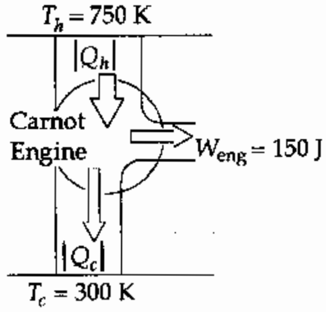

FIG. P22.21

(a) $|Q_h| = \frac{W_{\text{eng}}}{e_S} = \frac{150\text{ J}}{0.700} = \boxed{214\text{ J}}$

$|Q_c| = |Q_h| - W_{\text{eng}} = 214\text{ J} - 150\text{ J} = \boxed{64.3\text{ J}}$

(b) $|Q_{h,\text{net}}| = 214\text{ J} - 250\text{ J} = \boxed{-35.7\text{ J}}$

$|Q_{c,\text{net}}| = 64.3\text{ J} - 100\text{ J} = \boxed{-35.7\text{ J}}$

The net flow of energy by heat from the cold to the hot reservoir without work input, is impossible.

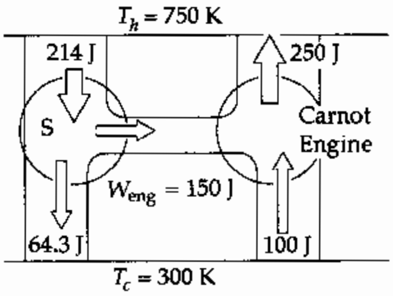

FIG. P22.21(b)

(c) For engine *S*: $|Q_c| = |Q_h| - W_{\text{eng}} = \frac{W_{\text{eng}}}{e_S} - W_{\text{eng}}$.

so $W_{\text{eng}} = \frac{|Q_c|}{\frac{1}{e_S} - 1} = \frac{100\text{ J}}{\frac{1}{0.700} - 1} = \boxed{233\text{ J}}$.

and $|Q_h| = |Q_c| + W_{\text{eng}} = 233\text{ J} + 100\text{ J} = \boxed{333\text{ J}}$.

(d) $|Q_{h,\text{net}}| = 333\text{ J} - 250\text{ J} = \boxed{83.3\text{ J}}$

$W_{\text{net}} = 233\text{ J} - 150\text{ J} = \boxed{83.3\text{ J}}$

$|Q_{c,\text{net}}| = \boxed{0}$

The output of 83.3 J of energy from the heat engine by work in a cyclic process without any exhaust by heat is impossible.

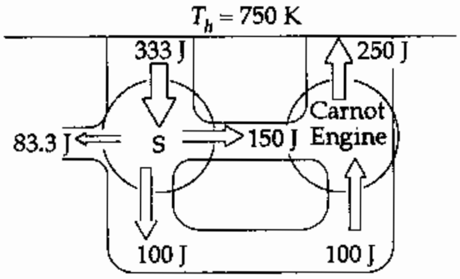

FIG. P22.21(d)

(e) Both engines operate in cycles, so $\Delta S_S = \Delta S_{\text{Carnot}} = 0$.

For the reservoirs, $\Delta S_h = -\frac{|Q_h|}{T_h}$ and $\Delta S_c = +\frac{|Q_c|}{T_c}$.

Thus, $\Delta S_{\text{total}} = \Delta S_S + \Delta S_{\text{Carnot}} + \Delta S_h + \Delta S_c = 0 + 0 - \frac{83.3\text{ J}}{750\text{ K}} + \frac{0}{300\text{ K}} = \boxed{-0.111\text{ J/K}}$.

A decrease in total entropy is impossible.

P22.22 (a) First, consider the adiabatic process $D \to A$:

$P_D V_D^\gamma = P_A V_A^\gamma$ so $$P_D = P_A\left(\frac{V_A}{V_D}\right)^\gamma = 1\,400\text{ kPa}\left(\frac{10.0\text{ L}}{15.0\text{ L}}\right)^{5/3} = \boxed{712\text{ kPa}}.$$

Also $$\left(\frac{nRT_D}{V_D}\right)V_D^\gamma = \left(\frac{nRT_A}{V_A}\right)V_A^\gamma$$

or $$T_D = T_A\left(\frac{V_A}{V_D}\right)^{\gamma-1} = 720\text{ K}\left(\frac{10.0}{15.0}\right)^{2/3} = \boxed{549\text{ K}}.$$

Now, consider the isothermal process $C \to D$: $T_C = T_D = \boxed{549\text{ K}}$.

$$P_C = P_D\left(\frac{V_D}{V_C}\right) = \left[P_A\left(\frac{V_A}{V_D}\right)^\gamma\right]\left(\frac{V_D}{V_C}\right) = \frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}$$

$$P_C = \frac{1\,400\text{ kPa}(10.0\text{ L})^{5/3}}{24.0\text{ L}(15.0\text{ L})^{2/3}} = \boxed{445\text{ kPa}}$$

Next, consider the adiabatic process $B \to C$: $P_B V_B^\gamma = P_C V_C^\gamma$.

But, $P_C = \frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}$ from above. Also considering the isothermal process, $P_B = P_A\left(\frac{V_A}{V_B}\right)$.

Hence, $P_A\left(\frac{V_A}{V_B}\right)V_B^\gamma = \left(\frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}\right)V_C^\gamma$ which reduces to $V_B = \frac{V_A V_C}{V_D} = \frac{10.0\text{ L}(24.0\text{ L})}{15.0\text{ L}} = \boxed{16.0\text{ L}}$.

Finally, $P_B = P_A\left(\frac{V_A}{V_B}\right) = 1\,400\text{ kPa}\left(\frac{10.0\text{ L}}{16.0\text{ L}}\right) = \boxed{875\text{ kPa}}$.

State	P(kPa)	V(L)	T(K)
A	1 400	10.0	720
B	875	16.0	720
C	445	24.0	549
D	712	15.0	549

(b) For the isothermal process $A \to B$: $\Delta E_{\text{int}} = nC_V\Delta T = \boxed{0}$

so $Q = -W = nRT\ln\left(\frac{V_B}{V_A}\right) = 2.34\text{ mol}(8.314\text{ J/mol}\cdot\text{K})(720\text{ K})\ln\left(\frac{16.0}{10.0}\right) = \boxed{+6.58\text{ kJ}}$.

For the adiabatic process $B \to C$: $Q = \boxed{0}$

$\Delta E_{\text{int}} = nC_V(T_C - T_B) = 2.34\text{ mol}\left[\frac{3}{2}(8.314\text{ J/mol}\cdot\text{K})\right](549 - 720)\text{ K} = \boxed{-4.98\text{ kJ}}$

and $W = -Q + \Delta E_{\text{int}} = 0 + (-4.98\text{ kJ}) = \boxed{-4.98\text{ kJ}}$.

continued on next page

For the isothermal process $C \to D$: $\Delta E_{\text{int}} = nC_V \Delta T = \boxed{0}$

and $Q = -W = nRT \ln\left(\frac{V_D}{V_C}\right) = 2.34 \text{ mol}(8.314 \text{ J/mol}\cdot\text{K})(549 \text{ K})\ln\left(\frac{15.0}{24.0}\right) = \boxed{-5.02 \text{ kJ}}$.

Finally, for the adiabatic process $D \to A$: $Q = \boxed{0}$

$$\Delta E_{\text{int}} = nC_V(T_A - T_D) = 2.34 \text{ mol}\left[\frac{3}{2}(8.314 \text{ J/mol}\cdot\text{K})\right](720 - 549) \text{ K} = \boxed{+4.98 \text{ kJ}}$$

and $W = -Q + \Delta E_{\text{int}} = 0 + 4.98 \text{ kJ} = \boxed{+4.98 \text{ kJ}}$.

Process	Q(kJ)	W(kJ)	ΔE_{int} (kJ)
$A \to B$	+6.58	−6.58	0
$B \to C$	0	−4.98	−4.98
$C \to D$	−5.02	+5.02	0
$D \to A$	0	+4.98	+4.98
ABCDA	+1.56	−1.56	0

The work done *by* the engine is the negative of the work input. The output work W_{eng} is given by the work column in the table with all signs reversed.

(c) $e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{-W_{ABCD}}{Q_{A\to B}} = \frac{1.56 \text{ kJ}}{6.58 \text{ kJ}} = 0.237$ or $\boxed{23.7\%}$

$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{549}{720} = 0.237$ or $\boxed{23.7\%}$

P22.23 $(\text{COP})_{\text{refrig}} = \frac{T_c}{\Delta T} = \frac{270}{30.0} = \boxed{9.00}$

P22.24 $(\text{COP})_{\text{heat pump}} = \frac{|Q_c| + W}{W} = \frac{T_h}{\Delta T} = \frac{295}{25} = \boxed{11.8}$

P22.25 (a) For a complete cycle, $\Delta E_{\text{int}} = 0$ and $W = |Q_h| - |Q_c| = |Q_c|\left[\frac{|(Q_h)|}{|Q_c|} - 1\right]$.

We have already shown that for a Carnot cycle (and only for a Carnot cycle) $\frac{|Q_h|}{|Q_c|} = \frac{T_h}{T_c}$.

Therefore, $W = \boxed{|Q_c|\left[\frac{T_h - T_c}{T_c}\right]}$.

(b) We have the definition of the coefficient of performance for a refrigerator, $\text{COP} = \frac{|Q_c|}{W}$.

Using the result from part (a), this becomes $\text{COP} = \boxed{\frac{T_c}{T_h - T_c}}$.

P22.26 $\text{COP} = 0.100\text{COP}_{\text{Carnot cycle}}$

or $\frac{|Q_h|}{W} = 0.100\left(\frac{|Q_h|}{W}\right)_{\text{Carnot cycle}} = 0.100\left(\frac{1}{\text{Carnot efficiency}}\right)$

$$\frac{|Q_h|}{W} = 0.100\left(\frac{T_h}{T_h - T_c}\right) = 0.100\left(\frac{293\text{ K}}{293\text{ K} - 268\text{ K}}\right) = 1.17$$

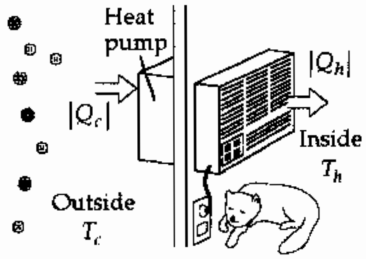

FIG. P22.26

Thus, 1.17 joules of energy enter the room by heat for each joule of work done.

P22.27 $(\text{COP})_{\text{Carnot refrig}} = \frac{T_c}{\Delta T} = \frac{4.00}{289} = 0.013\,8 = \frac{|Q_c|}{W}$

$\therefore W = \boxed{72.2\text{ J}}$ per 1 J energy removed by heat.

P22.28 A Carnot refrigerator runs on minimum power.

For it: $\frac{Q_h}{T_h} = \frac{Q_c}{T_c}$ so $\frac{Q_h/t}{T_h} = \frac{Q_c/t}{T_c}$.

Solving part (b) first:

(b) $$\frac{Q_h}{t} = \frac{Q_c}{t}\left(\frac{T_h}{T_c}\right) = (8.00\text{ MJ/h})\left(\frac{298\text{ K}}{273\text{ K}}\right) = \left(8.73\times10^6\text{ J/h}\right)\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = \boxed{2.43\text{ kW}}$$

(a) $$\frac{W}{t} = \frac{Q_h}{t} - \frac{Q_c}{t} = 2.43\text{ kW} - \frac{8.00\times10^6\text{ J/h}}{3\,600\text{ s/h}} = \boxed{204\text{ W}}$$

P22.29 $e = \frac{W}{Q_h} = 0.350$ $\qquad W = 0.350Q_h$

$Q_h = W + Q_c$ $\qquad Q_c = 0.650Q_h$

$$\text{COP(refrigerator)} = \frac{Q_c}{W} = \frac{0.650Q_h}{0.350Q_h} = \boxed{1.86}$$

***P22.30** To have the same efficiencies as engines, $1 - \frac{T_{\text{cp}}}{T_{\text{hp}}} = 1 - \frac{T_{\text{cr}}}{T_{\text{hr}}}$ the pump and refrigerator must operate between reservoirs with the same ratio $\frac{T_{\text{cp}}}{T_{\text{hp}}} = \frac{T_{\text{cr}}}{T_{\text{hr}}}$, which we define as r. Now $\text{COP}_{\text{p}} = 1.50\text{COP}_{\text{r}}$ becomes $\frac{T_{\text{hp}}}{T_{\text{hp}} - T_{\text{cp}}} = \frac{3}{2}\frac{T_{\text{cr}}}{T_{\text{hr}} - T_{\text{cr}}}$ or $\frac{T_{\text{hp}}}{T_{\text{hp}} - rT_{\text{hp}}} = \frac{3}{2}\frac{rT_{\text{hr}}}{T_{\text{hr}} - rT_{\text{hr}}}$, $\frac{2}{1-r} = \frac{3r}{1-r}$, $r = \frac{2}{3}$.

(a) $$\text{COP}_{\text{r}} = \frac{r}{1-r} = \frac{\frac{2}{3}}{1-\frac{2}{3}} = \boxed{2.00}$$

(b) $$\text{COP}_{\text{p}} = \frac{1}{1-r} = \frac{1}{1-\frac{2}{3}} = \boxed{3.00}$$

(c) $$e = 1 - r = 1 - \frac{2}{3} = \boxed{33.3\%}$$

Section 22.5 Gasoline and Diesel Engines

P22.31 (a) $P_iV_i^\gamma = P_fV_f^\gamma$

$$P_f = P_i\left(\frac{V_i}{V_f}\right)^\gamma = \left(3.00\times10^6\text{ Pa}\right)\left(\frac{50.0\text{ cm}^3}{300\text{ cm}^3}\right)^{1.40} = \boxed{244\text{ kPa}}$$

(b) $W = \int_{V_i}^{V_f} P\,dV \qquad P = P_i\left(\frac{V_i}{V}\right)^\gamma$

Integrating,

$$W = \left(\frac{1}{\gamma-1}\right)P_iV_i\left[1-\left(\frac{V_i}{V_f}\right)^{\gamma-1}\right] = (2.50)\left(3.00\times10^6\text{ Pa}\right)\left(5.00\times10^{-5}\text{ m}^3\right)\left[1-\left(\frac{50.0\text{ cm}^3}{300\text{ cm}^3}\right)^{0.400}\right]$$

$$= \boxed{192\text{ J}}$$

P22.32 Compression ratio $= 6.00$, $\gamma = 1.40$

(a) Efficiency of an Otto-engine $e = 1-\left(\frac{V_2}{V_1}\right)^{\gamma-1}$

$$e = 1-\left(\frac{1}{6.00}\right)^{0.400} = \boxed{51.2\%}.$$

(b) If actual efficiency $e' = 15.0\%$ losses in system are $e - e' = \boxed{36.2\%}$.

P22.33 $e_{\text{Otto}} = 1-\frac{1}{(V_1/V_2)^{\gamma-1}} = 1-\frac{1}{(6.20)^{(7/5-1)}} = 1-\frac{1}{(6.20)^{0.400}}$

$e_{\text{Otto}} = 0.518$

We have assumed the fuel-air mixture to behave like a diatomic gas.

Now $e = \frac{W_{\text{eng}}}{Q_h} = \frac{W_{\text{eng}}/t}{Q_h/t}$

$$\frac{Q_h}{t} = \frac{W_{\text{eng}}/t}{e} = 102\text{ hp}\frac{746\text{ W/1 hp}}{0.518}$$

$$\frac{Q_h}{t} = \boxed{146\text{ kW}}$$

$$Q_h = W_{\text{eng}} + |Q_c|$$

$$\frac{|Q_c|}{t} = \frac{Q_h}{t} - \frac{W_{\text{eng}}}{t}$$

$$\frac{|Q_c|}{t} = 146\times10^3\text{ W} - 102\text{ hp}\left(\frac{746\text{ W}}{1\text{ hp}}\right) = \boxed{70.8\text{ kW}}$$

P22.34 (a), (b) The quantity of gas is

$$n=\frac{P_AV_A}{RT_A}=\frac{\left(100\times10^3\text{ Pa}\right)\left(500\times10^{-6}\text{ m}^3\right)}{(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})}=0.020\,5\text{ mol}$$

$$E_{\text{int},A}=\frac{5}{2}nRT_A=\frac{5}{2}P_AV_A=\frac{5}{2}\left(100\times10^3\text{ Pa}\right)\left(500\times10^{-6}\text{ m}^3\right)=\boxed{125\text{ J}}$$

In process AB, $P_B=P_A\left(\frac{V_A}{V_B}\right)^{\gamma}=\left(100\times10^3\text{ Pa}\right)(8.00)^{1.40}=\boxed{1.84\times10^6\text{ Pa}}$

$$T_B=\frac{P_BV_B}{nR}=\frac{\left(1.84\times10^6\text{ Pa}\right)\left(500\times10^{-6}\text{ m}^3/8.00\right)}{(0.020\,5\text{ mol})(8.314\text{ J/mol}\cdot\text{K})}=\boxed{673\text{ K}}$$

$$E_{\text{int},B}=\frac{5}{2}nRT_B=\frac{5}{2}(0.020\,5\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(673\text{ K})=\boxed{287\text{ J}}$$

so $\Delta E_{\text{int},AB}=287\text{ J}-125\text{ J}=\boxed{162\text{ J}}=Q-W_{\text{out}}=0-W_{\text{out}}$ $\quad W_{AB}=\boxed{-162\text{ J}}$

Process BC takes us to:

$$P_C=\frac{nRT_C}{V_C}=\frac{(0.020\,5\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(1\,023\text{ K})}{62.5\times10^{-6}\text{ m}^3}=\boxed{2.79\times10^6\text{ Pa}}$$

$$E_{\text{int},C}=\frac{5}{2}nRT_C=\frac{5}{2}(0.020\,5\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(1\,023\text{ K})=\boxed{436\text{ J}}$$

$$E_{\text{int},BC}=436\text{ J}-287\text{ J}=\boxed{149\text{ J}}=Q-W_{\text{out}}=Q-0$$

$$Q_{BC}=\boxed{149\text{ J}}$$

In process CD:

$$P_D=P_C\left(\frac{V_C}{V_D}\right)^{\gamma}=\left(2.79\times10^6\text{ Pa}\right)\left(\frac{1}{8.00}\right)^{1.40}=\boxed{1.52\times10^5\text{ Pa}}$$

$$T_D=\frac{P_DV_D}{nR}=\frac{\left(1.52\times10^5\text{ Pa}\right)\left(500\times10^{-6}\text{ m}^3\right)}{(0.020\,5\text{ mol})(8.314\text{ J/mol}\cdot\text{K})}=\boxed{445\text{ K}}$$

$$E_{\text{int},D}=\frac{5}{2}nRT_D=\frac{5}{2}(0.020\,5\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(445\text{ K})=\boxed{190\text{ J}}$$

$$\Delta E_{\text{int},CD}=190\text{ J}-436\text{ J}=\boxed{-246\text{ J}}=Q-W_{\text{out}}=0-W_{\text{out}}$$

$$W_{CD}=\boxed{246\text{ J}}$$

and $\Delta E_{\text{int},DA}=E_{\text{int},A}-E_{\text{int},D}=125\text{ J}-190\text{ J}=\boxed{-65.0\text{ J}}=Q-W_{\text{out}}=Q-0$

$$Q_{DA}=\boxed{-65.0\text{ J}}$$

continued on next page

For the entire cycle, $\Delta E_{\text{int, net}} = 162\text{ J} + 149 - 246 - 65.0 = \boxed{0}$. The net work is

$$W_{\text{eng}} = -162\text{ J} + 0 + 246\text{ J} + 0 = \boxed{84.3\text{ J}}$$
$$Q_{\text{net}} = 0 + 149\text{ J} + 0 - 65.0\text{ J} = \boxed{84.3\text{ J}}$$

The tables look like:

State	T(K)	P(kPa)	$V(\text{cm}^3)$	E_{int} (J)
A	293	100	500	125
B	673	1 840	62.5	287
C	1 023	2 790	62.5	436
D	445	152	500	190
A	293	100	500	125

Process	Q(J)	output W(J)	ΔE_{int} (J)
AB	0	−162	162
BC	149	0	149
CD	0	246	−246
DA	−65.0	0	−65.0
ABCDA	84.3	84.3	0

(c) The input energy is $Q_h = \boxed{149\text{ J}}$, the waste is $|Q_c| = \boxed{65.0\text{ J}}$, and $W_{\text{eng}} = \boxed{84.3\text{ J}}$.

(d) The efficiency is: $e = \frac{W_{\text{eng}}}{Q_h} = \frac{84.3\text{ J}}{149\text{ J}} = \boxed{0.565}$.

(e) Let f represent the angular speed of the crankshaft. Then $\frac{f}{2}$ is the frequency at which we obtain work in the amount of 84.3 J/cycle:

$$1\,000\text{ J/s} = \left(\frac{f}{2}\right)(84.3\text{ J/cycle})$$
$$f = \frac{2\,000\text{ J/s}}{84.3\text{ J/cycle}} = 23.7\text{ rev/s} = \boxed{1.42 \times 10^3\text{ rev/min}}$$

Section 22.6 **Entropy**

P22.35 For a freezing process,

$$\Delta S = \frac{\Delta Q}{T} = \frac{-(0.500\text{ kg})(3.33 \times 10^5\text{ J/kg})}{273\text{ K}} = \boxed{-610\text{ J/K}}.$$

P22.36 At a constant temperature of 4.20 K,

$$\Delta S = \frac{\Delta Q}{T} = \frac{L_v}{4.20\text{ K}} = \frac{20.5\text{ kJ/kg}}{4.20\text{ K}}$$
$$\Delta S = \boxed{4.88\text{ kJ/kg}\cdot\text{K}}$$

P22.37 $\Delta S = \int_i^f \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mc\,dT}{T} = mc\ln\left(\frac{T_f}{T_i}\right)$

$\Delta S = 250\text{ g}(1.00\text{ cal/g}\cdot°\text{C})\ln\left(\frac{353}{293}\right) = 46.6\text{ cal/K} = \boxed{195\text{ J/K}}$

***P22.38** (a) The process is $\boxed{\text{isobaric}}$ because it takes place under constant atmospheric pressure. As described by Newton's third law, the stewing syrup must exert the same force on the air as the air exerts on it. The heating process is not adiabatic (energy goes in by heat), isothermal (T goes up), isovolumetic (it likely expands a bit), cyclic (it is different at the end), or isentropic (entropy increases). It could be made as nearly reversible as you wish, by not using a kitchen stove but a heater kept always just incrementally higher in temperature than the syrup. The process would then also be eternal, and impractical for food production.

(b) The final temperature is

$$220°\text{F} = 212°\text{F} + 8°\text{F} = 100°\text{C} + 8°\text{F}\left(\frac{100 - 0°\text{C}}{212 - 32°\text{F}}\right) = 104°\text{C}.$$

For the mixture,

$$Q = m_1c_1\Delta T + m_2c_2\Delta T = (900\text{ g }1\text{ cal/g}\cdot°\text{C} + 930\text{ g }0.299\text{ cal/g}\cdot°\text{C})(104.4°\text{C} - 23°\text{C})$$
$$= 9.59 \times 10^4\text{ cal} = \boxed{4.02 \times 10^5\text{ J}}$$

(c) Consider the reversible heating process described in part (a):

$$\Delta S = \int_i^f \frac{dQ}{T} = \int_i^f \frac{(m_1c_1 + m_2c_2)dT}{T} = (m_1c_1 + m_2c_2)\ln\frac{T_f}{T_i}$$
$$= [900(1) + 930(0.299)](\text{cal/}°\text{C})\left(\frac{4.186\text{ J}}{1\text{ cal}}\right)\left(\frac{1°\text{C}}{1\text{ K}}\right)\ln\left(\frac{273+104}{273+23}\right)$$
$$= (4\,930\text{ J/K})0.243 = \boxed{1.20 \times 10^3\text{ J/K}}$$

***P22.39** We take data from the description of Figure 20.2 in section 20.3, and we assume a constant specific heat for each phase. As the ice is warmed from −12°C to 0°C, its entropy increases by

$$\Delta S = \int_i^f \frac{dQ}{T} = \int_{261\text{ K}}^{273\text{ K}} \frac{mc_{\text{ice}}dT}{T} = mc_{\text{ice}} \int_{261\text{ K}}^{273\text{ K}} T^{-1}dT = mc_{\text{ice}} \ln T\Big|_{261\text{ K}}^{273\text{ K}}$$

$$\Delta S = 0.027\,0\text{ kg}(2\,090\text{ J/kg}\cdot{}^\circ\text{C})(\ln 273\text{ K} - \ln 261\text{ K}) = 0.027\,0\text{ kg}(2\,090\text{ J/kg}\cdot{}^\circ\text{C})\left(\ln\left(\frac{273}{261}\right)\right)$$

$$\Delta S = 2.54\text{ J/K}$$

As the ice melts its entropy change is

$$\Delta S = \frac{Q}{T} = \frac{mL_f}{T} = \frac{0.027\,0\text{ kg}\left(3.33\times 10^5\text{ J/kg}\right)}{273\text{ K}} = 32.9\text{ J/K}$$

As liquid water warms from 273 K to 373 K,

$$\Delta S = \int_i^f \frac{mc_{\text{liquid}}dT}{T} = mc_{\text{liquid}} \ln\left(\frac{T_f}{T_i}\right) = 0.027\,0\text{ kg}(4\,186\text{ J/kg}\cdot{}^\circ\text{C})\ln\left(\frac{373}{273}\right) = 35.3\text{ J/K}$$

As the water boils and the steam warms,

$$\Delta S = \frac{mL_v}{T} + mc_{\text{steam}} \ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = \frac{0.027\,0\text{ kg}\left(2.26\times 10^6\text{ J/kg}\right)}{373\text{ K}} + 0.027\,0\text{ kg}(2\,010\text{ J/kg}\cdot{}^\circ\text{C})\ln\left(\frac{388}{373}\right) = 164\text{ J/K} + 2.14\text{ J/K}$$

The total entropy change is

$$(2.54 + 32.9 + 35.3 + 164 + 2.14)\text{ J/K} = \boxed{236\text{ J/K}}.$$

We could equally well have taken the values for specific heats and latent heats from Tables 20.1 and 20.2. For steam at constant pressure, the molar specific heat in Table 21.2 implies a specific heat of $(35.4\text{ J/mol}\cdot\text{K})\left(\frac{1\text{ mol}}{0.018\text{ kg}}\right) = 1\,970\text{ J/kg}\cdot\text{K}$, nearly agreeing with $2\,010\text{ J/kg}\cdot\text{K}$.

Section 22.7 Entropy Changes in Irreversible Processes

P22.40 $\Delta S = \frac{Q_2}{T_2} - \frac{Q_1}{T_1} = \left(\frac{1\,000}{290} - \frac{1\,000}{5\,700}\right)\text{ J/K} = \boxed{3.27\text{ J/K}}$

P22.41 The car ends up in the same thermodynamic state as it started, so it undergoes zero changes in entropy. The original kinetic energy of the car is transferred by heat to the surrounding air, adding to the internal energy of the air. Its change in entropy is

$$\Delta S = \frac{\frac{1}{2}mv^2}{T} = \frac{750(20.0)^2}{293}\text{ J/K} = \boxed{1.02\text{ kJ/K}}.$$

P22.42 $c_{\text{iron}} = 448\ \text{J/kg}\cdot{}^\circ\text{C}$; $c_{\text{water}} = 4\,186\ \text{J/kg}\cdot{}^\circ\text{C}$

$Q_{\text{cold}} = -Q_{\text{hot}}$: $4.00\ \text{kg}(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(T_f - 10.0^\circ\text{C}) = -(1.00\ \text{kg})(448\ \text{J/kg}\cdot{}^\circ\text{C})(T_f - 900^\circ\text{C})$

which yields $T_f = 33.2^\circ\text{C} = 306.2\ \text{K}$

$$\Delta S = \int_{283\ \text{K}}^{306.2\ \text{K}} \frac{c_{\text{water}} m_{\text{water}}\, dT}{T} + \int_{1\,173\ \text{K}}^{306.2\ \text{K}} \frac{c_{\text{iron}} m_{\text{iron}}\, dT}{T}$$

$$\Delta S = c_{\text{water}} m_{\text{water}} \ln\left(\frac{306.2}{283}\right) + c_{\text{iron}} m_{\text{iron}} \ln\left(\frac{306.2}{1\,173}\right)$$

$$\Delta S = (4\,186\ \text{J/kg}\cdot\text{K})(4.00\ \text{kg})(0.078\,8) + (448\ \text{J/kg}\cdot\text{K})(1.00\ \text{kg})(-1.34)$$

$$\Delta S = \boxed{718\ \text{J/K}}$$

P22.43 Sitting here writing, I convert chemical energy, in ordered molecules in food, into internal energy that leaves my body by heat into the room-temperature surroundings. My rate of energy output is equal to my metabolic rate,

$$2\,500\ \text{kcal/d} = \frac{2\,500 \times 10^3\ \text{cal}}{86\,400\ \text{s}}\left(\frac{4.186\ \text{J}}{1\ \text{cal}}\right) = 120\ \text{W}.$$

My body is in steady state, changing little in entropy, as the environment increases in entropy at the rate

$$\frac{\Delta S}{\Delta t} = \frac{Q/T}{\Delta t} = \frac{Q/\Delta t}{T} = \frac{120\ \text{W}}{293\ \text{K}} = 0.4\ \text{W/K} \sim \boxed{1\ \text{W/K}}.$$

When using powerful appliances or an automobile, my personal contribution to entropy production is much greater than the above estimate, based only on metabolism.

P22.44 (a) $$V = \frac{nRT_i}{P_i} = \frac{(40.0\ \text{g})(8.314\ \text{J/mol}\cdot\text{K})(473\ \text{K})}{(39.9\ \text{g/mol})(100 \times 10^3\ \text{Pa})} = 39.4 \times 10^{-3}\ \text{m}^3 = \boxed{39.4\ \text{L}}$$

(b) $$\Delta E_{\text{int}} = nC_V\Delta T = \left(\frac{40.0\ \text{gm}}{39.9\ \text{g/mol}}\right)\left[\frac{3}{2}(8.314\ \text{J/mol}\cdot\text{K})\right](-200^\circ\text{C}) = \boxed{-2.50\ \text{kJ}}$$

(c) $W = 0$ so $Q = \Delta E_{\text{int}} = \boxed{-2.50\ \text{kJ}}$

(d) $$\Delta S_{\text{argon}} = \int_i^f \frac{dQ}{T} = nC_V \ln\left(\frac{T_f}{T_i}\right)$$

$$= \left(\frac{40.0\ \text{g}}{39.9\ \text{g/mol}}\right)\left[\frac{3}{2}(8.314\ \text{J/mol}\cdot\text{K})\right]\ln\left(\frac{273}{473}\right) = \boxed{-6.87\ \text{J/K}}$$

(e) $$\Delta S_{\text{bath}} = \frac{2.50\ \text{kJ}}{273\ \text{K}} = \boxed{+9.16\ \text{J/K}}$$

The total change in entropy is

$$\Delta S_{\text{total}} = \Delta S_{\text{argon}} + \Delta S_{\text{bath}} = -6.87\ \text{J/K} + 9.16\ \text{J/K} = +2.29\ \text{J/K}$$

$\Delta S_{\text{total}} > 0$ for this irreversible process.

P22.45 $\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) = R\ln 2 = \boxed{5.76\ \text{J/K}}$

There is $\boxed{\text{no change in temperature}}$.

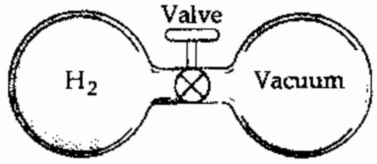

FIG. P22.45

P22.46 $\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) = (0.044\,0)(2)R\ln 2$

$\Delta S = 0.088\,0(8.314)\ln 2 = \boxed{0.507\ \text{J/K}}$

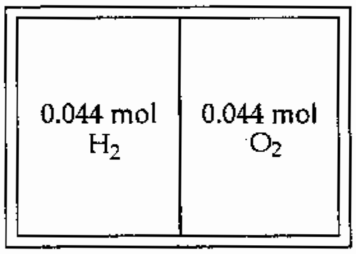

FIG. P22.46

P22.47 For any infinitesimal step in a process on an ideal gas,

$dE_{\text{int}} = dQ + dW$: $\quad dQ = dE_{\text{int}} - dW = nC_V dT + PdV = nC_V dT + \frac{nRTdV}{V}$

and $\quad \frac{dQ}{T} = nC_V\frac{dT}{T} + nR\frac{dV}{V}$

If the whole process is reversible, $\quad \Delta S = \int_i^f \frac{dQ_r}{T} = \int_i^f\left(nC_V\frac{dT}{T} + nR\frac{dV}{V}\right) = nC_V\ln\left(\frac{T_f}{T_i}\right) + nR\ln\left(\frac{V_f}{V_i}\right)$

Also, from the ideal gas law, $\quad \frac{T_f}{T_i} = \frac{P_fV_f}{P_iV_i}$

$$\Delta S = (1.00\ \text{mol})\left[\frac{3}{2}(8.314\ \text{J/mol}\cdot\text{K})\right]\ln\left(\frac{(2.00)(0.040\,0)}{(1.00)(0.025\,0)}\right) + (1.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})\ln\left(\frac{0.040\,0}{0.025\,0}\right)$$

$$= \boxed{18.4\ \text{J/K}}$$

P22.48 $\Delta S = nC_V\ln\left(\frac{T_f}{T_i}\right) + nR\ln\left(\frac{V_f}{V_i}\right)$

$$= (1.00\ \text{mol})\left[\frac{5}{2}(8.314\ \text{J/mol}\cdot\text{K})\right]\ln\left(\frac{2P\cdot 2V}{PV}\right) + (1.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})\ln\left(\frac{2V}{V}\right)$$

$$\Delta S = \boxed{34.6\ \text{J/K}}$$

Section 22.8 Entropy on a Microscopic Scale

P22.49 (a) A 12 can only be obtained $\boxed{\text{one}}$ way $6+6$

(b) A 7 can be obtained $\boxed{\text{six}}$ ways: $6+1,\ 5+2,\ 4+3,\ 3+4,\ 2+5,\ 1+6$

P22.50 (a) The table is shown below. On the basis of the table, the most probable result of a toss is $\boxed{\text{2 heads and 2 tails}}$.

(b) The most ordered state is the least likely state. Thus, on the basis of the table this is $\boxed{\text{either all heads or all tails}}$.

(c) The most disordered is the most likely state. Thus, this is $\boxed{\text{2 heads and 2 tails}}$.

Result	Possible Combinations	Total
All heads	HHHH	1
3H, 1T	THHH, HTHH, HHTH, HHHT	4
2H, 2T	TTHH, THTH, THHT, HTTH, HTHT, HHTT	6
1H, 3T	HTTT, THTT, TTHT, TTTH	4
All tails	TTTT	1

P22.51 (a)

Result	Possible Combinations	Total
All red	RRR	1
2R, 1G	RRG, RGR, GRR	3
1R, 2G	RGG, GRG, GGR	3
All green	GGG	1

(b)

Result	Possible Combinations	Total
All red	RRRRR	1
4R, 1G	RRRRG, RRRGR, RRGRR, RGRRR, GRRRR	5
3R, 2G	RRRGG, RRGRG, RGRRG, GRRRG, RRGGR, RGRGR, GRRGR, RGGRR, GRGRR, GGRRR	10
2R, 3G	GGGRR, GGRGR, GRGGR, RGGGR, GGRRG, GRGRG, RGGRG, GRRGG, RGRGG, RRGGG	10
1R, 4G	RGGGG, GRGGG, GGRGG, GGGRG, GGGGR	5
All green	GGGGG	1

Additional Problems

P22.52 The conversion of gravitational potential energy into kinetic energy as the water falls is reversible. But the subsequent conversion into internal energy is not. We imagine arriving at the same final state by adding energy by heat, in amount mgy, to the water from a stove at a temperature infinitesimally above 20.0°C. Then,

$$\Delta S = \int \frac{dQ}{T} = \frac{Q}{T} = \frac{mgy}{T} = \frac{5\,000\ \text{m}^3\left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(50.0\ \text{m})}{293\ \text{K}} = \boxed{8.36\times10^6\ \text{J/K}}.$$

P22.53 (a) $\mathcal{P}_{\text{electric}} = \frac{H_{ET}}{\Delta t}$ so if all the electric energy is converted into internal energy, the steady-state condition of the house is described by $H_{ET} = |Q|$.

Therefore, $$\mathcal{P}_{\text{electric}} = \frac{Q}{\Delta t} = \boxed{5\,000\text{ W}}$$

(b) For a heat pump, $$(\text{COP})_{\text{Carnot}} = \frac{T_h}{\Delta T} = \frac{295\text{ K}}{27\text{ K}} = 10.92$$

$$\text{Actual COP} = 0.6(10.92) = 6.55 = \frac{|Q_h|}{W} = \frac{|Q_h|/\Delta t}{W/\Delta t}$$

Therefore, to bring 5 000 W of energy into the house only requires input power

$$\mathcal{P}_{\text{heat pump}} = \frac{W}{\Delta t} = \frac{|Q_h|/\Delta t}{\text{COP}} = \frac{5\,000\text{ W}}{6.56} = \boxed{763\text{ W}}$$

P22.54 $|Q_c| = mc\Delta T + mL + mc\Delta T =$

$$|Q_c| = 0.500\text{ kg}(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(10^\circ\text{C}) + 0.500\text{ kg}\left(3.33\times10^5\text{ J/kg}\right) + 0.500\text{ kg}(2\,090\text{ J/kg}\cdot{}^\circ\text{C})(20^\circ\text{C})$$

$$|Q_c| = 2.08\times10^5\text{ J}$$

$$\frac{|Q_c|}{W} = \text{COP}_c(\text{refrigerator}) = \frac{T_c}{T_h - T_c}$$

$$W = \frac{|Q_c|(T_h - T_c)}{T_c} = \frac{\left(2.08\times10^5\text{ J}\right)\left[20.0^\circ\text{C} - (-20.0^\circ\text{C})\right]}{(273 - 20.0)\text{ K}} = \boxed{32.9\text{ kJ}}$$

P22.55 $$\Delta S_{\text{hot}} = \frac{-1\,000\text{ J}}{600\text{ K}}$$

$$\Delta S_{\text{cold}} = \frac{+750\text{ J}}{350\text{ K}}$$

(a) $$\Delta S_U = \Delta S_{\text{hot}} + \Delta S_{\text{cold}} = \boxed{0.476\text{ J/K}}$$

(b) $$e_c = 1 - \frac{T_1}{T_2} = 0.417$$

$$W_{\text{eng}} = e_c|Q_h| = 0.417(1\,000\text{ J}) = \boxed{417\text{ J}}$$

(c) $$W_{\text{net}} = 417\text{ J} - 250\text{ J} = 167\text{ J}$$

$$T_1\Delta S_U = 350\text{ K}(0.476\text{ J/K}) = \boxed{167\text{ J}}$$

***P22.56** (a) The energy put into the engine by the hot reservoir is $dQ_h = mcdT_h$. The energy put into the cold reservoir by the engine is $|dQ_c| = -mcdT_c = (1-e)dQ_h = \left[1-\left(1-\frac{T_c}{T_h}\right)\right]mcdT_h$. Then

$$-\frac{dT_c}{T_c} = \frac{dT_h}{T_h}$$

$$\int_{T_c}^{T_f} -\frac{dT}{T} = \int_{T_h}^{T_f} \frac{dT}{T}$$

$$-\ln T\Big|_{T_c}^{T_f} = \ln T\Big|_{T_h}^{T_f}$$

$$\ln\frac{T_c}{T_f} = \ln\frac{T_f}{T_h}$$

$$T_f^2 = T_cT_h$$

$$T_f = (T_hT_c)^{1/2}$$

(b) The hot reservoir loses energy $|Q_h| = mc(T_h - T_f)$. The cold reservoir gains $|Q_c| = mc(T_f - T_c)$. Then $|Q_h| = W_{eng} + |Q_c|$.

$$W_{eng} = mc(T_h - T_f) - mc(T_f - T_c)$$

$$= mc\left(T_h - \sqrt{T_hT_c} - \sqrt{T_hT_c} + T_c\right)$$

$$= mc\left(T_h - 2\sqrt{T_h}\sqrt{T_c} + T_c\right) = mc\left(\sqrt{T_h} - \sqrt{T_c}\right)^2$$

P22.57 (a) For an isothermal process, $Q = nRT\ln\left(\frac{V_2}{V_1}\right)$

Therefore, $Q_1 = nR(3T_i)\ln 2$

and $Q_3 = nR(T_i)\ln\left(\frac{1}{2}\right)$

For the constant volume processes, $Q_2 = \Delta E_{int,2} = \frac{3}{2}nR(T_i - 3T_i)$

and $Q_4 = \Delta E_{int,4} = \frac{3}{2}nR(3T_i - T_i)$

The net energy by heat transferred is then

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

or $Q = \boxed{2nRT_i \ln 2}$.

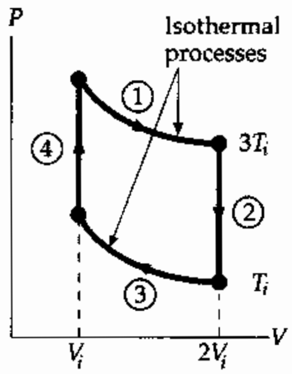

FIG. P22.57

(b) A positive value for heat represents energy transferred into the system.

Therefore, $|Q_h| = Q_1 + Q_4 = 3nRT_i(1 + \ln 2)$

Since the change in temperature for the complete cycle is zero,

$$\Delta E_{int} = 0 \text{ and } W_{eng} = Q$$

Therefore, the efficiency is $e_c = \frac{W_{eng}}{|Q_h|} = \frac{Q}{|Q_h|} = \frac{2\ln 2}{3(1+\ln 2)} = \boxed{0.273}$

P22.58 (a) $\frac{W_{eng}}{t} = 1.50 \times 10^8 \text{ W}_{(\text{electrical})}$, $Q = mL = \left[\frac{\frac{W_{eng}}{t}}{0.150}\right]\Delta t$,

and $L = 33.0 \text{ kJ/g} = 33.0 \times 10^6 \text{ J/kg}$

$$m = \left[\frac{W_{eng}/t}{0.150}\right]\frac{\Delta t}{L}$$

$$m = \frac{(1.50 \times 10^8 \text{ W})(86\,400 \text{ s/day})}{0.150(33.0 \times 10^6 \text{ J/kg})(10^3 \text{ kg/metric ton})} = \boxed{2\,620 \text{ metric tons/day}}$$

(b) Cost = (\$8.00/metric ton)(2 618 metric tons/day)(365 days/yr)

Cost = $\boxed{\$7.65 \text{ million/year}}$

(c) First find the rate at which heat energy is discharged into the water. If the plant is 15.0% efficient in producing electrical energy then the rate of heat production is

$$\frac{|Q_c|}{t} = \left(\frac{W_{eng}}{t}\right)\left(\frac{1}{e} - 1\right) = (1.50 \times 10^8 \text{ W})\left(\frac{1}{0.150} - 1\right) = 8.50 \times 10^8 \text{ W}.$$

Then, $\frac{|Q_c|}{t} = \frac{mc\Delta T}{t}$ and

$$\frac{m}{t} = \frac{\frac{|Q_c|}{t}}{c\Delta T} = \frac{8.50 \times 10^8 \text{ J/s}}{(4\,186 \text{ J/kg}\cdot{}^\circ\text{C})(5.00^\circ\text{C})} = \boxed{4.06 \times 10^4 \text{ kg/s}}.$$

P22.59 $e_c = 1 - \frac{T_c}{T_h} = \frac{W_{eng}}{|Q_h|} = \frac{\frac{W_{eng}}{\Delta t}}{\frac{|Q_h|}{\Delta t}}$: $\quad \frac{|Q_h|}{\Delta t} = \frac{\wp}{(1 - T_c/T_h)} = \frac{\wp T_h}{T_h - T_c}$

$|Q_h| = W_{eng} + |Q_c|$: $\quad \frac{|Q_c|}{\Delta t} = \frac{|Q_h|}{\Delta t} - \frac{W_{eng}}{\Delta t}$

$$\frac{|Q_c|}{\Delta t} = \frac{\wp T_h}{T_h - T_c} - \wp = \frac{\wp T_c}{T_h - T_c}$$

$|Q_c| = mc\Delta T$: $\quad \frac{|Q_c|}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)c\Delta T = \frac{\wp T_c}{T_h - T_c}$

$$\frac{\Delta m}{\Delta t} = \frac{\wp T_c}{(T_h - T_c)c\Delta T}$$

$$\frac{\Delta m}{\Delta t} = \frac{(1.00 \times 10^9 \text{ W})(300 \text{ K})}{200 \text{ K}(4\,186 \text{ J/kg}\cdot{}^\circ\text{C})(6.00^\circ\text{C})} = \boxed{5.97 \times 10^4 \text{ kg/s}}$$

P22.60 $e_c = 1 - \frac{T_c}{T_h} = \frac{W_{eng}}{|Q_h|} = \frac{\frac{W_{eng}}{\Delta t}}{\frac{|Q_h|}{\Delta t}}$ $\qquad \frac{|Q_h|}{\Delta t} = \frac{\wp}{\left(1 - \frac{T_c}{T_h}\right)} = \frac{\wp T_h}{T_h - T_c}$

$$\frac{|Q_c|}{\Delta t} = \left(\frac{|Q_h|}{\Delta t}\right) - \wp = \frac{\wp T_c}{T_h - T_c}$$

$|Q_c| = mc\Delta T$, where c is the specific heat of water.

Therefore, $\qquad \frac{|Q_c|}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right) c\Delta T = \frac{\wp T_c}{T_h - T_c}$

and $\qquad \frac{\Delta m}{\Delta t} = \boxed{\frac{\wp T_c}{(T_h - T_c)c\Delta T}}$

P22.61 (a) $35.0°\text{F} = \frac{5}{9}(35.0 - 32.0)°\text{C} = (1.67 + 273.15)\text{ K} = 274.82\text{ K}$

$98.6°\text{F} = \frac{5}{9}(98.6 - 32.0)°\text{C} = (37.0 + 273.15)\text{ K} = 310.15\text{ K}$

$\Delta S_{\text{ice water}} = \int \frac{dQ}{T} = (453.6\text{ g})(1.00\text{ cal/g}\cdot\text{K}) \times \int_{274.82}^{310.15} \frac{dT}{T} = 453.6 \ln\left(\frac{310.15}{274.82}\right) = 54.86\text{ cal/K}$

$\Delta S_{\text{body}} = -\frac{|Q|}{T_{\text{body}}} = -(453.6)(1.00)\frac{(310.15 - 274.82)}{310.15} = -51.67\text{ cal/K}$

$\Delta S_{\text{system}} = 54.86 - 51.67 = \boxed{3.19\text{ cal/K}}$

(b) $(453.6)(1)(T_F - 274.82) = (70.0 \times 10^3)(1)(310.15 - T_F)$

Thus,

$$(70.0 + 0.453\,6) \times 10^3 T_F = [(70.0)(310.15) + (0.453\,6)(274.82)] \times 10^3$$

and $T_F = 309.92\text{ K} = 36.77°\text{C} = \boxed{98.19°\text{F}}$

$\Delta S'_{\text{ice water}} = 453.6 \ln\left(\frac{309.92}{274.82}\right) = 54.52\text{ cal/K}$

$\Delta S'_{\text{body}} = -(70.0 \times 10^3)\ln\left(\frac{310.15}{309.92}\right) = -51.93\text{ cal/K}$

$\Delta S'_{\text{sys}} = 54.52 - 51.93 = \boxed{2.59\text{ cal/K}}$ which is less than the estimate in part (a).

P22.62 (a) For the isothermal process AB, the work on the gas is

$$W_{AB} = -P_A V_A \ln\left(\frac{V_B}{V_A}\right)$$

$$W_{AB} = -5(1.013\times10^5\text{ Pa})(10.0\times10^{-3}\text{ m}^3)\ln\left(\frac{50.0}{10.0}\right)$$

$$W_{AB} = -8.15\times10^3\text{ J}$$

where we have used $1.00\text{ atm} = 1.013\times10^5\text{ Pa}$

and $1.00\text{ L} = 1.00\times10^{-3}\text{ m}^3$

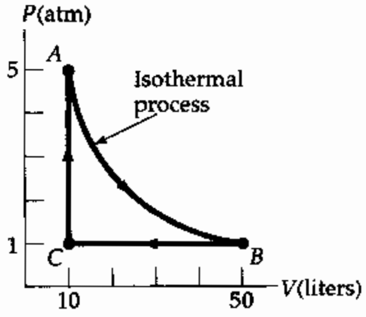

FIG. P22.62

$$W_{BC} = -P_B\Delta V = -(1.013\times10^5\text{ Pa})\left[(10.0-50.0)\times10^{-3}\right]\text{m}^3 = +4.05\times10^3\text{ J}$$

$$W_{CA} = 0 \text{ and } W_{\text{eng}} = -W_{AB} - W_{BC} = 4.11\times10^3\text{ J} = \boxed{4.11\text{ kJ}}$$

(b) Since AB is an isothermal process, $\Delta E_{\text{int},AB} = 0$

and $Q_{AB} = -W_{AB} = 8.15\times10^3\text{ J}$

For an ideal monatomic gas, $C_V = \frac{3R}{2}$ and $C_P = \frac{5R}{2}$

$$T_B = T_A = \frac{P_B V_B}{nR} = \frac{(1.013\times10^5)(50.0\times10^{-3})}{R} = \frac{5.05\times10^3}{R}$$

Also, $T_C = \frac{P_C V_C}{nR} = \frac{(1.013\times10^5)(10.0\times10^{-3})}{R} = \frac{1.01\times10^3}{R}$

$$Q_{CA} = nC_V\Delta T = 1.00\left(\frac{3}{2}R\right)\left(\frac{5.05\times10^3 - 1.01\times10^3}{R}\right) = 6.08\text{ kJ}$$

so the total energy absorbed by heat is $Q_{AB} + Q_{CA} = 8.15\text{ kJ} + 6.08\text{ kJ} = \boxed{14.2\text{ kJ}}$.

(c) $$Q_{BC} = nC_P\Delta T = \frac{5}{2}(nR\Delta T) = \frac{5}{2}P_B\Delta V_{BC}$$

$$Q_{BC} = \frac{5}{2}(1.013\times10^5)\left[(10.0-50.0)\times10^{-3}\right] = -1.01\times10^4\text{ J} = \boxed{-10.1\text{ kJ}}$$

(d) $$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{Q_{AB}+Q_{CA}} = \frac{4.11\times10^3\text{ J}}{1.42\times10^4\text{ J}} = 0.289 \text{ or } \boxed{28.9\%}$$

***P22.63** Like a refrigerator, an air conditioner has as its purpose the removal of energy by heat from the cold reservoir.

Its ideal COP is $$\text{COP}_{\text{Carnot}} = \frac{T_c}{T_h - T_c} = \frac{280\text{ K}}{20\text{ K}} = 14.0$$

(a) Its actual COP is $$0.400(14.0) = 5.60 = \frac{|Q_c|}{|Q_h| - |Q_c|} = \frac{|Q_c/\Delta t|}{|Q_h/\Delta t| - |Q_c/\Delta t|}$$

$$5.60\left|\frac{Q_h}{\Delta t}\right| - 5.60\left|\frac{Q_c}{\Delta t}\right| = \left|\frac{Q_c}{\Delta t}\right|$$

$$5.60(10.0\text{ kW}) = 6.60\left|\frac{Q_c}{\Delta t}\right| \text{ and } \left|\frac{Q_c}{\Delta t}\right| = \boxed{8.48\text{ kW}}$$

(b) $|Q_h| = W_{\text{eng}} + |Q_c|$: $$\frac{W_{\text{eng}}}{\Delta t} = \left|\frac{Q_h}{\Delta t}\right| - \left|\frac{Q_c}{\Delta t}\right| = 10.0\text{ kW} - 8.48\text{ kW} = \boxed{1.52\text{ kW}}$$

(c) The air conditioner operates in a cycle, so the entropy of the working fluid does not change. The hot reservoir increases in entropy by

$$\frac{|Q_h|}{T_h} = \frac{(10.0\times10^3\text{ J/s})(3\,600\text{ s})}{300\text{ K}} = 1.20\times10^5\text{ J/K}$$

The cold room decreases in entropy by

$$\Delta S = -\frac{|Q_c|}{T_c} = -\frac{(8.48\times10^3\text{ J/s})(3\,600\text{ s})}{280\text{ K}} = -1.09\times10^5\text{ J/K}$$

The net entropy change is positive, as it must be:

$$+1.20\times10^5\text{ J/K} - 1.09\times10^5\text{ J/K} = \boxed{1.09\times10^4\text{ J/K}}$$

(d) The new ideal COP is $$\text{COP}_{\text{Carnot}} = \frac{T_c}{T_h - T_c} = \frac{280\text{ K}}{25\text{ K}} = 11.2$$

We suppose the actual COP is $0.400(11.2) = 4.48$

As a fraction of the original 5.60, this is $\frac{4.48}{5.60} = 0.800$, so the fractional change is to $\boxed{\text{drop by }20.0\%}$.

P22.64 (a) $$W = \int_{V_i}^{V_f} P\,dV = nRT\int_{V_i}^{2V_i}\frac{dV}{V} = (1.00)RT\ln\left(\frac{2V_i}{V_i}\right) = \boxed{RT\ln 2}$$

(b) $\boxed{\text{The second law refers to cycles.}}$

P22.65 At point A, $P_iV_i = nRT_i$ and $n = 1.00$ mol

At point B, $3P_iV_i = nRT_B$ so $T_B = 3T_i$

At point C, $(3P_i)(2V_i) = nRT_C$ and $T_C = 6T_i$

At point D, $P_i(2V_i) = nRT_D$ so $T_D = 2T_i$

The heat for each step in the cycle is found using $C_V = \frac{3R}{2}$ and $C_P = \frac{5R}{2}$:

$$Q_{AB} = nC_V(3T_i - T_i) = 3nRT_i$$
$$Q_{BC} = nC_P(6T_i - 3T_i) = 7.50nRT_i$$
$$Q_{CD} = nC_V(2T_i - 6T_i) = -6nRT_i$$
$$Q_{DA} = nC_P(T_i - 2T_i) = -2.50nRT_i$$

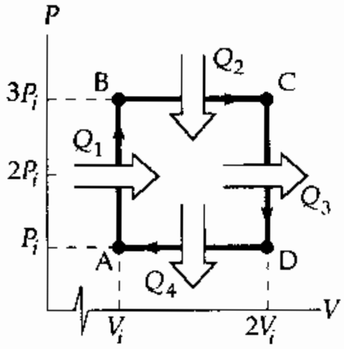

FIG. P22.65

(a) Therefore, $Q_{\text{entering}} = |Q_h| = Q_{AB} + Q_{BC} = \boxed{10.5nRT_i}$

(b) $Q_{\text{leaving}} = |Q_c| = |Q_{CD} + Q_{DA}| = \boxed{8.50nRT_i}$

(c) Actual efficiency, $e = \frac{|Q_h| - |Q_c|}{|Q_h|} = \boxed{0.190}$

(d) Carnot efficiency, $e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{T_i}{6T_i} = \boxed{0.833}$

***P22.66** $\Delta S = \int_i^f \frac{dQ}{T} = \int_i^f \frac{nC_P dT}{T} = nC_P \int_i^f T^{-1} dT = nC_P \ln T\Big|_{T_i}^{T_f} = nC_P(\ln T_f - \ln T_i) = nC_P \ln\left(\frac{T_f}{T_i}\right)$

$$\Delta S = nC_P \ln\left(\frac{PV_f}{nR}\frac{nR}{PV_i}\right) = \boxed{nC_P \ln 3}$$

***P22.67** (a) The ideal gas at constant temperature keeps constant internal energy. As it puts out energy by work in expanding it must take in an equal amount of energy by heat. Thus its entropy increases. Let P_i, V_i, T_i represent the state of the gas before the isothermal expansion. Let P_C, V_C, T_i represent the state after this process, so that $P_iV_i = P_CV_C$. Let P_i, $3V_i$, T_f represent the state after the adiabatic compression.

Then $P_CV_C^\gamma = P_i(3V_i)^\gamma$

Substituting $P_C = \frac{P_iV_i}{V_C}$

gives $P_iV_iV_C^{\gamma-1} = P_i(3^\gamma V_i^\gamma)$

Then $V_C^{\gamma-1} = 3^\gamma V_i^{\gamma-1}$ and $\frac{V_C}{V_i} = 3^{\gamma/(\gamma-1)}$

continued on next page

The work output in the isothermal expansion is

$$W=\int_i^C PdV=nRT_i\int_i^C V^{-1}dV=nRT_i\ln\left(\frac{V_C}{V_i}\right)=nRT_i\ln\left(3^{\gamma/(\gamma-1)}\right)=nRT_i\left(\frac{\gamma}{\gamma-1}\right)\ln 3$$

This is also the input heat, so the entropy change is

$$\Delta S=\frac{Q}{T}=nR\left(\frac{\gamma}{\gamma-1}\right)\ln 3$$

Since $C_P=\gamma C_V=C_V+R$

we have $(\gamma-1)C_V=R,\ C_V=\dfrac{R}{\gamma-1}$

and $C_P=\dfrac{\gamma R}{\gamma-1}$

Then the result is $\boxed{\Delta S=nC_P\ln 3}$

(b) The pair of processes considered here carry the gas from the initial state in Problem 66 to the final state there. Entropy is a function of state. Entropy change does not depend on path. Therefore the entropy change in Problem 66 equals $\Delta S_{\text{isothermal}}+\Delta S_{\text{adiabatic}}$ in this problem. Since $\Delta S_{\text{adiabatic}}=0$, the answers to Problems 66 and 67 (a) must be the same.

P22.68 Simply evaluate the maximum (Carnot) efficiency.

$$e_C=\frac{\Delta T}{T_h}=\frac{4.00\text{ K}}{277\text{ K}}=\boxed{0.014\,4}$$

The proposal does not merit serious consideration.

P22.69 The heat transfer over the paths *CD* and *BA* is zero since they are adiabatic.

Over path *BC*: $Q_{BC}=nC_P(T_C-T_B)>0$

Over path *DA*: $Q_{DA}=nC_V(T_A-T_D)<0$

Therefore, $|Q_c|=|Q_{DA}|$ and $Q_h=Q_{BC}$

The efficiency is then

$$e=1-\frac{|Q_c|}{Q_h}=1-\frac{(T_D-T_A)C_V}{(T_C-T_B)C_P}$$

$$e=1-\frac{1}{\gamma}\left[\frac{T_D-T_A}{T_C-T_B}\right]$$

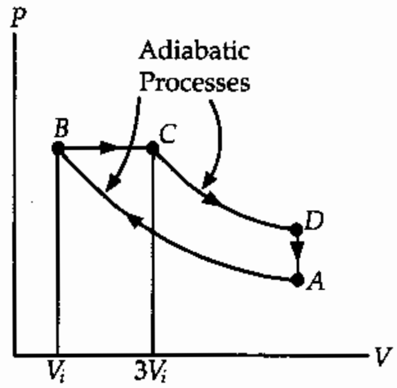

FIG. P22.69

P22.70 (a) Use the equation of state for an ideal gas

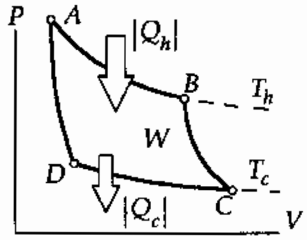

FIG. P22.70

$$V = \frac{nRT}{P}$$

$$V_A = \frac{1.00(8.314)(600)}{25.0(1.013\times10^5)} = \boxed{1.97\times10^{-3}\ \text{m}^3}$$

$$V_C = \frac{1.00(8.314)(400)}{1.013\times10^5} = \boxed{32.8\times10^{-3}\ \text{m}^3}$$

Since AB is isothermal, $P_A V_A = P_B V_B$

and since BC is adiabatic, $P_B V_B^\gamma = P_C V_C^\gamma$

Combining these expressions, $V_B = \left[\left(\frac{P_C}{P_A}\right)\frac{V_C^\gamma}{V_A}\right]^{1/(\gamma-1)} = \left[\left(\frac{1.00}{25.0}\right)\frac{(32.8\times10^{-3}\ \text{m}^3)^{1.40}}{1.97\times10^{-3}\ \text{m}^3}\right]^{(1/0.400)}$

$$V_B = \boxed{11.9\times10^{-3}\ \text{m}^3}$$

Similarly, $V_D = \left[\left(\frac{P_A}{P_C}\right)\frac{V_A^\gamma}{V_C}\right]^{1/(\gamma-1)} = \left[\left(\frac{25.0}{1.00}\right)\frac{(1.97\times10^{-3}\ \text{m}^3)^{1.40}}{32.8\times10^{-3}\ \text{m}^3}\right]^{(1/0.400)}$

or $V_D = \boxed{5.44\times10^{-3}\ \text{m}^3}$

Since AB is isothermal, $P_A V_A = P_B V_B$

and $P_B = P_A\left(\frac{V_A}{V_B}\right) = 25.0\ \text{atm}\left(\frac{1.97\times10^{-3}\ \text{m}^3}{11.9\times10^{-3}\ \text{m}^3}\right) = \boxed{4.14\ \text{atm}}$

Also, CD is an isothermal and $P_D = P_C\left(\frac{V_C}{V_D}\right) = 1.00\ \text{atm}\left(\frac{32.8\times10^{-3}\ \text{m}^3}{5.44\times10^{-3}\ \text{m}^3}\right) = \boxed{6.03\ \text{atm}}$

Solving part (c) before part (b):

(c) For this Carnot cycle, $e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{400\ \text{K}}{600\ \text{K}} = \boxed{0.333}$

(b) Energy is added by heat to the gas during the process AB. For the isothermal process, $\Delta E_{\text{int}} = 0$.

and the first law gives $Q_{AB} = -W_{AB} = nRT_h \ln\left(\frac{V_B}{V_A}\right)$

or $|Q_h| = Q_{AB} = 1.00\ \text{mol}(8.314\ \text{J/mol}\cdot\text{K})(600\ \text{K})\ln\left(\frac{11.9}{1.97}\right) = 8.97\ \text{kJ}$

Then, from $e = \frac{W_{\text{eng}}}{|Q_h|}$

the net work done per cycle is $W_{\text{eng}} = e_c|Q_h| = 0.333(8.97\ \text{kJ}) = \boxed{2.99\ \text{kJ}}$.

P22.71 (a) $\boxed{20.0°C}$

(b) $\Delta S = mc\ln\frac{T_f}{T_1} + mc\ln\frac{T_f}{T_2} = 1.00\text{ kg}(4.19\text{ kJ/kg}\cdot\text{K})\left[\ln\frac{T_f}{T_1} + \ln\frac{T_f}{T_2}\right] = (4.19\text{ kJ/K})\ln\left(\frac{293}{283}\cdot\frac{293}{303}\right)$

(c) $\Delta S = \boxed{+4.88\text{ J/K}}$

(d) $\boxed{\text{Yes}}$. Entropy has increased.

ANSWERS TO EVEN PROBLEMS

P22.2 (a) 667 J; (b) 467 J

P22.4 (a) 30.0%; (b) 60.0%

P22.6 55.4%

P22.8 77.8 W

P22.10 (a) 869 MJ; (b) 330 MJ

P22.12 197 kJ

P22.14 546°C

P22.16 33.0%

P22.18 (a) 5.12%; (b) 5.27 TJ/h; (c) see the solution

P22.20 453 K

P22.22 (a), (b) see the solution; (c) 23.7%; see the solution

P22.24 11.8

P22.26 1.17 J

P22.28 (a) 204 W; (b) 2.43 kW

P22.30 (a) 2.00; (b) 3.00; (c) 33.3%

P22.32 (a) 51.2%; (b) 36.2%

P22.34 (a), (b) see the solution; (c) $Q_h = 149$ J; $|Q_c| = 65.0$ J; $W_{eng} = 84.3$ J; (d) 56.5%; (e) 1.42×10^3 rev/min

P22.36 4.88 kJ/kg·K

P22.38 (a) isobaric; (b) 402 kJ; (c) 1.20 kJ/K

P22.40 3.27 J/K

P22.42 718 J/K

P22.44 (a) 39.4 L; (b) −2.50 kJ; (c) −2.50 kJ; (d) −6.87 J/K; (e) +9.16 J/K

P22.46 0.507 J/K

P22.48 34.6 J/K

P22.50 (a) 2 heads and 2 tails; (b) All heads or all tails; (c) 2 heads and 2 tails

P22.52 8.36 MJ/K

P22.54 32.9 kJ

P22.56 see the solution

P22.58 (a) 2.62×10^3 tons/d; (b) \$7.65 million/yr; (c) 4.06×10^4 kg/s

P22.60 $\dfrac{\wp T_c}{(T_h - T_c)c\Delta T}$

P22.62 (a) 4.11 kJ; (b) 14.2 kJ; (c) 10.1 kJ; (d) 28.9%

P22.64 see the solution

P22.66 $nC_P \ln 3$

P22.68 no; see the solution

P22.70 (a)

	P, atm	V, L
A	25.0	1.97
B	4.14	11.9
C	1.00	32.8
D	6.03	5.44

(b) 2.99 kJ; (c) 33.3%